CERN IdeaSquare Journal of Experimental Innovation, 2019; 3(2): 62-69
DOI: https://doi.org/10.23726/cij.2019.958

IDEASQUARE COFFEE-PAPER

Here is the thing I wasn't looking for!

In addition to its main purpose of publishing experimental innovation research related results, CIJ also publishes more light, inspirational food-for-thought intended "IdeaSquare Coffee Papers". These pieces are collaborative efforts prepared by visiting researchers from various walks of life visiting or staying at CERN IdeaSquare premises. The identity of the contributing authors is kept anonymous (although known) but helpful hints can be found in the literature references. Editors of this section are Dr. Markus Nordberg and Dr. Valeria Brancolini.

EXECUTIVE SUMMARY

This time the experimental innovation team of IdeaSquare explores serendipity in a serendipitous way.

INTRODUCTION

It is very difficult to overestimate the role that serendipity plays in relation to scientific discoveries and innovation. Plenty of excellent scholarship literature and studies exist in this respect[1]. However, never afraid of the possibility of re-inventing the wheel again, the IdeaSquare innovation team set itself the challenge of understanding serendipity in a serendipitous way. In other words, "serendipitizing[2]" the approach to understand serendipity. With this daunting task in mind, the team decided not to explicitly force itself upon any specific research path but to let it start in a fortuitous way and be carried away wherever the wind of serendipity was blowing.

THE SERENDIPITOUS BIRTH OF SERENDIPITY

As all our previous investigations, it all started in the course on a hot summer afternoon. During a casual coffee discussion about the origin of language in general, and words in particular, one member of the IdeaSquare innovation team told the story about the word serendipity.

As far as the legend goes, once upon a time, the King of Serendip[3] (an ancient name for Sri Lanka) had three sons who refused to succeed him. Thus, he expelled them. They began to travel around the world always discovering unintentionally new things and solving riddles, by a mixture of accident and sagacity. After many adventures, they returned to Serendip and eventually succeeded their father[4].

This tale, known as *The Three Princes of Serendip*, has been part of the ancient folklore in Persia and India and referred in the book Hasht Bihisht (Eight Paradises) by the Persian poet Amir Khusrau (1253-1325)[5]. His book served as a source for the compilation of tales collected by Cristoforo Armeno, a great admirer of the Persian culture, written in Italian and published in Venice in 1557 by Michele Tramezzino[6].

In 1719, Louis (or Jean), chevalier de Mailly sometimes credited as the originator of the clue-driven detective novel, made an adaptation in French of the *Voyage and Adventures of the Three Princes of Serendip*[7]. Horace Walpole (1717-1797), 4th Earl of Orford, art historian, man of letters, antiquarian and Whig politician, read Mailly's adaptation and the story stuck in his memory[8]. At the age of twenty-two, Walpole set out on a grand tour of Europe, spending much of his

[1] O. Yaqub, Serendipity: Towards a taxonomy and a theory, *Research Policy, Volume 47, 2018, Pages 169-179.*
[2] Please do not bother to look for this word in a dictionary.
[3] https://en.wikipedia.org/wiki/Names_of_Sri_Lanka
[4] https://en.wikipedia.org/wiki/The_Three_Princes_of_Serendip
[5] https://en.wikipedia.org/wiki/Amir_Khusrow
[6] https://it.wikipedia.org/wiki/Cristoforo_Armeno
[7] https://en.wikipedia.org/wiki/Chevalier_de_Mailly
[8] https://en.wikipedia.org/wiki/Horace_Walpole

trip immersed in the lively cultural life of Florence. There he stayed at Sir Horace Mann's (1706-1786) house, British Minister to the Court of Tuscany[9]. Walpole told him about his deep admiration for a painting portraying Bianca Capello who was an Italian noblewoman mistress, and afterward the second wife, of Francesco I de Medici[10]. Fourteen years later, Mann arranged to send the portrait to Walpole, but without its original frame. Walpole then needed to find a drawing of the coat of arms of the Capello family for decorating the painting appropriately. Luckily he discovered it in an old Venetian book published in 1578. He wrote to Mann thanking him for sending the portrait and referred to his unexpected discovery by coining the word SERENDIPITY in 1754:

[…] This discovery indeed is almost of that kind which I call serendipity, a very expressive word, which as I have nothing better to tell you, I shall endeavour to explain to you: you will understand it better by the derivation than by the definition. I once read a silly fairy tale, called The Three Princes of Serendip: as their highnesses travelled, they were always making discoveries, by accidents and sagacity, of things which they were not in quest of […][11].

After hearing this story, now being both amused and captivated, the IdeaSquare innovation team agreed that whatever serendipity is, it should have a lot to do with "fortuitous encounters" … the more, the better … and since encounters directly relate to collisions, it was necessary to investigate in more detail those ones. Obviously, CERN was the right place for this, but unfortunately for the IdeaSquare innovation team, the LHC and its experiments were not accessible at that moment. Thus, it was necessary to look for handier alternatives. Nothing really came to anybody's mind so it was decided that the best alternative was to forget about it and head for a nice and refreshing beer to fight the warm evening. While entering the nearby pub everyone noticed the billiard table…and the *Aha moment* kicked it: billiard table equals collisions!

LET YOUR CUE DO THE WORK

The billiard game seems to date back to the 1340s and was originally played outside and reminiscent to croquet[12]. King Louis XI of France (1461–1483) has been reported to own the first known indoor billiard table. The popularity of billiards grew extensively and by 1727, it was played in almost every Paris café. Over in England, by 1600, the game was so familiar to the public that even Shakespeare mentioned it in Antony and Cleopatra.

Despite the attractiveness of the game itself, what the IdeaSquare innovation team was really is the so-called Mathematical Billiards also known as Dynamical[13]. These are ideal systems for studying and discovering results about collisions[14] and how they are connected to serendipity.

Everybody knows the main ideal of a simplified billiard game setup (Fig. 1). It includes a flat billiard table (generally rectangular), a ball (normally more than one but let us simplify…) and something to hit it to get the ball in motion (normally a cue). For simplicity, we assume that there are no pockets or friction affecting the ball movement.

Let us now strike our billiard ball on our frictionless table. It will never stop bouncing. Imagine then that you leave for a long trip and return years later to check the movement of the ball. Would it have settled into some repeating bouncing pattern? Or would it be continually tracing new paths in a never-ending exploration on the billiard table?

You may wonder why our IdeaSquare team was up to when considering these questions … maybe a metaphor might clarify. Imagine you are the billiard ball and the billiard table is the world (Fig.1). You receive a serendipitous kick that allows you to wander on the table pretty much in the same way as the three princes of Serendip did around the world. The more places you visit the more serendipitous encounters you benefit from, therefore the more chances to make unexpected connections, and with the help of your sagacity, as the princes, more innovative discoveries.

Extremely excited by this fortuitous encounter with billiard tables, the IdeaSquare innovation team continued its billiard research journey.

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[9] https://en.wikipedia.org/wiki/Sir_Horace_Mann,_1st_Baronet
[10] https://en.wikipedia.org/wiki/Bianca_Cappello
[11] *Chase, Chance, and Creativity: The lucky art of novelty*, J.M. Austin, The MIT Press, 2003.
[12] https://en.wikipedia.org/wiki/Cue_sports
[13] http://mathworld.wolfram.com/Billiards.html
[14] https://en.wikipedia.org/wiki/Dynamical_billiards

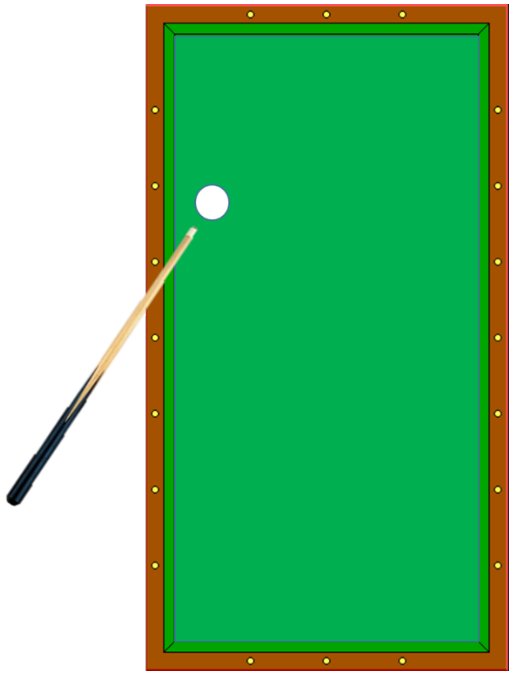

Fig. 1. You (the white ball) are about to receive a serendipitous first kick (the cue) to explore the world (the table). Will bouncing make you wander around the table? Would you rather set in a boring periodic motion?

"WITH A LITTLE CHAOS FROM MY FRIENDS"

All skilled billiard players (and trained physicists like us, at Ideasquare) know that if you shoot a ball so it hits the wall at right angles, it will bounce between opposite points on the rectangular table forever[15]. But what mathematicians know, is that this regular behaviour is actually uncommon. Instead, proven mathematical results state that for a vast majority of initial directions, non-periodic trajectories will result and eventually some of them will cover the area of the entire table[16]. This is the case when you are dealing with so-called optimal dynamic tables such as one with a pentagon shape (Fig. 2).

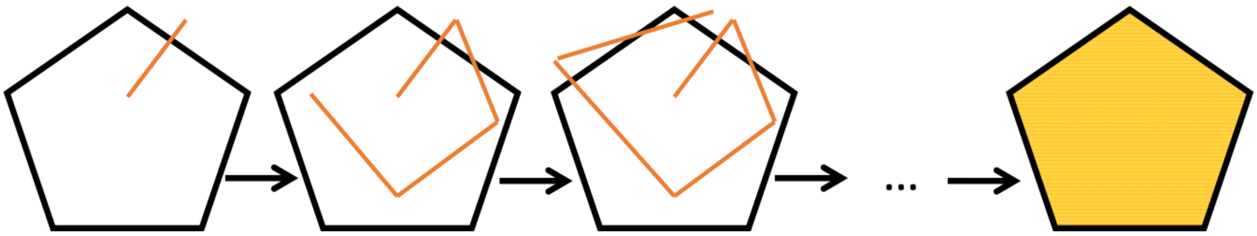

Fig. 2. Example of a billiard ball trajectory inside a regular pentagon. The trajectory ends up densely covering the whole pentagon. (Modified from http://dynamicsofpolygons.org/).

Which table shapes permit every non-periodic trajectory of the ball to cover the entire table? Well, only those ones having a lot of symmetry, such as regular polygons or tables made from multiple squares "glued" together, and some simple triangles. Mathematicians are trying to find more examples but so far unsuccessfully[17].

With these thoughts in mind, some of the members of the IdeaSquare innovation team started to play billiard. It is necessary to clarify at this point that for some of them, this was the first time they got in contact with the game. Thus, not much time elapsed until one of balls accidentally flew over the table boundaries[18]. This event triggered a heated discussion

[15] Again considering no friction.

[16] https://plus.maths.org/content/chaos-billiard-table

[17] https://imaginary.org/snapshot/billiards-and-flat-surfaces (See the wonderful article by Diana Davies, Billiards and Flat Surfaces).

[18] It was not documented whether or not the beer played a role.

about the properties of boundless billiards. Or, going along with the previous metaphor… what happens if we were to wander boundlessly?

UNFOLD YOURSELF

Mathematicians have a clever trick to make a billiard table boundless. They call it unfolding the table. First, they "glue" two tables together and make sure that the ball can keep going straight instead of bouncing[19].

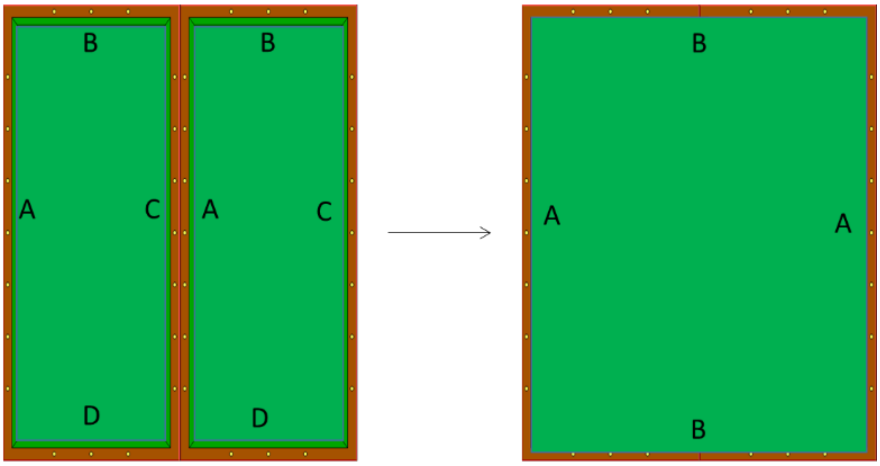

Fig. 3. Trick used by mathematicians to create boundless billiard tables.

Second, they identify the top and bottom edges, and the left and right ones (Fig. 3). Considering the example of tables, as shown in Fig. 3, when the ball moves in the direction of, let us say, right edge C, it will hit the right edge A of the new unfolded table. Similarly, when the ball moves towards the bottom edge D, it hits the bottom edge B in the new unfolded table. This mathematical "gluing" trick might seem dubious but, the more senior members of the IdeaSquare innovation team were actually familiar with it due to their young age and their old videogames such as "Pac-Man"or "Snake" where, when entering the right wall, one noticed the character re-emerging from the left and vice versa.

This trick allows to easily identifying non-periodic ball bouncing trajectories in a rectangular billiard table by thinking of them as a *single straight line* on graph paper expanding over identical copies of the original rectangular billiard table (Fig. 4). Suppose that we draw a straight line, whose slope is an irrational number[20] representing one of these trajectories. Then it will never cross two different horizontal (or vertical) edges at the same point: if it did, then the slope between those corresponding points would be a ratio of two integers. But we chose the slope to be irrational, so this can't happen. Hence, if we hit the ball with an irrational slope on the original billiard table, its trajectory will be non-periodic after theoretically infinite bounces and eventually covering all the points of the table. That led the IdeaSquare team to consider this: A bit if "irrationality"[21] generates serendipity.

[19] By iterating this process North, South, East and West indefinitely, you can end up with an infinite table.
[20] An irrational number cannot be expressed as a ratio between two numbers and it cannot be written as a simple fraction. For example the famous number pi (π).
[21] Some people like referring to this a "creative chaos".

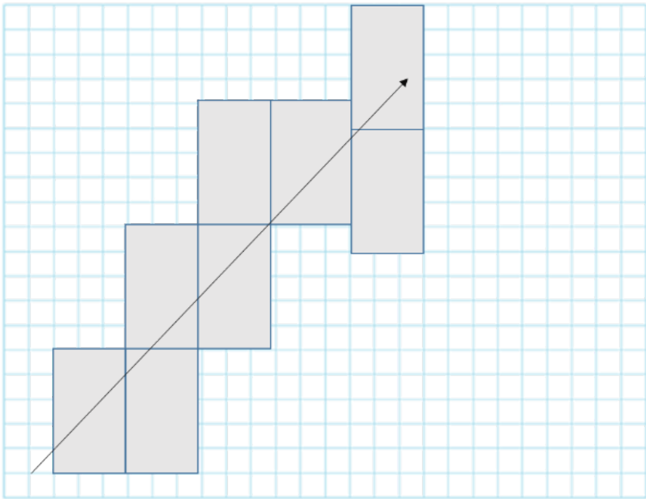

Fig. 4 A non-periodic trajectory. The grey areas represent unfolded copies of the same billiard table. The arrow represents the ball trajectory. The figure is for illustration purposes only since the slope is not an irrational number in the drawing.

After these zany thoughts, the sugar-deficient members of the IdeaSquare innovation team were in the need for a sugar dose. Our local pub is very scarce on food but as luck would have it, at least some sugary donuts were available.

DONUTS. IS THERE ANYTHING THEY CAN'T DO?[22]

According to a 1916 article in the Washington Post on June 22nd 1847, Captain Hanson Gregory, then a 16-year-old crewman aboard a schooner transporting limestone, allegedly poked a hole in a lump of dough with a round pepper tin, inventing the world's first modern donuts[23]. Although Captain Gregory died in 1921 by then Adolph Levitt, a Russian refugee in the US, invented the automatic donut-making machine[24]. The Donut Era started!

Although acknowledging Gregory's method for making donuts it is possible to make them mathematically in a slightly more elegant way[25]. Actually, an unfolded billiard table like the one in Fig. 3 is an excellent way to start if you imagine that it can be stretched as a rubber band.

Gluing both copies of edge A to each other (Fig. 3), you get a cylinder. When you wrap the cylinder around to glue both copies of edge B to each other, you get in fact the donut (Fig. 5).

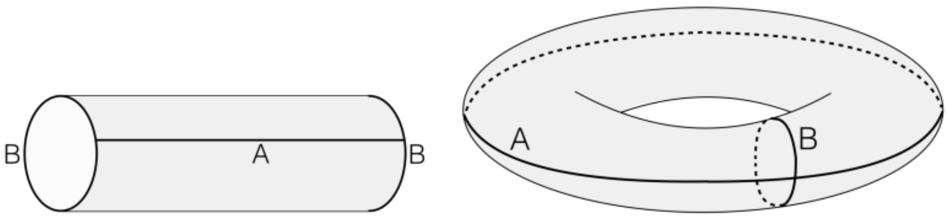

Fig. 5. An unfolded billiard table transformed into a donut thaks to mathematics. (taken from Diana Davies, Billiards and Flat Surfaces wonderful article at https://imaginary.org/snapshot/billiards-and-flat-surfaces).

Actually, mathematicians have a more technical name for a donut: *torus*[26]. In this way, any serendipitious trajectory you decide to follow across a donut (or torus) world, will have a corresponding one in an unfolded billiard table and vice

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22 Attributed to Homer Simpson (according to Matt Groening).
23 https://en.wikipedia.org/wiki/Doughnut
24 https://www.smithsonianmag.com/history/the-history-of-the-doughnut-150405177/
25 And hygienically better too!
26 https://en.wikipedia.org/wiki/Torus

versa. Mathematicians call these trajectories in the donut *geodesics*[27]. And they know a lot about them. For example, if a trajectory never closes itself in the torus, it will correspond to a trajectory that will cover the entire billiard table[28].

Extending the above line of thinking to our metaphor of serendipity, it is favoured by changing the point of view, like changing our point of view from a flat billiard table to donut or vice versa.

TO LIGHT A CANDLE IS TO CAST A SHADOW[29]

The hot summer afternoon turned into a thunderstorm which caused the lights of the pub to go off. Since the pub owner saw that his best clients[30] where in the middle of a trepidant billiard game, he offered to light up the table with some candles. Candles have inspired many fairy tales and world records too. Accoording to the Guiness book of World Records, a team of 100 people worked together to make a cake, individually placed 72,585 candles on top and lit them with 60 blowtorches to celebrate meditation teacher Sri Chinmoy's 85th birthday. Since there were far too many candles for anyone to blow out in the traditional manner, they were put out with CO_2 fire extinguishers which ensured that the cake was still edible[31].

The IndeaSquare innovation team billiard game proceeded under the candle light with some of their contestants complaining that illumination was not perfect. This raised the apparently innocent question:

Imagine a room with mirrored walls. If a candle is placed at some location in the room, will it illuminate every other point in it?

A little bit of serendipitous thinking will lead you to conclude that this is the same question as asking:

Imagine a billiard table. If a ball is hit at some location on the table and bounces infinitely, will its trajectory, visit every single point of it?

The two questions are indeed the same since light rays travel in straight lines and as they hits a mirror, they reflect back at the same angle as the one of incidence. In previous sections, we learnt that for tables of certain shape (e.g. regular polygons) it is possible to find such trajectories but would that be true for any table shape?

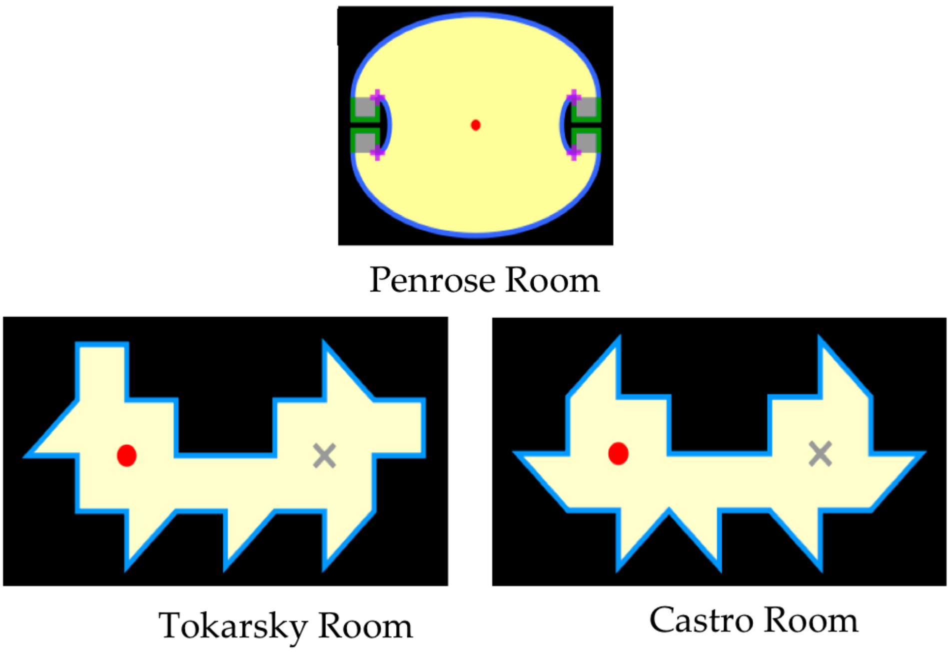

Fig. 6. Pernrose, Tokarsky and Castro rooms (Modified from Wikipedia https://en.wikipedia.org/wiki/Illumination_problem).

Roger Penrose[32] was the first to solve this problem in 1958 by showing a room with curved walls with always-dark regions if illuminated only by a single point source. The problem also interested George Tokarsky who in 1995 showed

[27] For the real Math funs we recommend http://www.rdrop.com/~half/math/torus/geodesics.xhtml
[28] https://plus.maths.org/content/billiards-donuts
[29] Ursula K. Le Guin, https://en.wikipedia.org/wiki/Ursula_K._Le_Guin
[30] Guess who? Obviously the IdeaSquare innovation team!
[31] http://www.guinnessworldrecords.com/news/2016/12/video-record-blasted-as-72-585-candles-burn-on-one-birthday-cake-453929
[32] https://en.wikipedia.org/wiki/Roger_Penrose

a polygonal 26-sided room with a "dark spot" which is not illuminated from another point in the room, even allowing for repeated reflections. David Castro in 1997, improved Tokarsky's solution by showing a 24-sided room with the same properties[33].

Nevertheless the answer to this question was given by Lelièvre, Monteil, and Weiss drawing upon the ground-breaking work of the great mathematician Mariam Mirzakhani[34]. They proved that if the room is polygonal with each angle a rational multiple of π, then no matter where the candle is placed, a finite number of locations in the room will not be illuminated. The translation to the world of billiards would be that if the billiard table is polygonal with all angles rational multiples of π, you can hit a billiard ball and it will reach every point on the table, except for possibly some finite number of them.

Maryam Mirzakhani was awarded the Fields Medal, becoming the first woman to receive math's highest honor. It will be hard to overstate her as a human being as well as a top mathematician. Perhaps the best description is the one that Kasra Rafi, Maryam's friend since school years, said about her: "Everything she touched she made better", which concerned things much broader than just mathematics[35]. Before tragically dying of cancer in 2017 she contemplated not just one billiard table, but the universe of all possible ones. Mirzakhani won the Fields Medal to a large extent for her discoveries about orbits in what mathematicians call *moduli spaces*. Moduli space is a "sort of trick" mathematicians have for grouping all objects of a given kind, so they can be studied in relation to one another. For example, let us say, that you want to study all the lines on a plane passing through a single point. That will be an awful lot of lines extending to infinite! But if you realize that each line crosses a circle drawn around that point in two opposite places, you can instead study points on a ring which is more manageable. We did in fact learnt this trick while considering trajectories in billiard tables. Instead of handling infinitely bouncing ball trajectories, we unfolded the billiard tables and studied the slopes of straight lines.

Curiously, the term moduli space is sometimes used in physics to refer specifically to the vacuum expectation values of a set of scalar fields. Thus, the Higgs field has a vacuum expectation value of 246 GeV underlining the Higgs mechanism of the Standard Model[36]…

At this point the electricity in the pub came back on again and the IdeaSquare innovation team realised that it was by now late. Luckily the thunderstorm was over and each one of the members could reach home safely.

TO BE SERENDIPITOUSLY CONTINUED…

The serendipitous journey towards serendipity has taken the IdeaSquare innovation team from the origin of the words itself all the way through the nice story of the three princes of Serendip, the game of billiard, donoughts, candles to finally end with the Higgs field. Along the way, the team learned that:

- The more prolific your serendipity journey is, the more chances you will have to come across interesting things.
- A little bit of chaos does not hurt; in fact, it makes your journey more exciting and successful.
- If you want your journey to be successful, you should contemplate things from different angles.

Sometimes, boundary conditions (like the way the world is shaped) can put limits to serendipity. But it will be the alert and prepared who will use the moduli space trick to reshape the world and with a single candle illuminate all space.

Let us finish this coffee paper with a quote by Maryam Mirzakhani, which, in her great style defines serendipity:

[33] https://en.wikipedia.org/wiki/Illumination_problem
[34] https://en.wikipedia.org/wiki/Maryam_Mirzakhani
[35] Obituary Maryam Mirzakhani (1977–2017) by Anton Zorich, EMS Newsletter March 2018.
[36] https://en.wikipedia.org/wiki/Moduli_space

 *Maryam Mirzakhani*

I don't have any particular recipe [for developing new proofs] ... It is like being lost in a jungle and trying to use all the knowledge that you can gather to come up with some new tricks, and with some luck, you might find a way out.

Cheers.