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Realistic Approach to Beam Dynamics Simulation with Synchrotron Radiation in High Energy Circular Lepton Colliders

S.A. Glukhov, E.B. Levichev
BINP SB RAS, Novosibirsk, Russia

Abstract
In extremely high energy circular lepton colliders, correct consideration of synchrotron radiation is important for beam dynamics simulation. We developed a fast precise effective method to track particles in a realistic lattice when the radiation effects are distributed along the orbit [1]. In the present paper we study an effect of decreasing dynamic aperture due to radiation from quadrupole lenses in the FCC-ee lepton collider.

Keywords
Synchrotron radiation; simulation; CUDA.

1 SR simulation techniques

1.1 Concentrated SR losses

A simple way to simulate SR in a circular lattice is to apply the following transformation to the coordinates of all particles once per turn at arbitrary azimuth s_0 [2] (the formulae are simplified for the case of flat lattice without betatron coupling)

$$\begin{array}{l} x \mapsto a_x(x-\eta_x\delta)+\eta_x\delta+b_x\hat r_1 \\ p_x \mapsto a_x(p_x-\eta_x'\delta)+\eta_x'\delta+b_x(\hat r_2-\alpha_x\hat r_1)/\beta_x \\ y \mapsto a_y y+b_y\hat r_3 \\ p_y \mapsto a_y p_y+b_y(\hat r_4-\alpha_y\hat r_3)/\beta_y \\ \delta=\Delta E/E_0 \mapsto e^{-\frac{T_0}{2\tau_\delta}}\delta+\sigma_\delta\sqrt{1-e^{-\frac{T_0}{\tau_\delta}}}\,\hat r_5 \end{array} , \quad (1)$$

where

$$a_u = e^{-\frac{T_0}{2\tau_u}} , \quad b_u = \sqrt{\varepsilon_u\beta_u\left(1-e^{-\frac{T_0}{\tau_u}}\right)} ,$$

E_0 — reference energy, T_0 — revolution period, σ_δ — energy spread, τ_u — damping times ($u = x, y$), ε_u — emittances, β_u, α_u, η_x and η_x' — optical functions at s_0, and $\hat r_1 \dots \hat r_5$ — random values with standard distribution.

1.2 Distributed SR losses

There is a more natural way of SR simulation consisting in distribution of the corresponding coordinate transformations over the whole lattice. One of such techniques is described in [3]. The method used in the present paper was developed on the basis of it and described in [1].

Let us consider a dipole magnet of a length L, bending angle θ, quadrupole gradient k_1 and rotation angles φ_1, φ_2 for the entrance and exit pole faces respectively. When an electron with a relativistic factor γ enters the dipole with an initial horizontal coordinate x_0 and energy deviation δ_0, it follows an arc with a radius $\rho = L/\theta$ and radiates N energy quanta. N has a Poisson distribution with a mean value of $\bar N$:

$$\bar N = \frac{5\sqrt3}{6}\alpha\theta\gamma_0\left(1+k_1\rho x_0\right)\left(1+h^* x_0\right) ,$$

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where

$$h^* = \frac{1}{\rho} - \frac{\tan\varphi_1 + \tan\varphi_2}{L},$$

α is the fine structure constant. The energy radiated in each quantum is

$$\Delta_i\delta = -\frac{3\lambda_e}{2\rho}{\gamma_0}^2\,(1+\delta_0)^2\,(1+k_1\rho x_0)\,y_i\,, \quad i = 1\ldots N\,,$$

where λ_e is the reduced electron Compton wavelength; $y_i \in SR$, which means that y_i has the so-called SR-distribution whose distribution density function is closely related to the well-known SR spectral power density function [1]. With a sufficient accuracy this distribution can be generated in the following way: let ξ have a uniform distribution over $[0;1]$ segment, then

$$f(\xi) = C\,(-\ln{(1-\xi^a)})^{3/a} \in SR\,,$$

where $C = 0.5770254$, $a = 2.535609$.

1.3 Distributed SR losses in dipoles

Energy deviation due to SR photon emission affects particle motion in the bending plane. In a flat lattice all bends are horizontal, and hence x and p_x are expected to change along with δ. The radiation damping in the magnet in both transversal planes is proportional to the magnet's contribution to the I_2 integral; the squared quantum excitation amplitude is proportional to the contribution to I_{5x}. The equilibrium distribution of the horizontal coordinates is Gaussian, and thus we can apply transformations (1) to x and p_x in each bending magnet separately, assuming that the addition due to quantum excitation in each magnet is also Gaussian. So, all radiation acts in the magnet can be simulated at once at its exit pole face. Finally, the following transformation should be applied to the coordinates of each particle after tracking through each bending magnet

$$\begin{array}{l} x \mapsto e^{c_{1x}\Delta\delta}(x-\eta_x\delta) + \eta_x(\delta+\Delta\delta) + c_{2x}\hat{r}_1\sqrt{\Delta^2\delta}\,, \\ p_x \mapsto e^{c_{1x}\Delta\delta}(p_x-\eta'_x\delta) + \eta'_x(\delta+\Delta\delta) + c_{2x}\frac{\hat{r}_2-\alpha_x\hat{r}_1}{\beta_x}\sqrt{\Delta^2\delta}\,, \\ y \mapsto e^{c_{1y}\Delta\delta}y\,, \quad p_y \mapsto e^{a_y\Delta\delta}p_y\,, \quad \delta \mapsto \delta+\Delta\delta\,, \end{array} \tag{2}$$

where

$$\begin{array}{l} \Delta\delta = \sum_{i=1}^{N}\Delta_i\delta\,, \quad \Delta^2\delta = \sum_{i=1}^{N}(\Delta_i\delta)^2\,, \quad \Delta_i\delta \in SR \\ c_{1x,1y} = \dfrac{3T_0}{2\tau_{x,y}r_e{\gamma_0}^3 I_2}\,, \\ c_{2x} = \sqrt{\dfrac{24\sqrt{3}}{55}\dfrac{\varepsilon_x\beta_x\langle H_x\rangle}{\alpha{\gamma_0}^5{\lambda_e}^2 I_{5x}}\left(1-e^{-\frac{T_0}{\tau_x}}\right)}\,, \end{array}$$

I_2 and I_{5x} — radiation integrals, $\langle H_x\rangle$ — horizontal dispersion invariant averaged over the magnet, β_x, α_x, η_x and η'_x — horizontal optical functions at the exit pole of the magnet, $\hat{r}_1$ and $\hat{r}_2$ — random values with standard distribution. Quantum excitation in the vertical plane can be simulated once per turn, as in (1).

Distributed energy losses lead to variation of the equilibrium beam energy $\langle\delta\rangle$ along the lattice: it drops in bending magnets and rises in RF cavities. This is the so-called sawtooth effect, which leads to the closed orbit distortions. It can be cured by a variation of magnetic field in beamline elements in proportion to changing equilibrium energy (magnet tapering). Besides, in the simulations the following transformation should be applied to the horizontal coordinates of each particle after each dipole:

$$\begin{array}{l} x \mapsto x + \rho(1-\cos\theta)\,\Delta\langle\delta\rangle\,, \\ p_x \mapsto p_x + \sin\theta\,\Delta\langle\delta\rangle\,, \end{array}$$

where $\Delta\langle\delta\rangle$ is the variation of equilibrium energy deviation in the dipole.

1.4 Distributed SR losses in quadrupoles

A particle follows a curved trajectory and therefore emits SR photons not only in dipoles but also in other beamline elements. SR in strong final focus quadrupoles may affect particle dynamics significantly, especially at high energy. The simplest way to study this effect is to consider each strong quadrupole as a "variable strength dipole" with parallel pole faces and no quadrupole gradient. This fictitious dipole acts in both transversal planes and has different bending angles and radii of curvature on each turn for each particle. These values will be different for horizontal and vertical planes:

$$\theta_x = |p_{x1} - p_{x0}| \,, \quad \theta_y = |p_{y1} - p_{y0}| \,, \quad \rho_{x,y} = L/\theta_{x,y} \,,$$

where p_{x0} and p_{y0} are the transversal momenta at the entrance pole face, and p_{x1} and p_{y1} are the transversal momenta at the exit pole face of the quadrupole. So, radiation in both transversal planes should be simulated independently:

$$\begin{aligned} &\bar{N}_{x,y} = \frac{5\sqrt{3}}{6}\alpha\theta_{x,y}\gamma_0 \,, \quad N_{x,y} \in Poisson\left(\bar{N}_{x,y}\right) \,, \\ &(\Delta_i\delta)_{x,y} = -\frac{3\lambda_e}{2\rho_{x,y}}{\gamma_0}^2\left(1+\delta_0\right)^2 y_i \,, \quad i = 1\ldots N_{x,y} \,, \\ &y_i \in SR \,, \end{aligned}$$

$$\begin{aligned} \Delta\delta &= \textstyle\sum_{i=1}^{N_x}\left(\Delta_i\delta\right)_x + \sum_{i=1}^{N_y}\left(\Delta_i\delta\right)_y \,, \\ \Delta^2\delta &= \textstyle\sum_{i=1}^{N_x}\left(\left(\Delta_i\delta\right)_x\right)^2 + \sum_{i=1}^{N_y}\left(\left(\Delta_i\delta\right)_y\right)^2 \,. \end{aligned}$$

Then transformation (2) should be applied.

2 Simulation results for FCC-ee

The simulation technique described above was implemented as part of TrackKing simulation program [4]. FCC-ee is a 100-km e+e- collider with a beam energy of 45–175 GeV. Simulations were performed for a preliminary version of 175 GeV FCC-ee lattice with 4 different algorithms: without SR, with concentrated SR, with distributed SR and tapering, and with distributed SR, tapering and SR in all quadrupoles. The dynamic apertures (DAs) in units of beam sizes are shown in Fig. 1.

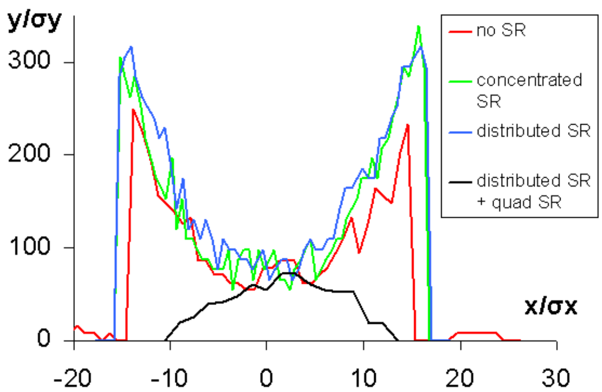

Fig. 1: DAs with different SR simulation modes

3 Discussion

Several effects can be noted in the results presented above. Firstly, the DA slightly decreases when SR is switched off. The cause is that in such a case initially unstable particles do not damp towards a stable phase space region and are thus lost eventually. This case is not a concern because it is only hypothetical. Secondly, results for concentrated and distributed simulations of SR in dipoles are in good agreement. And finally one can see that SR in quadrupoles reduces the DA significantly.

J.M. Jowett was the first to describe the latter effect [5]. The explanation is that synchrotron motion of particles with large betatron amplitudes becomes unstable due to SR losses in quadrupoles. It is not a single turn effect because energy radiated by a particle from quadrupoles during one turn (15 MeV) is only 5% of the equilibrium beam energy spread. Fig. 2 shows phase trajectories of synchrotron and horizontal betatron motion for an ensemble of on-energy particles with an initial horizontal deviation of $12.5\,\sigma_x$; the vertical motion is not excited and SR in quadrupoles is switched on. As one can see, strong synchrotron oscillations with an amplitude of up to $7\,\sigma_\delta$ are induced. During the first few synchrotron periods, particles that reached the energy acceptance boundary are lost, and then the others are damped towards a stable region. Fig. 3 shows phase tractories of synchrotron and horizontal betatron motion for

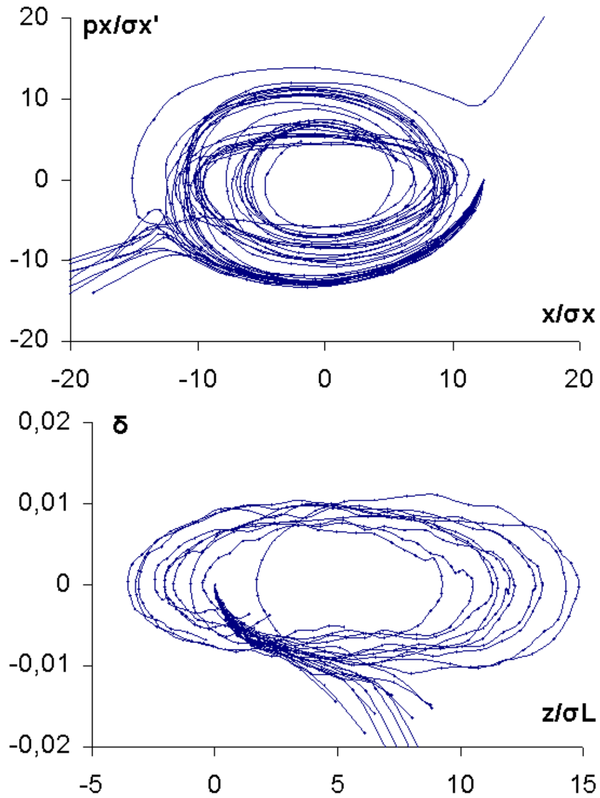

Fig. 2: Phase trajectories for particles with $x_0 = 12.5\,\sigma_x$; SR in quadrupoles is switched on

particles with the same initial conditions but with SR in quadrupoles switched off. In that case there is no sign of particle losses because strong synchrotron oscillations are not induced. Therefore, the effect of the DA shrinking due to SR in quadrupoles is highly non-equilibrium. So, it cannot be fully described in terms of radiation integrals because they are applied to an equilibrium beam state only.

The maximum induced energy deviation is reached after one quarter of synchrotron period and can be estimated in the following way:

$$\langle \Delta\delta \rangle = \frac{1}{4\nu_s} \frac{\oint \langle U_q(s) \rangle \, ds}{E_0} ,$$

where U_q is the energy radiated from quadrupoles and $\langle \ldots \rangle$ means averaging over beam particles. The

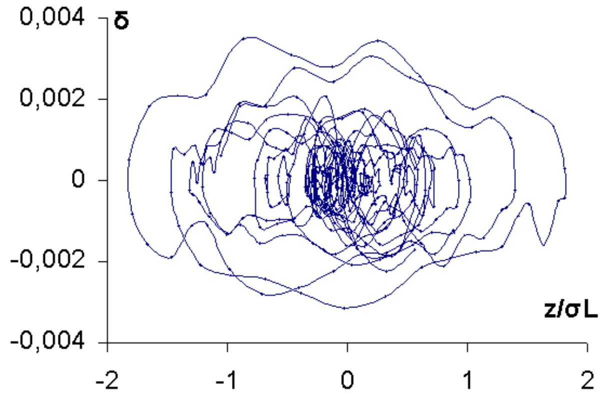

Fig. 3: Phase trajectories for particles with $x_0 = 12.5\,\sigma_x$; SR in quadrupoles is switched off

energy radiated by a single particle from one quadrupole of a length L_q and strength K_1 is

$$U_q = \frac{C_\gamma}{2\pi} E_0^4 \frac{(K_1 L_q x)^2}{L_q} .$$

Numerical estimations for the given lattice can be obtained using the following substitution:

$$x = n\sqrt{\varepsilon_x \beta_x} ,$$

where n is the initial horizontal coordinate expressed in horizontal beam sizes. Finally, the radiation-induced energy deviation for the lattice under consideration is the following:

$$\langle \Delta\delta \rangle = 0.58\% \left(\frac{n}{10}\right)^2 \quad (\approx 0.91\% \text{ for } x_0 = 12.5\,\sigma_x) .$$

It is in accordance with Fig. 2.

4 Conclusion

Conventional SR simulation techniques with SR concentrated at one azimuth is applicable even to lattices with extremely high radiation energy loss rate and tapering. SR in quadrupoles is also important for such lattices, but it can be taken into account only using distributed SR simulation techniques. Results of FCC-ee lattice simulations show that SR in quadrupoles reduces the DA significantly because of induced synchrotron ocsillations of particles with large initial transversal amplitude. A large energy deviation can be reached during the first synchrotron period. If the particle is not lost after that, then it forgets its initial conditions and remains stable.

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