

Beam Energy Measurement by Resonant Depolarization Method at VEPP-4M

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Abstract

Experiments on high precision mass measurement of particles require precise beam energy calibration. The most accurate method of beam energy measurement is the resonant depolarization technique. This article describes the beam energy measurement at the VEPP-4M storage ring using this method together with a Touschek polarimeter. The accuracy achieved is about 10^{-6} . More than thousand energy calibrations were used in the KEDR detector for the precise experiments on the measurement of J/ψ , $\psi(2S)$, $\psi(3770)$, D^+ , D^0 meson and τ lepton masses.

Keywords

Beam energy measurement; resonant depolarization; Touschek, intra-beam scattering; Compton backscattering; polarized beam; VEPP-4M; KEDR.

1 Introduction

High precision measurements in high energy physics requires accurate knowledge of the initial state of the colliding particles. In particular, precise measurement of the mass of elementary particles requires precision measurement of the beam energy. The most accurate method of beam energy calibration is the resonant depolarization (RD) technique. It is based on the measurement of the spin precession frequency, which is associated with the Lorentz factor of the beam and the well known normal and anomalous part of the magnetic moment of the electron due to the Thomas precession. The spin precession frequency is determined from the frequency of resonant destruction of beam polarization. The polarization degree can be measured via intra-beam scattering (Touschek), Compton backscattering or the synchrotron radiation process.

The VEPP-4M [1] accelerator complex (Fig. 1) with the KEDR [2] detector is designed for experiments in charm-tau physics in the $E=1-5.5$ GeV energy range. High precision mass measurement experiments on J/ψ $\psi(2S)$ [3], $\psi(3770)$ [4], D^0 , D^+ [5] mesons and τ lepton [6] were done using the resonant depolarization method. The main advantage of the resonant depolarization method is the best accuracy (10^{-6} for VEPP-4M) among other methods. However, it requires a polarized beam and a special run to prepare a polarized beam and perform an energy measurement. During a luminosity run the beam energy is interpolated with an accuracy of 10 – 30 keV using NMR and temperature sensors in the storage ring.

2 Resonant depolarization method

The method of resonant depolarization has forty years of successful history. It was first proposed in [7] and applied in experiments on Φ meson mass measurement [8] at Budker Institute of Nuclear Physics (Novosibirsk, Russia) in 1975. Later on it was used in the high precision mass measurement of K^+ [9]

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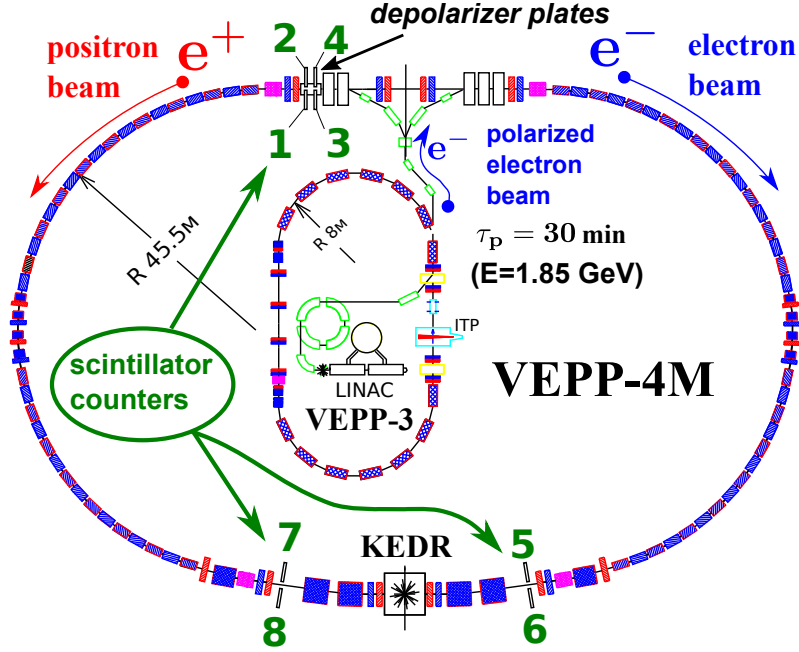


Fig. 1: The VEPP-4M storage ring with the KEDR detector. The location of eight scintillator counters of the Touschek polarimeter at VEPP-4M are labeled by numbers.

mesons at VEPP-2M with the OLYA detector, K^0 [10] and ω [11] mesons at VEPP-2M with the CMD detector, J/ψ and $\psi(2S)$ [12, 13] at VEPP-4 with the OLYA detector, $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ mesons at VEPP-4 with the MD-1 detector, at CESR with the CUSB detector [14] and at DORIS-II with the ARGUS and Crystal Ball detectors [15], Z boson [16] at LEP with the OPAL, DELPHI, L3 and ALEPH detectors. It was also used in the recent storage ring lattice calibration at SPEAR3 and AS [17] as well.

The spin evolution is described by the following equation:

$$\frac{d\vec{s}}{dt} = 2\mu \frac{\vec{s} \times \vec{B}'}{\gamma} + (\gamma - 1) \frac{\vec{s} \times [\vec{v} \times \dot{\vec{v}}]}{v^2}, \quad (1)$$

where \vec{s} is the spin direction; \vec{v} is the electron velocity; γ is the Lorentz factor; μ is the magnetic moment of the electron. The first part of the equation corresponds to dynamic spin rotation in a magnetic field B' in the electron rest frame. The second term corresponds to the pure relativistic kinematic effect, which was discovered by Thomas [18] and is called the Thomas precession. Bargmann, Michel, and Telegdi have derived the full relativistic equation [19]. In a storage ring it gives the spin precession with a frequency Ω :

$$\Omega = \omega_0 \left(1 + \frac{E}{m_e} \frac{\mu'}{\mu_0} \right), \quad (2)$$

where ω_0 is the revolution frequency. It depends on $a = \mu'/\mu_0$, the anomalous and normal parts of the electron magnetic moment ratio (magnetic moment anomaly), the beam energy E and the electron mass m_e ¹. The electron mass and magnetic moment anomaly are known with great accuracy: $\delta a \approx 2.3 \times 10^{-10}$ and $\delta m_e \approx 2.2 \times 10^{-8}$ [20], which allows one to calibrate the beam energy through measuring the spin precession frequency. Earlier experiments of ψ and Υ meson mass measurement were revised [21] after improvement of the electron mass measurement accuracy. Now the beam energy accuracy is limited only by the width of the spin line which is formed by the beam orbit disturbance in the presence of field

¹Here and further the system of units with $\hbar = 1$ and $c = 1$ is used

quadratic non-linearity and by magnetic field fluctuations. For example, the spin line width is about 5×10^{-7} at VEPP-4M.

The spin precession frequency is determined by the moment of resonant destruction of the beam polarization in an external electromagnetic field with ω_d . The resonant condition is $\Omega = n\omega_0 \pm \omega_d$, where $n \in \mathbb{Z}$. Thus one can calculate the beam energy as follows:

$$E = (440.6484431 \pm 0.0000097) [\text{MeV}] \times \left(n - 1 \pm \frac{\omega_d}{\omega_0} \right), \quad n \in \mathbb{Z}. \quad (3)$$

3 Radiative polarization

A polarized beam is obtained by the well known effect of radiative polarization, which was described by Sokolov and Ternov in 1963 [22]. The synchrotron radiation (SR) intensity of electrons in the storage ring has a small part of spin flip contribution: $W^{\uparrow\downarrow} \approx W_0 \frac{4}{3} (\omega_c/E)^2$, where W_0 is the total SR intensity and ω_c is the SR critical photon energy. It results in beam polarization with the characteristic time

$$\tau_p = \frac{8\sqrt{3}}{15} \frac{\lambda_C}{c} \frac{1}{\alpha\gamma^2} \left(\frac{H_0}{H} \right)^3, \quad (4)$$

and maximum polarization degree $P_0 = 8\sqrt{3}/15 \approx 92.4\%$. Here λ_C is the Compton wavelength of the electron; $H_0 = m_e^2 c^2 / e\hbar \approx 4.41 \times 10^{13}$ Gs is the Schwinger magnetic induction; H is the magnetic field of the storage ring; $\alpha \approx 1/137$ is the fine structure constant. Radiative polarization was first seen at VEPP-2 [7] in 1970 and at the ACO storage ring [23] in 1971. Then it became the main method of obtaining polarized beams for storage rings. A storage ring has a number of depolarizing spin resonances which satisfy the condition for the spin tune $\nu = \Omega/\omega_0 - 1 = k + l\nu_x + m\nu_y + n\nu_s$, where $k, l, m, n \in \mathbb{Z}$; ν_x, ν_y and ν_s are the horizontal betatron, vertical betatron and synchrotron tunes, respectively. Depolarizing resonances reduce the maximum polarization degree and polarization time by the factor $G = \tau_d/(\tau_p + \tau_d)$. The polarization process is described by the following dependence:

$$P(t) = P \left(1 - e^{-t/\tau} \right), \quad (5)$$

where $P = P_0 G$ is the reduced maximum polarization degree and $\tau = \tau_p G$ is the relaxation time of polarization. Calculation of the G-factor shows the possibility of obtaining polarization at ψ and Υ energies except of the $\Upsilon(4S)$ resonance. It is impossible to polarize a beam in VEPP-4M at low energies $E=1.5-2$ GeV because of the large polarization time (~ 100 hours). The beam is polarized in the VEPP-3 storage ring and injected into VEPP-4M. The non-planar injection channel results in a degradation of the vertical beam polarization. A 2.5 T·m pulsed solenoid is used to increase the polarization degree of the positrons [24]. The threshold energy $E = 1777$ MeV of τ lepton production is in a close vicinity to the $\nu = 4$ integer spin resonance, so there are special efforts were made [6, 25] for obtaining polarization.

4 Depolarizer device

The beam is depolarized by the TEM wave which is created by two matched striplines (Fig. 2). They are connected to a high frequency generator with tunable frequency, which is computer controlled. The depolarization time is:

$$\frac{1}{\tau_d} = 2 \frac{(2\mu' H l_d 2\pi\omega_0)^2}{c^2 \Delta\omega_d} |F^\nu|^2, \quad (6)$$

where H is the magnetic field of the TEM wave; l_d is the depolarizer length; $\Delta\omega_d \approx \sqrt{\dot{\omega}_d/2\pi}$ is the width of the depolarizer frequency, which depends on the frequency scan speed $\dot{\omega}_d$. F^ν is the spin response function, which depends on the depolarizer azimuth in the storage ring and the beam energy. The amplitude of the TEM wave H_0 is adjusted to get $\tau_d \sim 1$ s. The frequency generator and the VEPP-4M revolution frequency are stabilized by a rubidium atomic clock with stability of 10^{-10} .

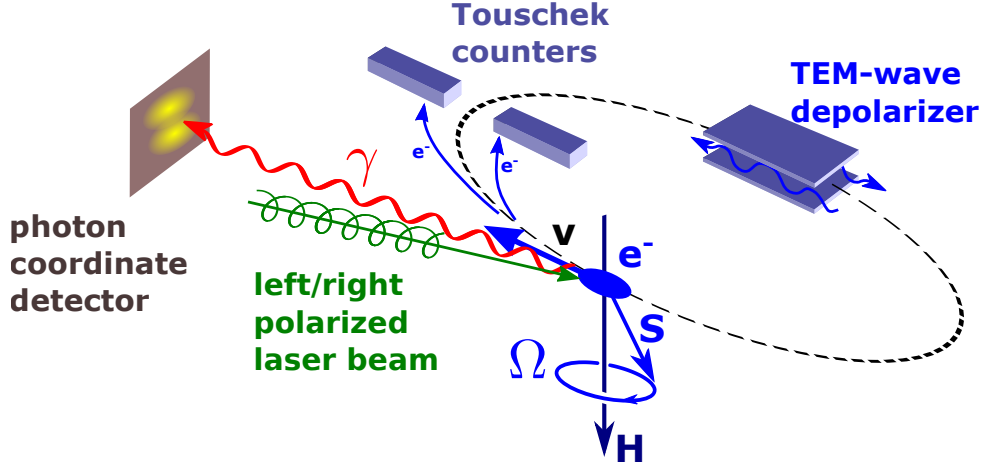


Fig. 2: Scheme of the resonant depolarization method with Touschek and Compton polarimeters

5 Polarization measurement

There are several methods of polarization measurement in storage rings. One of the most effective methods at lower energies ($\lesssim 2$ GeV) is the Touschek polarimeter, which is based on the polarization dependence of the intra-beam scattering intensity. Another method for higher energies is the Compton backscattering polarimeter, which relies on up-down asymmetry of the Compton backscattering of circularly polarized photons on vertically polarized electrons. The principal scheme of the resonant depolarization method with the Touschek and the Compton polarimeters is shown on Figure 2. The third method is based on the polarization dependence of the synchrotron radiation intensity.

5.1 Intra-beam scattering

The well known process of intra-beam scattering, which is called the Touschek (AdA) effect [26], is the main factor which limits the beam life time. In 1968 Baier and Khoze noticed [27] that the beam life time depends on beam polarization. This effect can be used for polarization measurement. In 1970 G. Tumaikin suggested [7] to use scintillator counters for the measurement of the intensity of intra-beam scattering. The problem associated with intra-beam scattering is the fast intensity and polarization contribution, decreasing at higher energies.

Pioneer work [27] assumed non-relativistic approximation with a flat beam. The calculation in [28] uses a two-dimensional beam; [29] takes into account relativistic effects, which become sufficient at energies of $\gtrsim 5$ GeV, and in [30] the Coulomb effects are concerned. Thus the intra-beam scattered intensity for a 2D relativistic (in the bunch rest frame) beam with the only assumption of small momentum transition $\varepsilon = \Delta p/p \ll 1$ has the following form:

$$\frac{dN}{dt} = -\frac{2\pi r_e^2 c N^2}{\gamma^2 V_b \varepsilon^2 \delta_x \delta_y} (I_1 + I_2 P^2) \stackrel{v \rightarrow 1}{\approx} -\frac{4\pi r_0^2 c N^2}{\gamma^2 V_b \varepsilon^2} \left(1 - \frac{\varepsilon P^2}{2\delta_x \delta_y}\right), \quad (7)$$

where $I_{1,2}$ are some integrals [30]; r_e is the classical electron radius; N is the number of particles in a beam; γ is the Lorentz factor; V_b is the effective beam volume; δ_x and δ_y are the horizontal and vertical momentum spreads in the $m_e c$ units, respectively; $\varepsilon = \Delta p/p$ is the relative momentum transition after scattering; v is the velocity of the electrons in the center of mass frame of the interacting particles. To calculate the count rate for a specific storage ring one has to take into account the betatron oscillation, location and distance to the beam orbit from the counters. Such calculations for VEPP-4M were done in [31]. The polarization effect $\Delta = \dot{N}_{pol}/\dot{N}_{unpol} - 1$ is about 3.5 % for $E = 1.85$ GeV and 2.5 % for $E = 1.5$ GeV, where \dot{N}_{pol} and \dot{N}_{unpol} are the count rate from the polarized and unpolarized beams, respectively. The intra-beam scattering intensity and polarization effect Δ decrease quickly at higher

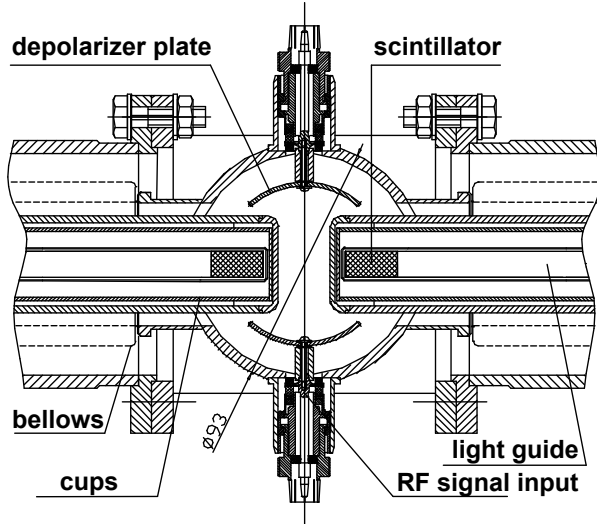


Fig. 3: Profile of the Touschek polarimeter with scintillator counters and depolarizer plates.

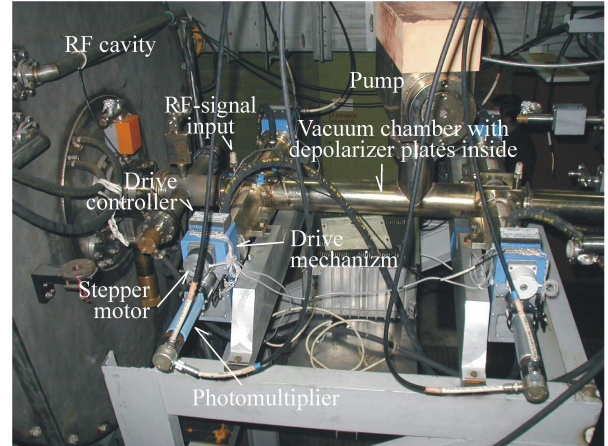


Fig. 4: View of the Touschek polarimeter.

energies. In the ultra relativistic limit $v \rightarrow 1$ the count rate behaves as $\dot{N} \propto E^{-5}$ and has the polarization contribution $\Delta \propto E^{-4}$. The measurement of the energy dependence of the count rate is described in [32]. At the Υ mesons energy region the Touschek count rate and the depolarization effect are expected to be 10 kHz and 0.1 %, respectively, for 10 mA beam current. This is a reason to use another method for the polarization measurement.

5.2 Compton backscattering

For higher energies $E \gtrsim 4$ GeV, the Compton backscattering polarimeter is useful. This method was first proposed in 1968 by Baier and Khoze [33]. The first realization of the laser polarimeter was done in 1979 at SPEAR [34]. Later on this method was applied at VEPP-4 [35], CESR [14], DORIS [15] and LEP [16].

The Compton cross section of the scattering of circularly polarized photons on polarized electrons in the rest frame is described by the following equation:

$$d\sigma = d\sigma_0 - \frac{r_e^2}{2} \left(\frac{\omega'}{\omega} \right)^2 \frac{1 + \cos\theta}{m} \left(\vec{k}' - \vec{k} \cos\theta \right) \vec{P} V d\Omega, \quad (8)$$

where \vec{P} is the polarization of the electron; V is the Stokes parameter of the circular polarization of the initial photon; $d\sigma_0$ is the Compton cross section of the unpolarized particle; ω and ω' are the initial and final photon energy in the electron rest-frame; \vec{k} and \vec{k}' are the initial and final photon momentum; θ is the azimuthal angle between the scattered and backward photon directions. The total up-down asymmetry is

$$A = \frac{N_{up} - N_{down}}{N_{up} + N_{down}} \approx -\frac{3}{4} \frac{\omega_0 E}{m_e^2} V P_{\perp}, \quad (9)$$

where P_{\perp} is the vertical electron polarization. A laser [14–16, 34, 35] or a synchrotron [36–40] can be used as a source of photons. The asymmetry is proportional to the initial photon energy ω_0 . This effect depends on the source of polarized photons and its value varies from $\sim 1\%$ to $\sim 10\%$.

6 Touschek polarimeter at VEPP-4M

The Touschek polarimeter at VEPP-4M has eight scintillator counters located at different locations around the VEPP-4M storage ring (Fig. 1). The counters are grouped into pairs inside and outside

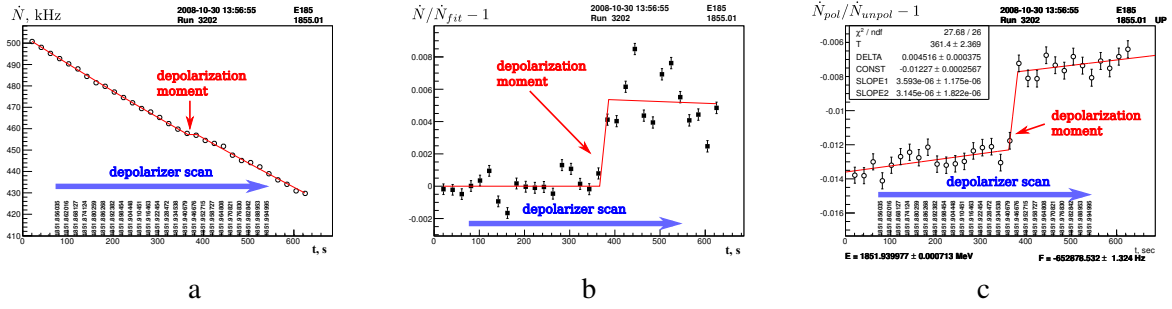


Fig. 5: (a) Count rate behavior during depolarizer frequency scan. It is hard to recognize the depolarization jump due to the wide range of count rate changes. (b) Count rate behavior with slope dependence removed. There is clear evidence of the beam depolarization, but count rate instabilities still exist. (c) Count rate from the polarized beam normalized on the unpolarized one. Instabilities of count rates are suppressed.

the beam orbit and can be moved inside the vacuum chamber in order to adjust their distance to the beam orbit and optimize the count rate (Fig. 3, 4). They have a water cooling shield to protect from synchrotron radiation heating. The typical count rate is about 50 kHz/mA^2 per counter. The total count rate of all counters is about 1–2 MHz. The counters share the same amount of Touschek electrons, and thus if a counter is moving, then the others change their count rate. Two counters are used in routine energy calibrations in order to simplify the counter position tuning and reduce the overall energy calibration time.

The data acquisition system (DAQ) is based on the CAMAC electronic standard and allows to measure the count rate from four electron or positron bunches with coincidence of counters in each pair. Using the coincidence allows to suppress the background from inelastic scattering on the residual gas because the Touschek events have two scattered electrons, which go inside and outside the beam orbit. Because of the features of particle tracking inside VEPP-4M, there turned out to be many events when only one electron from a scattered pair hit a counter. The probability of both counters hit is around 20% of that of only one counter hit. The high rate of scattered electrons results in a high random coincidence rate.

Figure 5.a shows the sum of count rates during a depolarizer frequency scan. Because of the wide range of the count rate variation it is hard to recognize a depolarization jump. When the slope of the count rate is removed by a special fit in an assumption of no depolarization, the depolarization jump is seen. However, there are count rate instabilities, which may complicate the depolarization moment determination (Fig. 5.b). This issue is solved using the following compensation technique. Two polarized and unpolarized bunches are prepared. After injection into VEPP-4M their currents are aligned to a level of 1% by step by step kicking of the inflector plates. The value $\Delta = \dot{N}_{pol}/\dot{N}_{unpol} - 1$ of relative difference of count rate from polarized and unpolarized bunches is under observation. This effectively suppresses the beam orbit or beam size instabilities, as one can see in Figure 5.c.

7 Laser polarimeter at VEPP-4M

It is planned to use a laser polarimeter at VEPP-4M for polarization measurement at energies around Υ mesons. A general view of the measurement scheme is shown on Figure 6. A solid state Nd:YLF pulse laser with a wavelength of 527 nm is used as a source of initial photons. The circular polarization is prepared by the KD*P Pockels cell or via rotation of the $\lambda/4$ phase plate. The left-right polarization switching has a frequency of about 1 kHz for the Pockels and 1 Hz for the phase plate. The motorized expander and mirrors focus the laser beam on an electron bunch at the azimuth between the quadrupole lenses and the bending magnet where a minimum vertical angular spread of $60 \mu\text{rad}$ at 5 GeV is expected. The Compton backscattered photons are detected by the two-dimensional coordinate detector based on

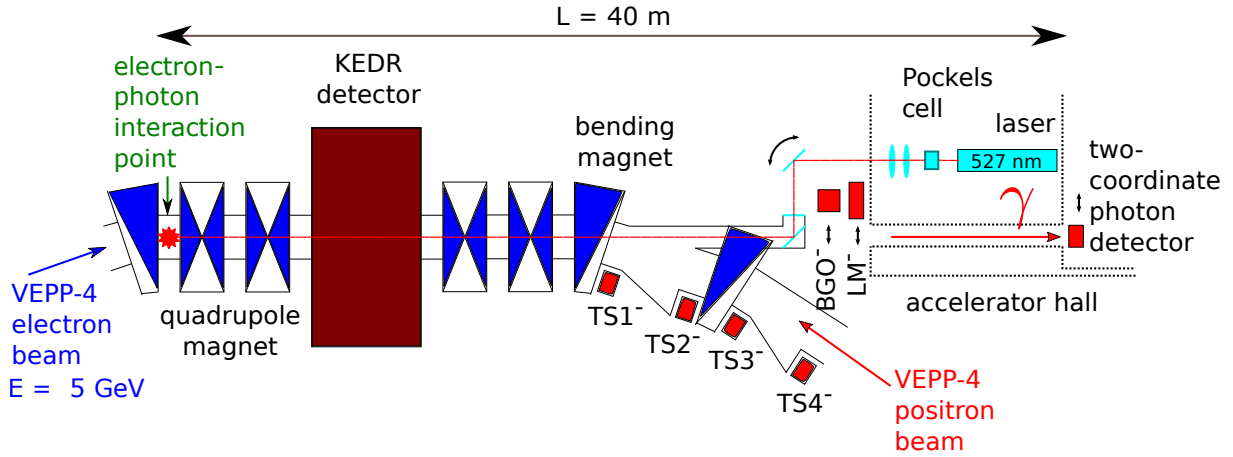


Fig. 6: Layout of the laser polarimeter at VEPP-4M

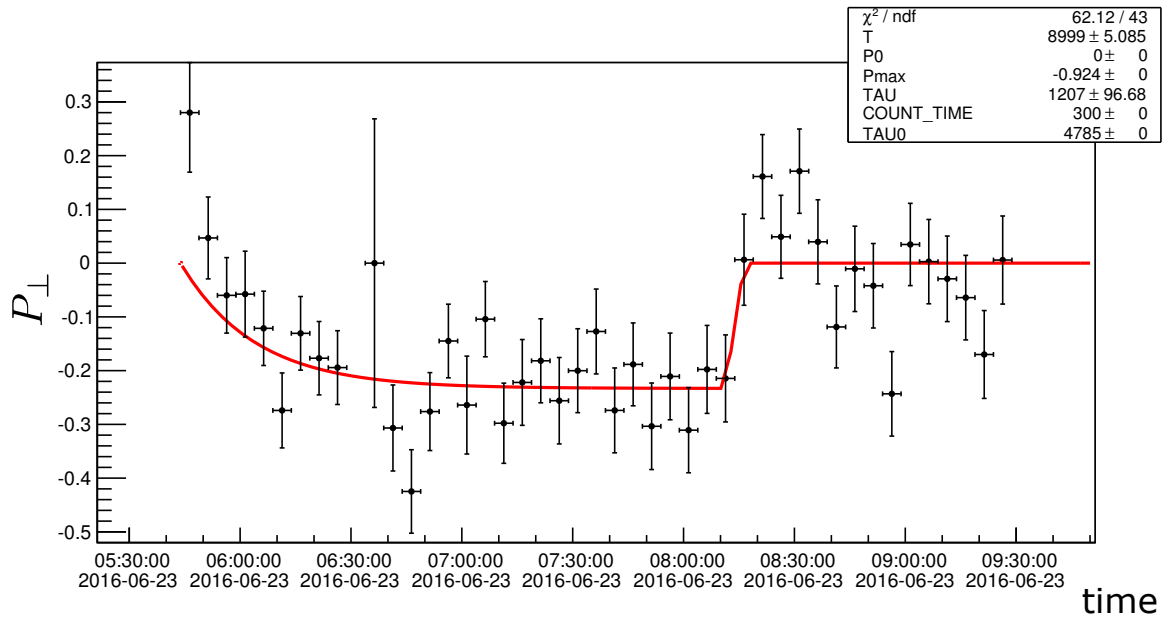


Fig. 7: Observation of the radiative polarization by the laser polarimeter at $E = 4.1$ GeV. P_{\perp} is the vertical electron polarization.

GEM [41] with a 12 mm thick lead photon converter. First experiments with this polarimeter were done in 2016. With a 2 kHz pulse repetition rate and a $180 \mu\text{J}$ pulse, the energy counting rate is about 600 Hz/mA. The radiative polarization measured is presented on Figure 7.

The polarization time measured is 36 minutes, and the polarization degree is $41 \pm 4\%$. The system requires improvement on the mirror quality and focusing. A new GEM detector is being designed and will be created. We expect the count rate to be about 16 kHz for a 10 mA beam current. It allows to measure the beam energy with an accuracy of 10^{-6} .

8 Energy calibration

During experiments on J/ψ , $\psi(2S)$ [3], $\psi(3770)$ [4], D^{\pm} , and D^0 [5] mass measurement, the following procedure is used. Energy calibration is performed *a*) at the beginning and at the end of the resonance energy scan point; *b*) after a VEPP-4M magnetic cycle; *c*) after a large ($\Delta H/H \gtrsim 10^{-4}$) change in

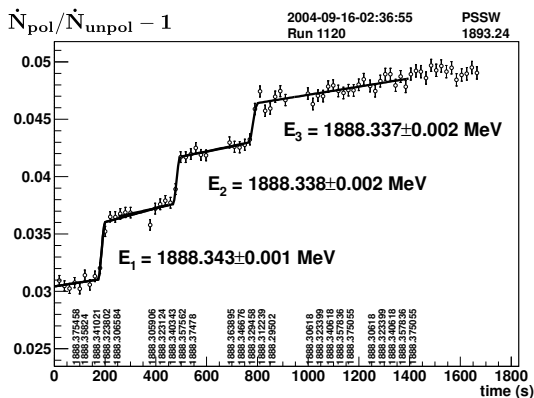


Fig. 8: Triple energy calibration on the same polarized bunch. The differences in energy calibrations are connected with the VEPP-4M energy drift and fluctuation.

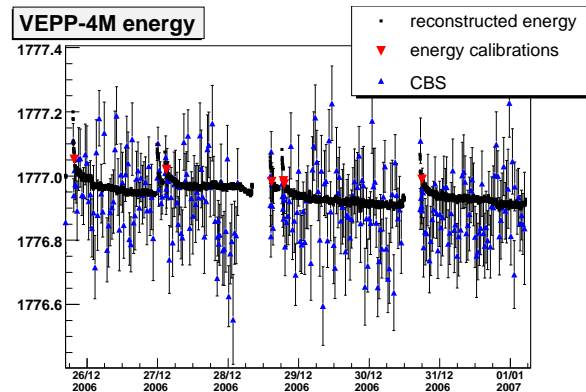


Fig. 9: Energy calibration by resonant depolarization method (red), by Compton backscattering method (blue) and interpolated energy via NMR and temperature parameters (black).

the magnetic field; *d*) after a large ($\gtrsim 1^\circ\text{C}$) change in the cooling water temperature. The magnetic field is measured by the nuclear magnetic resonance (NMR) method every two minutes. Power supply stabilization by NMR measurement feedback was done [42].

A single beam is prepared and polarized in VEPP-3 for 1.5 hours at 1.5 GeV and 1 hour at 1.85 GeV. Successful polarization requires control of the vertical and horizontal betatron tunes to avoid depolarizer resonances and depends on the working energy. After polarized beam injection into VEPP-4M and adjustment of the VEPP-4M betatron tunes, the second unpolarized beam is prepared and injected into VEPP-4M. Their currents are adjusted with an accuracy of 1%. The counters are moved to their working position to achieve a count rate of around 100 kHz per each counter. Final current alignment is done through count rate measurement from each bunch. An electrostatic separation in VEPP-4M should be turned off to satisfy beam orbit conditions during luminosity run.

In the τ mass measurement experiment in [6, 25], the polarized beam is prepared in a different way due to close vicinity to the $\nu = 4$ spin resonance. The beam is polarized at an energy of 1850 MeV in VEPP-3; after injection into VEPP-4M the beam energy is lowered to the τ threshold; after 30 minutes of magnetic field relaxation, resonant depolarization is done by common way. Between RD energy calibrations the energy is controlled by the alternative Compton backscattering edge method [43].

The beam energy uncertainty defines the depolarizer scan mode. At the beginning of the mass measurement scan we know the energy with an accuracy of around 1 MeV. The depolarizer has a 10 keV/s (~ 20 Hz/s) scan speed to cover the 4 MeV energy region. The subsequent energy calibration, already tied with the NMR measurement and energy can be predicted with accuracy $\lesssim 0.2$ MeV and thus the depolarizer scan speed is 0.3 keV/s and covers the 400 keV energy range. With a scan speed of 10 keV/s the energy calibration accuracy is about 10–20 keV, thus the energy calibration is repeated with a new polarized beam and a scan speed of 0.3 keV/s. The depolarizer amplitude is adjusted to get a depolarization time of about 1 s and partial beam depolarization that allows to perform two energy measurements with the same polarized bunch. The second scan is done in the opposite direction to determine the systematic error associated with the depolarization process for a finite spin line width. Another reason for the second opposite scan is to exclude depolarization on the side 50 Hz (25 keV) spin resonance caused by a 50 Hz magnetic field pulsation. The efficiency of the Touschek polarimeter and the method of partial depolarization is shown on Figure 8 via triple energy calibration on the same polarized bunch.

During and between energy calibration, the magnetic field is measured by the NMR method; the beam orbit is measured by beam position monitors; the temperatures of the cooling water, magnets

and wall of the tunnel where the ring is located are measured. This VEPP-4M parameters are used to interpolate the accelerator energy between energy calibrations [3]. An example of energy interpolation in the τ mass experiment is shown of Figure 9. The accuracy of interpolation depends on the energy calibration rate and the operation mode of the accelerator and was about 10–30 keV.

9 Conclusion

The resonant depolarization method for beam energy measurement is the most precise method of beam energy calibration. The high efficiency Touschek polarimeter at VEPP-4M is used in high precision mass measurement of J/ψ , $\psi(2S)$, $\psi(3770)$, $D^{+,0}$ mesons and τ lepton masses. More than three thousand energy calibrations were done. The Touschek polarimeter is used now for new D meson mass measurements with the KEDR detector. The VEPP-4M laser polarimeter will be used in new measurements of the mass of the Υ mesons. The resonant depolarization technique could also be applied in new τ mass measurements at the future Super Charm- τ factory [44].

10 Acknowledgment

This work is supported by the Russian Science Foundation under grant 14-50-00080. Part of the laser polarimeter equipment is provided by the Russian Foundation for Basic Research under grants 14-02-01011-a and 15-02-03114-a.

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