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Mixed QCD-EW corrections to Drell–Yan processes

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Abstract
We review the status of NNLO QCD–electroweak corrections to W- and Z-boson production at hadron colliders. We outline the application of the pole approximation to compute the dominant corrections, which arise from the combination of the QCD corrections to the production with electroweak corrections to the decay of the W/Z boson. We compare these results to simpler approximations based on naive products of NLO QCD and electroweak correction factors or leading-logarithmic approximations for QED final-state radiation.

Keywords
Electroweak vector bosons, higher-order corrections

1 Introduction

The Drell–Yan (DY)-like production of W and Z bosons, $\mathrm{pp}/\mathrm{p\bar{p}} \to V \to \mathrm{l_1 \bar{l}_2} + X$, is one of the most important class of "standard-candle" processes at hadron colliders and allows for precision tests of the Standard Model, as highlighted by the first LHC measurement of the W-boson mass M_W with an accuracy of 19 MeV [1], which may be reduced by a factor of two in the future. First LHC measurements of the forward-backward asymmetry in Z-production have appeared [2] that allow to extract the effective weak mixing angle, where the ultimate precision of the LHC might be competitive with that of LEP.

The sophisticated level of the current theoretical description of the DY-like production of W or Z bosons is reviewed in Ref. [3], where further references can be found. QCD corrections at next-to-next-to-leading-order (NNLO) accuracy are combined with higher-order soft-gluon effects through analytic resummation or matching to QCD parton showers. Electroweak (EW) corrections at NLO are supplemented with leading multi-photon final-state corrections and universal higher-order weak corrections. Compared to QCD corrections, the EW corrections lead to new features such as loop diagrams connecting initial and final states, the need for a consistent treatment of the finite vector-boson width, and the appearance of photon-induced processes. The NLO QCD and EW corrections are shown in Fig. 1 for the distributions of the transverse mass of the lepton pair ($M_{\mathrm{T},\nu\mathrm{l}}$), and the lepton transverse momentum ($p_{\mathrm{T,l}}$) in W production and the lepton-invariant-mass (M_{ll}) and $p_{\mathrm{T,l}}$ for Z production. The EW corrections significantly distort the distributions near the Jacobian peaks at $M_{\mathrm{T},\nu\mathrm{l}} \approx M_\mathrm{W}$ and $p_{\mathrm{T,l}} \approx M_\mathrm{V}/2$ and lead to large corrections in the invariant-mass spectrum in Z production due to photonic final-state corrections shifting the reconstructed value of M_{ll} away from the resonance $M_{\mathrm{ll}} = M_\mathrm{Z}$ to lower values. While the QCD corrections are moderate for the $M_{\mathrm{T},\nu\mathrm{l}}$ and M_{ll} distributions, they become extremely large for the $p_{\mathrm{T,l}}$ distributions above threshold due to the recoil of the vector boson against the real emission of a jet, and require all-order soft-gluon resummation for a consistent description.

The next challenges to improve fixed-order predictions for the DY processes are given by the N^{3}LO QCD corrections and the mixed NNLO QCD–EW corrections of $\mathcal{O}(\alpha_\mathrm{s}\alpha)$. In this contribution we provide an overview of the relevance and the current status of the $\mathcal{O}(\alpha_\mathrm{s}\alpha)$ corrections and outline our recent computation of the dominant corrections at this order for resonant vector-boson production using the so-called *pole approximation* [4, 5]. We compare these results to naive multiplicative combinations of NLO QCD and EW corrections, and to leading-logarithmic approximations of photon radiation.

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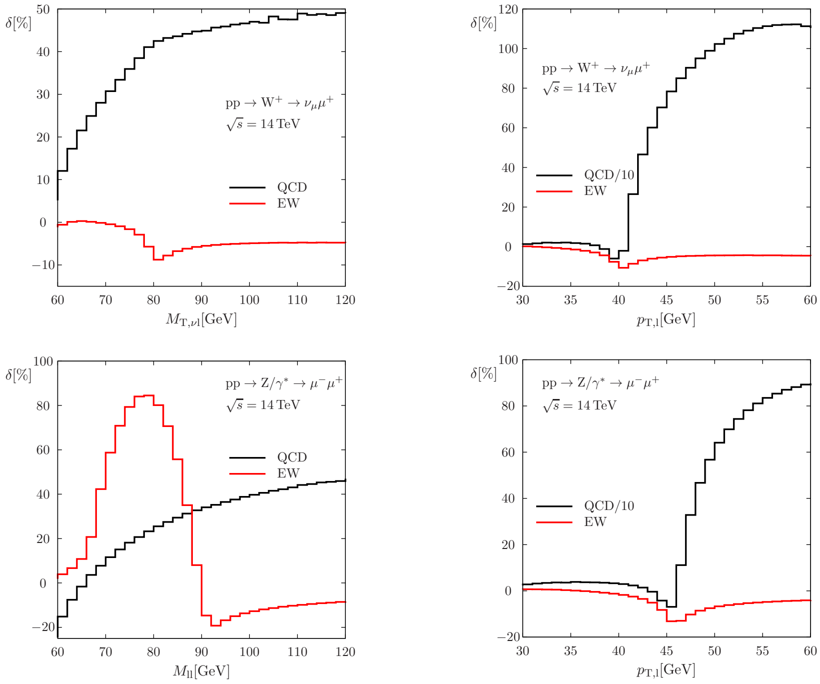

Fig. 1: NLO QCD and EW corrections relative to LO. Above: distributions in the transverse-mass (left) and transverse-lepton-momentum (right) for W^+ production at the LHC. Below: distributions in the invariant-mass (left) and transverse-lepton-momentum (right) for Z production at the LHC. (Taken from Ref. [4].)

2 Mixed QCD–EW corrections

The mixed QCD–EW NNLO corrections are expected to be particularly relevant in two regimes: First, at large invariant masses of the lepton pair the EW corrections are enhanced by so-called Sudakov logarithms, and the size of the $\mathcal{O}(\alpha_s\alpha)$ effects can be estimated to exceed the scale uncertainty of the NNLO QCD result [6]. On the other hand, observables for precision measurements dominated by the vector-boson resonance can show a percent-level sensitivity to the $\mathcal{O}(\alpha_s\alpha)$ corrections, resulting e.g. in an effect on the M_W determination of about 15 MeV [5, 7]. Therefore these corrections must be brought under theoretical control to match the precision goals of the LHC. Efforts are being made towards a full NNLO computation of the $\mathcal{O}(\alpha_s\alpha)$ corrections, which involves complicated multi-scale two-loop integrals [8] and requires a method for the cancellation of infra-red singularities to combine the two-loop corrections with the $\mathcal{O}(\alpha)$ EW corrections to $\mathrm{W/Z}+\mathrm{jet}$ production, the $\mathcal{O}(\alpha_s)$ QCD corrections to $\mathrm{W/Z}+\gamma$ production (see references in Ref. [5]), and the double-real corrections [9]. Awaiting the completion of these computations, the impact of the $\mathcal{O}(\alpha_s\alpha)$ corrections can be estimated by a naive multiplicative combination of the NLO QCD and EW corrections. In a more sophisticated approach, the fixed-order NLO QCD and EW corrections are matched to a QCD parton shower and a generator for final-state photon radiation (FSR) so that the virtual NLO corrections and the first emitted photon or gluon are treated exactly, while further emissions are generated in the collinear approximation. A careful treatment of the vector-boson resonance is required in order not to introduce spurious effects at $\mathcal{O}(\alpha_s\alpha)$ [7, 10].

2.1 Dominant mixed QCD–EW corrections in the pole approximation

A well-established method for the calculation of the EW corrections to precision observables dominated by the production of a resonant W or Z boson is provided by the pole approximation (PA). The PA is based on a systematic expansion of the cross section about the pole of the gauge-boson resonance and splits the corrections into factorizable and non-factorizable contributions. The former can be separately attributed to the production and decay of the gauge boson, while the latter link the production and decay

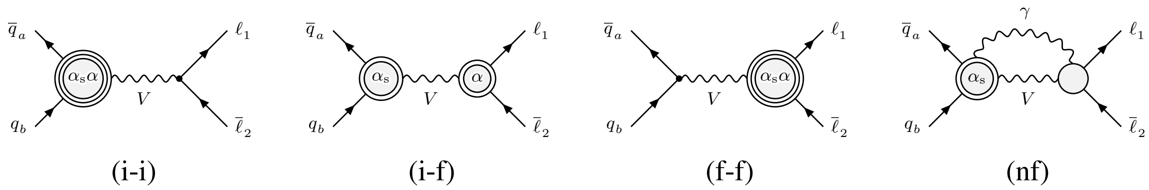

Fig. 2: Contributions to the mixed NNLO QCD–EW corrections in the PA illustrated by generic two-loop amplitudes: factorizable corrections of initial–initial (i-i), initial–final (i-f), and final–final type (f-f), and non-factorizable corrections (nf). Simple circles symbolize tree structures, double (triple) circles one-loop (two-loop) corrections.

subprocesses by the exchange of soft photons. The application of the PA to the NLO EW corrections shows agreement with the known full result up to fractions of 1% near the resonance [4, 11]. Motivated by this quality of the PA at NLO, in Refs. [4, 5] we have extended this method to the calculation of the $\mathcal{O}(\alpha_s\alpha)$ corrections in the resonance region, which are classified into the four types of contributions shown in Fig. 2 for the case of the double-virtual corrections:[1]

- (i-i) The initial–initial factorizable corrections are given by two-loop $\mathcal{O}(\alpha_s\alpha)$ corrections to on-shell W/Z production and the corresponding one-loop real–virtual and tree-level double-real contributions. Results for individual ingredients are known, but a consistent combination using a subtraction scheme for infrared singularities at $\mathcal{O}(\alpha_s\alpha)$ has not been performed yet.
- (i-f) The factorizable initial–final corrections consist of the $\mathcal{O}(\alpha_s)$ corrections to W/Z production combined with the $\mathcal{O}(\alpha)$ corrections to the leptonic W/Z decay and provide the numerically dominant contribution. The main results of their computation [5] are presented below.
- (f-f) Factorizable final–final corrections arise from the $\mathcal{O}(\alpha_s\alpha)$ counterterms of the lepton–W/Z-vertices. They yield a relative correction below 0.1% [5], so that they are phenomenologically negligible.
- (nf) The non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections are given by soft-photon corrections connecting the initial state, the intermediate vector boson, and the final-state leptons, combined with QCD corrections to W/Z-boson production. Their numerical effect is below 0.1% [4], so that for phenomenological purposes the $\mathcal{O}(\alpha_s\alpha)$ corrections can be factorized into terms associated with initial-state and/or final-state corrections and their combination.

The (i-i)-contributions are the only currently missing $\mathcal{O}(\alpha_s\alpha)$ corrections within the PA. Results of the PA at $\mathcal{O}(\alpha)$ show that observables such as the $M_{\mathrm{T},\nu l}$ distribution for W production or the M_{ll} distributions for Z production are extremely insensitive to photonic initial-state radiation (ISR) [4] and also do not receive overwhelmingly large QCD corrections. Therefore we do not expect significant initial–initial NNLO $\mathcal{O}(\alpha_s\alpha)$ corrections to such distributions. On the other hand, we expect class (i-f) to capture the dominant $\mathcal{O}(\alpha_s\alpha)$ effects, since it combines two types of corrections that are sizeable at NLO and deform the shape of differential distributions. Therefore our default prediction for $\sigma^{\mathrm{NNLO}_{\mathrm{s}\otimes\mathrm{ew}}}$ is given by the sum of the factorizable (i-f) corrections, $\Delta\sigma^{\mathrm{NNLO}_{\mathrm{s}\otimes\mathrm{ew}}}_{\mathrm{prod}\times\mathrm{dec}}$, and the full NLO QCD and EW corrections, $\Delta\sigma^{\mathrm{NLO}_\mathrm{s}} + \Delta\sigma^{\mathrm{NLO}_{\mathrm{ew}}}$. All contributions are consistently evaluated with NLO PDFs.

Figure 3 shows the numerical results for the relative $\mathcal{O}(\alpha_s\alpha)$ corrections for the $M_{\mathrm{T},\nu l}$ and the $p_{\mathrm{T,l}}$ distributions for W^+ production at the LHC. For Z production, the results for the M_{ll} distribution and a transverse-lepton-momentum ($p_{\mathrm{T,l^+}}$) distribution are displayed in Figure 4. We consider isolated ("bare") muons using the setup and input parameters of Ref. [5]. The corresponding corrections for "dressed leptons" recombined with collinear photons are similar, but typically smaller by a factor of two [5]. The figures show results for the following approximations:

[1] For each class of contributions apart from the (f-f) corrections, also the associated real–virtual and double-real corrections have to be computed, obtained by replacing one or both of the labels α and α_s in the blobs in Fig. 2 by a real photon or gluon, respectively, and including corresponding crossed partonic channels, e.g. with quark–gluon initial states.

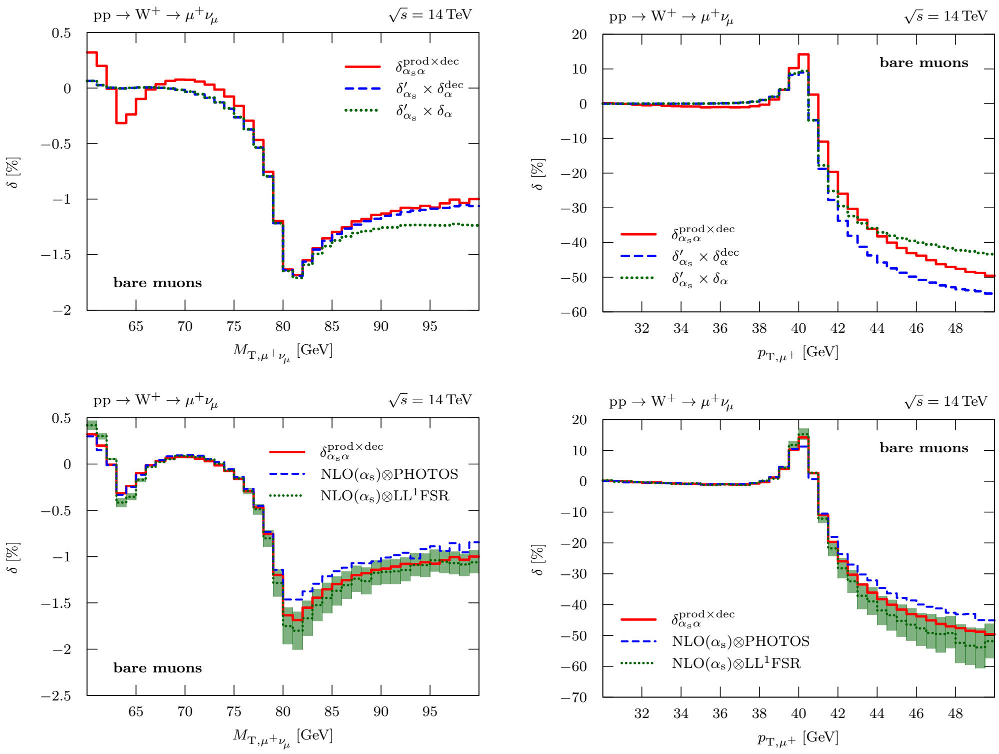

Fig. 3: Relative corrections of $\mathcal{O}(\alpha_s\alpha)$ induced by factorizable initial-state QCD and final-state EW contributions for the $M_{\mathrm{T},\nu l}$ (left) and $p_{\mathrm{T},l}$ (right) distributions for W^+ production at the LHC. The default prediction $\delta^{\mathrm{prod}\times\mathrm{dec}}_{\alpha_s\alpha}$ is compared to the naive products of the NLO correction factors δ'_{α_s} and δ_α (upper plots) and the combination of initial-state QCD with photonic FSR from PHOTOS or the structure-function ($\mathrm{LL}^1\mathrm{FSR}$) approach (lower plots). (Taken from Ref. [5].)

Pole approximation: Our default prediction of the (i-f) corrections, $\delta^{\mathrm{prod}\times\mathrm{dec}}_{\alpha_s\alpha} \equiv \Delta\sigma^{\mathrm{NNLO}_{s\otimes\mathrm{ew}}}_{\mathrm{prod}\times\mathrm{dec}}/\sigma^{\mathrm{LO}}$.

Naive products: The product $\delta'_{\alpha_s}\delta_\alpha$ of the QCD and EW correction factors,

$$\delta'_{\alpha_s} \equiv \Delta\sigma^{\mathrm{NLO}_s}/\sigma^{\mathrm{LO}}, \qquad \delta_\alpha \equiv \Delta\sigma^{\mathrm{NLO}_{\mathrm{ew}}}/\sigma^{0}. \tag{1}$$

Note that the LO prediction σ^{LO} (σ^0) is evaluated with LO (NLO) PDFs. The NLO EW corrections are defined in two different versions: based on the full $\mathcal{O}(\alpha)$ correction (δ_α), and on the dominant EW final-state correction of the PA ($\delta^{\mathrm{dec}}_\alpha$).

LL-FSR: The full NLO QCD corrections to W/Z production are combined with a leading-logarithmic (LL) approximation for FSR obtained using structure-functions [12] or PHOTOS [13]. To obtain the strict $\mathcal{O}(\alpha_s\alpha)$ corrections, only a single photon emission is generated in the LL approximation.

The difference of the two naive product versions gives an error-estimate of the PA, in particular of the missing (i-i) corrections. Deviations between our default prediction $\delta^{\mathrm{prod}\times\mathrm{dec}}_{\alpha_s\alpha}$ and the product approximations can be attributed to the double-real corrections, which do not simply factorize into a product of two NLO factors due to the interplay of recoil effects from jet and photon emission [5]. In contrast, both FSR approximations take this effect properly into account, but neglect subdominant finite contributions.

For the $M_{\mathrm{T},\nu l}$ distribution for W^+ production (left plots in Figure 3), the $\mathcal{O}(\alpha_s\alpha)$ corrections amount to $\approx -1.7\%$ around the resonance, which is about an order of magnitude smaller than the NLO EW corrections. Both variants of the naive product provide a good approximation to the full result in the region around and below the Jacobian peak, which is dominated by resonant W production. This is consistent with the insensitivity of $M_{\mathrm{T},\nu l}$ to photonic ISR already seen at NLO [4]. For larger $M_{\mathrm{T},\nu l}$, the

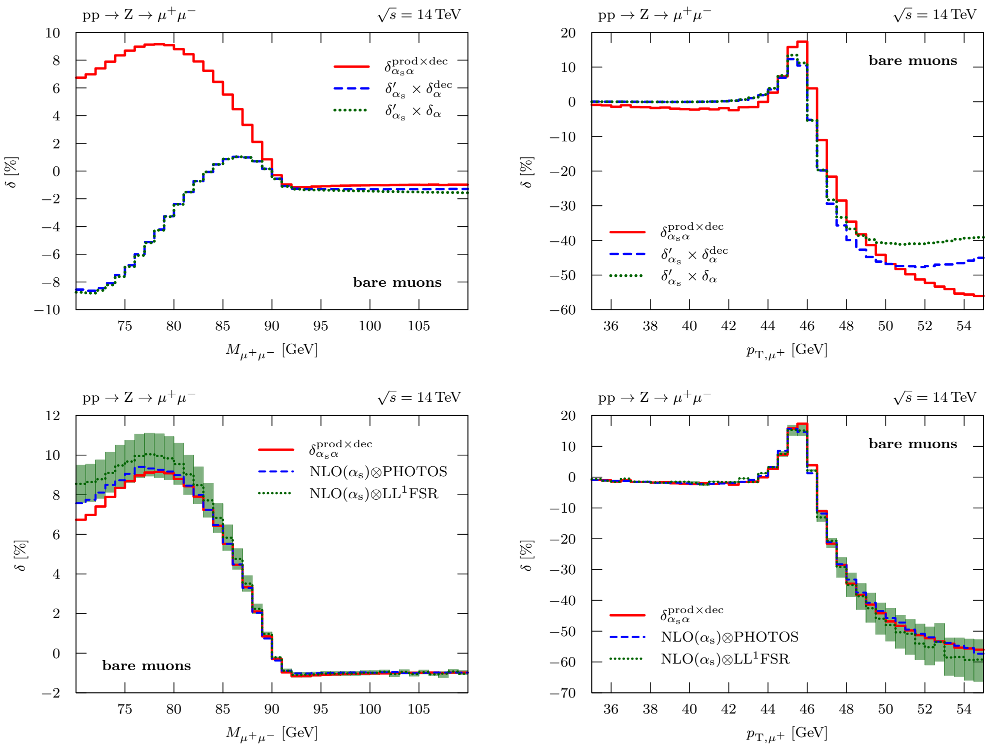

Fig. 4: As Fig. 3 but for the lepton-invariant-mass distribution (left) and a transverse-lepton-momentum distribution (right) for Z production at the LHC. (Taken from Ref. [5].)

product $\delta'_{\alpha_s}\delta_\alpha$ using the full NLO EW correction factor deviates from the other curves, which signals that effects beyond the PA become more important, but remain at the per-mille level for $M_{\mathrm{T},\nu l} \lesssim 90$ GeV. The two FSR approximations show good agreement and improve over the naive product approximations. For the structure-function approach (denoted by LL^{1}FSR), the intrinsic uncertainty of the LL approximation is illustrated by the band width resulting from varying the QED scale Q within the range $M_V/2 < Q < 2M_V$ for $V = \mathrm{W}, \mathrm{Z}$.

The corrections to the $p_{\mathrm{T,l}}$ distributions (right plots in Figures 3 and 4) are small far below the Jacobian peak, but rise to about 15% (20%) on the Jacobian peak at $p_{\mathrm{T,l}} \approx M_{\mathrm{V}}/2$ for the case of the W^+ boson (Z boson) and then drop to -50%. As in the NLO QCD results of Fig. 1, the large corrections above the Jacobian peak arise since recoil due to real QCD radiation shifts events with resonant W/Z bosons above the Jacobian peak. This effect implies a larger impact of the double-real emission corrections, which are not captured correctly by the naive product ansatz that deviates from the full result $\delta^{\mathrm{prod}\times\mathrm{dec}}_{\alpha_s\alpha}$ by 5–10% at the Jacobian peak, where the PA is expected to be the most accurate. The differences of the two versions of the naive products furthermore indicates the potential impact of the missing $\mathcal{O}(\alpha_s\alpha)$ (i-i) corrections.[2] The description of the $p_{\mathrm{T,l}}$ distributions is improved by the combination of the full NLO QCD corrections with LL photon emission, but some differences remain for W-production.

In the M_{ll} distribution for Z production (left plots in Figure 4), corrections up to 10% are observed below the resonance. This is consistent with the large NLO EW corrections from photonic final-state radiation (FSR) seen in Figure 1. The naive products $\delta'_{\alpha_s}\delta^{(\mathrm{dec})}_\alpha$ approximate the full (i-f) corrections $\delta^{\mathrm{prod}\times\mathrm{dec}}_{\alpha_s\alpha}$ reasonably well for $M_{\mathrm{ll}} \geq M_{\mathrm{Z}}$ but have the wrong sign already slightly below the resonance. The reason for this failure is that the appropriate QCD correction factor for the events that are shifted below the resonance by photonic FSR is given by its value at the resonance, whereas the naive product

[2] These deviations should be interpreted with care, since the peak region $p_{\mathrm{T,l}} \approx M_{\mathrm{V}}/2$ corresponds to the kinematic onset for $V+$ jet production where fixed-order predictions break down and QCD resummation is required for a proper description.

ansatz simply multiplies the corrections locally. In contrast, the two FSR approximations model the M_{ll} distribution correctly within their uncertainty.

3 Conclusions

The precision-physics program in Drell–Yan-like W- and Z-boson production at the LHC requires a further increase in the accuracy of the theoretical predictions, where the mixed QCD–electroweak corrections of $\mathcal{O}(\alpha_s\alpha)$ represent the largest component of fixed-order radiative corrections after the well established NNLO QCD and NLO electroweak corrections. In this contribution, we have reviewed the construction of the pole approximation for evaluating the $\mathcal{O}(\alpha_s\alpha)$ corrections to Drell–Yan processes [4], and summarized our numerical results [5] for the dominant factorizable corrections, which arise from the combination of sizeable QCD corrections to the production with large EW corrections to the decay subprocesses. Naive product approximations fail to capture these corrections in distributions that are sensitive to QCD initial-state radiation and therefore require a correct treatment of the double-real-emission part of the NNLO corrections. Naive products also fail to capture observables that are strongly affected by a redistribution of events due to final-state real-emission corrections, such as the invariant-mass distribution in Z production. A combination of the NLO QCD corrections and a collinear approximation of real-photon emission through a QED structure-function approach or a QED parton shower such as PHOTOS provides a significantly better agreement with our results. In order to reduce ambiguities due to the leading-logarithmic accuracy of these approaches, a consistent matching to the full NLO EW correction is mandatory, as emphasized recently also in Ref. [7].

Acknowledgements

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