LANDAU DAMPING WITH ELECTRON LENSES

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Abstract

Electron lenses provide an incoherent betatron tune spread for Landau damping of transverse coherent beam instabilities. We investigated the effect of the transverse electron beam size and shape for Landau damping with an electron lens. Another point of interests is Landau damping provided by a pulsed electron lens with homogeneous transverse beam profile. This type of electron lens is developed for space-charge compensation in SIS18.

INTRODUCTION

Impedance driven transverse beam instabilities in hadron synchrotrons are damped by either an active feedback system or passive mitigation via Landau damping [1] due to dedicated Landau octupole magnets.

For high energy and high-intensity synchrotrons a number of proposals of alternative sources of Landau damping have been proposed [2, 3]. In this contribution, we are comparing stability boundaries from dispersion relations for an electron lens proposed in [2] with our simulation results. In addition, we will analyse Landau damping from a pulsed electron lens [4].

We compare the stability boundaries obtained from the dispersion relations with the ones obtained from particle tracking simulations using an effective impedance.

Table 1: LHC and FCC-hh parameters

	LHC	FCC
Circumference, C [km]	27	100
Beam energy, E [GeV]	7	50
Average beta function, β_{avg} [m]	72	140
Betatron tune, Q_x	59.31	111.31
Betatron tune, Q_y	63.32	109.32
Synchrotron tune, Q_s	$2.2 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$
Number of octupoles, N_{oct}	168	≈ 4200

Landau damping due to octupole magnets

From the dispersion relations, it is possible to obtain an estimation of the stability area due to Landau damping for the given tune spread. For octupole magnets the incoherent betatron tune spread is linear with amplitude:

$$\Delta Q_x = a_{xx} J_x/\epsilon_x + a_{yx} J_y/\epsilon_x,$$
$$a_{xx} \propto N_{oct} I_{\text{oct}} \epsilon_{\text{n}}/\gamma^2$$

where I_{oct} is octupole current, ϵ_{n} – normalized emittance, γ – relativistic gamma, N_{oct} – number of octupoles.

Assuming that FCC-hh would use LHC-like octupoles one can obtain for the parameters given in table 1 that FCC-hh would need ≈ 25 times the number of octupoles currently used in the LHC operation to obtain the same order of incoherent betatron tune spread. (See Fig. 1.)

For this betatron tune spread we estimate the stability of the beam in the FCC-hh due to Landau damping from the dispersion relation. (See Fig. 2.) For the case of the rigid mode and two-dimensional betatron tune spread dependent on the transverse amplitudes the dispersion relation has been derived by F. Ruggiero and J.S. Berg [5]:

$$1 = -\Delta Q_{\text{coh}} \int_0^\infty dJ_x \int_0^\infty dJ_y \frac{J_x \dfrac{\partial \psi(J_x, J_y)}{\partial J_x}}{Q - Q_x(J_x, J_y)}, \tag{1}$$

where J_x, J_y – transverse action variable, $\psi(J_x, J_y)$ – particle distribution function.

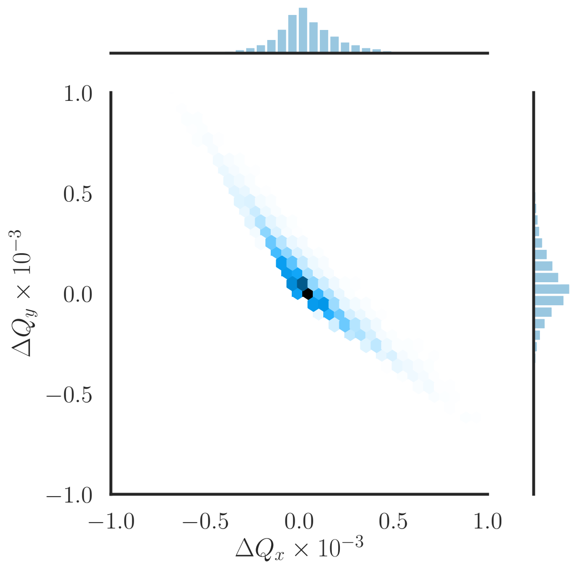

Figure 1: Tune spread for LHC-like octupoles with $Q_s = 1.2 \cdot 10^{-3}$, $\xi_{x,y} = 0$ and rms tune spread $\delta Q_{\text{rms}} \approx 2.1 \cdot 10^{-4}$.

Landau damping due to an electron lens

Electron lenses have been proposed as a potential Landau damping source [2]. An electron lens creates a non-linear dependence of the incoherent betatron tunes on the transverse amplitudes:

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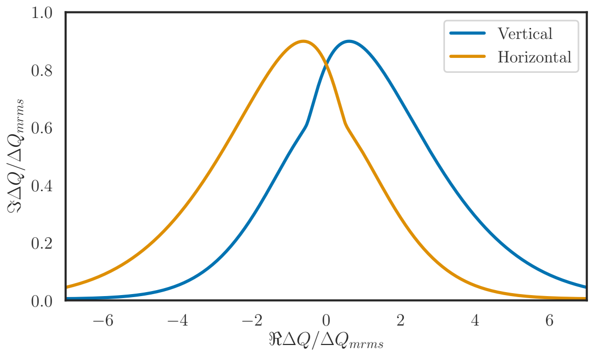

Figure 2: Stability diagram for LHC-like octupoles in vertical and horizontal plane.

$$\Delta Q_x \propto \delta Q_{\max} \iint \left(\frac{\int_0^r j_e(r')r'dr'}{r^2} \right) \left(\frac{\sin^2 \phi_x}{(2\pi)^2} \right) d\phi_x \, d\phi_y, \tag{2}$$

where $\delta Q_{\max}$ – maximal tune shift, $r^2 = 2J_x \sin^2 \phi_x + 2J_y \sin^2 \phi_y$, $j_e(r)$ – transverse current density distribution of an electron beam, ϕ_x, ϕ_y – betatron phases.

In this contribution, we will focus on the case of Gaussian current distribution of $j_e(r)$. Tune shift from the electron lens is at it largest for particles with $J_x, J_y = (0, 0)$ and it's decreasing with larger amplitude resulting in a diamond shape for the electron and proton beams of equal size. For this case maximal tune shift will be expressed as (See Fig. 3):

$$\delta Q_{\max} = \frac{I_e}{I_A} \frac{m_e}{m_p} \frac{\sigma_x^2}{\sigma_e^2} \frac{L_e}{4\pi\epsilon_n} \frac{1 \pm \beta_e}{\beta_e}, \tag{3}$$

where I_e is electron beam current, $I_A = \frac{m_e c^3}{e}$ 17kA – Alfven current, m_e, m_p – masses of electron and proton, σ_e, σ_x – electron and proton beam rms sizes, β_e – relativistic beta of the electron beam. (It is assumed here that $\beta_p \approx 1$ for the proton beam.) [2]

Let us fix the maximum tune shift $\delta Q_{\max}$ and vary the electron beam size σ_e. To give an intuition of how the tune distribution will be changing let us look at two asymptotic cases. First, let $\sigma_e \ll \sigma_p$. In this situation, most particles will experience a weak Coulomb force and their tunes will be close to the unperturbed tune. Second, if $\sigma_e \gg \sigma_p$, every particle of the proton beam is lying on the center of the electron beam, thus these particles will be experiencing the linear part of the force corresponding to $\delta Q_{\max}$. With these examples it is clear that changing the electron beam size leads to a redistribution of particles between $(0, 0)$ and $(\delta Q_{\max}, \delta Q_{\max})$. We can study how this affects the stability area using the dispersion relation from Eq. 1. This leads us to plot from Fig. 4, from which we can conclude that decreasing the rms size of the electron beam σ_e changes the stability area significantly towards a enlargement of the stable area for positive real coherent tune shifts. Increasing σ_e leads only to marginal benefits and $\sigma_e = 0.9\sigma_x$ is the optimal size ratio from the analytical estimation.

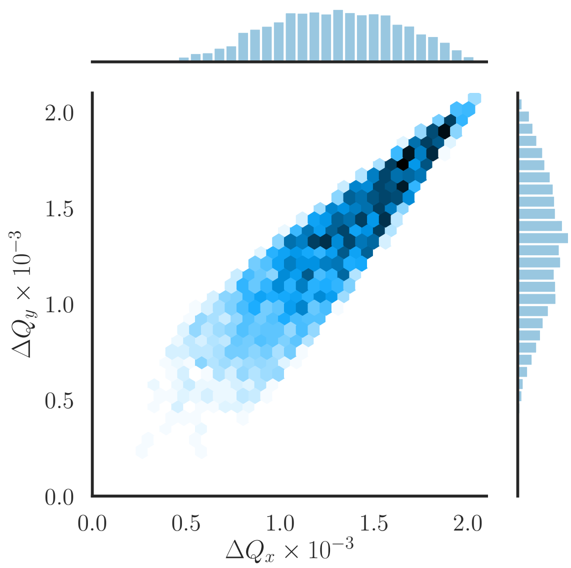

Figure 3: Tune spread from an electron lens with $\sigma_e = \sigma_x$, $\Delta Q^{\max}_{x,y} = 2 \cdot 10^{-3}$ with $Q_s = 1.2 \cdot 10^{-3}$, $\xi_{x,y} = 0$ and rms tune spread $\delta Q_{\text{rms}} \approx 3.5 \cdot 10^{-4}$.

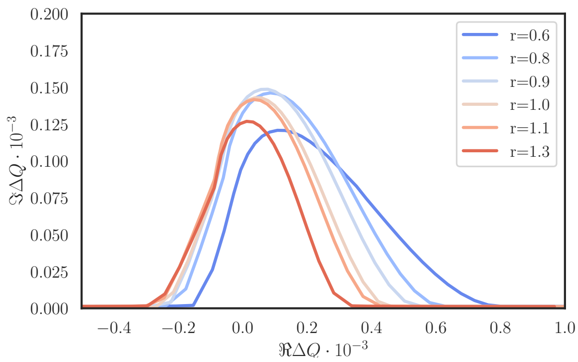

Figure 4: Stability diagram with electron lens beam sizes σ_e from, $r = 0.6 - 1.3$ of proton beam sizes σ_x.

Pulsed electron lens

For space-charge compensation, a pulsed electron lens has been proposed for SIS18 at GSI [4]. This type of an electron lens also creates incoherent betatron tune spread (Fig. 5), that depends on the longitudinal action J_z. This is similar to Radio Frequency Quadrupole proposed as a source of Landau damping for FCC-hh [3]. For tune spread depending on the longitudinal amplitude we are using the J.S.Berg and F.Ruggiero dispersion relation [5]:

$$1 = \int_0^\infty \Delta Q_{\text{coh}} \frac{\psi(J_z)|J_z|^m}{Q_x - Q(J_z) - mQ_s} dJ_z, \tag{4}$$

where J_z – longitudinal action variable, m – mode number (in this contribution we are always assuming $m = 0$).

In the next section, we are going to establish a method to reconstruct stability diagrams from particle tracking simulation and compare the results from simulation to the solution of the dispersion relations. Stability diagrams in this study are normalised by rms tune spread to compare all three sources as if they had the same rms tune spread.

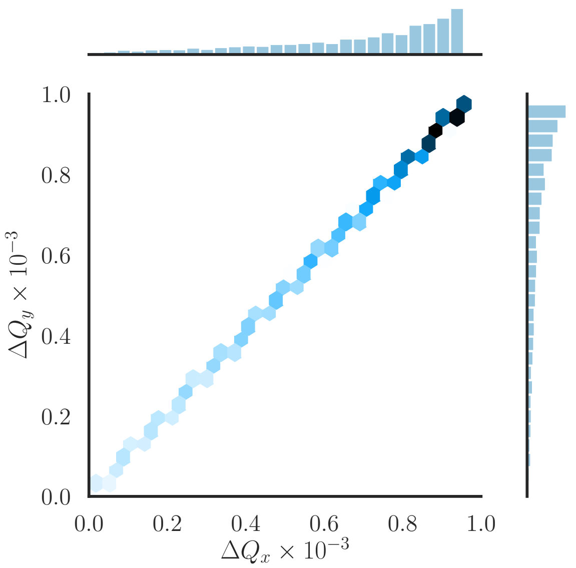

Figure 5: Tune spread from an pulsed electron lens with $\sigma_e = 4\sigma_x$, $\Delta Q^{\max}_{x,y} = 10^{-3}$ with $Q_s = 1.2 \cdot 10^{-3}$, $\xi_{x,y} = 0$, $\delta Q_{\rm rms} \approx 2.4 \cdot 10^{-4}$. Transverse distribution is assumed to be homogeneous.

STABILITY DIAGRAM RECONSTRUCTION

It is necessary to confirm with particle tracking simulations that the actual stability area would correspond to the one obtained from an analytical estimation of dispersion relations. Typically, this is done with Beam Transfer Functions(BTFs) both in experiments and in simulations. We will instead employ an effective impedance model to excite a rigid head-tail mode with a given $\Re\Delta Q$, $\Im\Delta Q$ similar to what has been done in [6]. Using this model allows us to follow the particle offset evolution and intrabunch motion during the simulation for all selected coherent tune shifts $(\Re\Delta Q, \Im\Delta Q)$ and determine individually for each pair the stability of the beam due to Landau damping. This method corresponds to the use of a transverse damper as an impedance source to excite an instability experimentally.

The effective impedance [7]

$$\Delta Q = -\frac{i\beta c^2}{2Q\omega_0^2 E_0}\frac{eI_0}{C}Z^{\perp}$$

, where C is the machine circumference, I is the beam current; is implemented as a kick in the tracking code

$$\Delta x' = 4\pi\Im\Delta Q_{\rm coh}\bar{x}' + 4\pi\Re\Delta Q_{\rm coh}\frac{\bar{x}}{\hat{\beta}_x} \qquad (5)$$

$\bar{x}'$ and $\bar{x}$ are the offset and its derivative averaged over all particles in the bunch and taken at the position of the kick. Within the above implementation the kick we only excite rigid ($k = 0$) bunch modes, which allows a direct comparison to the dispersion relations given by Eq. 1 and Eq. 4.

Reconstructed stability diagrams for octupoles and electron lens

The PyHEADTAIL particle tracking code with ≈ 30000 macroparticles is used to track the development or damping of the instability driven by the effective impedance for FCC-hh parameters. In order to determine if for given parameters the beam is stable or not, we used two simple criteria. First, if the maximal beam offset observed during simulation is greater than a given threshold ($5\mu m$), the point is considered to be unstable. Second, if the beam offset evolution is exponential, it is also considered to be unstable even if the amplitude did not reach the threshold during the run.

In Fig. 6 we can observe the reconstructed stability diagram for LHC-like octupoles with rms tune spread $\Delta Q_{\rm rms} \approx 2.1 \cdot 10^{-4}$ showed in Fig. 1. From the simulations, we are only showing coherent tune shifts for which the beam is stable. The agreement between the theoretical stability boundary and one obtained in particle simulation is achieved.

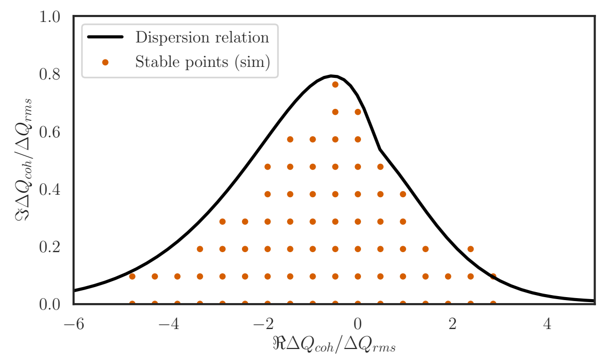

Figure 6: Stability diagram reconstruction for LHC-like octupoles using FCC parameters for the vertical plane($\xi_{x,y} = 0$, $Q_s = 1.2 \cdot 10^{-3}$), $\Delta Q_{\rm rms} = 2.1 \cdot 10^{-4}$. Solid line is the theoretical stability boundary and each point is obtained from particle tracking simulation.

In Fig. 7 we present the result of reconstructing stability boundary for an electron lens with $\delta Q_{\max} = 0.002$ and $\delta Q_{\rm rms} \approx 3.5 \cdot 10^{-4}$ and matched size $\sigma_e = \sigma_x$. This demonstrates that electron lens is a source of betatron tune spread that leads to Landau damping. The stability area normalised by the rms tune spread for electron lens is smaller than for octupole magnets but electron lens can achieve larger rms tune spread by scaling $\Delta Q_{\max}$ up to 0.01 [2].

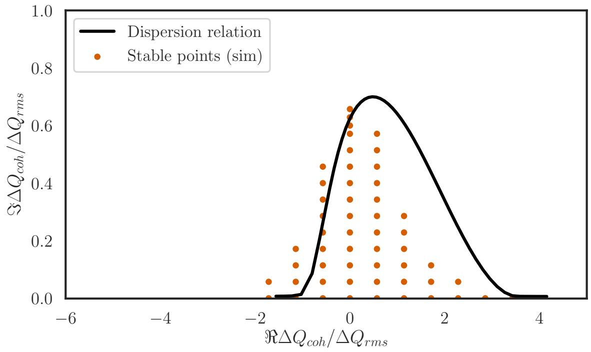

Figure 7: Stability diagram reconstruction for electron lens matched to the proton beam size($\xi_{x,y} = 0$, $Q_s = 1.2 \cdot 10^{-3}$). Solid line is the theoretical stability boundary and each point is obtained from particle tracking simulation.

Let us compare results from solving dispersion relation from Eq. 4 to the results of the simulation with a pulsed lens with homogeneous transverse profile. Electron beam sizes was chosen to be $\sigma_e = 4\sigma_x$ in order to ensure that tune spread during the simulation runs comes from the longitudinal amplitude only. In Fig. 8 we can see that similarly to an RFQ [8, 9] stability boundary is asymmetric with respect to $\Re\Delta Q_{coh} = 0$ line but contrary to the RFQ case pulsed electron lens provides the same stability diagram in both x and y planes. It is possible to overcome this asymmetry by trying a different transverse beam profile for the pulsed lens or combining it with octupoles. We have achieved a qualitative agreement between our simulation with the pulsed electron lens and existing dispersion relation theory.

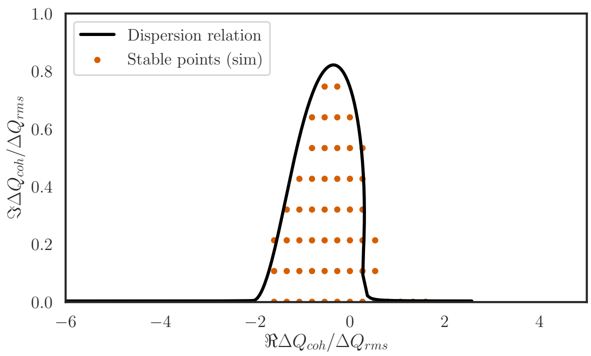

Figure 8: Stability diagram reconstruction for pulsed electron with $\sigma_e = 4\sigma_x$, $\Delta Q^{\max}_{x,y} = 10^{-3}$ with $Q_s = 1.2 \cdot 10^{-3}$, $\xi_{x,y} = 0$, $\delta Q_{\rm rms} \approx 2.4 \cdot 10^{-4}$. Transverse distribution is assumed to be homogeneous. Solid line is the theoretical stability boundary and each point is obtained from particle tracking simulation.

CONCLUSION

For an effective instability model stability diagrams were reconstructed from particle tracking simulation. We only account for rigid bunch oscillations and zero chromaticity. Additionally, the tracking studies account for the dynamic change in beam size and betatron tune spread over the course of the simulation, which can affect Landau damping[10]. In both [6, 10] it has been shown that the agreement between particle tracking and dispersion relations, as expected, is not perfect and the latter overestimates the stability boundary in comparison to the former, which agrees with our results.

For a standard electron lens, we showed that relative transverse size of the electron beam σ_e can serve as an additional knob to adjust stability boundary due to Landau damping. According to the dispersion relation, the relative beam size slightly smaller than 1.0 ($\sigma_e/\sigma_x \in (0.8, 1.0)$) provides a marginal benefit over the matched beams.

Our simulations show that both DC and pulsed electron lenses serve as a source of betatron tune spread that leads to the stabilisation of the beam due to Landau damping. It has been demonstrated that electron lens with an incoherent betatron tune spread of the same order as octupoles provides similar stability area in the complex tune shift space. Importantly, betatron rms tune spread from electron lens even with the modest maximal tune shift of $\Delta Q_{\max} = 0.002$ is already two times larger than the tune spread from ≈ 4200 LHC-like octupoles.

OUTLOOK

Further research into the discrepancy between DC electron lens simulation and dispersion relation results is necessary.

Firstly, for SIS18/SIS100 at GSI the incoherent space charge tune spread is significant. We plan to use the same method to study the stability boundary in the presence of space charge. Additionally, this method could be used to estimate the stability boundary for a combination of octupoles and RFQ or pulsed electron lens.

Secondly, the study of higher-order modes is especially necessary for the pulsed electron lens and the RFQ because dispersion relation in Eq.4 has a $|J_z|^m$ dependence on head-tail mode number m that implies a significant difference between stability diagrams for different modes.

REFERENCES

[1] Lev Landau. “On Oscillations of electron plasma.” In: *J. Phys. USSR* 10.25 (1946).

[2] V. Shiltsev et al. “Landau Damping of Beam Instabilities by Electron Lenses.” In: *Physical Review Letters* 119.13 (Sept. 2017), p. 134802.

[3] A. Grudiev. “Radio frequency quadrupole for Landau damping in accelerators.” In: *Physical Review Special Topics - Accelerators and Beams* 17.1 (Jan. 2014), p. 011001.

[4] O. Boine-Frankenheim and W.D. Stem. "Space charge compensation with pulsed electron lenses for intense ion beams in synchrotrons." In: *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 896 (July 2018), pp. 122–128.

[5] J. Scott Berg and Francesco Ruggiero. "Stability diagrams for Landau damping." In: *Proceedings of the 1997 Particle Accelerator Conference (Cat. No.97CH36167)*. Vol. 2. IEEE, 1998, pp. 1712–1714.

[6] V. Kornilov, O. Boine-Frankenheim, and I. Hofmann. "Stability of transverse dipole modes in coasting ion beams with nonlinear space charge, octupoles, and chromaticity." In: *Physical Review Special Topics - Accelerators and Beams* 11.1 (2008), p. 014201.

[7] K. Y. Ng. *Physics of Intensity Dependent Beam Instabilities*. WORLD SCIENTIFIC, Dec. 2005.

[8] M. Schenk et al. "Analysis of transverse beam stabilization with radio frequency quadrupoles." In: *Physical Review Accelerators and Beams* 20.10 (Oct. 2017), p. 104402.

[9] M. Schenk et al. "Experimental stabilization of transverse collective instabilities in the LHC with second order chromaticity." In: *Physical Review Accelerators and Beams* 21.8 (Aug. 2018), p. 084401.

[10] V. Kornilov and O. Boine-Frankenheim. "Landau damping due to octupoles of non-rigid head–tail modes." In: *Nuclear Instruments and Methods in Physics Research, Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 951 (2020), p. 163042.