Physics modelling and numerical simulation of beam-ion interaction in HEPS

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The beam-ion interaction is one of the potential limitations of beam performance in ultra-low emittance electron rings. The High Energy Photon Source (HEPS), under construction in Beijing, is one example for which the beam-ion effect has to be carefully evaluated. In this paper, we will introduce the beam-ion interaction models applied in HEPS. Based on these models, a new numerical simulation code is developed. Currently, the code includes modules such as ionization, beam-ion interaction, synchrotron radiation damping, quantum excitation, bunch-by-bunch feedback etc. Settings such as weak-strong, strong-strong and arbitrary number of interaction points can be launched in the code. It will be shown that the beam instability excited by the beam-ion interaction can be effectively suppressed by the bunch-by-bunch feedback system.

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I. INTRODUCTION

The beam-ion interaction, a two-stream effect coupled by the nonlinear Coulomb force, may pose an operation risk in high intensity, low emittance electron rings. This effect has been observed in many existing machines such as ALS [1, 2], PLS [3], SSRF [4] CESR-TA [5], SOLEIL [6], etc. The accumulated ions, derived from ionization between electron particles and residual gas molecules, interact with the electron beam particles resonantly, causing coherent and incoherent electron beam deformation such as beam centroid oscillation, beam rms emittance growth, rms beam sizes increase, energy spread blow up, and even a possible beam loss.

In the previous studies, the beam-ion effect [7–9] is divided into two circumstances known as the ion trapping effect and the fast ion effect. In the ion trapping study, ions are usually assumed to be constant as an "equilibrium" state, which means the transient electron beam induced ionization is not taken into account. In the fast ion effect study, on the other hand, the ions are considered to be cleaned turn by turn. The ions generated in the first turn do not disturb the beam performance in the second turn, which means the memory of the ionization is erased turn by turn. Thus, the beam-ion trapping studies can be considered as an effect in a long term sense and the fast ion effect in a transient sense. In both cases, the ions generated by the leading bunches oscillate transversely and resonantly disturb the motions of the subsequent bunches – a coupled bunch instability.

Generally, the methodologies to mitigate the beam-ion effect are: (1) adjust the beam filling pattern by including empty buckets long enough in the bunch train; (2) get rid of ions with certain accelerator elements; (3) cure the beam-ion instability by introducing a feedback system before it grows [10]. The first approach can extensively reduce the number of accumulated ions. With sufficiently large empty gaps, the trapped ions would drift to large amplitudes, where they may get lost on the pipe or form a diffuse ion halo. However, this approach is a partial solution since the disturbed bunch can not erase the memory of its prior interaction with the ions. The beam deformation by the beam-ion interaction will accumulate and the influence could appear eventually. The second and the third approaches both require extra hardware, which brings in new sources of lattice impedance. However, the bunch-by-bunch feedback system is a versatile [11] technique, since it can be also adopted to suppress the beam instabilities due to impedances.

In the code developed by our group, the ion-trapping and fast ion effects are not distinguished but treated consistently. The code is bascially based on the "quasi-strong-strong" model, in which electron particles and ions are both represented by multiple macroparticles. Modules of ionization, beam-ion interaction, synchrotron radiation damping, quantum excitation and bunch-by-bunch feedback are also established. The HEPS [12] lattice is adopted as an example, to show the beam-ion interaction in simulation. This paper is organized as follows. In section II, the physical process and models of beam-ion interaction will be discussed briefly. The logic flow and basic numerical simulation approaches used will be given. In section III, the beam-ion instability and its mitigation with a bunch-by-bunch feedback system in HEPS will be given. A brief discussion and conclusion are given in Section IV.

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II. PHYSICS MODEL AND NUMERICAL APPROACHES USED IN SIMULATION.

A. Basic model of beam-ion interaction

Denote P and T as the residual gas pressure and temperature, the molecular density n can be obtained,

$$PN_A = nRT, \tag{1}$$

where R and N_A are the ideal gas constant and the Avogadro number. Denoting $\sum$ as the ionization collision cross-section and N_b as the number of electron particles passing by, the number of ions generated due to beam-residual gas collision per unit length is

$$\lambda = \sum nN_b. \tag{2}$$

For simplicity, we assume the beam-ion interaction is localized at the lumped interaction locations and only affects the beam transversely. In addition, ions generated do not move longitudinally. When the beam bunches pass through the interaction points one by one, new ions will be randomly generated within the sizes of the electron bunches passing by. The beam filling pattern decides the interval time of the generation of the new ions. The accumulated ions, generated due to the former passed bunches, interact with the passing beam bunch and thereafter freely drift in transverse until the next beam bunch comes. Meanwhile, some ions might get lost on the pipe. Due to the ion generation and loss mechanisms, a dynamical quasi-equilibrium ion distribution can be foreseen finally.

Figure 1 shows the logic flow of the simulations. S_i represents the lumped interaction point. When one electron bunch passes by, the transverse momentum and position of the accumulated ions are updated according to the time interval from itself to next coming bunch. As to the bunched electron particles, after the momentum kicks induced by the accumulated ions, taking into account the effect of synchrotron radiation and quantum excitation, they are transferred to the next interaction point S_{i+1} by applying a linear transport matrix $M(S_{i+1}|S_i)$.

B. Coulomb interaction

In general, the motion equations of the ith accumulated ion $\vec{X}_i$ and the kth electron particle in the jth bunch $\vec{x}_{k;j}$ can be expressed as

$$\frac{d^2\vec{X}_i}{dt^2} + K_i(s)\vec{X}_i + \sum_{k=0}^{N_j} \vec{F}_C(\vec{X}_i - \vec{x}_{k;j}) = 0$$

$$\frac{d^2\vec{x}_{k;j}}{ds^2} + K_e(s)\vec{x}_{k;j} + \sum_{i=0}^{N_i} \vec{F}_C(\vec{x}_{k;j} - \vec{X}_i) = 0, \tag{3}$$

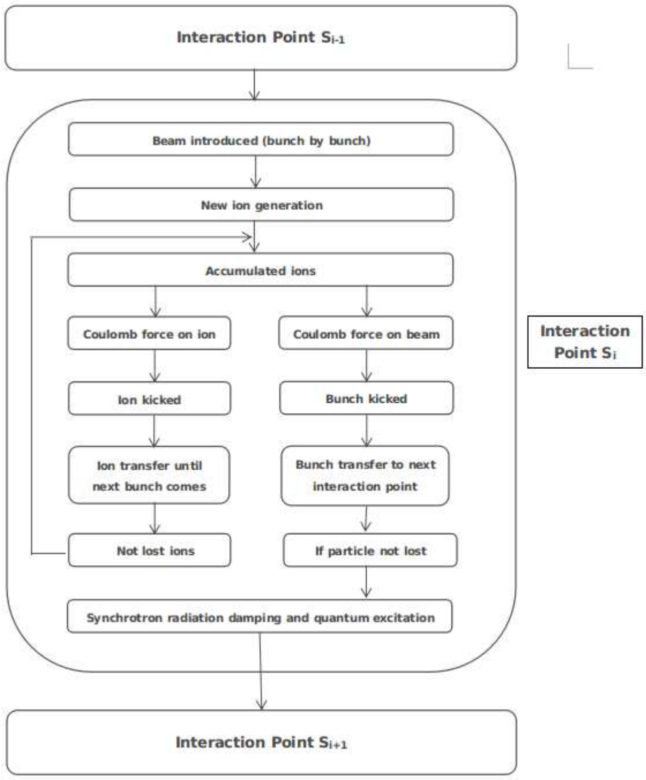

FIG. 1. Logic flow of the beam-ion interaction in simulation at the interaction points S_i. The ion generation, beam-ion interaction, beam and ion loss assertion, effect of synchrotron radiation and quantum excitation and beam transportation are sequentially calculated.

where $\vec{F}_C$ is the Coulomb force between the ions and electron particles, $K_i(s)$ and $K_e(s)$ represent the lattice focusing strength on ion and electron particle. In our model, since the bunched beam is assumed to follow the Gaussian distribution, the field generated by a beam bunch at spatial location (x, y), with respect to the bunch centre, is obtained with the 2D Bassetti-Erskine formula [13] ,

$$\begin{aligned} E_{C,y} + iE_{C,x} = &\frac{n_b}{2\epsilon_0\sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}}\{w(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}) \\ &- \exp(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}) \\ &+ w(\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2-\sigma_y^2)}})\}\delta(s_i), \end{aligned} \tag{4}$$

where n_b is the line density of the electron beam, $w(z)$ is the complex error function, i is the complex unit, σ_x and σ_y are the rms bunch size in x and y direction respectively. Substituting Eq. 4 into Eq. 3, the explicit momentum kick on ions is

$$\Delta p_{i,y} + i\Delta p_{i,x} = \frac{2n_b r_e m_e c}{\gamma_e}(E_{C,y} + iE_{C,x}), \tag{5}$$

where r_e is the classical electron radius, m_e is the electron mass, γ_e is the relativistic factor of electron beam. The accumulated ion momentum change induced by the electron bunch passed by can be obtained by integrating Eq. 5 along the length of adjacent electron bunches.

As to the space charge potential well generated by the ions, since the ion distribution is usually not a Gaussian type, the Bassetti-Erskine formula is not suitable any more. A self-consistent particle-in-cell (PIC) [14] solver or ion density profile fitting [15] is needed to ensure a better resolution. In our model, a compromise approach is applied–the "quasi-strong-strong" model. The ion distribution is truncated at 10 rms bunch sizes. The rms and centroid information of the truncated ion distribution are substituted in the Bassetti-Erskine formula to get the Coulomb potential produced by the ions. Although this approach is not strictly correct, it still shows the main features of the bunched beam and can explore this complex coupled dynamics in a reasonable computing time.

C. Beam transportation

We employ the accelerator coordinate $\boldsymbol{x} = (x, p_x, y, p_y, z, p_z)$ to describe the motion of particles. Here x, y, z are horizontal, vertical and longitudinal coordinates respectively, while p_x, p_y, p_z are the corresponding momenta normalized by the total momentum of a reference particle. Following Hirata's BBC code [16], in general the transportation consists of the following steps.

1. *From accelerator coordinates to normalized coordinates*

The transformation from accelerator variable $\boldsymbol{x}$ to normalized variable $\boldsymbol{X}$ can be written as

$$\boldsymbol{X} = \boldsymbol{BRHx}, \tag{6}$$

here $\boldsymbol{H}$ is the dispersion matrix characterized by the transverse dispersion functions D_x, D_{px}, D_y, D_{py}; $\boldsymbol{R}$ is the Teng matrix representing the coupling between the horizontal and vertical planes; $\boldsymbol{B}$ represents the Twiss matrix. More details can also be found in Ref. [17].

In our study, the dynamic is limited to the transverse plane, furthermore, the beam-ion interaction is assumed to take place in the dispersion and coupling free region so that the $\boldsymbol{H}$ and $\boldsymbol{R}$ are further degenerated to the unit matrix.

2. *Synchrotron radiation and quantum excitation*

With the Synchrotron radiation and quantum excitation effects, the transportation in the normalized coordinates is

$$\begin{pmatrix} \boldsymbol{X_1} \\ \boldsymbol{X_2} \end{pmatrix} = \lambda_x \begin{pmatrix} \boldsymbol{X_1} \\ \boldsymbol{X_2} \end{pmatrix} + \sqrt{\epsilon_x (1-\lambda_x^2)} \begin{pmatrix} \boldsymbol{\hat{r}_1} \\ \boldsymbol{\hat{r}_2} \end{pmatrix}, \tag{7}$$

$$\begin{pmatrix} \boldsymbol{X_3} \\ \boldsymbol{X_4} \end{pmatrix} = \lambda_y \begin{pmatrix} \boldsymbol{X_3} \\ \boldsymbol{X_4} \end{pmatrix} + \sqrt{\epsilon_y (1-\lambda_y^2)} \begin{pmatrix} \boldsymbol{\hat{r}_3} \\ \boldsymbol{\hat{r}_4} \end{pmatrix}. \tag{8}$$

Here $\hat{r}$'s are independent Gaussian random variables with unit variance, $\lambda_i = \exp(-1/T_i)$ with T_i the damping time in units of the number of turns.

3. *From normalized coordinates to accelerator coordinates*

The coordinates transformation from normalized variable $\boldsymbol{X}$ to accelerator variable $\boldsymbol{x}$,

$$\boldsymbol{x} = \boldsymbol{H}^{-1}\boldsymbol{R}^{-1}\boldsymbol{B}^{-1}\boldsymbol{X}. \tag{9}$$

4. *Beam transportation from interaction point S_i to S_{i+1}*

The beam transportation is modelled by a linear map

$$\boldsymbol{x(S_{i+1})} = M(S_{i+1}|S_i)\boldsymbol{x(S_i)}. \tag{10}$$

D. Bunch-by-bunch feedback system

In time domain, Eq. 11 is the general form of a FIR filter

$$\Theta_n = \sum_{k=0}^{N} a_k x_{n-k}, \tag{11}$$

where a_k represents the filter coefficient, x_{n-k} and Θ_n are the input and output of the filter, corresponding to beam position data at the $(n-k)$th turn and kick strength on the beam at the nth turn. The number of the input data $N+1$ is defined as taps. Following the approaches shown in Ref. [18], the time domain least square fitting (TDLSF) method is used to get the filter coefficients a_k.

In the code, the beam momentum change by the bunch-by-bunch feedback at the nth turn is modelled as

$$\Theta_{x,n} = K_x \sum_{k=0}^{N} a_{k,x} x_{n-k},$$
$$\Theta_{y,n} = K_y \sum_{k=0}^{N} a_{k,y} y_{n-k}, \tag{12}$$

here x_{n-k} and y_{n-k} are the beam centroids of the kth previous turn at the pickup. The beam motion transfer function in one turn including feedback is

$$\left[\begin{pmatrix} x_{n+1} \\ x'_{n+1} \\ y_{n+1} \\ y'_{n+1} \end{pmatrix}\right] = M_0 \left[\begin{pmatrix} x_n \\ x'_n \\ y_n \\ y'_n \end{pmatrix} + \begin{pmatrix} 0 \\ \Theta_{x,n} \\ 0 \\ \Theta_{y,n} \end{pmatrix}\right], \tag{13}$$

where M_0 is the one turn matrix at the kicker.

TABLE I. HEPS Lattice Parameters

Parameters	Values
Energy	6 GeV
Circumference	1360.4 m
Nominal emittance	34.2 pm
Working points	114.14/106.23
Number of super-periods	24
Average betatron function	4.5/8.1 m
Number of RF buckets	756
Beam current	200 mA
SR damping time (x/y)	2386/4536 turns
rms beam size (x/y)	12.4/5.26 μm
Ion species	CO
Gas pressure	1 nTorr
Gas temperature	300 K

III. SIMULATION STUDY OF BEAM-ION INTERACTION IN HEPS.

HEPS is a 1.3 km ultra-low emittance electron storage photon source being built in Beijing, China. The main parameters of the HEPS lattice are listed in Tab. I. Carbon monoxide (CO) with a temperature 300 K and pressure 1 nTorr is assumed as the main leaked gas. In the following study, the total electron beam current 10 mA is adopted to evaluate the beam-ion effect. To save computing time, one beam-ion interaction point is set per turn. However, the setting of multi-interaction points is necessarily to investigate due to the variation of the betatron and dispersion functions. The beam filling pattern is one continuous bunch train following 76 empty bunch gaps. The synchrotron radiation damping and quantum excitation [17] are both taken into account.

As to the bunch-by-bunch feedback system, constrained by the maximum kicker power 1 KW, a 9-taps FIR filter is designed to launch the signal processing. The FIR filter coefficients, frequency response of phase and gain of the 9-taps filters is shown in Fig. 2. For clarity, the pickup and kicker are assumed to be located at the same place with zero dispersion, which means the phase responses at target tunes are -90 degrees.

In the weak-strong simulation, a comparison at the 5000th turn with and without feedback is explicitly shown in Fig. 3. When the bunch-by-bunch feedback is turned on, the maximum bunch action $\sqrt{J_y}$ is well maintained around 0.1 rms beam size, Fig. 3a; the bunch oscillations due to beam-ion interaction are well eliminated, Fig. 3b; the power spectrum of bunch oscillations is roughly one order of magnitude smaller Fig. 3c. The position of the unstable bunch modes does not shift since the intrinsic beam-ion interaction is not violated.

As to the strong-strong simulations, Fig. 4 shows the emittance evolution as the function of tracking turns when the bunch-by-bunch feed back is turned on and off. As suspected, the beam emittance in y direction is well suppressed. More simulations and discussion in detail can be found in Ref. [19].

IV. CONCLUSIONS

In this paper, we have discussed the beam-ion instability and its mitigation by the bunch-by-bunch feedback system. To study the beam-ion interaction consistently, a simulation code is developed including modules such as ionization, beam-ion interaction, synchrotron radiation damping, quantum excitation and bunch-by-bunch feedback. As an example, the lattice parameters of the HEPS project are adopted to show the influence of the beam-ion instability and its mitigation.

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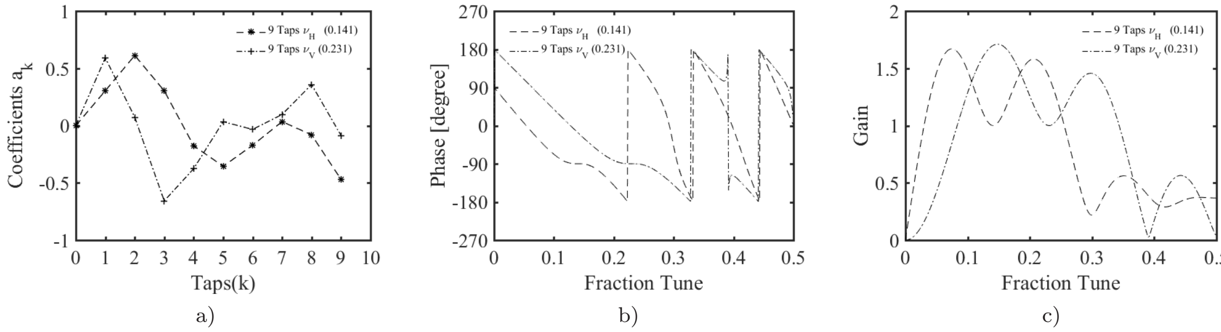

FIG. 2. FIR filter coefficients (a), frequency response of phase (b) and gain (c) of the 9-taps filters used in a bunch-by-bunch feedback system. The horizontal and vertical target tunes are 0.141 and 0.231.

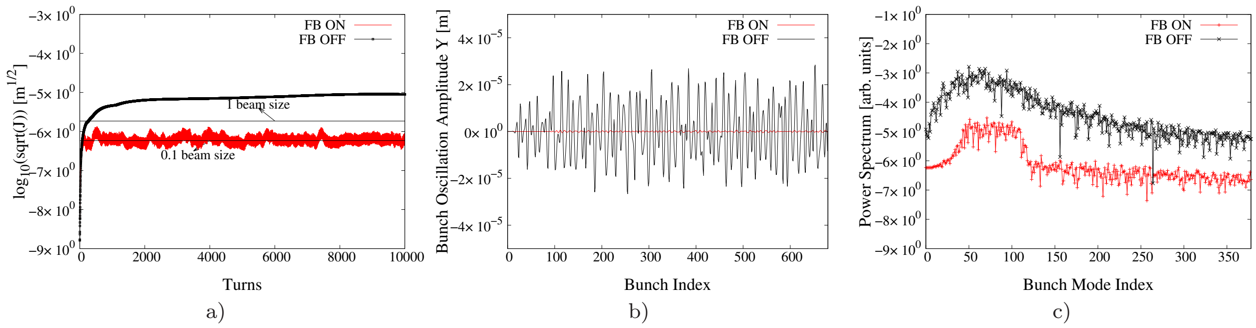

FIG. 3. The red and black curve show the maximum beam actions (a), the beam bunches oscillations (b) and the related coupled bunch modes power spectrum (c) in vertical plane with and without bunch-by-bunch feedback at the 5000th turns. The results are obtained from the "weak-strong" model taking the synchrotron radiation damping into account.

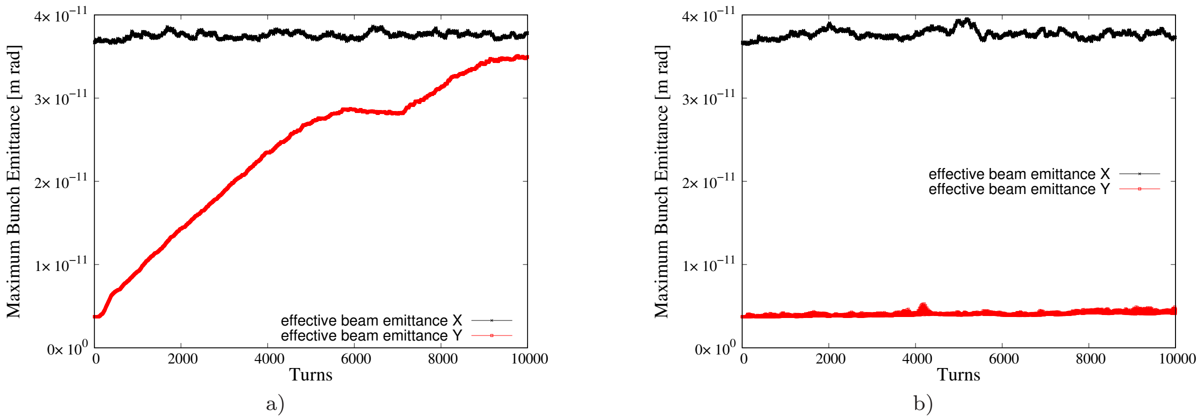

FIG. 4. The maximum bunch emittance references to the ideal orbit as function of passing turns without (a) and with (b) bunch-by-bunch feedback; The synchrotron radiation damping is taken into account. The beam current is 10 mA and the simulation results are given by "quasi-strong-strong" model.

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