VLASOV SOLVERS AND SIMULATION CODE ANALYSIS FOR MODE COUPLING INSTABILITIES IN BOTH LONGITUDINAL AND TRANSVERSE PLANES

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Abstract

Two Vlasov solvers for the longitudinal and transverse planes are used to study the frequency shift of coherent oscillation modes and possible mode coupling instability for the two cases of a proton bunch interacting with either a constant inductive or a broad-band resonator impedance. In parallel to this approach, a new method to study the coherent frequency shift from the results of simulation codes is presented. Comparisons between the two methods are discussed, as well as simple analytical formulae (valid in the "long-bunch" regime), which clearly reveal how to mitigate these instabilities.

INTRODUCTION

Starting from the Vlasov equation and using a decomposition on the low-intensity eigenvectors, as proposed by Laclare and Garnier [1,2], the effect of a transverse damper in the transverse plane was added and a new Vlasov solver code was developed, called GALACTIC (for GArnier-LAclare Coherent Transverse Instabilities Code), which helped to shed light on the destabilising effect of resistive transverse dampers such as in the CERN LHC [3,4]. A similar approach can be used in the longitudinal plane, leading to GALACLIC (for GArnier-LAclare Coherent Longitudinal Instabilities Code), which helped to understand the details of the mode coupling behind some longitudinal microwave instabilities [5,6].

The purpose of this paper is to compare the results from the Vlasov solvers and the ones from macroparticle tracking simulation codes. In the first section devoted to the longitudinal plane, the results from GALACLIC are compared to the ones obtained from the macroparticle tracking simulation code SBSC [7] (as well as BLonD [8] and MuSIC [9]) for the two cases of Constant Inductive (CI) and Broad-Band Resonator (BBR) impedances above transition, assuming a "Parabolic Line Density" (PLD) longitudinal distribution [1]. In the second section devoted to the transverse plane, GALACTIC is compared to the macroparticle tracking simulation code PyHEADTAIL [10] for the case of a BBR impedance, assuming a "Water-Bag" (WB) longitudinal distribution [1]. In the third section, simple analytical formulae are provided, which clearly reveal the different mitigation methods.

LONGITUDINAL

In the case of a PLD longitudinal distribution, the effect of the Potential-Well Distortion (PWD) is given by (with Q_s and Q_{s0} the intensity-dependent and low-intensity synchrotron tunes and Q the coherent synchrotron tune)

$$\frac{Q}{Q_{s0}} = \frac{Q}{Q_s} \times F_{PWD} \quad \text{with} \quad F_{PWD} = \frac{Q_s}{Q_{s0}} = \frac{1}{\sqrt{1-\frac{3}{4}x}} \tag{1}$$

where x is a normalised parameter proportional to the bunch intensity given by

$$x = \frac{\mathrm{Im}\left[\frac{Z_l(p)}{p}\right]_{p=0} 4\, I_b}{\pi^2 B^3 \hat{V}_T\, h \cos\phi_s} . \tag{2}$$

Here, the simplified case of a constant shape of the longitudinal distribution was assumed and $Z_l(p)/p$ is the longitudinal impedance (at the bunch spectrum line p), $I_b = N_b\, e\, f_0$ the bunch current (with e the elementary charge, N_b the number of charges and f_0 the revolution frequency), $B = f_0 \tau_b$ the bunching factor with τ_b the full ($4\,\sigma$) bunch length, $\hat{V}_T$ the total (effective) peak voltage, h the harmonic number and ϕ_s the RF phase of the synchronous particle ($\cos\phi_s > 0$ below transition and $\cos\phi_s < 0$ above). It is important to note that B, $\hat{V}_T$ and ϕ_s depend on the bunch intensity due to the PWD. The cases of CI and BBR impedances, above transition and taking into account PWD, are depicted on Figs. 1 and 2 respectively, considering the same numerical values as the ones used for the SBSC simulations discussed below.

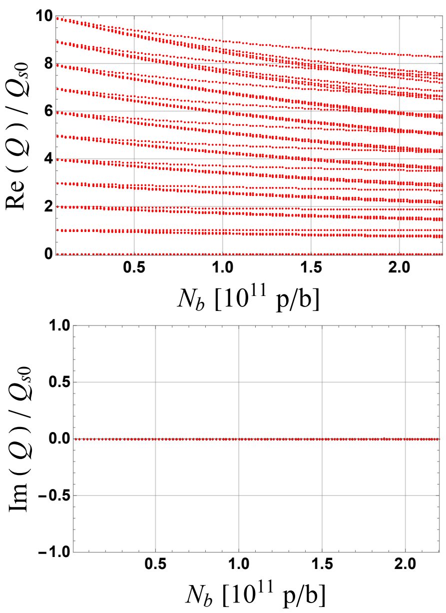

Figure 1: Normalised (to the low-intensity synchrotron tune) mode-frequency shifts from GALACLIC in the case of a CI impedance, above transition, taking into account the PWD and for a PLD longitudinal distribution, with the parameters mentioned below: (upper) real part and (lower) imaginary part.

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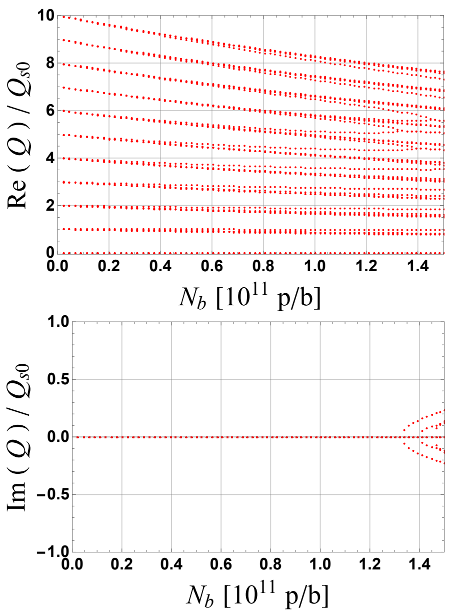


Figure 2: Normalised (to the low-intensity synchrotron tune) mode-frequency shifts from GALACLIC in the case of a BBR impedance (with a quality factor of 1 and a resonance frequency f_r such that $f_r\,\tau_b = 2.7$), above transition, taking into account the PWD and for a PLD longitudinal distribution, with the parameters mentioned below: (upper) real part and (lower) imaginary part.

The SBSC code is a macroparticle tracking code (Single-Bunch Simulation Code) for the longitudinal plane. The beam and machine parameters used for the benchmarks of this paper are the following (close to the CERN SPS case): the relativistic mass factor is γ = 27.73, the relativistic mass factor at transition is γ_{tr} = 22.77, the machine circumference is C = 6911 m, the peak RF voltage is $\hat{V}_{RF}$ = 6 MV, the harmonic number is h = 462 (instead of 4620 used in the CERN SPS, to be in a linear RF system and not mix other possible effects from the nonlinearities of the longitudinal phase space), the full bunch length (4 σ) is τ_b = 2.7 ns and the low-intensity synchrotron tune is $Q_{s0} = 3.26 \times 10^{-3}$. As concerns the impedance, a BBR model is considered, with a quality factor of 1, a resonance frequency f_r such that $f_r\,\tau_b$ = 2.7 (f_r = 1 GHz) and Im $[\,Z_l(p)\,/p\,] = 8.67\,\Omega$ at low frequency. The case of a CI impedance corresponds to the case where the resonance frequency f_r tends to infinity.

The initial stationary distribution, taking into account collective effects for protons, has been obtained with BLonD and a good agreement has been reached between SBSC and BLonD (and MuSiC), as can be seen from Fig. 3 revealing clearly the intensity threshold of the longitudinal "microwave instability" at ~ 1.2×10^{11} p/b for the case of the BBR impedance. In the case of CI impedance, no instability is observed as predicted from GALACLIC (see Fig. 1): a real part of the impedance is needed for mode coupling to take place. A similar result is also obtained for the transverse plane. It is worth noting also from Fig. 3 that a perfect agreement has been obtained between GALACLIC and the macroparticle simulation codes for the bunch lengthening due to the PWD (see red point).

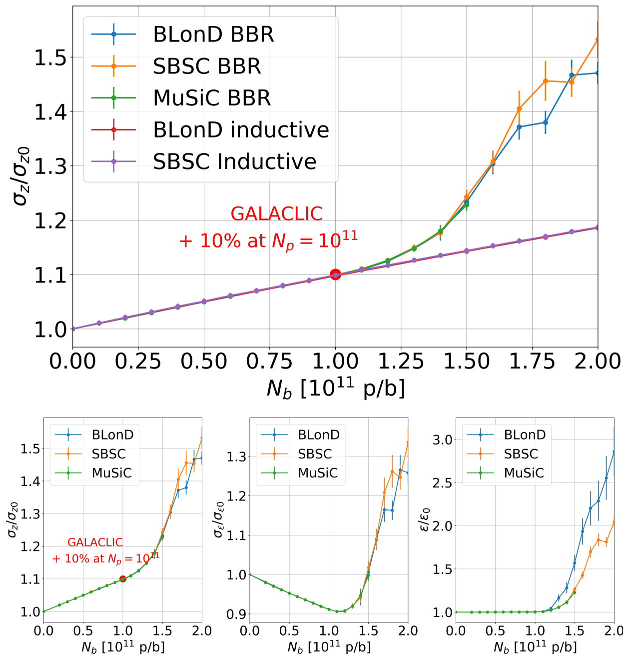


Figure 3: Simulation results from BLonD, SBSC and MuSIC codes with the parameters mentioned above: (upper) evolution of the normalised rms bunch length vs. bunch intensity for the cases of BBR and CI impedances; (lower) evolution of the normalised rms bunch length, energy spread and longitudinal emittance vs. bunch intensity for the case of the BBR impedance.

To analyse this instability in more detail, a new mode analysis was implemented for the post-processing of the results obtained through macroparticle tracking simulations, by computing

$$M_{n,l} = \left(\int_{-\infty}^{+\infty} z^n \lambda(z;t) dz \right)^{1/n}$$
$$\simeq \left(\int_{-\infty}^{+\infty} z^n \lambda_0(z) dz \right)^{1/n} \left(1 + K_n e^{j\Omega_n t}\right) \qquad (3)$$

with $\lambda(z;t)$ the total longitudinal distribution, $\lambda_0(z)$ the stationary distribution, Ω_n the coherent (angular) frequency of the n^{th} mode and K_n a time constant parameter depending on machine parameters, the mode pattern and its amplitude. The important feature of Eq. (3) is that its dependence on time is only related to the coherent (complex) frequency. Indeed $M_{n,l}$ oscillates at frequency Re$[\Omega_n]$ with a time amplitude dependence proportional to Im$[\Omega_n]$. Therefore, by evaluating Eq. (3) turn after turn, and by doing its FFT (Fast Fourier Transform), we obtain the (complex) frequency of the n^{th} mode. If we sum the spectra of the first lowest modes, we obtain the result of Fig. 4 for both cases of a CI impedance and the BBR model. In the bottom figure, we have also represented the intensity threshold deduced from Fig. 3. Some mode coupling could be guessed but this is not easy to say from Fig. 4 alone.

Superimposing the plots from GALACLIC and SBSC, as shown in Fig. 5, a good agreement is obtained for both cases of a CI impedance and the BBR model, even if for the latter some slight shift is observed for the higher-order modes. This would need to be investigated in more detail in the future but it should be reminded that the simplest model of PWD was used here, which assumes that the shape of the longitudinal distribution does not change and that only the bunch length changes with the bunch intensity. The model could be refined in the future to take into account the variation of the bunch profile with intensity. However, the agreement seems already sufficiently good to state that the longitudinal "microwave instability" observed in Fig. 3 is a Longitudinal Mode Coupling Instability (LMCI) of high-order modes (6 and 7).

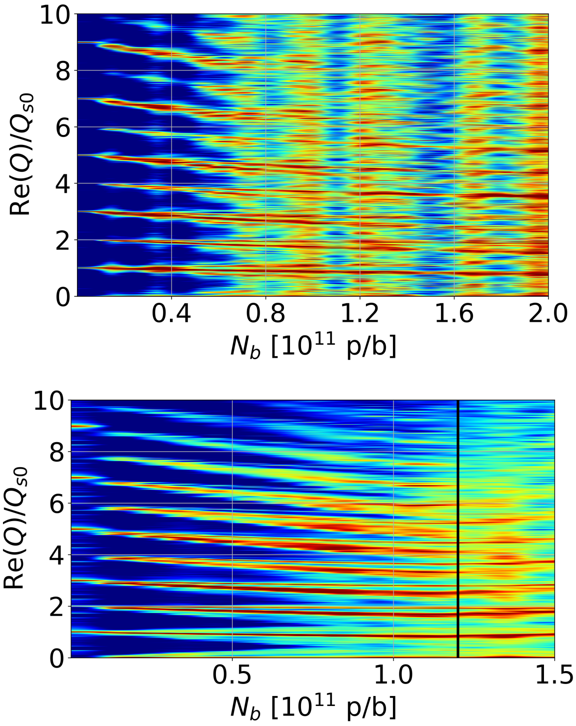
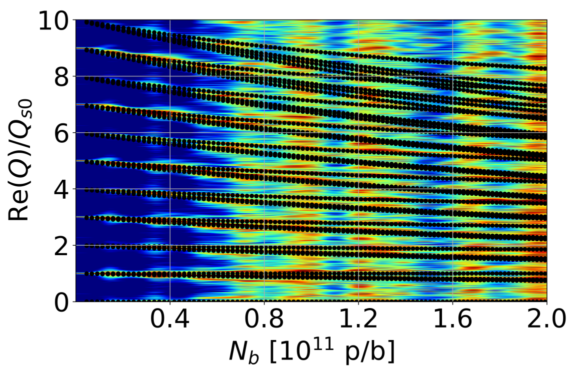


Figure 4: Real part of the normalised mode-frequency shifts from the SBSC tracking code, using the new mode analysis described in Eq. (3), for the case of CI (upper) and BBR (lower) impedance.



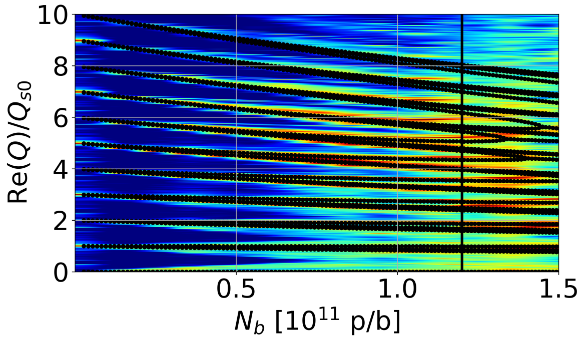


Figure 5: Real part of the normalised mode-frequency shifts: comparison between GALACLIC (black lines) and SBSC for the cases of CI (upper) and BBR (lower) impedances.

TRANSVERSE

A similar detailed comparison in the transverse plane, between GALACTIC and the PyHEADTAIL macroparticle tracking code [10], revealed an excellent agreement as can be observed in Figs. 6 and 7 for the case of a BBR impedance and assuming a WB longitudinal distribution. As already mentioned above, there is no instability in the case of a CI impedance as a real part of the impedance is needed for mode coupling to take place.

As done for the longitudinal plane, a new mode analysis was implemented for the post-processing of the results obtained through macroparticle tracking simulations, by computing

$$M_{n,\perp} = \left(\int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} z^n \rho(z,y;t)\, y\, dz \right)^{1/n} \simeq A\left(1 + K_n' e^{j\Omega_n t}\right) \qquad (4)$$

Here A is a constant depending on the stationary distribution, and K_n' a time constant parameter depending on some machine parameters, the mode pattern and amplitude. As for Eq. (3), $M_{n,\perp}$ oscillates at the coherent (angular) frequency $\mathrm{Re}[\Omega_n]$ with a time amplitude dependence proportional to $\mathrm{Im}[\Omega_n]$. The FFT of $M_{n,\perp}$ highlights the (complex) frequency of the n^{th} mode, and by summing the lowest first modes, we obtain the results of Fig. 6.

Finally, the growth rates shown in Fig. 7 have been obtained from PyHEADTAIL by considering the betatron oscillations of the bunch center of mass turn after turn, and by using an exponential fit for its maximum amplitude.

SIMPLE FORMULAE AND POSSIBLE MITIGATIONS

In the "long-bunch" regime (where $2\, f_r\, \tau_b \gg 1$), simple analytical formulae can be obtained in both longitudinal and transverse planes, which correspond to the coasting-beam formulae with peak values [1], and with no dependence anymore on the synchrotron tune.

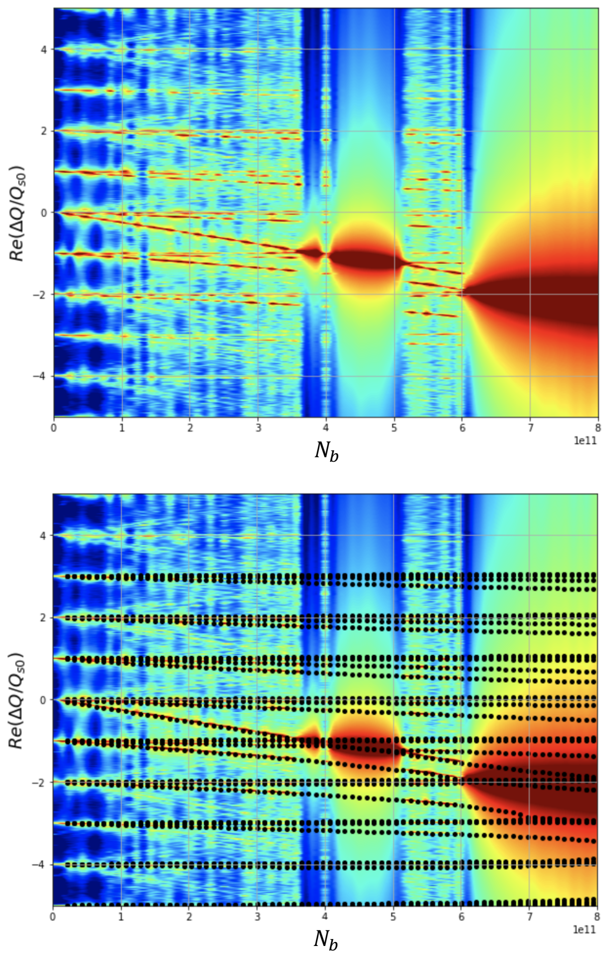


Figure 6: Real part of the normalised mode-frequency shifts: comparison between PyHEADTAIL (top) and GALACTIC (black dots, bottom) for the case of a BBR impedance (with a resonance frequency f_r such that $f_r\,\tau_b = 2.7$) and assuming a WB longitudinal distribution.

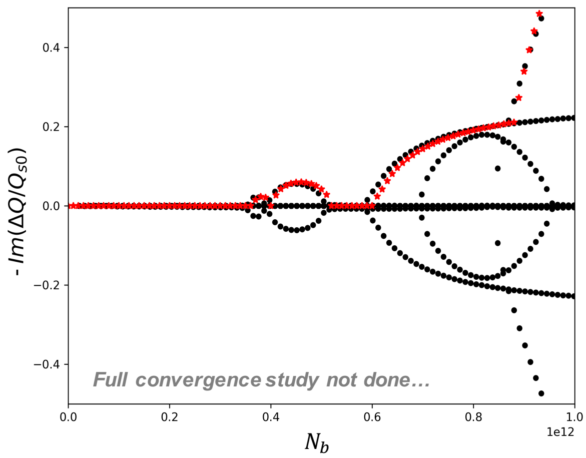


Figure 7: Imaginary part of the normalised mode-frequency shifts: comparison between PyHEADTAIL (red dots) and GALACTIC (black dots) for the case of a BBR impedance (with a resonance frequency f_r such that $f_r\,\tau_b = 2.7$) and assuming a WB longitudinal distribution.

In the longitudinal plane, the stability criterion corresponds to the Keil-Schnell-Boussard criterion (i.e. the Keil-Schnell criterion for coasting beams applied with peak values for bunched beams as proposed by Boussard) whose scaling is given by [1]

$$N_{b,th}^{l} \propto |\eta|\;\varepsilon_l\;\frac{\Delta p}{p_0}\;/\left|\frac{Z_l(p)}{p}\right| \tag{5}$$

where η is the slip factor (measuring the distance to transition), ε_l the longitudinal emittance and $\Delta p/p_0$ the longitudinal momentum spread. Therefore, to increase the longitudinal intensity threshold one needs to reduce the impedance and/or increase the slip factor (i.e. move further away from transition) and/or increase the longitudinal emittance and/or increase the momentum spread. Note that as it is the product between the longitudinal emittance and the momentum spread which matters (and as protons are considered in this paper), it is more effective to increase the momentum spread than increasing the bunch length. Indeed, increasing for instance the RF voltage, and assuming that the longitudinal emittance is preserved (as protons are considered), the momentum spread increases and therefore the longitudinal intensity threshold as well.

In the transverse (e.g. vertical y) plane, a similar criterion can be obtained, whose scaling is given by [11]

$$N_{b,th}^{y} \propto |\eta|\;\varepsilon_l\;Q_y\;f_r\;/\left|Z_y\right| \tag{6}$$

where Q_y is the vertical tune. Therefore, to increase the transverse (vertical) intensity threshold one needs to reduce the impedance and/or increase the slip factor (i.e. move further away from transition) and/or increase the longitudinal emittance and/or increase the vertical tune. Equation (6) was successfully used in the past to significantly increase the intensity threshold at the CERN SPS, even if the role of space charge still needs to be fully understood [12].

CONCLUSION

A good agreement has been reached between the GALACLIC Vlasov solver and the SBSC longitudinal macroparticle tracking code (as well as BLonD and MuSIC) for the two cases of CI and BBR impedances above transition, taking into account the simplest model of PWD (where the shift of the synchronous phase is neglected). For the BBR impedance model, the longitudinal "microwave instability" observed in Fig. 3 has been explained by a LMCI (see Fig. 5 lower), whose intensity threshold is very close to the Keil-Schnell-Boussard criterion. The scaling of the latter is shown in Eq. (5), which reveals how to increase the longitudinal intensity threshold.

An excellent agreement has also been reached in the transverse plane between the GALACTIC Vlasov solver and the PyHEADTAIL macroparticle tracking code for the case of a BBR impedance model, as can be observed from Figs. 6 and 7. In this case, the scaling of the intensity threshold is shown in Eq. (6), which also reveals how to increase the transverse intensity threshold.

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