SYNCHRONOUS PHASE SHIFT MEASUREMENTS FOR EVALUATION OF THE LONGITUDINAL IMPEDANCE MODEL AT THE CERN SPS

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Abstract

The High Luminosity LHC (HL-LHC) requires 2.3×10^{11} protons per bunch (ppb) at LHC injection. For the SPS, the injector to the LHC, this goal requires a doubling of the injected intensity to 2.6×10^{11} ppb. Longitudinal instabilities were observed in the SPS for intensities below the required 2.6×10^{11} ppb. Identifying, and ultimately mitigating, the impedance sources driving the instabilities requires an accurate impedance model. Here, we report on measurements of the synchronous phase shift with intensity and corresponding energy loss at the SPS injection. Using the loss factor to compute the energy loss from the measured bunch spectrum and the SPS impedance model leads to significant disagreements with measurements. This issue is investigated for the simplified case of a single resonator. However, simulating matched bunches using the SPS impedance model yields better agreement with measurements.

INTRODUCTION

The longitudinal impedance model of the SPS [1] is shown in Fig. 1 (blue curve). The dominant contribution at 200 MHz arises from the Travelling Wave Cavities (TWC) together with their Higher-Order Modes (HOM) at 630 MHz and 915 MHz. The peak at 800 MHz is due to the fourth-harmonic TWCs used as Landau cavities. Contributions above 1.2 GHz are caused mainly by the vacuum flanges and HOMs of the 800 MHz TWC. An extensive upgrade program is currently under implementation to reduce the machine impedance. It includes rearranging the sections of the main TWCs, damping of the 630 MHz HOM, and shielding of the vacuum flanges. The result (orange curve in Fig. 1) is a 20% reduction of the impedance at 200 MHz, 66% at 630 MHz, as well as a significant reduction in the impedance around 1.4 GHz.

One method to verify this complex impedance model, to identify missing impedance sources, or to see in future the result of upgrades, is to measure the synchronous phase shift of a bunch with intensity and to compare it to the model predictions [2–5]. Since any impedance source Z leads to an energy loss U, the bunch adjusts its phase ϕ w.r.t. to the RF to recuperate this energy loss

$$\frac{U}{eV_{\mathrm{RF}}} = \sin\phi \simeq \phi\,(N,\sigma) \ , \qquad (1)$$

where e denotes the elementary charge and V_{RF} the RF amplitude. Since ϕ is small, we linearize the sin-function here and in the following. We have emphasized that the bunch

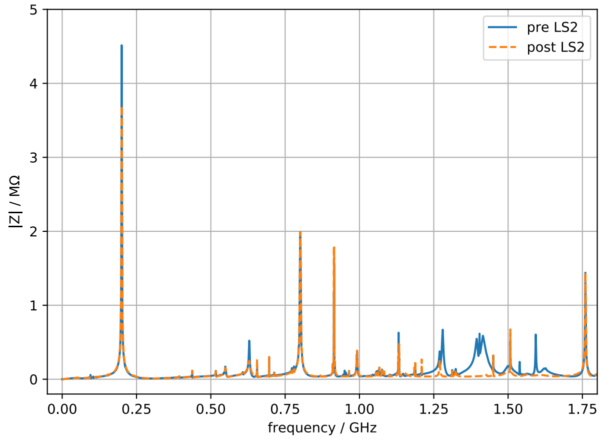

Figure 1: The longitudinal SPS impedance models before (blue) and after (orange) the impedance reduction campaign conducted during Long Shutdown 2 (LS2).

phase depends on the number of particles per bunch (ppb) N and the bunch shape, represented by the bunch length σ. By measuring the bunch phase ϕ, one can, thus, measure the lost energy U and compare to the impedance model prediction.

There are two possible methods to find out how the energy lost due to an impedance changes the bunch position. In the following, we normalize the energy loss by the number of particles, and denote it by an overbar, e.g. $\bar{U} = U/N$. The first is the (normalized) average energy loss $\bar{U}_\kappa$ and is given by [6]

$$\bar{U}_\kappa = e^2 N\kappa\,, \qquad (2)$$

with the loss factor κ defined as

$$\kappa = 2\int_0^\infty \mathrm{Re}Z(f)\,|\Lambda(f)|^2\,\mathrm{d}f\,. \qquad (3)$$

Here, $\Lambda(f)$ denotes the Fourier transform of the longitudinal line density $\lambda(\tau)$, normalized as $\int_{-\infty}^{\infty}\lambda(\tau)\mathrm{d}\tau = 1$. By using the wake function instead of the impedance Z, the energy loss $\bar{U}_\kappa$ can be rewritten in terms of the average induced voltage. Hence, it is related to the center-of-mass, or *mean* position of the bunch $\tau_{\mathrm{mean}} = \int_{-\infty}^{\infty}\tau\,\lambda(\tau)\mathrm{d}\tau$ as

$$\bar{U}_{\mathrm{mean}} = eV_{\mathrm{RF}}\,\omega_{\mathrm{RF}}\tau_{\mathrm{mean}}\,. \qquad (4)$$

The second method is to compute the phase shift $\delta\phi_{\mathrm{s}} = \phi_s - \phi_{s0}$ of the synchronous particle, i.e. the particle synchronous with the RF wave, due to potential well distortion. In first order perturbation theory, it is given as [7]

$$\delta\phi_{\mathrm{s}} = \frac{2e\,N}{V_{\mathrm{RF}}\cos\phi_{s0}}\int_0^\infty \mathrm{Re}\left[Z(f)\Lambda_0(f)\right]\,\mathrm{d}f\,. \qquad (5)$$

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The index '0' refers to the unperturbed, zero-intensity quantities, e.g. the unperturbed synchronous phase ϕ_{s0}. If the line density $\lambda(\tau)$ is symmetric, the spectrum $\Lambda(f)$ is real as well. This synchronous phase shift $\delta\phi_s$ indicates an energy loss of

$$\bar{U}_{\phi_s} = eV_{\mathrm{RF}}\,\delta\phi_s/N\,. \tag{6}$$

The position of the synchronous particle $\tau_s = \phi_s/\omega_{\mathrm{RF}}$ is not directly accessible to measurements. However, the synchronous particle, by definition, sits at the minimum of the potential well formed by the RF- and induced voltage. For a stable, i.e. time-independent, bunch, the minimum of the potential well coincides with the maximum or peak of the bunch profile. Hence, we can obtain the position of the synchronous particle by measuring the *peak* position τ_{peak} of the stable bunch profile. For a given profile, we obtain τ_{peak} by fitting a parabola to a window of ±0.5 ns around the profile maximum. The corresponding energy loss is then

$$\bar{U}_{\mathrm{peak}} = eV_{\mathrm{RF}}\,\omega_{\mathrm{RF}}\tau_{\mathrm{peak}}\,. \tag{7}$$

While we have access to the unperturbed spectrum Λ_0 in simulation, it is not measurable in practice, where we can only measure the bunch profile perturbed by the potential well distortion. We, thus, use the (measurable) perturbed spectrum $\Lambda(f)$ in Eq. (5) to obtain an approximate value for $\Delta\phi_s$ and the resulting energy loss $\bar{U}_{\phi_s,\mathrm{approx}}$.

Notice that both $\bar{U}_\kappa$ and $\bar{U}_{\phi_s}$ depend only on the real part of the impedance. Since both losses depend on the overlap of the impedance with the bunch spectrum, shorter bunches tend to have a higher energy loss, but in general the energy loss is non-monotonic. First, the bunch spectrum is non-monotonic since the proton bunches do not have a Gaussian shape. Second, the machine impedance is highly non-monotonic, see Fig. 1.

NARROW-BAND RESONATOR MODEL

To illustrate the above discussion, we first did a particle tracking simulation of a single bunch with the longitudinal tracking code BLonD [8]. Instead of the complex SPS impedance model, we only take a single resonator into account. The shunt impedance of 4.5 MΩ and quality factor $Q = 140$ give a rough estimate for the impedance of the 200 MHz TWC [3], while we vary the resonant frequency f_r. We fix the bunch intensity at 1×10^{11} ppb, which is slightly below the present nominal LHC intensity. The initial bunch distribution including four million macro-particles is generated from a binomial profile

$$\lambda_0(\tau) = A\left[1 - \left(\frac{2(\tau - \tau_{\mathrm{max}})}{\tau_L}\right)^2\right]^{\mu+1/2}, \tag{8}$$

and is matched to the RF bucket with intensity effects. We used $\mu = 2.4$ and the bunch length $4\sigma_{FWHM} \simeq 0.78\tau_L = 2.5$ ns. To take into account that the matching is not perfect, we let the bunch filament a little by tracking for 15 synchrotron periods (1000 turns) before 'measuring' the

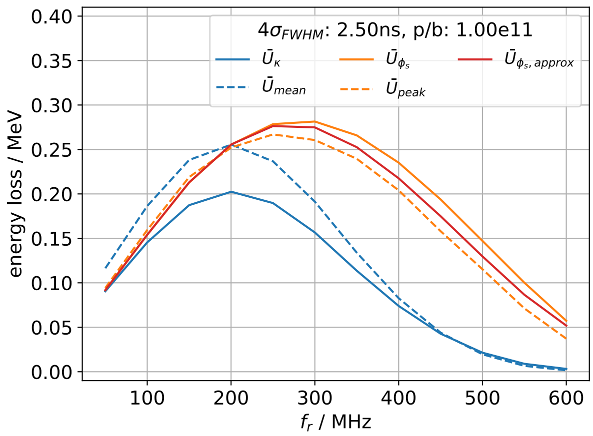

Figure 2: Simulated energy loss using a single resonator. Energy losses obtained from a bunch position are plotted as dashed lines, while those derived from the impedance model are plotted as solid lines.

bunch profiles during another 15 synchrotron periods. We compute the average bunch profile, and do the same data analysis as with the measured profiles.

Figure 2 shows the resulting simulated and calculated energy losses for different resonant frequencies. All losses decrease for higher resonant frequencies, as the overlap between bunch spectrum and impedance decreases. Since we have access to the unperturbed bunch spectrum Λ_0 in simulations, we can use equations (5) and (6) directly to compute the energy loss $\bar{U}_{\phi_s}$ causing the synchronous phase shift (orange curve). From the 'measured' peak position τ_{peak}, we obtain the corresponding energy loss $\bar{U}_{\mathrm{peak}}$ using Eq. (7) (orange dashed curve). The two curves are in agreement also with the approximate result (red curve), calculated from the 'measured' perturbed bunch spectrum Λ.

The blue curve shows the model prediction for the average energy loss of the bunch according to Eq. (2). This is compared to the energy loss obtained from the 'measured' mean bunch position τ_{mean} using Eq. (4) (blue dashed curve). We observe good agreement for resonant frequencies above 400 MHz, but significant deviations when the resonant frequency f_r is close to the RF frequency of ~200 MHz. At this frequency, the 'measured' $\bar{U}_{\mathrm{mean}}$ agrees with the energy loss from the synchronous phase shift.

For $f_r = f_{\mathrm{RF}}$, the results can be analyzed and explained by treating the narrow-band impedance as a δ-function centered at f_{RF}. In this case, the induced voltage just leads to an amplitude and phase shift of the RF voltage, rather than an asymmetric potential well distortion. Therefore, the total voltage seen by the beam is still symmetric around the synchronous particle. The matched, perturbed, bunch profile λ equals the unperturbed profile λ_0, but shifted by an amount given by Eq. (5). Since the profile λ is symmetric, the peak τ_{peak} and mean τ_{mean} positions coincide, which explains why $\bar{U}_{\mathrm{peak}}$ equals $\bar{U}_{\mathrm{mean}}$. Moreover, $\bar{U}_\kappa = \Lambda_0(f_{\mathrm{RF}})\bar{U}_{\phi_s}$ in this case. Using the analytic expression for the spectrum with the bunch parameters given above yields $\Lambda_0(f_{\mathrm{RF}}) \simeq 0.8$, which is in quantitative agreement with Fig. 2. For other resonance frequencies, the potential well becomes asymmetric, which leads to an asymmetric bunch profile. Now, the synchronous particle is no longer at the bunch center, and $\bar{U}_{\mathrm{peak}}$ disagrees with $\bar{U}_{\mathrm{mean}}$.

MEASUREMENTS

Measuring the phase between the bunch and the RF wave is difficult in practice. Instead, we employed a reference and a witness bunch [2]. The reference bunch had a fixed small intensity. The witness bunch followed at a sufficiently large distance so as not to be affected by the wake field of the reference bunch (1.5 µs in our case). We then recorded the bunch profiles, starting 500 ms after injection to give the bunches time to filament and reach a stable state. To measure the bunch intensity, we calibrated the integrated profile against the intensity of the DC Beam Current Transformer, see [9] for details. The relative phase between the bunches is determined from their bunch positions as

$$\Delta\phi = \omega_{\mathrm{RF}} \left[(t_w - t_r) \,\%\, T_{\mathrm{RF}} \right] , \tag{9}$$

where % is the modulo operation and $t_{r,w}$ denote the position of the *r*eference and *w*itness bunch, respectively. We then scanned the bunch intensity and length, while keeping the shape of reference and witness bunch the same. In practice, we considered two bunch shapes the same, if their bunch lengths do not differ by more than 5%. For a fixed bunch length, the absolute phase distance scales linearly with intensity, i.e.

$$\phi(N, \sigma = const) \simeq N \Delta\phi(\sigma)/\Delta N \,. \tag{10}$$

Finally, the normalized energy loss $\bar{U}(\sigma)$ is given by

$$\bar{U}(\sigma) \simeq eV_{\mathrm{RF}}\, \Delta\phi(\sigma)/\Delta N \,. \tag{11}$$

We used three methods to compute the bunch position from the measured bunch profile. As discussed above, two of them are the mean τ_{mean} and peak τ_{peak} positions. By fitting the profile to the line density in Eq. (8) we obtained the position of the maximum of the fitted profile τ_{fit}.

Figure 3 shows the measured phase difference obtained by using τ_{fit} in Eq. (9). It shows the expected qualitative behavior, i.e. an increasing phase shift for either increasing intensity at fixed bunch length, or decreasing bunch length at fixed intensity. To obtain the slope $\Delta\phi(\sigma)/\Delta N$, we binned the data according to bunch length (using a window of ±0.2 ns) and performed a linear fit. An example is shown in Fig. 4, together with the linear fit. The error in the fitted slope then directly transfers into the error of the energy loss $\bar{U}$. The corresponding value for the bunch length is the mean bunch length of all data points within that window, and their RMS gives the bunch length error. Notice that we do not consider an error in neither the bunch intensity, nor the RF voltage V_{RF}.

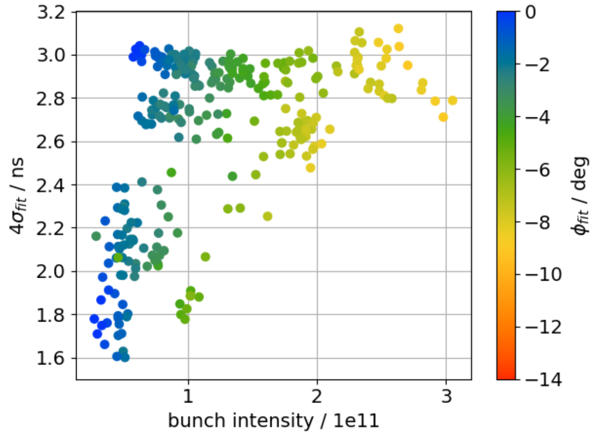

Figure 3: Measured phase shift for different bunch lengths and intensities. The bunch position and length $4\sigma_{\mathrm{fit}}$ were obtained by fitting a binomial to the measured profile.

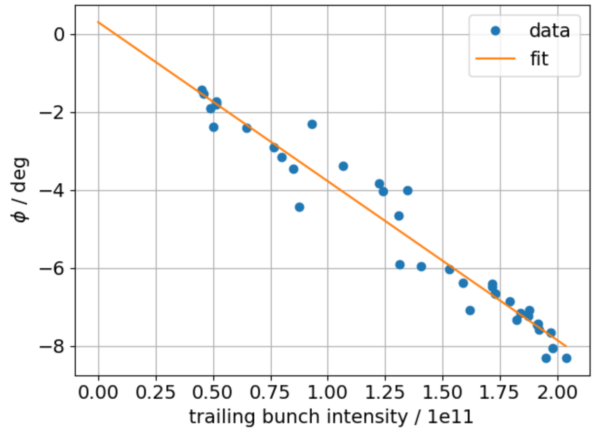

Figure 4: Data points of Fig. 3 in the interval $4\sigma_{\mathrm{fit}} = 2.44\,\mathrm{ns} \pm 0.2\,\mathrm{ns}$ together with the linear fit.

RESULTS

First, we use the peak position τ_{peak} in Eq. (9), and proceed as described in the previous section. The results for $\bar{U}_{\mathrm{peak}}$ are shown as the orange points in Fig. 5[1]. An energy loss in the order of $10\,\mathrm{keV}/10^{10}$ ppb was reported in [3], roughly agreeing with our $\bar{U}_{\mathrm{peak}}$. From the discussion of the narrow-band resonator model, $\bar{U}_{\phi_s,\mathrm{approx}}$ is the most relevant quantity to compare to and is shown as the red data points. We compute $\bar{U}_{\phi_s,\mathrm{approx}}$ for the measured bunch spectra Λ and the full longitudinal SPS impedance model in Fig. 1.

Surprisingly, we see a large discrepancy between the two. At face value, this would mean that the model significantly overestimates an impedance source. Since $\bar{U}_{\phi_s,\mathrm{approx}}$ exceeds $\bar{U}_{\mathrm{peak}}$ for all bunch lengths, it would suggest an overestimation in the low-frequency regime of the model. However, this impedance model was used many times in BLonD sim-

[1] Here, and in the following figures, the data points are joined to guide the eye.

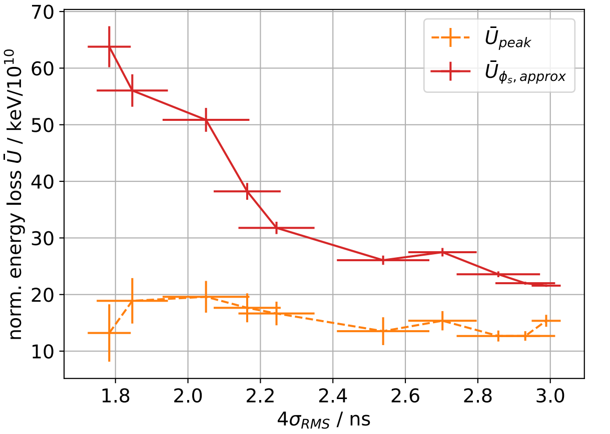

Figure 5: Comparison between the measured $\bar{U}_{\text{peak}}$ (orange, dashed) and the impedance model prediction $\bar{U}_{\phi_s,\text{approx}}$ (red, solid).

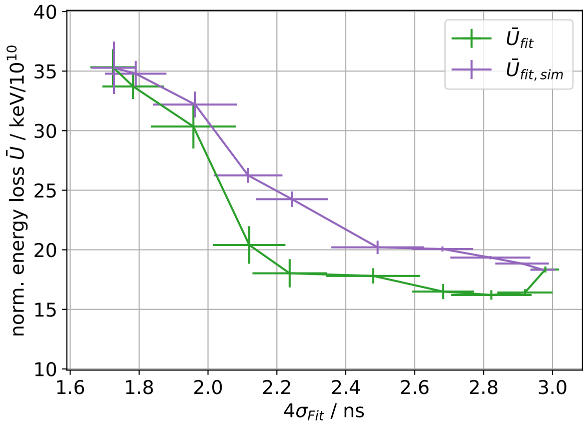

Figure 7: Energy loss obtained by fitting the measured profiles $\bar{U}_{\text{fit}}$ (green) and the energy loss $\bar{U}_{\text{fit,sim}}$ obtained by simulating bunches created from these fit parameters (purple).

ulations that successfully reproduced measured intensity effects in the SPS. This makes a large error in the impedance model unlikely, but, so far, we have not been able to find an error in the computation of $\bar{U}_{\phi_s,\text{approx}}$.

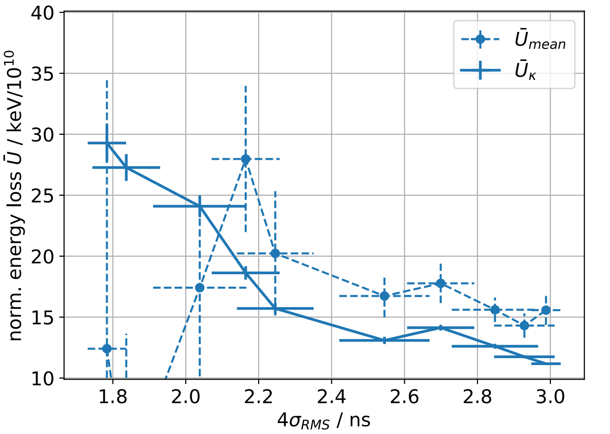

Figure 6: Comparison between the measured $\bar{U}_{\text{mean}}$ (blue, dashed) and the impedance model prediction $\bar{U}_{\kappa}$ (blue, solid).

As a second method, we compare $\bar{U}_{\text{mean}}$ obtained from τ_{mean} and compare to average energy loss $\bar{U}_{\kappa}$, see Fig. 6. For bunch lengths longer than 2.2 ns, the measured energy loss is above the impedance model prediction, but both have the same shape. Reminding the fact that the SPS impedance is dominated by the fundamental cavities, we are in a similar situation as discussed for the narrow-band resonator model. The simulation of the 'measured' energy loss was systematically above the model prediction. This is reaffirmed by the fact that longer bunches mainly sample the impedance of the 200 MHz TWCs and are less affected by the higher-frequency impedances. For smaller bunch length, the deviation increases, but the error bars increase significantly as well.

Finally, we fit the binomial line density in Eq. (8) to the measured profiles for the reference and witness bunch, thus obtaining τ_{fit}. The resulting $\bar{U}_{\text{fit}}$ is shown in Fig. 7 as the green data. To compare with the impedance model, we create matched distributions from these fit parameters and track them in BLonD, including the intensity effects due to the full SPS impedance model. The reference and witness bunches are tracked independently, thus ignoring any interaction between them. We also do not include the phase loop in simulations, which was active during the measurements. This is justified by the fact that the phase loop (before the upgrade during LS2) can only act on the reference bunch, and that we only consider the stable situation 500 ms after injection. As in the simulations for the narrow-band model, we track an initial 1000 turns to reach a stable situation, and then save the bunch profiles averaged over another 1000 turns. We again do a binomial fit to find $\tau_{\text{fit,sim}}$, and display the corresponding energy loss $\bar{U}_{\text{fit,sim}}$ as the purple data points in Fig. 7. Again, the simulated energy loss is above the measured one, but both agree for long and short bunches.

CONCLUSION

The narrow-band resonator model suggests to compare $\bar{U}_{\text{mean}}$ with $\bar{U}_{\kappa}$ and $\bar{U}_{\text{peak}}$ with $\bar{U}_{\phi_s,\text{approx}}$. However, these comparisons work less well for measured energy losses and their counterparts from the SPS impedance model (see Figs. 5 and 6). There are still a couple of error sources which were not considered. First, the data points used for the linear fits did not include errors on the bunch intensity, and the error in the RF voltage is not considered. Likely more important is the error on the bunch positions, as often the difference between τ_{mean} and τ_{peak} is only a few pico-seconds, compared to the bunch length of a few nano-seconds. Moreover, we found that the parabolic fit to find τ_{peak} fits the data very well, but the error on τ_{peak} depends on the absolute displacement of the bunch. This can give errors on the peak position well above 10 ns for a bunch that is only 2 ns long! The reason for this dependence is yet to be understood. At least, using 'brute force' simulations, based on the fit parameters of the measured bunch profiles, yield a reasonable agreement.

Other methods exist to compare a machine impedance model to a real one. For example, the quadrupole frequency shift with intensity was used to gain information on the reactive part of the SPS impedance model [10]. Another method is to inject long bunches with a small energy spread without an RF voltage. Before debunching, different impedance sources can drive instabilities that leave a 'finger print' on the bunch profile [11, 12]. In this case, one can also use the drift of the bunch center due to the energy loss as a measure of the mean energy loss $\bar{U}_\kappa$.

To validate the impedance model after LS2, injections of long bunches and measurements of both the synchronous phase and quadrupole frequency shifts are planned. The former method is sensitive to the impedance reduction at 1.4 GHz (due to the shielding of vacuum flanges), while measurements of the synchronous phase and quadrupole frequency shifts probe the low-frequency part of the machine impedance. The latter measurement can also readily extended and used as a measurement of the synchronous phase. It will be aided by the availability of automatic over-night parameter scans, yielding more data points with identical machine parameters. This would also help with quantifying the systematic errors arising from the imprecise knowledge of the beam intensity and RF voltage.

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