ADVANCED LANDAU DAMPING WITH RADIO-FREQUENCY QUADRUPOLES OR NONLINEAR CHROMATICITY

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Abstract

Landau damping is a powerful mechanism to suppress impedance-driven coherent instabilities in circular accelerators. In the transverse planes it is usually introduced by means of magnetic octupoles. We will discuss a method to generate the required incoherent betatron frequency spread through detuning with the longitudinal rather than the transverse amplitudes. The approach is motivated mainly by the high-brightness, low transverse emittance beams in future colliders where detuning with the transverse amplitudes from magnetic octupoles becomes significantly less effective. Two equivalent methods are under study: a radio-frequency quadrupole cavity and the nonlinear chromaticity. The underlying beam dynamics mechanisms are explained based on a recently extended Vlasov theory and relevant results are discussed for different longitudinal beam distributions under specific approximations. Finally, the analytical studies are benchmarked against numerical simulations employing a circulant matrix and a macroparticle tracking model.

INTRODUCTION

The use of radio frequency (rf) quadrupole cavities against coherent beam instabilities has first been discussed in [1, 2] to suppress coupled-bunch modes, and later in [3, 4] to raise the intensity threshold of the transverse mode-coupling instability (TMCI). Here, rf quadrupoles are considered to provide Landau damping of weak single-bunch head-tail modes [5–7]. Detailed theoretical, experimental, and simulation studies of the latter have been reported in [8–12] and a summary of relevant extracts thereof is given here.

The purpose of the rf quadrupole for Landau damping is to generate transverse quadrupolar kicks on the beam particles with a strength that depends on their longitudinal coordinate. Every particle feels a different focusing (defocussing) force as it passes through the device and hence experiences a change in its betatron frequencies depending on its longitudinal position within the bunch. The result is an incoherent betatron frequency spread which leads to Landau damping in the transverse planes. Other than for magnetic octupoles, the frequency spread from an rf quadrupole is dependent on the longitudinal amplitude spread within the bunch [7, 8]. It can be shown that nonlinear chromaticity can introduce an equivalent longitudinal amplitude dependent frequency spread (see e.g. [10]). This result will be used here to discuss the analytical studies.

Thanks to the orders of magnitude larger spread in the longitudinal compared to the transverse amplitudes of the beams in future hadron colliders, a longitudinal amplitude-dependent frequency spread can be produced very efficiently compared to magnetic octupoles. The differences are particularly important at increased beam energies and for reduced transverse emittances. In addition, the amount of frequency spread remains unaffected by beam manipulations in the transverse planes, such as beam halo cleaning through collimation, for example. Recently, it has also been demonstrated that transverse linear coupling can strongly reduce the incoherent betatron frequency distributions generated through detuning with the transverse amplitudes [13]. This can lead to a loss of Landau damping and requires an accurate correction of the linear coupling in future machines [14]. The shape and amount of frequency spread introduced through detuning with the longitudinal amplitude, on the other hand, is not affected by linear coupling [15]. It is hence expected that there is no loss of Landau damping in that case. Another effect that is currently under detailed investigation is transverse noise that can locally significantly reduce the stability diagrams generated by magnetic octupoles and hence lead to a loss of Landau damping [16]. It is believed that this effect will not be present for rf quadrupoles or nonlinear chromaticity thanks to the separation of the longitudinal amplitude space and the transverse planes where the frequency spread is created.

THEORY

Berg and Ruggiero developed the basic formalism for longitudinal amplitude dependent Landau damping in [17]. They also demonstrated that it differs to some extent from Landau damping introduced by octupole magnets. The theory has been further developed and thoroughly analyzed in [9]. Only the key equations and a summary of their interpretations are presented here.

The goal is to extend the existing Vlasov formalism by introducing a general variation $\Delta\omega_\beta(\delta)$ of the betatron frequency with arbitrary orders of chromaticity $\xi^{(n)}$

$$\Delta\omega_\beta(\delta) = \omega_{\beta,0} \sum_{n=1}^{m} \frac{\xi^{(n)}}{n!} \delta^n, \qquad (1)$$

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with

$$\xi^{(n)} = \frac{1}{\omega_{\beta,0}} \frac{\partial^n \omega_\beta}{\partial \delta^n}\bigg|_{\delta=0}, \tag{2}$$

$\omega_{\beta,0}$ the zero-amplitude betatron frequency, and δ the relative momentum deviation. One may, analogously, introduce a general variation of the betatron frequency with the longitudinal position $\Delta\omega_\beta(z)$ to describe the frequency spread from an rf quadrupole. The two approaches eventually lead to the same results. Here, we assume that the frequency spread is produced by nonlinear chromaticity.

Following the path laid out by Chao in [6], but using the general dependence of $\Delta\omega_\beta$ on $\xi^{(n)}$, one can derive an eigenvalue equation (details in [9])

$$\sigma_{lk} = -iK \sum_{l',\,k'=-\infty}^{\infty} \sigma_{l'k'}\, Z_\perp\left(k'\omega_0 + \Omega^{(l)}\right) \times \int_0^\infty \frac{r g_0(r)\, \overline{H_l^{k'}}(r)\, H_l^k(r)}{\Omega^{(l)} - \omega_{\beta,0} - l\omega_s - \langle\Delta\omega_\beta\rangle_\phi(r)}\, dr, \tag{3}$$

where

$$\sigma_{lk} = \int_0^\infty r R_l(r)\, H_l^k(r)\, dr. \tag{4}$$

K is a constant, $Z_\perp$ the dipolar impedance function, g_0 the longitudinal particle distribution, ω_0 the revolution frequency, ω_s the synchrotron frequency, $\Omega^{(l)}$ the complex coherent frequency of the l^{th} azimuthal mode, (r, ϕ) are polar coordinates in longitudinal phase space, and R_l the radial beam modes. The H_l^k functions can be perceived as generalized Bessel functions. They reduce to Bessel functions of the first kind for a purely linear chromaticity. The term $\langle\Delta\omega_\beta\rangle_\phi$ describes the betatron frequency spread introduced through detuning with the longitudinal amplitude r

$$\langle\Delta\omega_\beta\rangle_\phi(r) = \frac{1}{2\pi}\int_0^{2\pi} \Delta\omega_\beta\left[\delta(r,\phi)\right]\, d\phi. \tag{5}$$

This term appears in the denominator of the dispersion integral on the right hand side of Eq. (3) which demonstrates that it indeed provides Landau damping. One realizes that for odd orders of chromaticity the average frequency spread vanishes $\langle\Delta\omega_\beta\rangle_\phi(r) \equiv 0$. This result is independent of the longitudinal particle distribution. Hence, odd orders of chromaticity do not introduce Landau damping, at least for instabilities with rise times in the order of several synchrotron periods where the frequency spread averages to zero[1]. On the other hand, even orders of chromaticity introduce a frequency spread with longitudinal amplitude that does not average out over time which leads to Landau damping, similarly to an rf quadrupole operated (anti-) on-crest of the rf wave. There is yet another mechanism, however. Both odd and even orders of chromaticity introduce a change of the effective impedance and modify the complex frequencies of the coherent modes in that manner. This effect is described by the generalized Bessel functions introduced above. The H_l^k functions contain complex, chromaticity-dependent, phase terms which describe the alteration of the interaction of the beam with the impedance. The result is that the overlap sum over index k' in Eq. (3) between the $H_l^{k'}$ functions and the impedance changes. In time domain these chromatic phase terms can be interpreted as a change of the synchronicity between wake kicks, betatron, and synchrotron motion of the particles. They lead to a change of the coherent frequencies of all the modes. Note that such modification of the effective impedance is independent of frequency spread and there is no increase of the area of stability in the complex frequency space. Thus, this effect is not related to Landau damping.

[1] This is analogous to an rf quadrupole operated at the zero crossing of the rf wave studied in [3, 4] to increase the TMCI threshold.

SOLUTIONS

Solutions to the Vlasov Eq. (3) are determined for two different types of longitudinal particle distributions. The analytical results presented here are benchmarked against the PyHeadtail macroparticle tracking model and the BimBim circulant matrix solver [18–20].

Airbag beam

For the airbag model the beam particles are assumed to populate an infinitesimally thin elliptical shell in the longitudinal phase space, i.e. they all oscillate with the same longitudinal amplitude. As a result, the betatron frequency spread from nonlinear chromaticity or rf quadrupoles vanishes and hence there can be no Landau damping. In the weak-wake approximation considered here, azimuthal mode coupling can be neglected and one can solve the equations for all the azimuthal modes $l \in \mathbb{Z}$ independently of each other. For an airbag distribution the dispersion integral can be easily evaluated and one obtains the solutions

$$\Omega^{(l)} - \omega_{\beta,0} - l\omega_s - \langle\Delta\omega_\beta\rangle_\phi(\hat{z}) = -i\,\frac{Ne^2c}{2\omega_{\beta,0}T_0^2E_0}\sum_{k=-\infty}^{\infty} Z_\perp(\omega')\left|H_l^k(\hat{z})\right|^2, \tag{6}$$

where $\omega' = k\omega_0 + \omega_{\beta,0} + l\omega_s$. N denotes the bunch population, T_0 the revolution period, E_0 the beam energy, $\hat{z}$ the longitudinal amplitude of the airbag beam, e the elementary charge, and c the speed of light. We have obtained an explicit expression for the coherent frequency shift of every azimuthal mode. The detuning term $\langle\Delta\omega_\beta\rangle_\phi(\hat{z}) = \text{const.}$ is now independent of the longitudinal amplitude r and is identical for all the particles. As expected, the dispersion integral has disappeared from the equation which can be interpreted as the absence of Landau damping. Equation (6) is a generalization of Eq. (6.188) in [6] and is valid for arbitrary orders of chromaticity. It reduces to Chao's equation for a purely linear chromaticity as shown in [9].

The new formalism is first benchmarked against the well-known case of a purely linear chromaticity and a broad-band resonator impedance. The results are summarized in Fig. 1. The analytical calculations are given by the colored lines and represent the real (upper plot) and imaginary (lower plot)

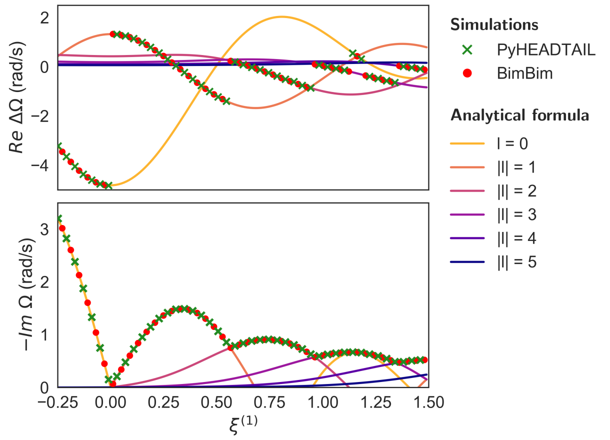

Figure 1: Real (top) and imaginary (bottom) coherent frequency shifts as a function of $\xi^{(1)}$ for an airbag model.

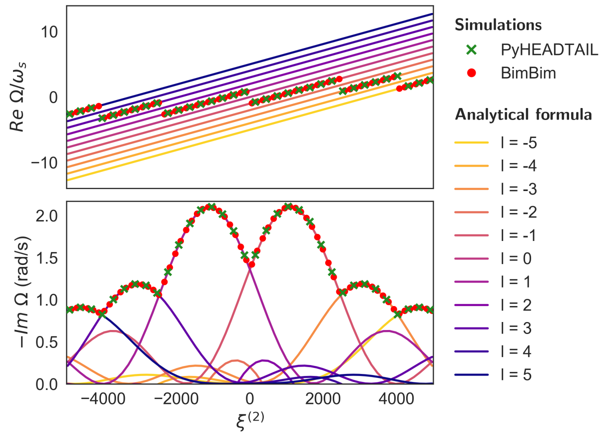

Figure 2: Real (top) and imaginary (bottom) coherent frequency shifts as a function of $\xi^{(2)}$ at fixed $\xi^{(1)}$ for an airbag model.

coherent frequency shifts of the six lowest-order azimuthal modes vs. $\xi^{(1)}$. The real part is measured with respect to the respective unstable synchrotron side band. It can be seen that the azimuthal modes for a specific positive and negative azimuthal number are identical. This will no longer be the case when introducing second-order chromaticity as described below. For $\xi^{(1)} < 0$ the most unstable mode is a head-tail mode zero (above transition). For increasing $\xi^{(1)} > 0$, the most unstable mode changes from azimuthal mode one through five. The outputs from BimBim (red) and PyHeadtail (green) after post-processing are shown on top of the analytical results. The three approaches are in excellent agreement which confirms that they all work well for the basic linear chromaticity case.

Figure 2 summarizes the more interesting case in presence of nonlinear chromaticity. The coherent frequency shifts obtained from analytical formula [Eq. (6)], PyHeadtail, and BimBim are shown as functions of $\xi^{(2)}$ for constant $\xi^{(1)}$. Similar to the case with linear chromaticity, $\xi^{(2)}$ changes the effective impedance and eventually, transitions to other, more unstable azimuthal modes occur. A major difference with respect to Fig. 1, however, is that the degeneracy in the azimuthal mode number is lifted. For a certain absolute value of the mode number, the modes with the two opposite signs are no longer identical. Additionally, the real part of the coherent frequency shift is dominated by the constant and real-valued $\langle \Delta\omega_\beta \rangle_\phi(\hat{z})$ which is the same for all the azimuthal modes. This is specific to the airbag beam and is again a result of the absence of a spread in longitudinal amplitude. As for the linear case, the theoretical predictions are in perfect agreement with both the tracking and circulant matrix models which confirms that the formalism developed above is indeed valid for the airbag beam.

Arbitrary distributions

The new theory describes the change of the effective impedance from nonlinear chromaticity very accurately and produces satisfying results for the airbag model. The next step is to introduce beam distributions where the particles exhibit a spread in their longitudinal amplitudes, for example Gaussian, to validate the theory also in presence of Landau damping. Unfortunately, Eq. (3) could not be solved exactly for the general case. To make the dispersion relation and the presence of Landau damping more apparent and to bring the equation into a form that can be solved and benchmarked against numerical models, strict assumptions are made on the shape of the transverse dipolar impedance instead

$$Z_\perp(\omega') = \begin{cases} Z_{k_0} \neq 0, & k = k_0, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Equation (3) then simplifies to

$$1 = -iKZ_{k_0} \int_0^\infty \frac{r g_0(r) \left| H_l^{k_0}(r) \right|^2}{\Omega^{(l)} - \omega_\beta(r) - l\omega_s} \, dr, \tag{8}$$

where $\omega_\beta(r) = \omega_{\beta,0} + \langle \Delta\omega_\beta \rangle_\phi$. Equation (8) is a dispersion relation. The formula for stability boundary diagrams can now be easily determined

$$\begin{aligned} \frac{1}{\Delta\Omega_{\text{lin}}^{(l)}} &= \frac{1}{\mathcal{N}} \int_0^\infty \frac{r g_0(r) \left| H_l^{k_0}(r) \right|^2}{\Omega^{(l)} - \omega_\beta(r) - l\omega_s} \, dr, \\ \mathcal{N} &= \int_0^\infty r g_0(r) \left| H_l^{k_0}(r) \right|^2 dr, \end{aligned} \tag{9}$$

where $\Delta\Omega_{\text{lin}}^{(l)} = \Omega_{\text{lin}}^{(l)} - \omega_{\beta,0} - l\omega_s$ and $\Omega_{\text{lin}}^{(l)}$ denotes the complex coherent frequency of the azimuthal mode l in absence of Landau damping. It can be demonstrated that the dispersion relation derived here is equivalent to the results found by Berg and Ruggiero in [17] (proof in [9]).

To benchmark the analytical model against PyHeadtail tracking simulations we assume a Gaussian beam distribution and define a scenario which fulfills best the approximations and assumptions made when deriving Eq. (9). The main assumption is to use a highly narrow-band resonator impedance to mimic the single-peak impedance approximation. This can be achieved by tuning the quality factor and the frequency of the resonator accordingly. The parameters were set to match the spectral maximum of the azimuthal mode zero while remaining small for all the other modes. It was verified that the error in both the real and imaginary coherent frequencies between the single-peak approximation and the simulation was less than ten percent.

Next, the dispersion relation in Eq. (9) is solved numerically to obtain the stability boundary diagrams in complex frequency space. The solutions are displayed in Fig. 3 for four different values of $\xi^{(2)}$, increasing in absolute value from top left to bottom right. The plots illustrate the increase of the stability boundary (black line, $-\text{Im}\ \Omega = 0$) and hence of the stable area (blue hatched region, $-\text{Im}\ \Omega \leq 0$) in complex frequency space. The coherent frequency shift of the unstable mode under consideration (red cross) is obtained from PyHeadtail simulations. It is demonstrated in [9] that the change of the effective impedance (chromatic effect) introduced by $\xi^{(2)}$ is negligible for this particular instability and that Landau damping is the dominant mechanism here. The unperturbed coherent frequency can hence be assumed to be independent of $\xi^{(2)}$. The colored lines in the figure refer to constant values of imaginary frequency shift ($-\text{Im}\ \Omega = \text{const.}$) and follow the distortion of the frequency space caused by the spread introduced by $\xi^{(2)}$. By means of these isolines one can read off the effective change of the imaginary frequency shift of the unstable mode as a function of frequency spread, or $\xi^{(2)}$. This illustrates the damping process: with increasing spread the imaginary part of the unstable mode is effectively reduced, meaning that the growth rate of the instability decreases. For $\xi^{(2)} \leq -9.6$, the area of stability has become large enough to include the unstable mode. At this point the instability is Landau damped. The final comparison of the imaginary frequency shifts, or instability growth rates, between stability diagram theory (red), obtained from the isolines in Fig. 3, and from PyHeadtail simulations (green) is shown in Fig. 4. They are both in excellent agreement with each other. Not only the stabilizing threshold for the amount of $\xi^{(2)}$ matches, but also the intermediate stages of $\xi^{(2)}$ show a remarkable agreement on the imaginary frequency shifts. This proves that the theory works successfully and that nonlinear chromaticity or rf quadrupoles indeed provide Landau damping. It should be pointed out, however, that the one-sidedness of the stability diagrams is a limitation of this method. A frequency spread from $\xi^{(2)} < 0$, for example, would only be able to Landau-damp the modes with $\text{Re}\ \Omega < 0$. The modes with $\text{Re}\ \Omega > 0$ could potentially be suppressed by means of a second, complementary method such as frequency spread from octupole magnets.

CONCLUSIONS

The existing Vlasov theory on transverse dipole modes has been extended to include the effects of nonlinear chromaticity up to arbitrary orders. This new formalism made it

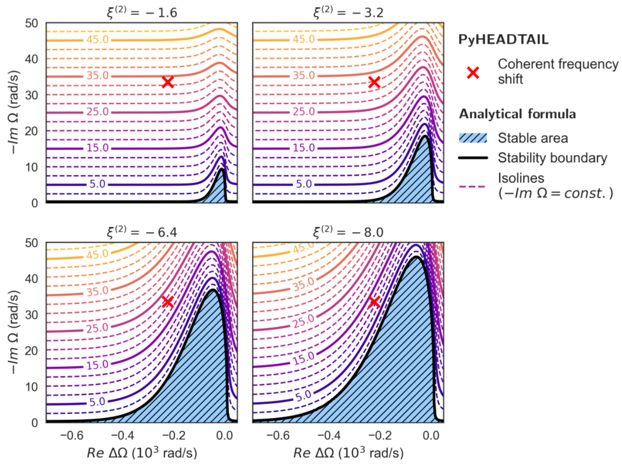

Figure 3: Stability boundary diagrams for four different values of $\xi^{(2)}$ increasing in absolute value from top left to bottom right.

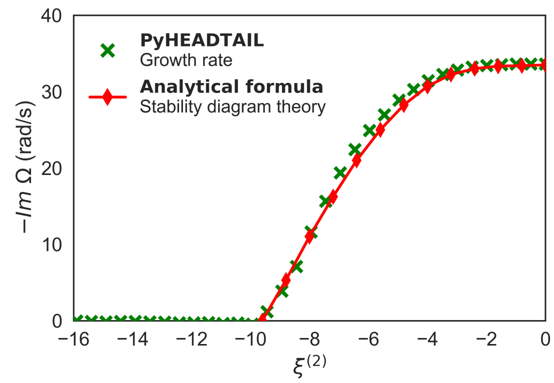

Figure 4: Stabilization of the head-tail mode zero vs. $\xi^{(2)}$ for a Gaussian beam. PyHeadtail simulations (green crosses) are shown together with analytical predictions calculated by means of stability diagram theory (red diamonds).

possible to confirm the hypothesis that nonlinear chromaticity and rf quadrupoles have two effects on the beam dynamics of transverse coherent modes: (1) they lead to a change of effective impedance; and (2) they introduce Landau damping thanks to the incoherent betatron frequency spread with longitudinal amplitude. The two mechanisms have been identified and studied separately using analytical formulae. In addition, the theory has been successfully benchmarked up to second-order chromaticity for an airbag model and a Gaussian beam. In the first case, there is no Landau damping due to the missing frequency spread from detuning with longitudinal amplitude. Analytical results have been validated both with a tracking model and a circulant matrix solver which revealed an outstanding agreement. For the Gaussian beam it has been demonstrated that, given the assumption of a strongly peaked impedance, analytical predictions from stability diagram theory are in excellent agreement with tracking simulations. This proves that detuning with longitudinal amplitude indeed provides Landau damping. The results are also in accordance with experiments and simulations that were carried out on the rf quadrupole and on nonlinear chromaticity in the Large Hadron Collider and provide the foundation for the interpretation of these results. The study also demonstrates, however, that beam stabilization with rf quadrupoles or nonlinear chromaticity is not easily evaluated analytically for arbitrary impedances. Macroparticle tracking simulations are instead the most accurate way to study these effects.

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REFERENCES

[1] Y. H. Chin, "Landau damping of a multi-bunch instability due to bunch-to-bunch tune spread", in Proc. of the Particle Accelerator Conference 1987 (1987), p. 1213.

[2] S. Sakanaka and T. Mitsuhashi, "Construction of an RF Quadrupole Magnet for Suppressing Transverse Coupled-Bunch Instabilities", in Proc. of the Particle Accelerator Conference 1991 (1991), p. 1836.

[3] V. V. Danilov, "Increasing the transverse mode coupling instability threshold by RF quadrupole", Phys. Rev. ST Accel. Beams **1**, 041301 (1998).

[4] E. A. Perevedentsev and A. A. Valishev, "Synchrobetatron dynamics with a radio-frequency quadrupole", in Proc. of the 8th European Particle Accelerator Conference, Paris, 2002 (EPS-IGA and CERN, Geneva, 2002), p. 1574.

[5] L. Landau, "On the vibrations of the electronic plasma", Zh. Eksp. Teor. Fiz. **16**, 574 (1946) [J. Phys. USSR **10**, 25 (1946)].

[6] A. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, Wiley Series in Beam Physics and Accelerator Technology (Wiley, New York, 1993).

[7] A. Grudiev, "Radio frequency quadrupole for Landau damping in accelerators", Phys. Rev. ST Accel. Beams **17**, 011001 (2014).

[8] M. Schenk et al., "Analysis of transverse beam stabilization with radio frequency quadrupoles", Phys. Rev. Accel. Beams **20**, 104402 (2017).

[9] M. Schenk et al., "Vlasov description of the effects of nonlinear chromaticity on transverse coherent beam instabilities", Phys. Rev. Accel. Beams **21**, 084402 (2018).

[10] M. Schenk et al., "Experimental stabilization of transverse collective instabilities in the LHC with second order chromaticity", Phys. Rev. Accel. Beams **21**, 084401 (2018).

[11] M. Schenk, "A novel approach to Landau damping of transverse collective instabilities in future hadron colliders", EPFL Thesis 9298, PhD thesis (Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland, 2019).

[12] K. Papke and A. Grudiev, "Design of an rf quadrupole for Landau damping", Phys. Rev. Accel. Beams **20**, 082001 (2017).

[13] L. R. Carver, X. Buffat, K. Li, E. Métral, and M. Schenk, "Transverse beam instabilities in the presence of linear coupling in the Large Hadron Collider", Phys. Rev. Accel. Beams **21**, 044401 (2018).

[14] E. Métral, S. Antipov, F. Antoniou, R. B. Appleby, G. Arduini, J. Baranco García, P. Baudrenghien, N. Biancacci, C. Bracco, R. Bruce, et al., *Update of the HL-LHC operational scenarios for proton operation*, tech. rep. CERN-ACC-NOTE-2018-0002 (CERN, Geneva, Switzerland, 2018).

[15] M. Schenk, L. R. Carver, A. Grudiev, K. Li, and E. Métral, *Tune footprints in presence of linear coupling*, *presented at* "74$^{\text{th}}$ HSC Meeting" (CERN, Geneva, Switzerland, Apr. 2016).

[16] S. Vik Furuseth et al., *MD3288: Instability latency with controlled noise*, tech. rep. CERN-ACC-NOTE-2019-0011 (CERN, Geneva, Switzerland, 2019).

[17] J. S. Berg and F. Ruggiero, "Stability Diagrams for Landau Damping", in Proc. of the Particle Accelerator Conference, Vancouver, Canada (1997).

[18] K. Li et al., "Code Development for Collective Effects", in Proc. of HB2016, Malmö, Sweden (July 2016), pp. 362 – 357.

[19] K. Li, "Code presentation: PyHEADTAIL", *presented at* 5th ABP-CWG Meeting, (CERN, Geneva, Switzerland, Nov. 2016).

[20] X. Buffat, "Transverse beams stability studies at the Large Hadron Collider", EPFL Thesis 6321, PhD thesis (Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland, 2015).