ON LANDAU DAMPING RESTORATION WITH ELECTRON LENSES IN SPACE-CHARGE DOMINATED BEAMS*

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Abstract

It is shown that the Lorentz forces of a low-energy, magnetically stabilized electron beam, or "electron lens", can introduce transverse nonlinear focusing sufficient for Landau damping of transverse beam instabilities in accelerators. Unlike other nonlinear elements, the electron lens can provide the frequency spread mainly at the desirable range of particle amplitudes, thus permitting to avoid the beam lifetime degradation.

INTRODUCTION

There are many impedance-driven collective instability phenomena in high intensity charged particle beams [1, 2] limiting the single-bunch and total beam intensities. Suppression of the collective instabilities can be obtained via the stabilizing effect of Landau damping [3] when the spectrum of incoherent frequencies $\omega_{x,y}$ overlap the frequencies of the unstable collective modes, thus allowing absorption of the collective energy by the resonant particles.

But the direct space charge forces in high intensity beam shift the incoherent frequencies away from the frequency of the zero head-tail mode leaving it exposed to instability. Similar effect happens with the Σ-mode in colliding beams.

To restore Landau damping the octupole magnets are commonly used with the transverse magnetic field $B_x + iB_y = O_3(x+iy)^3$ which generates the amplitude-dependent betatron frequency spread [4]. Damping by octupoles has several drawbacks: first of all, the corresponding frequency spread $\delta\omega_{x,y}$ scales with beam energy increase as $1/E^2$ due to increasing rigidity and smaller beam size, hence, one needs to increase strength of these magnets accordingly. Secondly, strong octupoles significantly reduce machine's dynamic aperture.

Another method involves beam-based feedback system which suppresses coherent motion of the beam or bunch centroid. Though generally effective, such feedback systems which act only on the modes with non-zero dipole moment, leaving the multitude of other "head-tail" modes unsuppressed [5]. Electron lenses [6] were shown to provide effective Landau damping [7] mechanism free of all the above listed drawbacks of other methods.

ELECTRON LENS

Over the years the electron lenses served for a number of purposes [8]. Here we discuss their use for restoration of Landau damping switched off by the beam space-charge (or by the beam-beam effect). In these cases the electron lens should provide a comparable tuneshift/tunespread raising the possibility of an adverse effect on the incoherent particle motion in the beam. There are a number of ideas how to avoid this, e.g. by making the optics with electron lens integrable.

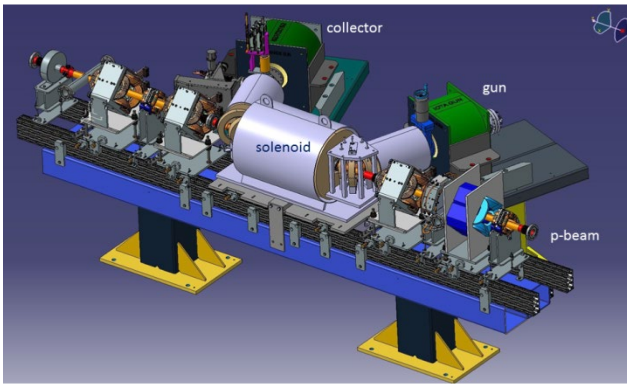

Figure 1: IOTA electron lens (courtesy of G. Stancari)

Such experiment is planned at the Integrable Optics Test Accelerator (IOTA) at Fermilab [9]. Figure 1 shows the lens being built for this purpose. By changing the gun cathode shape and voltage it is possible to form electron beam of different transverse profile and current and – by changing the solenoid magnetic field – to adjust the e-beam size.

Danilov-Nagaitsev Paradigm

In Ref. [10] V. Danilov and S. Nagaitsev proposed a recipe for building nonlinear integrable optics which in the simplest case can be described as follows:

- The lattice outside a special nonlinear insertion should be (almost) linear with phase advances being multiples of π.
- Beta-functions in the nonlinear insertion should be equal ($\beta_x = \beta_y = \beta_\perp$). In the particular case of an octupole-like nonlinearity its gradient $O_3(s)$ in order to preserve the Hamiltonian should be distributed along the path s as

$$O_3(s) \sim 1/\beta_\perp^3(s). \quad (1)$$

It can be expected that weak nonlinearities outside the special insertion will not break the KAM tori leaving the motion stable in a wide range of amplitudes.

In the case of a hollow electron beam which does not affect the beta-functions the longitudinal profiling can be achieved simply by variation of the solenoid magnetic field, while in the case of solid electron beam the situation is complicated by the effect of the lens itself on the beta-functions.

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HOLLOW E-LENS

A hollow electron lens was considered as the nonlinear element for creating a tunespread for Landau damping in the Recycler [11]. The lattice of the RR30 strait section (which housed the antiproton electron cooler in the past) was redesigned to have a 22.5 m drift with $\beta_x = \beta_y = \beta^*$ and $\alpha_x = \alpha_y = 0$ at its center. β^* was varied in the range 8.7÷15.5 m so that the phase advance was $(0.4\div0.6)\pi$ over the drift.

The transverse e-lens profile was chosen as

$$\rho(r) = \rho_0[\exp(-r^2/2\sigma_1^2) - \exp(-r^2/2\sigma_2^2)] \quad (2)$$

with $\sigma_{1,2} \sim \beta_\perp^{3/4}$ to produce a gradient satisfying Eq. 1.

The effect of the hollow lens with (constant) σ_2/σ_1=0.85 on the proton tunes is illustrated in Fig. 2.

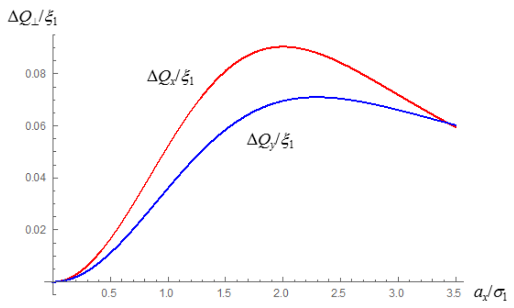

Figure 2: Tuneshifts as functions of relative horizontal amplitude a_x/σ_1 at a_y= 0. ξ_1 is the maximum tuneshift which the first term in Eq. 2 would produce alone.

Besides the electron lens, octupoles and special nonlinear magnets with strength also obeying Eq. 1 were simulated but incurred drastic reduction in dynamic aperture. On the contrary, the electron lens did not affect the dynamic aperture. The probable explanation is that the e-lens shifts both tunes in the same direction for all amplitudes thus avoiding some resonances.

Landau Damping by Hollow e-Lens

Figure 3 shows histograms of the spectral density of transverse oscillations in a bunched beam after receiving a kick in the presence of a hollow electron lens. The lens transverse dimensions were chosen such that the maximum tuneshift, δQ_{max}, was reached at oscillations amplitude of $3.4\sigma_{beam}$, σ_{beam} being the proton beam transverse r.m.s. size. With increasing δQ_{max} the spectral peak is shifting (but not as much) and widens testifying of increased Landau damping. The synchrotron tune for this example was $Q_s = 0.02{\cdot}Q_{SC}$ with Q_{SC} being the maximum absolute value of the space charge tuneshift.

The Runaway Effect

It was naively expected that while a hollow electron lens shifts the tunes of protons with large amplitudes and compensates for the space charge tuneshift, it will not affect the coherent tune making the overlap possible. Actually there is an appreciable coherent tuneshift as well so that the gap with incoherent tunes remains (A. Burov). This can be called a run-away effect. The explanation is that the maximum contribution to the dispersion integral comes from particles with $\sim\sigma_{beam}$ transverse amplitudes which do see the e-lens field in the considered case.

A hint of this effect can be seen in Fig. 3. Still in the case of a bunched beam where the head and tail experience much weaker space charge defocusing the overlap does happen providing Landau damping.

The situation is different in the case of the ~ rectangular RF bucket as well as for the head-on colliding beam.

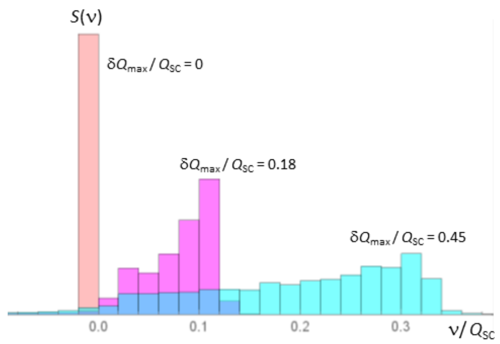

Figure 3: Spectral density of transverse oscillations in a bunch with space charge at indicated values of the maximum tuneshift due to a hollow electron lens.

GAUSSIAN E-LENS

Limited applicability of the hollow electron lens necessitates the consideration of a solid lens, here we limit ourselves to a Gaussian transverse profile. Since the runaway effect is caused by particles with transverse amplitudes $\geq \sigma_{beam}$, we will look at a narrow "pencil" electron lens beam with the size smaller than the size of the proton beam, $\sigma_{lens} < \sigma_{beam}$.

Figure 4 shows the total tuneshift (SC + e-lens) for three values of e-lens strength.

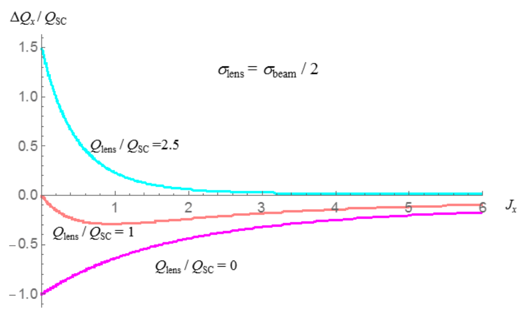

Figure 4: Tuneshift produced by space charge and a Gaussian e-lens with $\sigma_{lens} = \sigma_{beam}/2$ vs. the action variable J_x normalized by the beam emittance.

Coasting Beam

There is a limitation on the longitudinal bunch profile for which the eigenmode analysis of the Vlasov equation described in [12] can be applied. Besides Gaussian it can be constant which is a fair representation of a flat bunch in a multi-harmonic RF.

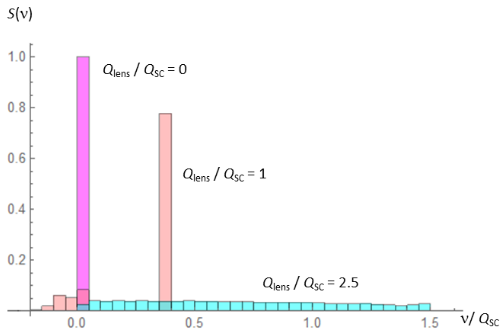

Figure 5: Spectral density of transverse oscillations in a coasting beam with space charge at indicated values of the maximum tuneshift due to a Gaussian electron lens.

The spectra for the same e-lens strength as in Fig. 4 are shown in Fig. 5. One can see that in order to intercept the runaway coherent tune the e-lens should be noticeably stronger than the space charge.

The absence of narrow peaks in the spectrum is a testimony of Landau damping but does not tell us how strong it is. For this purpose the technique of stability diagram can be employed. It shows the stability region in the plane of complex coherent tuneshift ζ produced by external impedances.

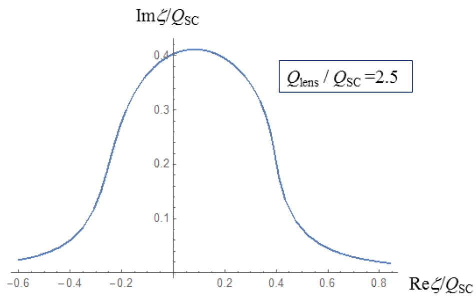

Figure 6: Stability diagram for coasting beam.

We use here the method based on the eigenmode analysis [12]. Figure 6 shows the stability diagram for the e-lens strength Q_{lens} / Q_{SC} =2.5. It proves that Gaussian lens can provide stability in the presence of large impedance no matter what the sign of its real part is (focusing or defocusing).

Bunched Beam

With longitudinally bell-shaped bunch (Gaussian in our study) there is no need to make the e-beam transverse size small compared with the proton beam size. The analysis presented below was performed for $\sigma_{lens} = \sigma_{beam}$.

Stability diagrams were estimated analytically and calculated using the method of [12] for a range of e-lens strength parameters and synchrotron tunes Q_s. The results are summarized in Fig. 7. As the measure of Landau damping efficiency the boundary value Λ = Imζ_{max} at Reζ = 0 was chosen.

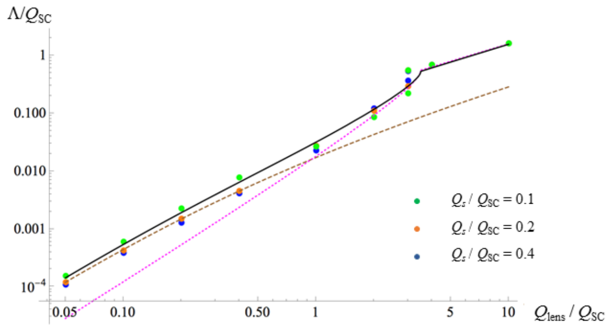

Figure 7: Landau damping rate vs e-lens strength normalized to Q_{SC}. Dots: the Vlasov model, dashed lines: analytical approximations for weak and medium-strong e-lens, solid black line: integration of the two approximations [12].

OUTLOOK

The presented results predict high efficiency of electron lenses in restoration of Landau damping. A few questions still remain open, such as dependence of the “runaway effect” on the thickness of the hollow e-lens, single-particle stability in the presence of a “pencil” electron beam, etc.

The analytical methods which we had used here should be complemented by numerical simulations and experimental studies planned at IOTA [9].

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