BNS damping*

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Abstract

Many years have passed since the invention of BNS damping, but it still attracts wide attention of accelerator physicists to implement this method in classical accelerators and future high gradient acceleration technique. In this article we recall initial background principles of the development of BNS damping.

SOME HISTORY AS AN INTRODUCTION

In the middle of 70s of the last century the director of the Institute of Nuclear Physics at Novosibirsk (nowadays Budker Institute of Nuclear Physics), Gersh Itskovich Budker introduced a new small group of researchers under the leadership of Vladimir Balakin to study a new type of an accelerator for colliding beams. He called this future machine as "SuperLinac". Nowadays we know such type of accelerators as "Linear Colliders". The main task of the group was to investigate the possibility to achieve a high gradient acceleration of order of a 1 MV/cm and analyze possible beam dynamics problems and find solutions for them. It happened that it was not the only task. In parallel, we worked also on the proton ironless ring of 10 GeV, which Andrei Mikhailovich (as we really called Budker) extremely wanted to build. This machine could give a possibility to study very exciting physics of heavy nucleolus, predicted by Spartak Timofeyevich Belyaev, who was at that time a president of the Novosibirsk State University.

However, all studies on linear colliding beams were kept in secrete and it was forbidden to show or publish the results outside the laboratory. What happened later? In several years, our group led by the outstanding physicist Vladimir Balakin successfully solved many important problems of the linear collider project. We have developed a technology of manufacturing high-gradient accelerating structure. Using this technology, we designed, manufactured, and tested a single S-band cavity. We have achieved almost 2 MV/cm in this cavity. We have understood main beam dynamics problems of acceleration of intense bunches of electrons and positrons in a high gradient linear accelerator using analytic and numerical approach. We have developed a numerical code for calculation of the electromagnetic fields interacting with a beam in the accelerating structure. It was may be the very first code in the world for the wake field calculations. We have found solutions for almost all of them. Unfortunately, Andrei Mikhailovich died in 1977 just before he became a sixty. A new director Alexander Nikolayevich Skrinsky continued the activity on linear colliding beams at the lab. The "SuperLinac" project got an official name: VLEPP (colliding linear electron positron beams), as analog to the names of the circular colliding beam facilities developed in the lab: VEPP-1, VEPP-2, VEPP-3 and VEPP-4. And finally, the results of theoretical and experimental studies were presented outside the lab. The VLEPP project was first presented at the International Symposium devoted to 60-year anniversary of G. I. Budker and All-union particle accelerator Conference at Dubna [1-3]. The results published in the Russian language were immediately translated to English at SLAC [4-6].

The most famous result of the linear colliders study at Novosibirsk became a new method of damping the transverse instability of a single bunch in a linear accelerator. This method got the name "BNS damping" by the first letters of the inventors Balakin, Novokhatski and Smirnov. The method was first published in 1978, however references in many other publications correspond to our 5-years later publication [7] in the proceedings of the12th International Conference on High Energy Accelerators at Fermilab (1983). And after more than 10 years BNS damping was successfully tested at the SLAC linear accelerator and implemented for operation of the first linear collider SLC [8].

In the following chapters we give more details starting with the description of the electromagnetic forces acting on the bunch particles moving in the accelerating structure. Then we present an equation for the bunch particle motion in the present of energy spread. We discuss the physics of BNS damping using a two-particle model. Then we present analytic solutions of a simplified equation of motion and numerical solutions for the VLEPP linear collider parameters. We analyze the efficiency of the method and make comparison with the Landau damping.

SINGLE BUNCH INSTABILITY

One of the main beam dynamics problems in a liner collider project was a transverse beam instability or beam break-up effect, discovered in operation of many linear accelerator including SLAC linac [9]. The transverse instability limits the intensity of the accelerating beams. High intensity beams are needed to achieve high luminosity of the beam collisions. Higher luminosity allows study of very rare events.

For the VLEPP project it was proposed to use a single high intensity short bunch. At that time, in comparison with a multi-bunch operation [10], it was not so much clear how a single short bunch interacts with an accelerating structure, what the beam break-up threshold can be? In the multi-bunch operation fields, exciting by passing by bunches accumulate in the structure. Accumulating fields are usually one or two eigen RF modes of the accelerating structure. A new bunch interacts with the field excited by the previously passed bunches. In a single bunch mode operation, all fields excited by the bunch particles are chasing the bunch because the field and the bunch particle moving with speed of light. There was a weak hope that a single

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bunch can have higher beam break-up threshold. To check it we need a reliable method for calculation of electromagnetic interaction of a charged bunch with a metal accelerating structure.

Electromagnetic fields and wake field code

For calculating electromagnetic forces, we used a wake field approach. Assuming that a bunch is moving with a constant speed along the structure in a straight line. Then we can easily describe the charge and current distributions in time domain needed for Maxwell equations. The boundary conditions are determined by geometry of the accelerating structure and can be very complicated. It is not so simple to solve Maxwell equations analytically, but we can use numerical methods for this. We developed a stable numerical scheme for solving Maxwell equations in the time domain in the azimuthally symmetrical structures. An example of calculated field distribution of a bunch moving in accelerating structure is shown in Fig. 1.

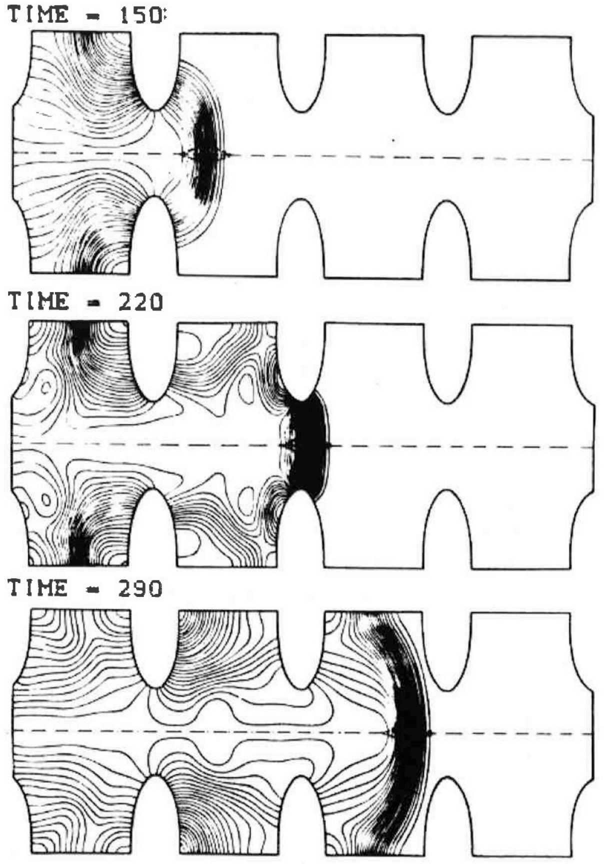

Figure 1: The dynamics of electric force lines of a charged bunch moving in "empty" periodic iris-loaded accelerating structure (1978).

This numerical scheme still in use and works very well for very short bunches [11-12]. In parallel with calculating of electromagnetic field components we calculate integrated forces (longitudinal and transverse) acting on the bunch particles. Then we average forces for a unit length of the structure.

$$F_{\parallel}(s,r_0)=\frac{1}{L}\int_{-L/2}^{L/2}E_z\left(z,r_0,t=\frac{z+s}{c}\right)dz \qquad L\to\infty$$
$$F_{\perp}(s,r_0)=\frac{1}{L}\int_{-L/2}^{L/2}\left(E_r\left(z,r_0,t=\frac{z+s}{c}\right)-H_{\varphi}\left(z,r_0,t=\frac{z+s}{c}\right)\right)dz \tag{1}$$

Transverse force description

The magnitude of the transverse force is determined by the structure geometry, by the shift of the bunch trajectory relative to the axes, by the bunch charge and the bunch length. Fig. 2 shows the distribution of the transverse force along the bunch for different bunch length.

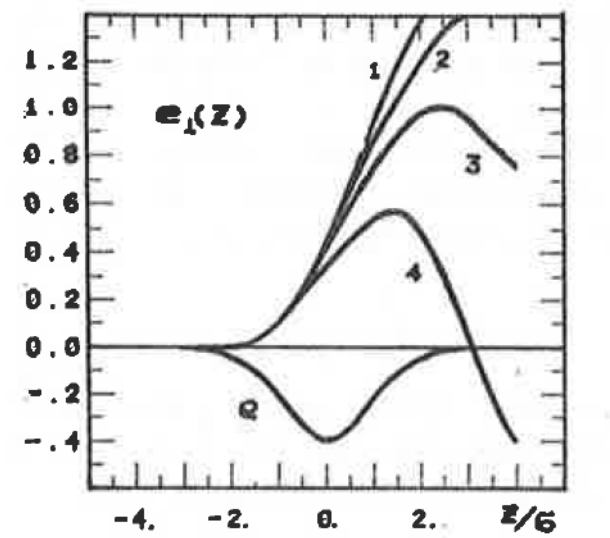

Figure 2: Transverse force acting on the bunch particles for bunches of different bunch length (1978). Line 1 is for a shorter bunch and line 4 is for a longer bunch.

Electromagnetic fields generated by a bunch in an accelerating structure have a defocusing action on the bunch particles if they travel off axes. The more leading particles are far away of axes the more effect on the following particles. Naturally, the force grows along the bunch. We may assume that a particle trajectory is not very far from the axes, so the dipole component of the excited field, which is proportional to the transverse coordinate, plays the main role. With our code we can calculate electromagnetic fields of a very short bunch. This gave us a possibility to derive an approximation for the Green's function. With a Green's function $g_{\perp}(s)$ we can present the dipole transverse force in the following way

$$F_{\perp}(s)=\frac{eQ}{4\pi\varepsilon_0 a_w^3}\int_{-\infty}^{s}\rho(\xi)X(\xi)g_{\perp}(\xi-s)d\xi \tag{2}$$

$X(\xi)$ is a bunch particle transverse coordinate and a_w is an effective parameter, characterizing the geometry of the accelerating structure. The Green's function is a dimensionless function in this presentation. The bunch charge longitudinal distribution is normalized:

$$\int_{-\infty}^{\infty}\rho(\xi)d\xi=1 \tag{3}$$

Equation for the transverse motion

Now using the transvers force description, we can write an equation for a bunch particle transverse motion in a linear accelerator with a FODO focusing system

$$\frac{\partial}{\partial \tau}\left(\frac{\gamma(\tau,s)}{\gamma_0}\frac{\partial}{\partial \tau}X(\tau,s)\right)+\frac{\gamma(\tau,s)}{\gamma_0}\mu(\tau)^2 X(\tau,s)= \\ =\int_{-\infty}^{s}\rho(\xi)X(\tau,\xi)g_{\perp}(s-\xi)d\xi \tag{4}$$

In this equation $\gamma(s)$ is a relativistic factor of a particle with a longitudinal position in the bunch s, γ_0 is the initial relativistic factor, which may depend upon the particle longitudinal position in the bunch. For simplicity of the equation we introduced a special parameter, a characteristic length of the accelerator where the transverse force produces a noticeable effect

$$L^* = a_w\sqrt{\frac{\gamma_0 mc^2}{\frac{eQ}{4\pi\varepsilon_0 a_w}}} \tag{5}$$

The characteristic length becomes smaller when the bunch charge increases. Also, in this equation we measure time and betatron wave vector (frequency) in characteristic length

$$t=\frac{\tau}{c}L^* \qquad \nu=\frac{\mu}{L^*} \tag{6}$$

It is interesting that solving the equation (4) for the case of a zero energy spread and strong focusing $\mu=\nu * L^* \geq 1$ we can find that characteristic length can be consider to be an instability growth rate parameter. The bunch emittance will grow exponentially as

$$\varepsilon \sim \exp\left(\sqrt{\frac{\tau}{\mu}}\right)=\exp\left(\frac{1}{L^*}\sqrt{\frac{ct}{\nu}}\right) \tag{7}$$

PHYSICS OF BNS DAMPING

With a very precise description of the forces acting on the bunch particles we have found immediately in the beam dynamics study a very strong transverse instability of a single bunch. We can describe it in the following way. The particles of the head of a bunch do not experienced action of the wake field and freely oscillate in the focusing lattice at the betatron frequencies. However, this oscillation produce a periodical force for the particles of the tail of the bunch because they experience the action of the wake field. As the frequency of the force and the frequency of free oscillations are the same, then the amplitude of oscillations of the tail particles will grow up in time because of the resonance. Considering that this action goes through the entire bunch we got an exponential growth of the amplitude of the particle oscillations.

An immediate solution for cancelling the oscillation growth is to destroy the resonance, that means to give different betatron frequencies to the particle of the bunch head and particles of the bunch tail. It can be done in many different ways, but a simple solution is to utilize the fact that the betatron oscillation frequency depends by virtue of the chromaticity on the energy of the beam particles. So, if we give different energy to the particles then we will have different betatron frequencies. The difference in energy along the bunch must have a definite sigh. Since the transverse wake field introduces defocusing force, then we need the additional chromatic focusing to compensate defocusing. That means that particles of the bunch tail must have smaller energy. By accelerating the bunch behind the crest of the accelerating field, the tail particles gain less energy than the head. Therefore, the tail particles are focused more by the quadrupoles than the head. The longitudinal wake field actually helps to increase the energy spread. The tail particles loss more energy due to the action of this field. With increasing of the particle energy during the acceleration, the energy difference can be reduced. The beam break up effect becomes smaller $\sim \gamma_0/\gamma$ and the bunch is now moved ahead of the crest to reduce the energy spread in the beam. We can see that that BNS damping does not require any additional accelerator elements like special focusing elements. It can be easily applied to any linear accelerator, just change the phases of the klystrons along the linac.

Two-particle model

We can get a reliable solution in a two-particle model. Assuming that we have particles with different betatron frequencies. The head particle has only oscillations due to the focusing system, but the tail particle has an additional force proportional to the transverse coordinate of the head particle

$$\frac{\partial^2}{\partial\tau^2}X_H(\tau)+\mu_H{}^2 X_H(\tau)=0. \\ \frac{\partial^2}{\partial\tau^2}X_T(\tau)+\mu_T{}^2 X(\tau)=X_H(\tau) \tag{8}$$

The solutions of these equations with an initial nonzero coordinate are

$$X_H(\tau)=\cos(\mu_H\tau) \\ X_T(\tau)=1+\frac{\cos(\mu_H\tau)-\cos(\mu_T\tau)}{\mu_H^2-\mu_T^2} \tag{9}$$

To keep the oscillation amplitude of tail particle we need the following condition for the difference of betatron frequencies

$$\frac{\Delta\mu}{\mu}\geq\frac{1}{2\mu^2} \tag{10}$$

Or

$$\frac{\Delta\nu}{\nu}=\frac{\Delta\mu}{\mu}\geq\frac{1}{2\mu^2}=\frac{1}{2\nu^2\left(L^*\right)^2}=\frac{1}{2\left(\nu a_W\right)^2}\frac{eQ/4\pi\varepsilon_0 a_W}{\gamma mc^2}$$

We can consider the minimum frequency spread needed to damp oscillation growth of the tail particle, to be the BNS damping condition.

$$\frac{\Delta\mu_{\min}}{\mu}=\frac{1}{2\mu^2} \tag{11}$$

Analytical solutions for many particles

The equation for the particle motion (4) is rather complicated for analytical solution but can be easily solved by using the numerical methods. However, some properties of BNS damping can be found based on the analytical solutions of a simplified equation with the following assumption:

No acceleration $\frac{d\gamma}{d\tau}=0$

Bunch longitudinal distribution is constant

$$\rho(s)=const[0,1]$$

Green's function is constant

$$g(s)=const$$

Linear distribution of energy along the bunch

$$\gamma(s)=\gamma_0\left(1-\frac{\Delta\gamma}{\gamma_0}s\right)$$

Correspondent linear distribution of the betatron frequency

$$\mu(s)=\mu_0\left(1+\frac{\Delta\mu}{\mu_0}s\right)\quad \frac{\Delta\mu}{\mu_0}=\frac{\Delta\gamma}{\gamma_0}$$

Simplified equation takes the following form

$$\frac{\partial^2}{\partial\tau^2}X(\tau,s)+\mu(s)^2X(\tau,s)=\frac{\gamma_0}{\gamma(s)}\int_{-\infty}^{s}X(\tau,\xi)d\xi \tag{12}$$

We found a way to solve this equation analytically using the Laplace transformation because this problem is a problem with initial conditions. Laplace transform

$$V(p,s)=\int_0^{\infty}X(\tau,s)e^{-p\tau}d\tau \tag{13}$$

Using it we can get an analytical solution in the Laplace presentation

$$V(p,s)=\frac{\gamma_0}{\gamma(s)}\frac{pX_0}{\mu^2(s)+p^2}\left(\frac{\mu^2(s)+p^2}{\mu_0^2+p^2}\right)^{\eta}\times \\ \times\left(1-\mu_0\int_{\mu_0}^{\mu(s)}\left(\frac{\mu^2(s)+p^2}{\mu_0^2+p^2}\right)^{-\eta}d\tau\right) \tag{14}$$

Immediately we found a parameter η, which gives the ratio between the BNS condition (12) and total frequency spread

$$\eta=\frac{1}{2\mu_0\Delta\mu}=\frac{\frac{1}{2\mu_0^2}}{\frac{\Delta\mu}{\mu_0}} \tag{15}$$

The inverse Laplace transform is easy to derive for integer values of η

- **η =0** There are no transverse forces. Just to check the model. Free oscillations with natural frequencies

$$X(s,\tau)=X_0\frac{\gamma_0}{\gamma(s)}\cos(\mu(s)\tau)$$

- **η =2** Instability. Resonant build-up of oscillations at a frequency of the "head" particle

$$X(s,\tau)=X_0\frac{\gamma_0}{\gamma(s)}\cos(\mu_0\tau)+ \\ +X_0\frac{\gamma_0}{\gamma(s)}\frac{\mu^2(s)-\mu_0^2}{2\mu_0^2}(\mu_0\tau)\sin(\mu_0\tau)$$

- **η =-1** Instability. Resonant build-up of oscillations at the natural frequencies

$$X(s,\tau)=X_0\frac{\gamma_0}{\gamma(s)}\cos(\mu(s)\tau)+ \\ +X_0\frac{\gamma_0}{\gamma(s)}\frac{\mu^2(s)-\mu_0^2}{2\mu_0^2}(\mu(s)\tau)\sin(\mu(s)\tau)$$

- **η =1** BNS damping. All particles oscillate at the frequency of the "head" particle without any growth

$$X(s,\tau)=X_0\frac{\gamma_0}{\gamma(s)}\cos(\mu_0\tau)$$

This nice behavior of the bunch is possible for other combinations of the Green's function and bunch distribution.

- **η =1/2** One more exciting solution. The amplitude of oscillation of the "tail" particles is going down in time

$$X(s,\tau)=X_0\frac{\gamma_0}{\gamma(s)}J_0\left(\frac{\mu(s)-\mu_0}{2}\tau\right)\cos(\mu(s)\tau)$$

J_0 is the Bessel function of zero order. The amplitude of oscillations goes down as a square root of time. Unfortunately, analysis shows that this interesting result holds only for the constant Green's function.

APPLICATION TO THE VLEPP PROJECT

Computer simulations with realistic Green's function and bunch distribution showed the same particle dynamics. Analysis of the longitudinal beam dynamics including longitudinal wake fields showed that it is possible to make a linear energy variation along the bunch. The results of the numerical integration of equation (4) exhibit the similar regularities of the transverse motion of the bunch particle. Fig. 3 gives the phase images (X X' plane) of particles along the bunch at different energy spread. The dependence of the bunch effective emittance upon the sign and the value of energy variation along the bunch is shown in Figure 4. One can see here the results of numerical simulation for the 100 GeV section of the accelerator VLEPP. As is seen, to suppress the transvers instability, an initial energy spread of 10% needs to be introduced. During the acceleration this linear spread can be decreased down to 3% and can reach a minimally achievable one on the final section. Without BNS damping the beam emittance could be many orders higher. The effect of application of BNS damping is is very strong.

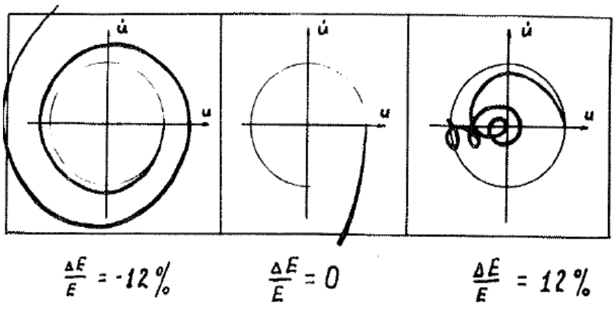

Figure 3: X' X phase plot for different energy spreads (1978)

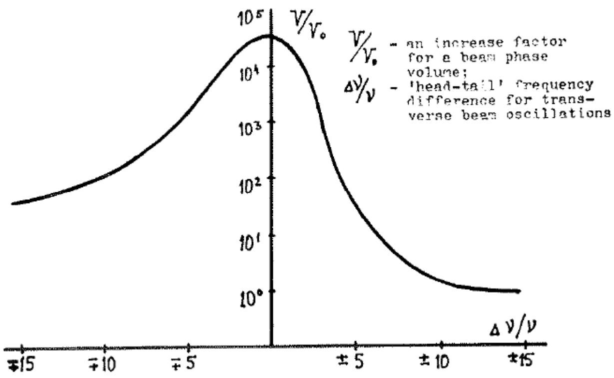

Figure 4: Relative emittance at the exit of the 100 GeV accelerator section versus the initial energy spread. With the initial energy spread of 12% the beam can reach at the section exit with a minimally achievable spread of 3% (1978)

EFFICIENCY AND COMPARISON WITH LANDAU DAMPING

We would like to mention that the efficiency of the BNS damping strongly depends upon the focusing system. For higher betatron frequencies less frequency spread is needed, and much higher intensity bunch can be accelerated without emittance growth. This is in general good for any linear accelerator to have more periods of transverse oscillations, made by the bunch particles on the accelerator length. Very important the sign of the linear energy spread along the bunch to completely damp the instability. For the opposite sign of the energy spread, the instability growth rate decreases, but instability cannot be fully damped. Such feature is very close to Landau damping, where betatron oscillation frequencies are not corelated with the longitudinal positions of particles in the bunch. Usually is it a random distribution of betatron frequencies. This is the main difference between the damping methods. Maybe it is possible to use Landau damping to decrease the instability growth, however a comparison with BNS damping showed that Landau damping is not so effective as BNS damping. Fig. 5 shows comparison of BNS damping and Landau damping. The function of Landau damping is mirrored for negative values of energy spread.

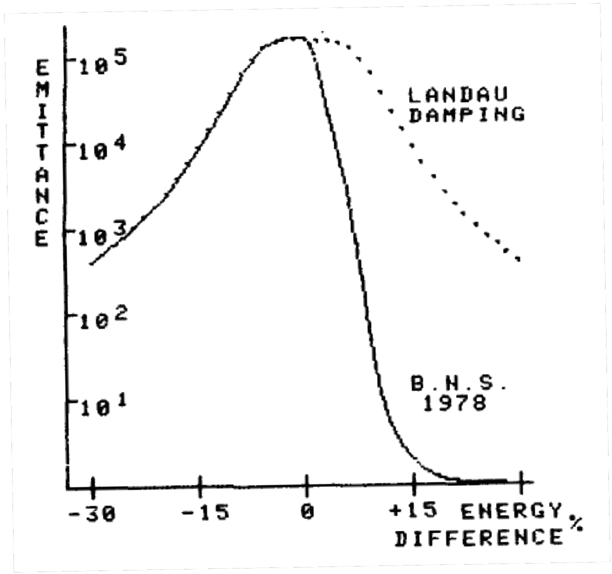

Figure5: Beam emittance upon the linear energy spread for BNS damping (a solid line) and for the case of Landau damping (a dotted line). For the negative values of energy spread the result form Landau damping is mirrored from positive values. One can see that Landau damping works something like BNS damping but for an opposite sign of the energy spread and cannot damp instability completely.

CONCLUSION

- BNS damping is a very efficient method for damping the transverse instability in a linear accelerator
- Naturally, it works in the multi-bunch regime as well.
- SLC, the first linear collider using this method increased luminosity several times.
- BNS damping was effectively used in the injector of intense beams for the SLAC PEP-II B-factory.
- In the linear accelerators BNS damping works much better than Landau damping.

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