RF scheme to mitigate longitudinal instabilities in the SPPC

Lin Hao Zhang†, Jing Yu Tang∗

Institute of High Energy Physics, CAS, Beijing 100049, China
University of Chinese Academy of Sciences, Beijing 100049, China
Spallation Neutron Source Science Center, Dongguan 523803, China

Abstract

A large collider project CEPC-SPPC is under study with a global effort and a leading role from IHEP, with CEPC (Circular Electron-Positron Collider, Phase I) to probe Higgs physics and SPPC (Super Proton-Proton Collider, Phase II) to explore new physics beyond the Standard Model. The key design goal for SPPC is to reach 75 TeV in center-of-mass energy with a circumference of 100 km. As an important part of the SPPC conceptual study, the study on the longitudinal dynamics at SPPC has been conducted, with a focus on the RF scheme to meet the requirements for luminosity and mitigate the relevant instabilities. The longitudinal instability bottleneck associated with the longitudinal dynamics is the loss of Landau damping. To mitigate the beam instabilities, a higher harmonic RF system (800 MHz) together with the basic RF system (400 MHz) to form a dual-harmonic RF system is found helpful, which also increases the luminosity by producing shorter bunches. The preliminary results indicate that longitudinal impedance threshold can be increased.

INTRODUCTION

Following the discovery of Higgs boson at the Large Hadron Collider (LHC) in 2012, which opens a brand new door to the unknown physics, new large colliders are being proposed and studied by the international high-energy community to explore the Higgs boson in depth and probe new physics beyond the Standard Model. In China, a two-stage project including two colliders of unprecedented scale – CEPC and SPPC (Circular Electron-Positron Collider & Super Proton-Proton Collider) was initiated, with CEPC (Phase I) focusing on the Higgs physics and SPPC (Phase II) being an energy frontier collider and a discovery machine which is far beyond the performance of the LHC [1]. The two colliders share the same tunnel of 100 km in circumference.

The key design goal for SPPC is to reach 75 TeV in center-of-mass energy [2]. In such a high-energy proton-proton collider, synchrotron radiation becomes non-negligible during the acceleration cycle, especially in the collision phase. In addition, as the bunch population of 1.5×10^{11} protons will be utilized to reach the nominal luminosity of 1×10^{35} cm^{-2}s^{-1}, beam instabilities become a major concern. Therefore, the longitudinal beam dynamics including the emittance radiation damping and controlled blow-up, instabilities and mitigations has been studied. In this paper, the longitudinal dynamics design and RF schemes to mitigate the longitudinal instabilities at SPPC are presented.

MAIN LONGITUDINAL INSTABILITIES IN SPPC

According to the phenomena observed and the experience accumulated during SPS and LHC design and operation [3-4], the main single bunch longitudinal collective effects requiring to be carefully considered for the next-generation proton collider like SPPC are intra-beam scattering, longitudinal microwave instability and loss of Landau damping, among which the latter plays a critical role. Concerning the mitigation of the longitudinal instabilities, more attention should be paid to the increase of Landau damping.

Intra-beam Scattering

Intra-beam Scattering (IBS) is a multiple small-angle Coulomb scattering of charged particles within a bunch. It normally leads to two effects in proton or ion accelerators: redistribution of beam momenta and impact on beam quality due to transverse and longitudinal emittance blow-up. In SPPC, it is found that IBS is well under control during the whole acceleration cycle, provided that the longitudinal emittance at the collision is maintained at a high value in the presence of strong synchrotron radiation damping. The emittance blow-up is also needed for compressing the other instabilities.

Longitudinal Microwave Instability

The longitudinal microwave instability is normally caused by the higher frequency part of impedance. When this instability occurs, it will normally first cause a high-frequency structure on the bunch profile. In proton synchrotron the microwave instability is observed as a fast increase of the bunch length and thus of the longitudinal emittance [5]. The threshold of this instability can be estimated by the well-known Keil-Schnell-Boussard criterion [6],

\[ |Z_n| \leq \frac{2\pi\beta^2 E\sigma_{\perp}^2}{\epsilon \hat{I}}. \]  

Here, \( \hat{I} \) is peak beam current, \( \beta \) is Lorentz factor, \( E \) is the total energy, \( \sigma_{\perp} \) is the momentum spread, \( \eta \) is the slip factor and \( F \) is a form factor depending on the particle distribution. Usually, longitudinal microwave instability will not be worsened for larger hadron machines with the assumption of keeping the same bunch length and mo-
momentum spread [7]. In SPPC, the overall longitudinal impedance should be well controlled to avoid the instability. A reasonable assumption of 0.2 Ω is adopted in the studies, following 0.1 Ω at LHC which has a smaller circumference.

**Loss of Landau damping**

The Landau damping in the longitudinal phase plane comes from the spread in the synchrotron frequency due to the nonlinearity of the RF voltage. This is a natural stabilizing mechanism for different longitudinal instabilities. Here a particular instability concerning the single bunch instability, that was observed in SPS, is described, which is often called the loss of Landau damping and caused by the excitation due to reactive impedance overshadowing the Landau damping. Based on the well-known Sacherer dispersion relation, an approximate stable boundary can be given by $S > 4/\sqrt{m}|\Delta \omega_m|$ for Sacherer’s smooth distribution, where $m$ is the azimuthal mode which specifies the oscillation type of single bunch, like $m = 1$ for dipole mode, $m = 2$ for quadrupole mode, $S$ is the synchrotron frequency spread and $\Delta \omega_m$ is the total coherent frequency shift from the contribution of various impedance [8]. It is notable that the dispersion relation as well as the size of the stable region is related to the bunch distribution [9]. Using the stability criteria with $m = 1$ mode, the broad band impedance threshold, above which the single bunch instability will be generated, corresponds to the loss of Landau damping can be derived [10]:

$$\left|\frac{\text{Im} Z}{n}\right| \leq F \frac{E}{e \beta^2} \left(\frac{\Delta E}{E}\right)^2 \frac{\Delta \omega}{\omega_s} f_0 \tau.$$  \hspace{0.5cm} (2)

where $f_0$ is the revolution frequency, $I_b = eN_b f_0$ is the single bunch current with bunch intensity $N_b$, $\tau$ is the full bunch length, $\Delta \omega/\omega_s$ is the synchrotron frequency spread within the bunch and $\Delta E/E$ is the relative energy spread.

Given that the stable region will be bigger for the flat bunch distribution, it seems favourable to induce a dual-harmonic RF system to lengthen the bunch and to increase the frequency spread which is an effective cure for loss of Landau damping. However, it has been demonstrated that Landau damping is lost when the derivative of synchrotron frequency is zero outside the bunch center [11], which makes the dual harmonic RF system working in bunch-lengthening mode (BLM) not preferable at SPS [12]. In SPPC, this instability is also of major concern, and the corresponding measures are studied in detail as shown below.

**LONGITUDINAL DYNAMICS DESIGN FOR SPPC**

The main parameters related to the SPPC longitudinal dynamics are listed in Table 1. One of the main goals of the collider design is to achieve a high luminosity which is relevant to the beam current, kinetic energy, beam-beam parameter and the reduction factor due to the hourglass effect and the Piwinski angle [13]. The influence of hourglass effect can be neglected if $\beta^*/\sigma_z \gg 1$, with $\beta^*$ being the beta function at the collision point and $\sigma_z$ the RMS bunch length. This is the case for the SPPC. However, the reduction factor $F_{\text{ca}}$ with respect to different $\beta^*/\sigma_z$ and bunch spacings that is caused by the Piwinski angle is shown in Fig. 1. While $\beta^*$ is constrained by the overall design of the interaction point, it is the goal of the longitudinal dynamics design to provide a bunch as short as possible. There are two main constraints in limiting very short bunches, one from the intra-beam scattering (IBS) and the other from the longitudinal instabilities. In the SPPC baseline design, an RF system at 400 MHz is used. We are trying to improve the design by optimizing the RF system in order to have a better control over the above constraints or an even higher luminosity.

Table 1: Main RF and beam parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference, C</td>
<td>100</td>
<td>km</td>
</tr>
<tr>
<td>Injection/collision energy, $E$</td>
<td>2.1/37.5</td>
<td>TeV</td>
</tr>
<tr>
<td>Transition gamma, $\gamma_t$</td>
<td>99.21</td>
<td></td>
</tr>
<tr>
<td>Bunch intensity, $N_b$</td>
<td>1.5x10^11</td>
<td></td>
</tr>
<tr>
<td>Number of bunches, $n_b$</td>
<td>10080</td>
<td></td>
</tr>
<tr>
<td>Bunch spacing, $\Delta t$</td>
<td>25</td>
<td>ns</td>
</tr>
<tr>
<td>RF frequency, $f_{rf}$</td>
<td>400</td>
<td>MHz</td>
</tr>
<tr>
<td>RMS bunch length in collision, $\sigma_z$</td>
<td>7.55</td>
<td>cm</td>
</tr>
<tr>
<td>Harmonic number, $h$</td>
<td>133333</td>
<td></td>
</tr>
<tr>
<td>Energy loss in collision, $U_0$</td>
<td>1.48</td>
<td>MeV/turn</td>
</tr>
<tr>
<td>Longitudinal emittance damping time in collision, $\tau_f$</td>
<td>1.17</td>
<td>h</td>
</tr>
</tbody>
</table>

![Figure 1: Luminosity reduction factor due to the Piwinski angle for different $\beta^*/\sigma_z$ and bunch spacing.](image)

The growth time of the IBS effect is quite different in the different phases of the SPPC cycle and those at the injection and collision are shown in Fig. 2. At the injection, the longitudinal emittance between 1.5 eVs and 2.5 eVs seems to be a good choice because the IBS growth time, at least 20 h, is much greater than the injection duration of 840 s. However, in collision, the longitudinal emittance has to be increased to at least 6 eVs due to the long physics collision time as long as 14 h, and the short longi-
tudinal emittance damping time of 1.07 h. Therefore, a controlled emittance blow-up scheme during the acceleration cycle is crucial.

The longitudinal instability constraint mainly results from the loss of Landau damping. With Eq. (2), a simplified formula corresponding to loss of Landau damping can be derived:

$$\left| \text{Im} Z \right| \leq \frac{3\pi^2}{n} \frac{h^2 V_{\text{rf}}}{32} \left( \frac{L}{C} \right)^5.$$

where \( h \) is the harmonic number, \( C \) is the ring circumference, \( L = 4\sigma_z \) is the full bunch length. Thus, the threshold on the longitudinal reactive impedance is determined by the bunch length at injection and in collision as shown in Fig. 3. At injection, the RMS bunch length is at least 8 cm in order to have the longitudinal impedance threshold greater than 0.2 \( \Omega \) and match with the upstream accelerator in the injector chain in the longitudinal phase plane. Thus, a lower RF voltage should be used. However, in collision, to reach the RMS bunch length that is required by the luminosity in the baseline design, an RF voltage of at least 32 MV is needed, and a similar limitation on the impedance threshold above 0.2 \( \Omega \) is maintained. Based on the above considerations, in the SPPC baseline design, the RF voltage is chosen to be 20 MV and the RMS bunch length is 9 cm at injection, whereas in collision, the RF voltage increases to 40 MV. The corresponding longitudinal reactive impedance threshold during the cycle is above 0.2 \( \Omega \). To obtain a shorter bunch length during collision than 7.55 cm, it will need a higher RF voltage but at the cost of a slight reduction in the longitudinal impedance threshold. Besides, we can also use a higher harmonic RF system of 800 MHz, or a dual harmonic RF system of 400 MHz and 800 MHz working in the bunch shortening mode, which will be illustrated below.

**RF SCHEMES TO MITIGATE INSTABILITIES**

In order to enhance the Landau damping in the SPPC, a large synchrotron frequency spread inside the bunch is required, which can be achieved by using either a higher harmonic RF system of 800 MHz, or a dual harmonic RF system of 400 MHz and 800 MHz. A controlled emittance blow-up is also needed to avoid too small emittance that naturally shrinks due to synchrotron radiation.

For the option of a single harmonic RF system at 800 MHz, due to the limit of the momentum filling factor below 0.8 during physics running to avoid beam loss, the RMS bunch length will be reduced to 5.56 cm or less from 7.55 cm in the case of 400 MHz, as illustrated in Fig. 4. In this study, the RMS bunch length was taken as 5.2 cm, which is beneficial to the luminosity, and the relevant parameters are: the longitudinal emittance 6.4 eVs, RF voltage 52 MV and RMS momentum spread 0.79 \times 10^{-4}. The corresponding longitudinal impedance threshold is about 1.6 times as high as that in the case of 400 MHz. However, the RF bucket is occupied too much which will have an adverse effect on the bunch storage and collision dynamics running of up to 14 h.

**Figure 4: Momentum filling factor and bunch length for 800 MHz**

A dual harmonic RF system can work in two different modes: the bunch lengthening mode (BLM) and bunch shortening mode (BSM) which are determined by the relative phase difference between the two harmonic systems. Fig. 5 shows the composed RF voltage curve, potential well and bucket for the SRF (Single 400 MHz), BSM and BLM. One can see that BLM has a larger bucket area and a smaller peak line density which is beneficial to reduce space charge effects. These are extremely beneficial in lower-energy high-intensity proton synchrotrons such as the p-RCS of the SPPC injector chain, while is negligible in the SPPC. However, as shown in Fig. 6, the BLM has relatively larger synchrotron frequency spread in the mode of \( k=0.4 \) and \( k=0.5 \) for the RMS bunch length 7.55 cm or less, which corresponds to the maximum phase extension is about 1.3 rad, but it will significantly decrease with small phase errors and introduce regions with zero derivative, which is disadvantageous for Landau damping. Furthermore, the average synchrotron tune is smaller, which is unfavorable for controlling the Transverse Mode Coupling Instability (TMCI) [14]. However,
both the synchrotron frequency spread and the average synchrotron tune at BSM are improved as compared with SRF. In addition, the bunch length is slightly shorter which is helpful to enhance the luminosity. Although the bucket area is relatively smaller, it can be cured by giving a larger voltage on the fundamental harmonic RF.

Another vital measure in the longitudinal beam dynamics is to blow up the longitudinal emittance in a controlled way. There are two main ways: (1) one injects a bandwidth limited RF phase noise from $\omega_{down}$ to $\omega_{up}$ into the fundamental RF system during the ramp-up through a phase loop, where $\omega_{up}$ is a little beyond the bunch central synchrotron frequency $\omega_z$ to fully influence the bunch core, but $\omega_{down}$ is the frequency of the bunch edge associated with the desired bunch emittance. The particles with synchrotron frequencies falling in this frequency range are excited. Outside the frequency range, there is no resonant excitation, thus the longitudinal motion remains unchanged. Then a controlled larger longitudinal emittance is obtained [15]. This method has been adopted by LHC, SPS and PSB [16-18] at CERN. (2) One adds a phase-modulated higher frequency RF to the fundamental RF, which will drive bunches towards resonant islands and cause the bunch density redistribution [19]. This method is used in PS [20]. The controlled longitudinal emittance plays a critical role in the reduction of the intra-beam scattering, counteracting of the synchrotron radiation damping, suppression of the collective instabilities and mitigation of the space charge effects.

**SUMMARY**

The potential key longitudinal instabilities in the SPPC are reviewed. In order to enhance the Landau damping for longitudinal microwave instability, loss of Landau damping, a dual harmonic RF system composed of 400 MHz and 800 MHz is proposed. In the bunch shortening mode, it can provide larger tune spread which is good for suppressing the instabilities and shorter bunch length which can enhance the luminosity.

**ACKNOWLEDGEMENTS**

We would like to thank the organizers of the 2019 MCBI workshop for providing us a good opportunity to learn the progress in the field and discuss with the other participants. Special thanks to Dr. Elena Shaposhnikova of CERN for beneficial discussions and comments on the longitudinal instabilities and mitigation schemes at LHC/SPS.

**REFERENCES**


