DIAGNOSTICS WITH QUADRUPOLAR PICK-UPS

A. Oeftiger*, GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt, Germany

Abstract

The spectrum of a quadrupolar pick-up gives access to coherent second-order modes of a beam in a non-invasive way. The signature of bunched beams not only features the first-order and second-order dipolar modes, but also most notably (1.) bands of even envelope modes, (2.) odd (skew) envelope modes and (3.) coherent dispersion modes. The odd (skew) envelope modes can provide information on linear coupling and mismatch thereof. The tunes of even envelope modes and the coherent dispersion modes shift significantly with space charge and can thus provide a measurement thereof.

The paper presents measurements of the various modes at the CERN Proton Synchrotron, demonstrates how the spectral amplitudes depend on the mismatch conditions, and suggests a new, alternative way to quantify space charge at high brightness conditions via the coherent dispersion mode.

INTRODUCTION

Quadrupolar pick-ups (QPU) in a beam line provide information about the coherent transverse second-order moments of a passing bunch of particles. These devices have often been used in studies measuring the beam emittance or injection mismatch of the optics functions [1–3], or the strength of space charge in synchrotrons [4–8]. The advantage is the non-invasive and thus non-destructive nature of measuring the quadrupolar moment S_{QPU} via induced currents in the four symmetrically arranged electrodes, in particular for inferring the transverse RMS emittances in comparison to destructive profile measurement methods like flying wire scans or secondary electron emission (SEM) grids. Typically, *time domain* oriented methods to determine the emittance from a measured quadrupolar moment demand well controlled experimental setups, where differential offsets in the quadrupolar moments need to be understood and controlled precisely while the strong dipolar component in the signal needs to be suppressed. These challenges could possibly be the main reason why QPUs are typically not yet used as beam diagnostics in regular operation. A technically less demanding and thus potentially more rewarding approach in a synchrotron is to profit from the *frequency domain* and measure the bunch eigenmodes, where only the frequency content and not the absolute values of S_{QPU} matter.

The quadrupolar spectrum of a circulating beam has a rich structure. Most often the two even transverse envelope modes of the oscillating $\sigma_{x,y}(s)$ are studied, most prominently for measuring the strength of space charge by determining the coherent tune shift of the envelope due to space charge defocusing. Past experiments mainly studied coasting beam conditions [4–7]. A relatively recent study extends this space charge measurement to bunched beams [8]: since the envelope oscillations due to injection mismatch typically decohere very rapidly in bunches, they are much more challenging to measure than in coasting beams. As an alternative, the study establishes the quadrupolar beam transfer function (Q-BTF) technique based on a transverse feedback system, which quadrupolarly excites the bunch in a frequency sweep while measuring the beam response in the QPU. The such measured “bands” of coherent envelope modes (due to varying defocusing by space charge depending on the longitudinal line charge density) reveal the maximum coherent envelope tune shift at the longitudinal peak line charge density.

While a quadrupolar kick exciting the beam sizes would intrinsically lead to RMS emittance growth $\Delta\epsilon/\epsilon_0$, the discussed Q-BTF approach lead to well constrained values $\Delta\epsilon/\epsilon_0 \leq 5\%$ (comparable to the impact of the flying wire scan technique). Nonetheless, this method remains principally destructive. It may be a good moment to reflect on simpler non-destructive measurements of space charge than via the envelopes $\sigma_{x,y}$, since they decohere so rapidly after injection for bunched beams (i.e. usual operation mode in most synchrotrons).

In this contribution, we turn our attention to other second-order beam moments in the QPU frequency spectrum, which may last much longer and would thus be more accessible for precision measurements of space charge detuning. The odd (skew or tilting) envelope modes are identified in measurements for the first time. In the perturbative space charge conditions of synchrotrons, these linear coupling modes do not exhibit significant scaling with space charge though, nonetheless they can provide a measure of linear coupling. Furthermore, we discuss the coherent dispersion mode, which represents the correlation between transverse and longitudinal degrees of freedom.The dispersion mode tune does shift with space charge, which could make it an interesting alternative candidate to the envelope modes in order to measure space charge. Also, the dispersion mode could provide new insights on beam dynamics regarding the topic of head-tail instabilities vs. space charge: head-tail instabilities correlate the transverse planes with the longitudinal plane, i.e. they naturally provide amplitude to the coherent dispersion mode.

The CERN Proton Synchrotron (PS) is equipped with a sensitive strip-line pick-up featuring a diode set-up for precise tune measurements (“Base-Band Q-meter”). Using the quadrupolar combination of its four electrode signals gives access to the quadrupolar spectrum. Based on this set-up, we can identify the various modes in the spectrum by varying the underlying beam dynamics such as dipolar mismatch etc. We structure this paper as follows: first we outline the relevant beam modes in the quadrupolar spectrum, before we turn to the PS measurements – starting from the dominant

* also at CERN, Geneva, Switzerland

dipolar modes, then investigating the odd envelope modes and finally discussing the coherent dispersion mode.

OVERVIEW QUADRUPOLAR SPECTRUM

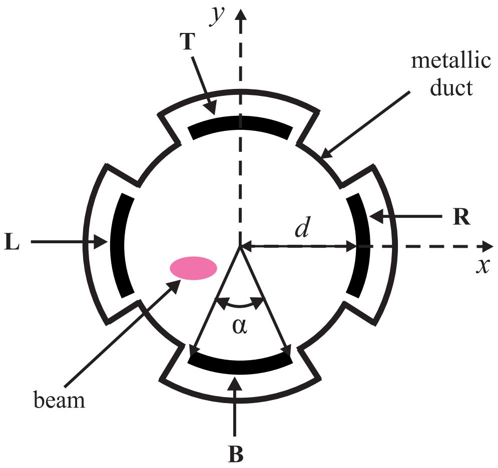


Figure 1: Schematics of four electrodes in quadrupolar pick-up, taken from Ref. [9]

Four electrodes, symmetrically arranged in 90° steps in the transverse plane around the passing beam as indicated in Fig. 1, pick up on the quadrupolar moment of the passing beam by combining their induced voltages as $S_{QPU} = (U_T + U_B) - (U_L + U_R)$ [9]. This quadratic signal

$$S_{QPU} \propto \langle y^2 \rangle - \langle x^2 \rangle \qquad (1)$$

contains contributions from all collective second-order modes coupling into the transverse plane, where x denotes the horizontal particle offset and y the vertical offset. $\langle \cdot \rangle$ denotes the expectation value summing over all particles in the beam.

Let us consider betatron motion x_β around a constant equilibrium orbit $\overline{x}$ with overlaid dispersive motion $D_x\delta$ for dimensionless momentum offset $\delta \doteq \frac{\Delta p}{p_0}$. D_x denotes the horizontal dispersion for this particle, $D_x = \partial x/\partial\delta$. The horizontal particle coordinate at the QPU location thus amounts to

$$x = \overline{x} + x_\beta + D_x\delta \quad , \qquad (2)$$

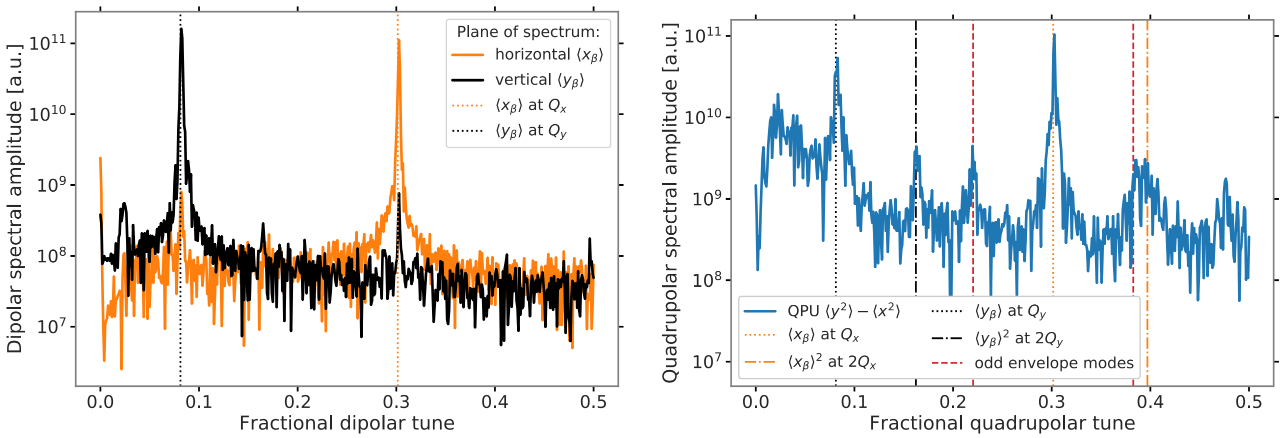
and likewise for the vertical plane. The collective quadratic signal thus splits up into

$$S_{QPU} \propto \underbrace{\left(\langle y_\beta^2\rangle - \langle x_\beta^2\rangle\right)}_{\left(\sigma_y^2 - \sigma_x^2 + \langle y_\beta\rangle^2 - \langle x_\beta\rangle^2\right)} + 2\cdot\overline{y}\,\langle y_\beta\rangle - 2\cdot\overline{x}\,\langle x_\beta\rangle \qquad (3)$$
$$+ \left\langle y_\beta\,\delta\right\rangle - \left\langle x_\beta\,\delta\right\rangle + \text{longitudinal } \delta \text{ terms} \quad .$$

On top, for skew components in the focusing channel, additional coupling terms appear involving $\sigma_x\sigma_y$ and a skew envelope term σ_{xy}^2 (first discussed by Chernin [10]).

The spectral content of S_{QPU} comprises all the corresponding eigenmodes for these coherent first- and second-order moments. Given the bare betatron tunes $Q_{x,y}$, we thus conceptually expect to see

- $Q_{x,y}$: dipolar transverse motion from $\overline{x}\cdot\langle x\rangle$, $\overline{y}\cdot\langle y\rangle$,
- $2Q_{x,y}$: dipolar transverse motion from $\langle x_\beta\rangle^2$, $\langle y_\beta\rangle^2$,
- $2Q_{x,y} - \Delta Q_{x,y}^{\mathrm{env,SC}}$: horizontal and vertical even envelope modes from $\sigma_{x,y}^2$,
- $Q_{x,y} - \Delta Q_{x,y}^{\mathrm{disp,SC}}$: horizontal and vertical coherent dispersion mode from $\left\langle x_\beta\,\delta\right\rangle$, $\left\langle y_\beta\,\delta\right\rangle$, and
- $|Q_x - Q_y|, Q_x + Q_y$: odd envelope or Chernin modes from σ_{xy}^2, $\sigma_x\sigma_y$.



(a) Spectra of dipolar channels: horizontal and vertical dipole moment tunes Q_x, Q_y appear as peaks in the signal of the respective plane.

(b) Spectrum of quadrupolar channel: dipolar tunes appear in first (Q_x, Q_y) and second harmonic $(2Q_x, 2Q_y)$ besides the two odd envelope modes $Q_x - Q_y$ and $Q_x + Q_y$.

Figure 2: Spectra of the dipole and quadrupole moments, recorded for a single shot at injection with amplitudes to scale.

Note that this association of the two even envelope modes with a respective transverse degree of freedom is only valid for vanishing coupling. For full coupling, i.e. isotropic focusing $Q_x = Q_y$, the two modes become simultaneous anti-phase and in-phase oscillations of both planes, often called "anti-symmetric" and "breathing" quadrupole modes.

In general, the second-order modes are affected by coherent tune shifts due to direct space charge defocusing (unlike the first-order modes). This is indicated by $\Delta Q_{x,y}^{\mathrm{env,SC}}$ for the even envelope modes and by $\Delta Q_{x,y}^{\mathrm{disp,SC}}$ for the coherent dispersion mode.

The coherent tune shift of the odd envelope modes does not significantly depend on space charge under typical conditions of synchrotrons, as discussed by Aslaninejad and Hofmann [11], which is why we do not consider a corresponding ΔQ^{SC} term above. The odd envelope tunes coherently shift mainly due to the emittance ratio, which is rather small for the typically round PS beams.

EXPERIMENTAL SETUP IN PS

The CERN PS is equipped with two skew quadrupole families. One can hence adjust the strength of linear coupling in the machine. Until this year, the PS has typically been operated with maximised linear coupling to mitigate the appearing horizontal head-tail instability, as the coupling shares Landau damping between the two planes – this effect significantly weakens the otherwise strong horizontal instability [12].

In order to freely scrutinise the modes in the quadrupolar spectrum in the CERN PS, we prepare a short bunch of relatively low intensity ($N = 5 \times 10^{11}$ ppb) and very low space charge ($\Delta Q_{x,y}^{KV} \approx 0.01$ due to blown up transverse emittances) in the upstream PS Booster machine. The low intensity weakens the horizontal head-tail instability and we can thus freely choose the coupling strength without the necessity of using the (new) transverse feedback system for stabilisation during the first part of the cycle. To start with, the skew quadrupoles are adjusted for maximised linear coupling. The two transverse betatron tunes have been adjusted to a larger distance than the usual PS setting (at around $Q_x \approx 6.24$ and $Q_y = 6.21$) to enable us to clearly distinguish between horizontal and vertical first-order $Q_{x,y}$ and second-order $2Q_{x,y}$ dipolar modes.

Figure 2 compares the spectra for the horizontal, vertical and quadrupolar channel of the strip-line pick-up for a single shot. Each spectrum is recorded over 1024 turns, starting right after the injection bump closure in the PS to avoid any related tune shifts thereof [13, Fig. 20]. While the two transverse dipolar spectra mainly only contain the respective betatron tune peaks at Q_x and Q_y, the quadrupolar spectrum exhibits several distinct modes. Besides the first-order dipolar modes $Q_{x,y}$ one observes the second-order dipolar modes $2Q_{x,y}$ and the two odd envelope modes at $|Q_x \pm Q_y|$.

The low space charge conditions entail vanishing coherent dispersion mode tune shifts $\Delta Q_{x,y}^{disp,SC} \to 0$. The dispersion mode tunes are thus inseparable from the dominating first-order dipolar mode tunes at $Q_{x,y}$ under the present conditions.

The "longitudinal δ terms" in Eq. (3) contribute to the very low frequency part of the quadrupolar spectrum. Indeed, Fig. 2b shows significant amplitude in a broad area to the left of the vertical dipolar tune peak, which is absent in the dipolar spectra in Fig. 2a.

In the following we confirm the nature of the beam modes in the quadrupolar spectrum as seen in Fig. 2b. To this end, we discuss the panels of Fig. 3 showing spectograms of the quadrupolar pick-up signal over about 1000 turns starting at injection.

DIPOLAR MODES

For this first part we aim to carve out only the dipolar modes in the spectrum while suppressing all other modes. To this end, we operate at minimised linear coupling to suppress the odd coupling modes.

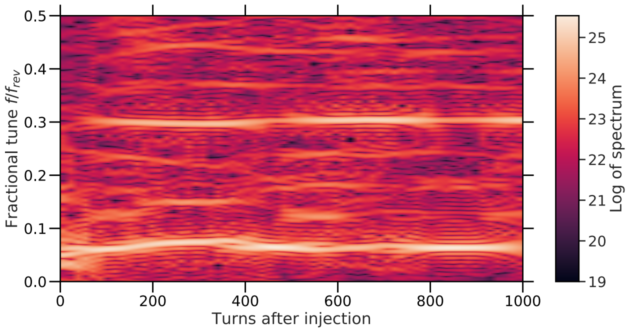
Figure 3a illustrates a typical optimised injection into the decoupled PS where dipolar injection mismatch has been minimised. One observes mainly the two transverse first-order dipolar modes at Q_x and Q_y (due to remaining small but finite dipole motion of the beam around the orbit). The first-order horizontal dipolar mode $\langle x_\beta \rangle$ oscillates at fractional horizontal tune $q_x = 0.3$ and the vertical dipolar mode $\langle y_\beta \rangle$ at fractional vertical tune $q_y = 0.06$. The second-order dipolar modes $\langle x_\beta \rangle^2$ and $\langle y_\beta \rangle^2$ at respective fractional tunes $1 - 2q_x = 0.4$ (mirrored at 0.5) and $2q_y = 0.12$ turn out to be very faint for the matched set-up, they almost vanish in the noise background.

By using a horizontal steering dipole magnet in the upstream transfer line for a horizontal dipolar injection mismatch, the beam performs significant dipole oscillations after injection for more than 1000 turns. Correspondingly, we can clearly observe both first-order $q_x = 0.3$ and second-order $1 - 2q_x = 0.4$ signals in Fig. 3b. At a closer look one can observe the PS injection bump closure leading to a shifting tune during the first 400 turns. The bump closure implied tune shift manifests as a downwards bent horizontal tune shift for the dipole mode, which correspondingly appears as an upwards bent shift in the mirrored second-order case. Applying the same mismatch only in the vertical plane yields the dominating first-order $q_y = 0.06$ and second-order $2q_y = 0.12$ signals in Fig. 3c – both bending upwards from the injection bump. In both transverse cases, the dipolar mismatch is also confirmed through large oscillation amplitudes in the usual beam position monitor (BPM) system of the PS.

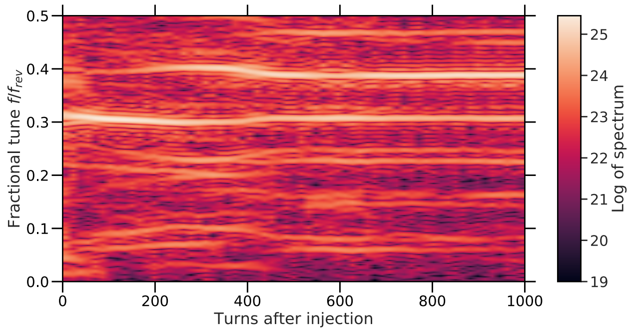
These dipolar signals are always present in the QPU spectrum with the first-order components $\langle x_\beta \rangle$ and $\langle y_\beta \rangle$ being the most dominant in the spectrum.

ODD ENVELOPE MODES

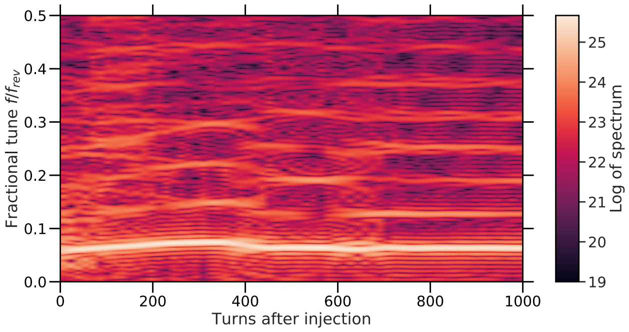
We return to the usual set-up of the PS with maximised linear coupling via the skew quadrupoles. Figure 3d shows a quadrupolar spectrogram with four distinct modes visible. They persist at least for several hundreds of turns. The two



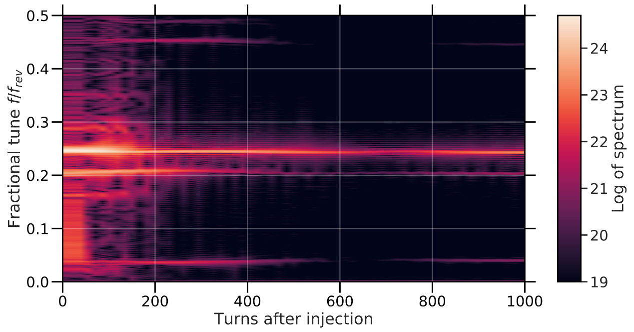
(a) Typical optimised injection with minimal mismatch, mainly the two dipolar fractional tunes q_x, q_y are visible.



(b) Intentional horizontal dipolar offset, the horizontal dipolar tune appears in first order q_x and mirrored second order $1 - 2q_x$.



(c) Intentional vertical dipolar offset, the vertical dipolar tune q_y dominates over the horizontal plane.



(d) Linear coupling mismatch via maximised skew component, the horizontal and vertical dipolar fractional tunes q_x, q_y appear together with the fainter odd envelope mode tunes at $q_x \pm q_y$.

Figure 3: Quadrupolar spectograms for various injection and lattice settings.

first-order dipolar modes $\langle x_\beta \rangle$ and $\langle y_\beta \rangle$ are again clearly visible at fractional tunes $q_x = 0.24$ and $q_y = 0.20$. Below we find the low-frequency odd envelope mode σ_{xy}^2 (corresponding to the difference resonance $|Q_x - Q_y|$) which oscillates at $q_x - q_y = 0.04$. On the other side we find the high-frequency odd envelope mode $\sigma_x \sigma_y$ to oscillate at $q_x + q_y = 0.44$, as expected for the sum resonance term. Since the transverse emittances are approximately equal, the coherent tune shift of the odd envelope modes vanishes [11].

Although the odd envelope mode tunes do not shift with space charge and can thus not provide a measurement thereof, they can still serve a useful purpose as they give access to measuring linear coupling: any injection mismatch with respect to linear coupling leads to amplitude in these modes. At the same time, the beams arrive mainly without transverse correlation from the upstream transfer line. Their oscillating tune can then readily be identified in the spectrum of a QPU along with the corresponding amplitude.

By adjusting the skew quadrupole families one can thus minimise the linear coupling experienced by the beam in a beam-based approach by simply minimising the amplitude of the two odd envelope modes. This suggested new approach turns out to be rather flexible and is not restricted to equal tunes – as opposed to the often employed "closest tune approach" [14], where one measures the tune distance $|C^-|$ between the two dipolar modes when setting equal tunes.

On the other hand, one can precisely measure coherent frequency shifts in the case of unequal transverse emittances since the odd modes persist for a sufficiently long time. For a horizontally 1.5 times larger emittance, the chart in Aslaninejad and Hofmann [11, Fig. 4] predicts a coherent tune shift on the order of $0.1\Delta Q_x^{KV}$. For a typical strong space charge beam at PS injection the incoherent KV tune shift amounts to $\Delta Q_x^{KV} \approx 0.1$. Thus the coherent tunes of the odd modes shift by $\Delta Q^{odd} \approx 0.01$, which should be well identifiable in future experiments and thus providate a means to measure the transverse emittance ratio.

COHERENT DISPERSION MODE

The impact of space charge on dispersion has first been discussed in the context of extended RMS envelope equations by Venturini and Reiser [15] and Lee and Okamoto [16]. To the author's knowledge, the first experimental evidence of the corresponding dispersion mode comes from the Q-BTF measurements at the CERN PS in Ref. [8]: the comparison between measurement and simulation identifies the peak shifted below $Q_x = 6.18$ as the coherent dispersion mode, as has been presented in Fig. 4 [8, Fig. 3].

The RF excitation with a quadrupolar kick in frequency sweep exhibits a dipolar feed-down component, coming from any finite residual offset of the beam centroid when passing through the kicker module (which has been minimised due to

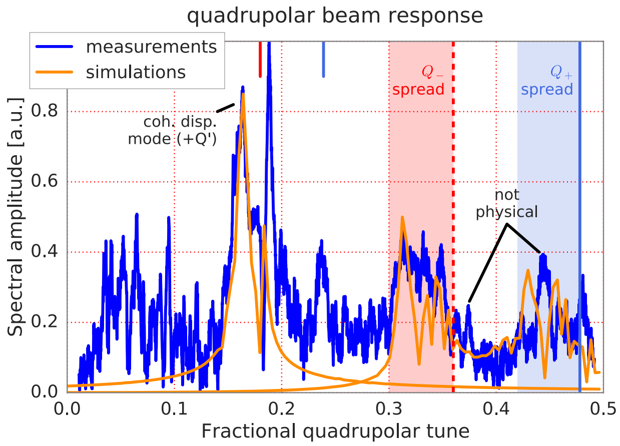


Figure 4: Comparison between measured beam frequency response and simulated eigenmodes, taken from [8, Fig. 3].

careful orbit adjustment to the centre of the kicker before the experiment). In the presence of finite chromaticity, dipolar excitation of the beam at the eigenfrequency of a head-tail mode of radial order k, i.e. at the kth synchrotron sideband of the dipolar tune $Q_{x,y} \pm k \cdot Q_s$, builds up amplitude in this head-tail mode. This approach has been successfully applied e.g. in Singh [17, Fig. 5.16], where dipolar RF excitation in frequency sweep mode individually excites the lowest order head-tail modes.

Non-rigid head-tail modes (for $k \neq 0$) correlate the longitudinal phase space with the transverse plane. In particular, the transverse amplitude exhibits nodes and maximum displacement depending on the longitudinal location z or, equivalently, the longitudinal momentum offset δ. This means nothing else than providing energy to the coherent dispersion mode $\langle x_\beta \, \delta \rangle$.

The dipolar feed-down of the quadrupolar excitation is thus the reason why (1.) the excited dipolar tunes become visible in the dipolar beam response which had been recorded along with the quadrupolar one [8, Fig. 1] and (2.) why also the dispersion modes become excited along with the even envelope bands.

As the PS injection plateau intrinsically exhibits the horizontal head-tail instability (for sufficient beam intensity) at natural chromaticity, the coherent horizontal dispersion mode can persist for a long time. This feature makes it a candidate for precise tune measurement. The space charge conditions for the above Q-BTF experiment were relatively low at $\Delta Q^{KV}_{x,y} \approx 0.02$ [8, Table 1] (about a factor 5 below the then operational LHC beams). While the even envelope mode bands displayed maximum tune shifts of around 0.06, the dispersion mode peak shifted only by about 0.01. In this weak space charge regime and at vanishing chromaticity (suppressing its corresponding widening effects on the even envelope mode band as described in Ref. [8]), the maximum tune shift of the even envelope mode can thus provide a good measurement of direct space charge in units of $\Delta Q^{KV}_{x,y}$.

When space charge becomes stronger (as for the LHC beams), the even envelope modes become shifted to much lower tunes: now the envelope band can easily overlap with the first-order dipolar mode peaks, which can complicate peak identification in the rather crowded fractional tune space. Under these circumstances it can become beneficial to measure space charge via its induced tune shift of the coherent dispersion mode: firstly, $\Delta Q^{disp,SC}$ shifts considerably less than the maximum even envelope mode tune $\Delta Q^{env,SC}$. Secondly, one measures a rather narrow peak as opposed to the broad band of even envelope modes implied by the longitudinally changing space charge conditions, which extend from weak shifts at the head and tail of the bunch to the maximum space charge tune shift at the peak line charge density. In the case of the Q-BTF measurement, the dispersion mode also featured a much better signal-to-noise ratio. These observations make the dispersion mode potentially much easier to identify compared to the even envelope modes, providing thus a viable alternative approach to measure direct space charge via the implied coherent tune shift $\Delta Q^{disp,SC}$.

CONCLUSION

The spectrum of the quadrupolar signal $S_{QPU} \propto \langle y^2 \rangle - \langle x^2 \rangle$, measured non-destructively in a four electrode pick-up, provides rich information on beam dynamics. Diagnostics can be extended to coherent second-order modes involving the transverse degrees of freedom x, y – beyond present tune measurement systems with BPMs measuring only the coherent first-order transverse modes at Q_x, Q_y. Literature on QPUs usually focusses on measurements based on the even envelope modes, also often simply called “quadrupole” modes.

The present work however scrutinises the other second-order modes typically found in the quadrupolar spectrum of a bunch, in particular due to mismatch after injection. Besides the first- and second-order dipolar modes, one encounters the odd envelope (or coherent skew) modes giving access to linear coupling and mismatch thereof. Further we have discussed the coherent dispersion modes, which measure correlation between transverse amplitude and longitudinal momentum. Energy in the coherent dispersion mode can be provided through excitation of a head-tail mode, e.g. by dipolar RF excitation at the corresponding synchrotron sideband of the coherent dipolar tune, or by a growing head-tail instability. The coherent dispersion mode can last much longer after injection than the even envelope mode (in particular for bunched beams). As the coherent dispersion mode also shifts with direct space charge, it makes for an alternative candidate to measure space charge besides the typical approach to quantify the even envelope mode shift due to space charge. This suggested new approach can be particularly beneficial under strong space charge conditions.

ACKNOWLEDGEMENTS

The author has the great pleasure to thank his CERN colleagues – in particular Simon Albright, Vincenzo Forte, Gerd Kotzian, Tom Levens and Haroon Rafique for support in the extensive measurement campaigns.

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