IMPACT OF COHERENT AND INCOHERENT BEAM-BEAM EFFECTS
ON THE BEAMS STABILITY

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Abstract

Various coherent instability mechanisms involving colliding beams are described together with techniques for their mitigation. They are illustrated with some examples based mostly on the recent experience at the Large Hadron Collider.

INTRODUCTION

The stability of the two beams in high energy particle colliders is heavily impacted by the electromagnetic forces that they exert on each other, the so-called beam-beam forces. The coherent oscillation of the two beams against each other may be driven unstable through a resonant mechanism or via an interaction with the machine impedance. Such instabilities are discussed in the next section. In other cases, the stabilising mechanism of single beam instabilities is jeopardised by the incoherent effects of beam-beam interactions. This occurs for example when the amplitude detuning driven by the beam-beam interaction is such that it compensates Landau damping for the head-tail instability. The description of these instability mechanisms involving coherent and incoherent beam-beam effects are briefly reviewed in the next two sections respectively. We then conclude with a summary of the corresponding mitigation techniques.

COHERENT BEAM-BEAM MODES

Resonant instability

The coupling of the two beams through the beam-beam interactions give rise to new modes of oscillation which may become unstable if their frequency matches a low order resonance driven by the lattice or by the beam-beam interactions themselves. In the simplest configuration of two symmetric beams colliding at a single interaction point, one finds two modes of oscillation corresponding to in and out of phase motion of the two beams at the Interaction Point (IP). The spectrum resulting from a self-consistent macro-particle simulation exhibiting these so-called σ- and π-modes is shown in Fig. 1. Their frequencies can be derived analytically [2], consequently it is rather straightforward to avoid resonant conditions. In more complex machines involving multiple bunches per beam, multiple IPs with asymmetric phase advances between them or even unequal revolution frequencies, the number of coherent beam-beam modes increases rapidly. Additionally, coherent synchro-betatron resonances may appear in colliders featuring collision with a significant synchro-betatron coupling due for example to a crossing angle between the beams or to the hourglass effect. Avoiding resonant conditions may become challenging and imposes constraints on the machine layout and the phase advances between IPs. The frequency of the coherent beam-beam modes and the resonant conditions can only be obtained analytically in some specific cases. Otherwise the rigid bunch model, the circulant matrix model [3, 4] or macro-particle simulations may be used to address more realistic machine configurations.

Such instabilities were a concern for asymmetric B factories which eventually were not constructed [5, 6]. Nowadays these instability mechanisms regained interest with proposals of asymmetric electron ion colliders [7–9] and high energy electron-positron colliders with crab waist [10]. In order to illustrate a mitigation technique with an example, Fig. 2 shows the instability prediction for a simplistic linearised model of the Electron-ion collider in China (EicC) [9]. The results for different ratios of the revolution

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Figure 1: Beam oscillation spectrum of two colliding beams experiencing two head-on beam-beam interactions at opposite azimuth in symmetric rings (top plot). The beams are round at the IP and the spectrum is obtained with self-consistent macro-particle simulations (COMBI [11]). The green curve represents a configuration with equal phase advances between the IPs, as illustrated on the bottom left plot. The purple curve correspond to a configuration where the phase advances were chosen such that the coherent mode frequencies are in the incoherent spectrum, at approximately 0.4 and 0.6 times the beam-beam tune shift. (Q1 = 1.31, Q1 = 0.405 and Q2 = 0.905).
frequency between the two beams shows the importance of a proper choice of machine layout for the mitigation of coherent beam-beam instabilities which may limit the intensity reach of the collider. Similarly other parameters such as the machines tune and the phase advance between IPs may be adjusted to mitigate further the instabilities.

**Mode coupling instability**

Even if resonant conditions are avoided, the coherent beam-beam modes may be driven unstable by the machine impedance, resulting in the mode coupling instability of colliding beams [3, 4, 13]. As for the transverse mode coupling instability (TMCI), this instability may be mitigated with chromaticity and/or an active feedback, yet they are not always sufficient to fully suppress it. Figure 3 shows an example of mode coupling instability involving the $\sigma$- and $\pi$-modes, but also higher order head-tail modes in the presence of synchro-betatron resonance driven by a beam-beam interaction with a crossing angle. The active feedback based on the average position of the bunch is effective to suppress the instability of the $\sigma$- and $\pi$-modes, but not the coupling of head-tail modes $+1$ and $+2$. An active feedback with intra-bunch capabilities would be needed to stabilise such modes. Alternatively to active feedbacks, Landau damping constitutes an efficient mitigation, it is further discussed after the following discussion on longitudinal instabilities.

**Longitudinal instabilities**

Most instabilities related to beam-beam interactions are in the transverse planes. Yet in the presence of a crossing angle, the beam-beam interaction leads to an energy change that depends on the longitudinal position of the particles [14]. We may expect that such a force generates longitudinal coherent
beam-beam modes. A correlation between the longitudinal oscillation of the two beams was observed in the LHC when the longitudinal emittance was let to shrink freely due to radiation damping, thus eventually losing longitudinal stability [15] (Fig. 4). Given the absence of limitation linked to this mechanism, the understanding of the so-called "Las Ketchup" instability remains limited.

**Intrinsic Landau damping**

Due to its non-linear nature, the beam-beam interactions have an impact on the amplitude detuning and consequently on Landau damping. For a head-on collision, the frequency of oscillation of the individual particles in the beam extends from the beam-beam tune shift to the machine bare tune [16], forming the so-called incoherent spectrum. Since the $\sigma$- and $\pi$-mode frequencies are outside of the incoherent spectrum, Landau damping is not expected to affect their stability [16]. If a resonant condition cannot be avoided or if the mode coupling instability cannot be fully mitigated, Landau damping of the coherent beam-beam modes may be restored with a proper choice of phase advance between the IPs. An example of such a mitigation is shown in Fig. 1, where the phase advances were adjusted such that the coherent modes are all inside the incoherent spectrum, based on prediction from the rigid bunch model implemented in BimBim. In the presence of synchro-betatron coupling, Landau damping from the synchrotron sidebands of the incoherent spectrum can be expected [16, 17]. Yet, currently, quantitative estimate of Landau damping in the presence of a given machine impedance is only obtained via macro-particle tracking simulations [18].

**LANDAU DAMPING OF THE HEAD-TAIL INSTABILITY**

When instabilities of coherent beam-beam modes have been effectively mitigated, the impact of the beam-beam interactions on the amplitude detuning may still affect Landau damping of classical single beam instabilities. In high energy hadron colliders such as the LHC, the head-tail instability driven mainly by the collimators impedance remain a concern through all the cycle. Therefore, the impact of long-range, offset or head-on beam-beam interactions on the amplitude detuning during the various operational phases such as the betatron squeeze, the collapse of the separation bump or even luminosity levelling requires a dedicated control.

**Beam-beam driven amplitude detuning**

Analytical expressions exist for the amplitude detuning generated by long-range and head-on beam-beam interactions [20, 21]. Based on those expressions, it is possible to estimate the impact of beam-beam interactions on Landau damping using the corresponding dispersion integral [22]. There exists configurations in which the beams collide with a small transverse offset between the beams (i.e., comparable to their r.m.s. transverse size) either transiently, e.g. when collapsing the separation bumps, or steadily, e.g. when levelling the luminosity or during Van der Meer scans. In such configurations, the estimation of the amplitude detuning is usually performed with single particle tracking codes. The tracking then serves as an input for the dispersion integral [23]. The amplitude detuning, and consequently the stability diagram, depends in a strongly non-linear manner on the offset between the beams in both transverse planes, the optical $\beta$ functions at the IP, the crossing and crab angles as well as on the beam intensity and emittances. It is therefore convenient to define a coherent stability factor that characterises the beam stability in order to ease the quantitative comparison between different configurations. We chose the highest ratio between the modulus of the...
complex tune shifts to their respective projections on the stability diagram. Consequently a stability factor larger than one indicates an unstable configuration. Figure 5 illustrates the complexity of this configuration with estimates for the HL-LHC for different octupole currents and crab angles. One observes that, for large separations between the beams at the IP (larger than 10σ, the r.m.s. beam size), the stability is dominated by the interplay between the arc octupole and the long-range interactions. In the convention chosen, the negative octupole current induces a negative direct detuning term which compensates the one of the long-range interactions, such that positive currents are favourable. The offset interaction at the two IPs further increases this difference for separations down to approximatively 6σ. For a lower separation, the additional spread from the offset interaction at the IPs increases significantly the stability diagram, except between approximatively 1.5 and 2.0 σ. This minimum of stability corresponds to the configuration when the particles oscillating with a low amplitude, i.e. the beam core, reaches the maximum of the beam-beam force which corresponds to a zero of the first order detuning term. In this configuration the stability is entirely determined by the higher order detuning terms. In the configuration considered here, we find that the negative octupole currents are favourable in absence of crab cavities, while the positive polarity remains favourable when the crossing angle is compensated with crab cavities.

At the LHC, instabilities observed while levelling the luminosity [23] and during Van der Meer scans [24] can be attributed to the loss of Landau damping with offset beams. They were mitigated by ensuring that the bunches colliding with an offset at one IP also collide head-on in another, thus restoring Landau damping. A proper choice of external detuning using dedicated non-linear magnets (e.g. octupoles) may also mitigate this instability, requiring a detailed understanding of the non-linear dynamics in the configuration considered. Such a control was demonstrated in a dedicated experiment at the LHC [25, 26]. In the same experiment, it was also shown that transient unstable configurations such as the one described in Fig. 5 can be acceptable if short enough with respect to the instability rise time, similarly to the crossing of transition in low energy machine.

The head-on beam-beam interaction is significantly more efficient than octupole magnets at providing Landau damping with a limited impact on the beam quality thanks to the large amplitude detuning generated for particles oscillating at a low amplitude which vanishes at high amplitude [23]. In some cases, the head-on beam-beam interaction can therefore become a mitigation of coherent instabilities. Future hadron collider projects feature a cycle with collision as early as possible in the cycle thus profiting from this stabilising force [27, 28]. Electron lenses mimicking the head-on beam-beam force were also proposed as an alternative to non-linear magnets to provide Landau damping [29].

**PACMAN linear coupling**

Most modern colliders feature several bunches which, in some cases, are non-uniformly distributed along the machine due to the need for long empty gaps for injection and extraction at the various steps of the injector chain as well as in the collider ring. As a result, different bunches may experience a different set of beam-beam interactions. Consequently, a given correction of a beam-beam driven effect with a global scheme cannot be made optimal for all bunches, it is the so-called PACMAN effect [30]. An important aspect for the beam stability is the presence of beam-beam interactions with a non-zero separation on a skew transverse plane leading to linear coupling, as the latter can significantly deteriorate Landau damping [31]. The impossibility to correct this contribution with a global scheme imposes constraints.

Figure 5: Coherent stability factor for different variations of the HL-LHC ultimate configuration [19] at the start of collision with offset beams in the two main IPs. The separation is in a different transverse plane in both IPs and varied simultaneously. The half-crossing angle φ is 250 μrad. The upper plot corresponds to a configuration without crab cavities and the lower plot to partial compensation of the crossing angle with crab cavities (φ_{CC} = 200 μrad).
Figure 6: Linear coupling measured with the AC-dipole method [32] at different stages of a cycle of the LHC with a fully filled machine (2700 bunches). The three bunches considered experience no beam-beam collision (blue), collision in the two main IPs (red) and all beam-beam interactions (green) as indicated by the legend. The flat top phase is characterised by weak beam-beam interactions, thus linear coupling is identical for all bunches. The squeeze of the $\beta$ function at the main IPs to 40 cm increases the strength of long-range interactions there, thus generating a difference in coupling between non-colliding and colliding bunches. This difference is further increased when squeezing down to 30 cm.

on the orbit control such that skew beam-beam interactions are avoided.

In the particular case of the LHC and HL-LHC, the crossing angles in all IPs are either horizontal or vertical by design. In reality various effects may result in skew beam-beam interactions. In some parts of the cycle the combination of crossing angle and parallel separation bumps can lead to skew long-range interactions. Additionally, the measurement of orbit misalignments within the common beam chamber is rather challenging, and roll angles in the order of $10^\circ$ may be expected [33]. Beam-beam induced orbit effects at other IPs may also add to the misalignment of the crossing angles with a PACMAN component [34]. The self-consistent code TRAIN [35] was recently updated to compute PACMAN linear coupling [34], showing results compatible with dedicated bunch-by-bunch measurement of linear coupling at the LHC (Fig. 6).

CONCLUSION

Already at the level of linear stability the coupling between additional degrees of freedom generated by the beam-beam interactions with respect to a single beam model generate a large variety of coherent resonances that have to be avoided. Mitigation of these instabilities usually involve the machine layout, the tunes in all degrees of freedom and the phase advance between IPs.

Mode coupling instabilities may occur when the coherent beam-beam modes interact with head-tail modes driven unstable by the machine impedance. They may be mitigated by a combination of the transverse feedback and chromaticity. Additionally, the machine tunes or the phase advance between IPs may be used to enhance Landau damping by controlling the mode frequencies with respect to the incoherent spectrum generated by the beam-beam interactions.

For both resonant and mode coupling instabilities, the rigid-bunch model and a 6D extension of it, the circulant matrix model, are mostly used for the understanding of linear stability. Analytical derivations of the impact of the non-linearities on the frequency of the coherent modes and their damping via Landau damping exist for some specific cases, but quantitative estimates in realistic configurations are mostly obtained with self-consistent macro-particle simulation.

Single beam instability mechanisms are affected by the amplitude detuning generated by the beam-beam interactions even if the proper measures are taken to fully suppress coherent beam-beam instabilities. The additional tune spread may be beneficial. The head-on interaction is particularly well suited to generate a large stability diagram thanks to its strong action on the beam core and vanishing for the beam tail. On the other hand in some configurations the amplitude detuning generated by the beam-beam interaction may compensate other sources and result in a loss of Landau damping. A detailed understanding of the non-linear dynamics including the machine non-linearities as well as the beam-beam interactions is required to ensure that the tune spread remains sufficient through the cycle. The mitigation of this mechanism for loss of Landau damping often requires a dedicated controlled source of detuning such as octupole magnets as well as operational procedures that avoids critical configurations and/or cross through the unstable configuration faster than the instability rise time.

REFERENCES


