Abstract

Instability thresholds are explored at the Large Hadron Collider (LHC) by means of the computation of the Landau Stability Diagram (SD). In the presence of diffusive mechanisms, caused by resonance excitations or noise, the SD can be reduced due to the modification of the particle distribution inside the beam. This effect can lead to a possible lack of Landau damping of the coherent modes previously damped by lying within the unperturbed SD area. The limitations derived from coherent instabilities in the LHC are crucial in view of future projects that aim to increase the performance of the LHC such as the High-Luminosity upgrade (HL-LHC) or Future Circular Collider (FCC). Simulation tools for the computation of the SD have been extended in order to take into account the incoherent effects from long particle tracking through the detailed model of the accelerator machine. The model includes among others beam-beam interactions and octupoles and the interplay between both is addressed. Finally the simulation results are compared to the Beam Transfer Function (BTF) measurements in the LHC.

INTRODUCTION

The beam coupling impedance drives the so-called head-tail instabilities [1] that are characterized by different modes of oscillations. Each mode is defined by a complex tune shift. The imaginary part is related with the rise time of the coherent instability and the real part with the coherent real tune shift. Coherent beam instabilities can be mitigated by chromaticity, transverse feedback and/or Landau damping. As long as the modes can be treated independently, the Landau damping is quantified by the dispersion integral for a given detuning \( \omega_{x,y}(J_x, J_y) \) and particle distribution \( \psi(J_x, J_y) \) as a function of the transverse actions \( J_x \) and \( J_y \) in each plane [2,3]:

\[
SD^{-1} = \frac{-1}{\Delta Q_{x,y}} = \int_0^\infty \int_0^\infty \frac{J_{x,y}}{\Omega - \omega_{x,y}(J_x, J_y)} d\psi dJ_x dJ_y,
\]

where \( \Delta Q_{x,y} \) are the complex tune shifts at the stability limit for each frequency \( \Omega \). The term \( \omega_{x,y}(J_x, J_y) \) is the amplitude detuning (tune spread) generated by any non-linearities, including the beam-beam (BB) interaction during collisions. The dispersion integral is the inverse of the SD that defines the stability limit in the complex plane of the coherent tune shifts. In order to be stabilized, the coherent impedance modes must lie inside the SD. In the LHC, the tune spread is mainly generated by the so-called Landau octupoles [3] and beam-beam interactions (long range and head-on) [4,5] when in collision.

In case of diffusive mechanisms and/or reduced dynamic aperture with particle losses or redistributions, the shape of the SD may be modified by the particle distribution changes. Hence it is important to characterize the impact of the particle distribution on Landau damping that may differ from the one evaluated for a Gaussian particle distribution. Models have been extended to include the particle distribution changes during long term tracking by using the SixTrack code [6], in the presence of a realistic lattice configuration and excited resonances. The detuning with amplitude \( \omega_{x,y}(J_x, J_y) \) in Eq. (1) is given by the tracking module of MAD-X [7,8] for different machine configurations and optics. In 2015 a Beam Transfer Function (BTF) system was installed in the LHC in order to measure the Landau damping of the proton beams. The BTF is proportional to the dispersion integral (Eq. (1)) and it is sensitive to particle distribution changes. It is a direct measurement of the SD [9]. The impact of incoherent effects on the SDs are presented together with BTF measurements for various machine configurations and beam-beam interactions.

IMPACT OF INCOHERENT EFFECTS ON STABILITY DIAGRAMS

The term \( d\psi/dJ_{x,y} \) in Eq. (1) is related to the particle distribution changes and may cause modifications of Landau damping w.r.t. the Gaussian distribution case. In order to take into account these effects, the models have been extended to compute the dispersion integral including the tracked particle distribution under the effects of a real lattice configuration by using the SixTrack code. At the first turn, a uniform distribution is generated, usually \( 10^6 \) particles, and tracked for \( 10^6 \) turns. The initial distribution is uniform between 0 to 6 \( \sigma \) (in units of the rms beam size) in both planes, corresponding to 0-18 \( J_{x,y} \) in terms of the (normalized) action variables. Before performing the integration, the re-distribution of the tracked particle distribution is weighted with a bi-dimensional exponential function. In the presence of strong machine non-linearities, particles may be subject to diffusion mechanisms due to excited resonances. SixTrack simulations have been performed for different octupole currents at the LHC injection energy (450 GeV) at collision tunes \( (Q_x \sim 64.31, Q_y \sim 59.32) \) for the normalized transverse beam emittance of \( \epsilon = 2.0 \) µm in both planes. Figure 1 shows the tracked particle distribution for an octupole cur-
recent $I_{\text{oct}} = 35$ A (Fig. 1a) together with the corresponding SD (Fig. 1b). The solid black line is the computed SD for the Gaussian distribution case. The blue and the red lines represent the computed SD from tracked particle distribution, in the horizontal and the vertical plane, respectively. The SD increases as a function of the octupole current due to the larger tune spread in the beams. No important effects are visible for low octupole strength, while particle distribution changes are observed for an octupole current of 35 A (Fig. 1a) due to the excited resonances. In this case, a cut in the horizontal SD is visible on the side of the negative coherent tune shifts (the blue line in Fig. 1b).

**BEAM TRANSFER FUNCTION MEASUREMENTS IN THE LHC**

The installation of the BTF system in the LHC was justified by the frequent coherent instabilities observed at flat top energy in the LHC despite the fact that predictions indicate sufficient Landau damping of the coherent impedance modes [5, 10, 11]. From dedicated measurements, the observed instabilities seemed to be caused by the loss of transverse Landau damping. Therefore, the transverse BTF system was installed in the LHC in 2015 in collaboration with the Beam Instrumentation team. During a BTF acquisition the chosen beam is safely excited, i.e. without causing losses or emittance blow up, at different betatron frequencies for a given number of turns. If the system is calibrated, the BTF gives a direct measurements of the SD, however, this is not the case for the LHC. Hence, in order to reconstruct the SD from BTF measurements, a fitting function is used [12]. The fitting function is parameterized as follows (assuming a reference case with linear detuning from the octupole magnets):

\[
\begin{align*}
\varphi(Q_{\text{meas}}) &= \varphi \left[ p_0 + p_1 \cdot (Q_{\text{model}} - Q_0) \right] \\
A(Q_{\text{meas}}) &= p_2 / p_1 \cdot A_{\text{model}}(Q_{\text{model}}).
\end{align*}
\]

The parameter $p_0$ gives the tune shifts w.r.t. the frequencies of the analytical detuning $(Q_{\text{model}} - Q_0)$ with $Q_0$ the model bare tune. The parameter $p_1$ gives the measured tune spread w.r.t. the expected one. The tune spread factor is evaluated from the measured phase and it does not depend on the calibration factor. The model is the dispersion integral computed by using the PySSD code [4] that includes the tune spread evaluated from the tracking module of MAD-X. The factor $p_2$ gives the missing calibration factor of the amplitude response w.r.t. the reference case $(A_{\text{model}})$. **Measurements with single beam**

Scans in octupole current have been performed at injection energy with collisions tunes $(Q_x \sim 64.31, Q_y \sim 59.32)$.
A linear increase of the detuning with amplitude is expected as a function of the octupole current. The BTFs were acquired on the single bunch of Beam 1 in both planes for a current of 0 A, 6.5 A, 13 A and 26 A. The BTFs are shown in Fig. 2. By increasing the octupole strength, the phase slope decreases while the amplitude width increases. For the largest octupole strength (26 A) the larger tune spread is observed in the horizontal plane (Fig. 2a) w.r.t. the vertical plane (Fig. 2b). This was not expected since the detuning with amplitude due to the octupole magnets is symmetric in the two planes as shown in Fig. 3 for the current of 26 A. Figure 4 shows the measured tune spread factor as a function of the octupole current. The tune spread factor is obtained by applying the fitting function (Eq. (2)) w.r.t. the analytical case for the octupole current of 6.5 A. The horizontal plane is represented by the blue line while the vertical plane by the green line. The black line represents the expectations. A larger tune spread is measured in both planes w.r.t expectations due to machine non-linearities at injection energy, consistently with optics measurements in 2015. The red shadow takes into account an error of ±20% for each case. As expected, the tune spread linearly increases as a function of the octupole current. In the vertical plane the linear behavior deviates for currents above 13 A, with a consequent reduction of the tune spread w.r.t. the horizontal plane. Particle losses were observed in the vertical plane related with the octupole current changes, as shown in Fig. 5, where Beam 1 losses are plotted at the primary collimators as a function of the octupole current (the dashed red line). In the presence of the transverse linear coupling, asymmetries in the tune spread are expected [13]. During the measurements the linear coupling was not corrected. The tune distribution in the presence of the linear coupling obtained by SixTrack tracking is shown in Fig. 6 where a cut in the vertical distribution is visible. The particle losses in the vertical plane due to a reduction of the dynamic aperture could explain the asymmetric behavior in the measured tune spread. The dynamic aperture of $\approx 3.5 \sigma$ was expected for a current of 26 A. In the presence of excited resonances and/or particle losses, increasing the tune spread inside the beams is not beneficial for Landau damping as demonstrated for the first time by BTF measurements in the LHC.

**Measurements with beam-beam interactions**

In order to measure the beam-beam long range contribution to Landau damping a crossing angle scan was performed at the end of the betatron squeeze. By reducing the crossing angle the separation of the beam-beam long range encounters reduces and their effects become stronger.

Measurements were acquired for positive octupole polarity, for which the tune spread from the long range beam-beam interactions adds up to the one given by the octupole magnets. Figure 7 shows the measured tune spread factor $p_1$ (Eq. (2)) as a function of the separation at the long range encounters w.r.t. the reference case of the nominal crossing angle ($370 \mu$rad). The black line represents expectations, and the blue line represents the measurements in the horizontal plane while the red line represents the measurements.
in the vertical plane. An unexpected behavior was found w.r.t. models: a larger tune spread was measured in the horizontal plane while a smaller one in the vertical plane except for the last measurement at 9σ separation for which a strong dependency on the working point was observed. The red star represents the measured tune spread in the vertical plane with a reduced tune of ∆Qy = −0.001. In this condition, BTF measurements show an important reduction of the tune spread as visible in Fig. 8. The blue line is the vertical BTF amplitude response at the beginning of the tune scan, the green line corresponds to the tune reduction of ∆Qy = −0.001 and the red line corresponds to the tune increase of ∆Qy = +0.001 to the recovered initial tune. In this last case, the BTF response is fully recovered, showing that it is a reproducible effect. The tune spread reduction was not expected from the models unless the transverse linear coupling is introduced in the simulations. In the presence of linear coupling, an overall reduction is expected in both planes but with a more significant reduction in the vertical plane, for which particles towards the diagonal are lost.

Figure 6: Two dimensional detuning up to 6σ particles obtained by SixTrack simulations for the octupole current of 26 A and in the presence of the linear coupling. The dashed red lines represent 3σ amplitude particles.

Figure 7: Measured tune spread factor as a function of the long range encounter separation at the IPs in units of the rms beam size. Measurements were acquired at the end of the betatron squeeze.

Figure 8: Vertical BTF response during the tune scan in the vertical plane. The blue line is the measured response at the beginning of the scan, the green line corresponds to the tune reduction of -0.001 and the red line are the measurements with the recovered initial tune value.

Figure 9: Two dimensional tune diagram (a) in the presence of the transverse linear coupling (2012 configuration) and corresponding tracked particle distribution (b). The color bar in the distribution plot represents the number of particles per bin.

with a consequent cut in the tune footprint. An example of this effect is shown in Fig. 9 where the tune footprints (Fig. 9a) have been evaluated for the 2012 LHC configuration. During the 2012 run several coherent instabilities were observed at the end of the betatron squeeze. The long range beam-beam contribution was important due to the high beam intensity (Np = 1.6 × 10^{11} p/bunch). The case without linear coupling corresponds to the blue color while the case with linear coupling corresponds to the red color. For this last case, as during BTF measurements, a negative
tune shift was also applied ($\Delta Q_y = -0.003$). The corresponding particle distribution obtained by SixTrack tracking is shown in Fig. 9b. The important deformation due to the linear coupling is visible in the tune footprint in red with a consequent reduction of the SD in the vertical plane as shown in Fig. 10 (the green line) where impedance modes are now at the edge of the stability. In the horizontal plane the increase of the SD is due to the clustering of the particle in the same direction that increases the contribution of the derivative $d\psi/dJ_{x,y}$ in the dispersion integral.

\[ \psi = \int Q \, d\psi \]

Figure 10: Stability diagrams at end of the betatron squeeze (2012 configuration) for positive octupole polarity and transverse linear coupling. The light blue line and the pink line represent the SD in the horizontal and vertical plane respectively with nominal tunes. The blue line and the green line represent the stability diagram in the horizontal and vertical plane, respectively, for a reduced tune in the vertical plane ($Q_y = 0.317$). The impedance coherent modes are included for a chromaticity of 2 units (red dots) and 10 units (blue dots).

**CONCLUSION**

The coherent stability not only depends on the tune spread in a beam but also on the particle distribution, as shown in Fig. 1 where the SD is deformed due to the particle distribution modification for a high octupole current of 35 A at injection energy (with collisions tunes). Beam Transfer Function measurements were acquired for different octupole strengths at injection energy. The results of these measurements showed the well reproducible tune spread in the horizontal plane and a much reduced tune spread in the vertical plane due to particle losses, as observed for the octupole currents above 13 A. The losses were due to the presence of the transverse linear coupling, that causes particle redistributions and reduces the dynamic aperture, expected to be less than 3.5 $\sigma$. For the first time it was demonstrated by BTF measurements in the LHC, that a larger tune spread is not beneficial for Landau damping if diffusive mechanisms and/or particle distribution changes are present due to a small dynamic aperture (less than 3.5 $\sigma$). Measurements acquired at the end of the betatron squeeze for different crossing angles showed the unexpected asymmetric behavior in terms of tune spread and tune shifts between the horizontal and vertical plane. This can be reproduced by including the transverse linear coupling in the models that induces particle losses and redistributions. In this case, both the tune footprint and the stability diagram are modified. The sharp cut is visible in SD in the vertical plane (Fig. 10) due to the deformed tune footprint in the same direction (the red color in Fig. 9) while in the horizontal plane the clustering of the particle in this direction contributes increasing the SD due to the increase of the derivative $d\psi/dJ_{x,y}$ in the dispersion integral (Eq. (1)).

REFERENCES


