IMPEDANCE THEORY AND MODELING

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Abstract

The "impedance" limits the performance of all the particle accelerators where the beam intensity (or beam brightness) is pushed, leading to beam instabilities and subsequent increased beam size and beam losses, and/or excessive (beam-induced RF) heating, which can deform or melt components or generate beam dumps. Each equipment of each accelerator has an impedance, which needs to be characterised and optimised. This impedance is usually estimated through theoretical analyses and/or numerical simulations before being measured through bench and/or beam-based measurements. Combining the impedances of all the equipment, a reliable impedance model of a machine can be built, which is a necessary step to be able to understand better the machine performance limitations, reduce the impedance of the main contributors and study the interplay with other mechanisms such as optics non-linearities, transverse damper, noise, space charge, electron cloud, beam-beam (in a collider), etc.

INTRODUCTION

As the beam intensity increases, the beam can no longer be considered as a collection of non-interacting single particles: in addition to the "single-particle phenomena", "collective effects" become significant [1-4]. At low intensity a beam of charged particles moves around an accelerator under the Lorentz force produced by the "external" electromagnetic fields (from the guiding and focusing magnets, RF cavities, etc.). However, the charged particles also interact with themselves (leading to space charge effects) and with their environment, inducing charges and currents in the surrounding structures, which create electromagnetic fields called wake fields. In the ultra-relativistic limit, causality dictates that there can be no electromagnetic field in front of the beam, which explains the term "wake". It is often useful to examine the frequency content of the wake field (a time domain quantity) by performing a Fourier transformation on it. This leads to the concept of impedance (a frequency domain quantity), which is a complex function of frequency.

If the wall of the beam pipe is perfectly conducting and smooth, a ring of negative charges (for positive charges travelling inside) is formed on the walls of the beam pipe where the electric field ends, and these induced charges travel at the same pace with the particles, creating the socalled image (or induced) current. But, if the wall of the beam pipe is not perfectly conducting or contains discontinuities, the movement of the induced charges will be slowed down, thus leaving electromagnetic fields (which are proportional to the beam intensity) mainly behind.

An ICFA mini-workshop on "Electromagnetic Wake

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Fields and Impedances in Particle Accelerators" was held in 2014 in Erice [5] to review the recent developments and main current challenges in this field. They concerned the computation, simulation and measurement of the (resistive) wall effect for cylindrical and non cylindrical structures, any number of layers, any frequency, any beam velocity and any material property (conductivity, permittivity and permeability); the electromagnetic characterization of materials; the effect of the finite length of a structure; the computation and simulation of geometrical impedances for any frequency; the computation and simulation (in time and frequency domains) of all the transverse impedances needed to correctly describe the beam dynamics (i.e. the usual driving or dipolar wake, the detuning or quadrupolar wake, the angular wake, the constant and nonlinear terms, etc.); the issue of the wake function needed (inverse Fourier transform of the impedance, response to a delta-function) vs. the wake potential obtained from electromagnetic codes (i.e. response to a usually Gaussian pulse); the simulation of all the complexity of equipment like kickers, collimators and diagnostics structures; etc.

This paper is structured as follows: the first section discusses some historical considerations, while the second one reviews some theoretical aspects. The third section analyses the numerical techniques and the fourth one the analytical computations. The fifth section examines in detail the particular and important case of the transverse (resistive) wall impedance in the presence of coatings (with a better or a worse conductor). Finally, the sixth section concludes this review.

HISTORICAL CONSIDERATIONS

The workshop discussed previously [5] was dedicated to A.M. Sessler, who passed away just before it on 17/04/2014 and who, together with V.G. Vaccaro, introduced the concept of impedance in particle accelerators.

The first mention of the impedance concept appeared on November 1966 in the CERN internal report "Longitudinal instability of a coasting beam above transition, due to the action of lumped discontinuities" by V.G. Vaccaro [6]. Then, a more general treatment of it appeared in February 1967 in the CERN yellow report "Longitudinal instabilities of azimuthally uniform beams in circular vacuum chambers of arbitrary electrical properties" by A.M. Sessler and V.G. Vaccaro [7]. The concept of wake field came two years later, in 1969, in the paper "The wake field of an oscillating particle in the presence of conducting plates with resistive terminations at both ends" by A.G. Ruggiero and V.G. Vaccaro [8]. This was the beginning of many studies, which took place over the last five decades, and today, impedances and wake fields continue to be an important field of activity, as concerns theory, simulation, bench and beam-based measurements.

SOME THEORETICAL ASPECTS

What needs to be computed are the wake fields at a distance z behind a source particle and their effects on the test or witness particles that compose the beam [1-3 and references therein. Additional information and references can be found there if not specified in this paper] (see Fig. 1). For a particle moving along a straight line with the speed of light, due to causality, the electromagnetic field scattered by a discontinuity on the beam pipe does not affect the charges, which travel ahead of it [1-4]. This field can only interact with the charges in the beam that are behind the particle, which generates the field. For short bunches, the time needed for the scattered fields to reach the beam on axis may not be negligible, and the interaction with this field may occur well downstream of the point where the field was generated. To find where the electromagnetic field produced by a leading charge reaches a trailing particle traveling at a distance Δs behind the leading one, let's assume that a discontinuity located on the surface of a pipe of radius b at coordinate s = 0 is passed by the leading particle at time t = 0 with the speed of light c (see Fig. 2). It can be deduced from Fig. 2 that

$$s^{2} = (ct)^{2} = (s - \Delta s)^{2} + b^{2}.$$
 (1)

Assuming that $\Delta s \ll b$, it can be shown from Eq. (1) that



Figure 1: Sketch of a vacuum chamber, which generates wake fields. Courtesy of G. Rumolo.



Figure 2: A wall discontinuity located at s = 0 scatters the electromagnetic field of a relativistic particle. When the

particle moves to location s, the scattered field arrives to point $s - \Delta s$. Courtesy of K. Bane and G. Stupakov [4].

The distance s given by Eq. (2) is called the catch-up distance. Only after the leading charge has traveled that far away from the discontinuity, a particle at point Δs behind it starts to feel the wake field generated by the discontinuity.

The computation of the wake fields is quite involved and two fundamental approximations are generally introduced: (i) the rigid-beam approximation (the beam traverses a piece of equipment rigidly, i.e. the wake field perturbation does not affect the motion of the beam during the traversal of the impedance. The distance of the test particle behind some source particle does not change) and (ii) the impulse approximation (as the test particle moves at a fixed velocity through a piece of equipment, the important quantity is the impulse, i.e. the integrated force, and not the force itself). Starting from the four Maxwell equations for a particle in the beam and taking the rotational of the impulse, it can be shown that for a constant relativistic velocity factor β (which does not need to be 1)

$$\vec{\nabla} \times \Delta \vec{p}(x, y, z) = 0, \qquad (3)$$

which is known as Panofsky-Wenzel theorem. This relation is very general, as no boundary conditions have been imposed. Only the two fundamental approximations have been made. Another important relation can be obtained when β is equal to 1 (taking the divergence of the impulse), which is

$$\dot{\nabla}_{\perp} \cdot \Delta \vec{p}_{\perp} = 0. \tag{4}$$

Considering the case of a cylindrically symmetric chamber (using the cylindrical coordinates r, θ , s) and as a source charge density (which can be decomposed in terms of multipole moments) a macro-particle of charge $Q = N_b e$ (with N_b the number of charges and e the elementary charge) moving along the pipe (in the sdirection) with an offset r = a in the $\theta = 0$ direction and with velocity $v = \beta c$, the whole solution can be written, for $\beta = 1$ (with q the charge of the test particle and L the length of the structure)

$$\upsilon \Delta p_{s}(r,\theta,z) = \int_{0}^{L} F_{s} ds = -q Q a^{m} r^{m} \cos m\theta \ W_{m}'(z),$$

$$\upsilon \Delta p_{r}(r,\theta,z) = \int_{0}^{L} F_{r} ds = -q Q a^{m} m r^{m-1} \cos m\theta \ W_{m}(z), \quad (5)$$

$$\upsilon \Delta p_{\theta}(r,\theta,z) = \int_{0}^{L} F_{\theta} ds = q Q a^{m} m r^{m-1} \sin m\theta \ W_{m}(z).$$

The function W_m is called the transverse (\perp) wake function and its derivative is called the longitudinal (//) wake function of azimuthal mode m. They describe the shock response (Green function) of the vacuum chamber environment to a δ -function beam which carries an *m*th moment. The integrals (on the left) are called wake potentials

(2)

(these are the convolutions of the wake functions with the beam distribution; here it is just a point charge). The Fourier transform of the wake function is called the impedance. As the conductivity, permittivity and permeability of a material depend in general on frequency, it is usually better (or easier) to treat the problem in the frequency domain, i.e. compute the impedance instead of the wake function. It is also easier to treat the case $\beta \neq 1$. Then, an inverse Fourier transform is applied to obtain the wake function in the time domain. Two important properties of impedances can be derived. The first is a consequence of the fact that the wake function is real, which leads to

$$\begin{bmatrix} Z_m^{\prime\prime}(\omega) \end{bmatrix}^* = Z_m^{\prime\prime}(-\omega),$$

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$$\begin{bmatrix} Z_m^{\perp}(\omega) \end{bmatrix}^* = Z_m^{\perp}(-\omega),$$
 (6)

where * stands for the complex conjugate and $\omega = 2 \pi f$ is the angular frequency. The second is a consequence of Panofsky-Wenzel theorem (with the wave number $k = \omega / \upsilon$)

$$Z_m^{\prime\prime}(\omega) = k \, Z_m^{\perp}(\omega). \tag{7}$$

Another interesting property of the impedances is the directional symmetry (Lorentz reciprocity theorem): the same impedance is obtained from both sides if the entrance and exit are the same. In the case of a cavity, an equivalent RLC circuit can be used (with three parameters which are the shunt impedance $R_{\rm sh}$, the inductance and the capacity). In a real cavity, these three parameters cannot be separated easily and some other related parameters are used, which can be measured directly such as the resonance frequency f_r , the quality factor Q (describing the width of the resonance) and the damping rate (of the wake). When the quality factor is low, the resonator impedance is called "broad-band", and this model (with Q = 1) was extensively used in the past in many analytical computations.

The situation is more involved in the case of non axisymmetric structures (due in particular to the presence of the quadrupolar wake field, see below) and for $\beta \neq 1$, as in this case some electromagnetic fields also appear in front of the source particle. In the case of axi-symmetric structures, a current density with some azimuthal Fourier component creates electromagnetic fields with the same azimuthal Fourier component. In the case of non axisymmetric structures, a current density with some azimuthal Fourier component may create an electromagnetic field with various different azimuthal Fourier components. If the source particle (1) and test particle (2) have the same charge, and in the ultra-relativistic case, the transverse wake potentials can be written (taking into account only the linear terms with respect to the source and test particles and neglecting the coupling terms)

$$\int_{0}^{L} F_{x} ds = -q^{2} \left[x_{1} W_{x}^{\text{driving}}(z) - x_{2} W^{\text{detuning}}(z) \right],$$

$$\int_{0}^{L} F_{y} ds = -q^{2} \left[y_{1} W_{y}^{\text{driving}}(z) + y_{2} W^{\text{detuning}}(z) \right],$$
(8)

where the driving term is used here instead of dipolar and detuning instead of quadrupolar (or incoherent) and where $x_{1,2}$ and $y_{1,2}$ are the horizontal and vertical coordinates of the source (1) and test (2) particles. In the frequency domain, Eq. (8) leads to the following generalized impedances

$$Z_{x}\left[\Omega\right] = x_{1} Z_{x}^{\text{driving}} - x_{2} Z^{\text{detuning}},$$

$$Z_{y}\left[\Omega\right] = y_{1} Z_{y}^{\text{driving}} + y_{2} Z^{\text{detuning}}.$$
(9)

Note that in the case $\beta \neq 1$, another quadrupolar term is also found. From Eqs. (8) and (9), the procedure to simulate or measure the driving and detuning contributions can be deduced. In the time domain, using some time-domain electromagnetic codes like for instance CST Particle Studio, the driving and detuning contributions can be disentangled. A first simulation with $x_2 = 0$ gives the driving part while a second one with $x_1 = 0$ provides the detuning part. It should be noted that if the simulation is done with $x_2 = x_1$, only the sum of the driving and detuning parts is obtained. The situation is more involved in the frequency domain, which is used for instance for impedance measurements on a bench. Two measurement techniques can be used to disentangle the transverse driving and detuning impedances, which are both important for the beam dynamics (this can also be simulated with codes like Ansoft-HFSS). The first uses two wires excited in opposite phase (to simulate a dipole), which yields the transverse driving impedance only. The second consists in measuring the longitudinal impedance, as a function of frequency, for different transverse offsets using a single displaced wire. The sum of the transverse driving and detuning impedances is then deduced applying the Panofsky-Wenzel theorem in the case of top/bottom and left/right symmetry. Subtracting finally the transverse driving impedance from the sum of the transverse driving and detuning impedances obtained from the one-wire measurement yields the detuning impedance only. If there is no top/bottom or left/right symmetry the situation is more involved and requires more measurements.

Finally, all the transverse impedances (dipolar or driving and quadrupolar or detuning) should be weighted by the betatron function at the location of the impedances, as this is what matters for the transverse beam dynamics.

NUMERICAL TECHNIQUES

Analytical computations are possible only if the structures are fairly simple. In practice this is often not the case and one has to rely on numerical techniques. First numerical wake field computations were performed in time domain by V. Balakin et al. in 1978 [9] and T. Weiland in 1980 [10]. As for highly relativistic bunches, due to causality, wake fields can catch up with trailing particles only after traveling the catch-up distance (see before), this motivated to compute wakes in linacs by using a mesh that moves together with the bunch: the moving mesh technique was introduced by K. Bane and T. Weiland in 1983 [11].

Nowadays many methods are available for beam coupling impedance simulations [3]:

Time Domain (TD) method,

- Frequency Domain (FD) method,
 - Eigenmode methods,

 \circ Methods based on beam excitation in FD, and the main ElectroMagnetic (EM) codes currently used are

- ABCI,
- Ansys HFSS,
- CST Studio (MAFIA),
- GdfidL,
- ECHO2D,
- ACE3P,
- Etc.

In TD, Finite Differences Time Domain (FDTD) and Finite Integration Technique (FIT) with leapfrog algorithm for the time stepping are used. More specialized techniques are the Boundary Element Method (TD-BEM), the Finite Volume method (FVTD), the Discontinuous Galerkin Finite Element Method (DG-FEM) or Implicit methods. The bunch length and the wake length are the two important parameters for TD impedance computations and the criterion for the time step is also referred to as the Courant-Friedrichs-Lewy (CFL) criterion. The TD simulations are suitable at medium and high frequencies, and particularly in perfectly conducting structures.

In FD, Eigenmode methods are used when high quality factor structures are under investigation and high accuracy is required.

The methods based on beam excitation in FD are well suited at low frequencies, where the CFL criterion poses a strong requirement on the time step. Due to the uncertainty principle, lower frequencies require computing longer wakes. As the time step is fixed by structure properties via the CFL criterion, this leads to the necessity to compute many time steps. The FD methods prevail for low frequency, low velocity of the beam and dispersively lossy materials.

The particularly difficult components to simulate are those, which combine elements of geometric wake fields and resistive elements (like tapered collimators or dielectric structures), the surface roughness and small random pumping slots (such as e.g. in Fig. 3).

Finally, the EM properties of some materials (vs. frequency) are not well known and should be measured with precision before performing simulations to allow for reliable results.

ANALYTICAL COMPUTATIONS

Analytical computations are usually used to compute the (resistive) wall impedance of multi-layered vacuum chambers, beam screens and collimators over a huge frequency range, as they are usually much faster and precise than simulation codes, which are facing several issues depending on the frequency range (number of mesh cells, etc.). Three formalisms are usually adopted:

- Transmission-line,
- Field matching,
- Mode matching.



Figure 3: Sketch of the LHC beam screen.

For a cylindrical beam pipe or two parallel plates, with any number of layers, any beam velocity and any electric conductivity, permeability and permittivity, the IW2D code was derived using field matching [12], which is valid when the length of the structure is (much) larger than the beam pipe radius (or half gap in the case of two parallel plates) [3].

In the LHC, the wall impedance of the numerous collimators (see Fig. 4) represents a significant fraction of the total machine impedance. The important parameters are the beam pipe radius (or half gap in the case of two parallel plates), the thickness of the different layers and the skin depth, which is plotted vs. frequency in Fig. 5 for three different materials. Assuming for simplicity first the case of a (round) LHC collimator, the transverse wall impedance is represented in Fig. 6, exhibiting three regimes of frequencies:

- Low-frequency or "inductive bypass" regime,
- Intermediate-frequency or "classical thickwall" regime,
- High-frequency regime.

Before discussing in detail the first two regimes, which are of interest for the LHC, let's have a closer look at the third (high-frequency) regime, which is zoomed in Fig. 7. A resonance is clearly revealed (the formula giving the resonance frequency is shown in Fig. 7, where Z_0 is the free-space impedance, σ_{DC} is the DC conductivity and τ is the relaxation time.), whose physical interpretation was provided by K. Bane [13]. The beam/wall interaction can be thought of occurring in two parts: first the beam loses energy to the high frequency resonator and then, on a longer scale, this energy is absorbed by the walls.



Figure 4: The numerous collimators of the LHC, whose distance to the beam is of few mm. Courtesy of S. Redaelli.



Figure 5: Skin depth versus frequency for different resistivities: stainless steel, graphite and copper (at room and cryogenic temperatures).



Figure 6: Transverse wall impedance for the case of a (round) LHC collimator.



Figure 7: Zoom of the third (high-frequency) regime of Fig. 6: real (in brown) and imaginary (in green) parts of the transverse impedance.

In the case of resistive elliptical beam pipes, Yokoya form factors [14] for the dipolar and quadrupolar impedances are usually used (see Fig. 8): these form factors are numbers to be multiplied to the results obtained with the circular geometry (with the height of the elliptical beam pipe h equal to the radius b of the circular beam pipe). However, it should be reminded that several assumptions were made as concerns both the frequency and the material. Generalised form factors have been deduced from IW2D for two parallel plates compared to the circular case and two examples are shown in Fig. 9 [15].



Figure 8: Yokoya form factors for dipolar and quadrupolar impedances in resistive elliptical pipes (compared to the circular case) [14]. The height of the elliptical beam pipe is h and the width is w (the case where w = h corresponds to the circular case).



Figure 9: Generalised form factors (compared to the circular case) from the IW2D code (mentioned as "this theory") for the case of graphite (upper) and hBN ceramic (lower) [15].

TRANSVERSE WALL IMPEDANCE AND COATINGS

Assuming a round beam pipe with a length of 1 m with only one layer going to infinity, the first two frequency regimes are depicted in Fig. 10 for two different beam pipe radii and conductivities.



Figure 10: Transverse wall impedance assuming a round beam pipe with a length of 1 m and only one layer going to infinity.

The effect of a copper coating inside a round beam pipe with a length of 1 m, one layer of stainless steel going to infinity and a radius of 20 mm, is represented in Fig. 11 [16,17]. It is shown that the imaginary part of the impedance is always reduced while a too high thickness of the coating can considerably increase the real part of the impedance at low frequencies (as the better conductor keeps the induced current closer to the beam).



Figure 11: Effect of a copper coating (at room temperature) inside a round beam pipe with a length of 1 m, one layer of stainless steel going to infinity and a radius of 20 mm. Ratio of the impedance (real and imaginary parts) with the coating of thickness (from top to bottom) 1 μ m and 1000 μ m to the impedance without the coating.

The effect of a copper coating inside a round beam pipe with a length of 1 m, one layer of graphite going to infinity and a radius of 2 mm, is represented in Fig. 12 [16,17]. As before (but with an effect, which is amplified), it is shown that the imaginary part of the impedance is always reduced while a too high thickness of the coating can considerably increase the real part of the impedance at low frequencies (as the better conductor keeps the induced current closer to the beam).



Figure 12: Effect of a copper coating (at room temperature) inside a round beam pipe with a length of 1 m, one layer of graphite going to infinity and a radius of 2 mm. Ratio of the impedance (real and imaginary parts) with the coating of thickness (from top to bottom) 1 μ m and 50 μ m to the impedance without the coating.

The effect of a graphite coating (e.g. as could be used to reduce the secondary emission yield and relevant electron cloud effects) inside a round beam pipe with a length of 1 m, one layer of copper (at room temperature) going to infinity and a radius of 20 mm, is represented in Fig. 13 [16,17]. In this case, it is shown that if the coating thickness is sufficiently small, the real part of the impedance does not change but only the imaginary part increases at high frequency. For larger coating thicknesses, even the real part of the impedance will be significantly higher for high frequencies, which could have detrimental effects for beam stability.



Figure 13: Effect of a graphite coating inside a round beam pipe with a length of 1 m, one layer of copper (at room temperature) going to infinity and a radius of 20 mm. Ratio of the impedance (real and imaginary parts) with the coating of thickness (from top to bottom) 1 μ m and 50 um to the impedance without the coating.

CONCLUSION

The first mention of the impedance concept appeared on November 1966 in the CERN internal report "Longitudinal instability of a coasting beam above transition, due to the action of lumped discontinuities" by V.G. Vaccaro [6]. This was the beginning of many studies, which took place over the last five decades, and today, impedances and wake fields continue to be an important field of activity, as concerns theory, simulation, bench and beam-based measurements. Furthermore, several extensions of the impedance concept appeared over the years for space charge, electron cloud and CSR (Coherent Synchrotron Radiation).

Even if the impedance is fifty-one years old, most of the particle accelerators do not have a sufficiently precise impedance model and there are still challenges for the future, such as e.g. with the new surface treatments, which need to be implemented to fight against electron cloud.

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