

# Contents

<b>Abstract</b>	<b>v</b>
<b>Editors' note</b>	<b>vii</b>
<b>Executive summary</b>	<b>1</b>
<b>A Introduction and overview</b>	
<i>A. Blondel, J. Gluza, S. Jadach, P. Janot, T. Riemann</i>	<b>3</b>
1 The FCC-ee electroweak factory . . . . .	3
2 What this theory report brings: an overview . . . . .	7
<b>B Precision calculations in the Standard Model</b>	<b>9</b>
1 $\alpha_{\text{QED, eff}}(s)$ for precision physics at the FCC-ee/ILC <i>F. Jegerlehner</i> . . . . .	9
1.1 $\alpha(M_Z^2)$ in precision physics (precision physics limitations) . . . . .	9
1.2 The ultimate motivation for high-precision SM parameters . . . . .	13
1.3 $R$ data evaluation of $\alpha(M_Z^2)$ . . . . .	14
1.4 Reducing uncertainties via the Euclidean split trick: Adler function controlled pQCD . . . . .	19
1.5 Prospects for future improvements . . . . .	23
1.6 The need for a space-like effective $\alpha(t)$ . . . . .	26
1.7 Conclusions . . . . .	28
1.8 Addendum: the coupling $\alpha_2$ , $M_W$ , and $\sin^2 \Theta_f$ . . . . .	28
2 Precision quantum chromodynamics <i>D. d'Enterria</i> . . . . .	38
2.1 Higher fixed-order pQCD corrections . . . . .	39
2.2 Higher-order logarithmic resummations . . . . .	40
2.3 Per-mille-precision $\alpha_s$ extraction . . . . .	42
2.4 High-precision non-perturbative QCD . . . . .	45
3 Inclusion of mixed QCD–QED resummation effects at higher orders <i>G.F.R. Sborlini</i> . . . . .	51
3.1 Introduction and motivation . . . . .	51
3.2 Splittings and PDF evolution . . . . .	51
3.3 Fixed-order effects: application to diphoton production . . . . .	52
3.4 Mixed resummation effects: Z boson production . . . . .	53
3.5 Conclusions . . . . .	54
4 CoLoRFulNNLO at work: a determination of $\alpha_S$ <i>A. Kardos, S. Kluth, G. Somogyi, Z. Trócsányi, Z. Tulipánt, A. Verbytskyi</i> . . .	57
4.1 Introduction . . . . .	57

4.2	Precision through higher orders . . . . .	58
4.3	Precision through small power corrections . . . . .	60
4.4	Conclusions . . . . .	62
5	Theoretical luminosity precision for the FCC-ee: overview of the path to 0.01% <i>B.F.L. Ward, S. Jadach, W. Płaczek, M. Skrzypek, S.A. Yost</i> . . . . .	65
6	$e^+e^- \rightarrow \gamma\gamma$ at large angles for FCC-ee luminometry <i>C.M. Carloni, M. Chiesa, G. Montagna, O. Nicrosini, F. Piccinini</i> . . . . .	71
6.1	Introduction . . . . .	71
6.2	Theoretical approach and numerical results . . . . .	72
6.3	Summary and outlook . . . . .	73
7	Prospects for higher-order corrections to W pair production near threshold in the EFT approach <i>C. Schwinn</i> . . . . .	77
7.1	Effective theory approach to W pair production . . . . .	78
7.2	Estimate of NNLO <sup>EFT</sup> corrections and beyond . . . . .	83
7.3	Summary and outlook . . . . .	86
8	Perspectives of heavy quarkonium production at the FCC-ee <i>Z.-G. He, B.A. Kniehl</i> . . . . .	89
8.1	Heavy quarkonium production through $e^+e^-$ annihilation . . . . .	90
8.2	Heavy quarkonium production in $\gamma\gamma$ collisions . . . . .	91
8.3	Summary and outlook . . . . .	94
9	Vertex functions in QCD—preparation for beyond two loops <i>J.A. Gracey</i> . . . . .	97
9.1	Introduction . . . . .	97
9.2	Current status . . . . .	98
9.3	Three-loop strategy . . . . .	101
9.4	Discussion . . . . .	103
10	Effective field theory approach to QED corrections in flavour physics <i>M. Beneke, C. Bobeth, R. Szafron</i> . . . . .	107
10.1	Introduction and motivation . . . . .	107
10.2	QED corrections in $B_q \rightarrow \ell^+\ell^-$ . . . . .	108
10.3	Summary and outlook . . . . .	113
11	Top pair production and mass determination <i>A. Maier</i> . . . . .	117
11.1	Introduction . . . . .	117
11.2	Effective theory framework . . . . .	117
11.3	Higher-order corrections . . . . .	117
11.4	Cross-section predictions . . . . .	119
12	Higgs boson decays: theoretical status <i>M. Spira</i> . . . . .	123

12.1	Introduction . . . . .	123
12.2	SM Higgs boson decays . . . . .	123
12.3	Uncertainties . . . . .	126
<b>C</b>	<b>Methods and tools</b>	<b>135</b>
1	Heritage projects, preservation, and re-usability concerns <i>S. Banerjee, M. Chrzaszcz, Z. Was, J. Zaremba</i> . . . . .	135
1.1	Common tools for all FCC design studies . . . . .	136
2	Scalar one-loop Feynman integrals in arbitrary space–time dimension $d$ – an update <i>T. Riemann, J. Usovitsch</i> . . . . .	139
2.1	Introduction . . . . .	139
2.2	Interests in the $d$ -dependence of one-loop Feynman integrals . . . . .	140
2.3	Mellin–Barnes representations for one-loop Feynman integrals . . . . .	143
2.4	The basic scalar one-loop functions . . . . .	149
2.5	The cases of vanishing Cayley determinant $\lambda_n = 0$ and of vanishing Gram determinant $G_n = 0$ . . . . .	154
2.6	A massive four-point function with vanishing Gram determinant . . . . .	154
2.7	Calculation of Gauss hypergeometric function ${}_2F_1$ , Appell function $F_1$ , and Saran function $F_S$ at arbitrary kinematics . . . . .	155
3	NNLO corrections in four dimensions <i>R. Pittau</i> . . . . .	163
3.1	Introduction . . . . .	163
3.2	FDR integration and loop integrals . . . . .	163
3.3	Keeping unitarity in the virtual component . . . . .	165
3.4	Keeping unitarity in the real component . . . . .	167
3.5	Results and conclusions . . . . .	167
4	Unsubtractions at NNLO <i>J.J. Aguilera-Verdugo, F. Driencourt-Mangin, J. Plenter, S. Ramírez-Uribe, G. Rodrigo, G.F.R. Sborlini, W.J. Torres Bobadilla, S. Tracz</i> . . . . .	169
4.1	Introduction . . . . .	169
4.2	Loop-tree duality . . . . .	170
4.3	Four-dimensional unsubtraction . . . . .	171
4.4	Unitarity thresholds and anomalous thresholds . . . . .	172
4.5	Conclusions . . . . .	173
5	Numerics for elliptic Feynman integrals <i>C. Bogner, I. Hönenmann, K. Tempest, A. Schweitzer, S. Weinzierl</i> . . . . .	177
6	Numerical multiloop calculations: sector decomposition and QMC integration in pySECDEC, <i>S. Borowka, G. Heinrich, S. Jahn, S.P. Jones, M. Kerner, J. Schlenk</i> . . . . .	185
6.1	Feynman integrals and sector decomposition . . . . .	185
6.2	Quasi-Monte Carlo integration . . . . .	187

6.3	Summary and outlook . . . . .	188
7	Analytics from numerics: five-point QCD amplitudes at two loops <i>S. Abreu, J. Dormans, F. Febres Cordero, H. Ita, B. Page</i> . . . . .	193
7.1	Introduction . . . . .	193
7.2	Amplitudes . . . . .	194
7.3	Simplifications for functional reconstruction . . . . .	195
7.4	Implementation and results . . . . .	197
7.5	Conclusion . . . . .	198
8	Recent developments in Kira <i>P. Maierhöfer, J. Usovitsch</i> . . . . .	201
8.1	Introduction . . . . .	201
8.2	Improved symmetrization . . . . .	201
8.3	Parallel simplification algorithms for coefficients . . . . .	201
8.4	Basis choice . . . . .	203
8.5	Conclusions . . . . .	203
9	Precision Monte Carlo simulations with WHIZARD <i>S. Braß, W. Kilian, T. Ohl, J. Reuter, V. Rothe, P. Stienemeier</i> . . . . .	205
10	FCC tau polarisation <i>S. Banerjee, Z. Was</i> . . . . .	211
11	Electron–positron annihilation processes in MCSANCee <i>A. Arbuzov, S. Bondarenko, Y. Dydyyshka, L. Kalinovskaya, L. Rumyantsev, R. Sadykov, V. Yermolchyk</i> . . . . .	213
11.1	Introduction . . . . .	213
11.2	Cross-section structure . . . . .	213
11.3	Numerical results and comparison . . . . .	214
11.4	Conclusion . . . . .	214
12	Global electroweak fit in the FCC-ee era <i>J. Erler, M. Schott</i> . . . . .	217
<b>D</b>	<b>SMEFT</b>	<b>221</b>
1	CoDEX: BSM physics being realised as SMEFT <i>S.D. Bakshi, J. Chakrabortty, S.K. Patra</i> . . . . .	221
1.1	Introduction . . . . .	221
1.2	The package in detail . . . . .	222
<b>E</b>	<b>Beyond the Standard Model (BSM)</b>	<b>231</b>
1	(Triple) Higgs coupling imprints at future lepton colliders <i>J. Baglio, C. Weiland</i> . . . . .	231
1.1	Triple Higgs coupling studies in an EFT framework . . . . .	231
1.2	Probing heavy neutral leptons via Higgs couplings . . . . .	232
1.3	Conclusions . . . . .	236

2	Exotic Higgs decays (and long-lived particles) at future colliders <i>J.F. Zurita</i>	241
2.1	Exotic Higgs decays: motivations and signatures	241
2.2	Long-lived particles (LLPs)	241
2.3	Exotic Higgs decays vis-à-vis current LHC data	242
2.4	Future experiments: HL-LHC, FCC, CEPC, LHeC	243
2.5	Conclusions	244
3	Precision predictions for Higgs decays in the (N)MSSM <i>F. Domingo, S. Heinemeyer, S. Paßehr, G. Weiglein</i>	247
3.1	Introduction	247
3.2	Higgs decays to SM particles in the $\mathcal{CP}$ -violating NMSSM	248
3.3	Discussion concerning the remaining theoretical uncertainties	258
	<b>Acknowledgements</b>	<b>267</b>