Executive summary

The main theoretical issues of the FCC-ee studies discussed in this report may be summarised as follows.

1. To adjust the precision of theory predictions to the experimental demands from the FCC-ee, an update of existing software and the development of new, independent software will be needed. This should include, in the first instance, solutions to the following issues:
   (a) factorisation to infinite order of multiphoton soft-virtual QED contributions;
   (b) resummations in Monte Carlo generators;
   (c) disentangling of QED and EW corrections beyond one loop, with soft-photon factorisation or resummation;
   (d) proper implementation of higher-loop effects, such as Laurent series around the Z peak;
   (e) further progress in methods and tools for multiloop calculations and Monte Carlo generators.

   Some discussions have been initiated in the 2018 report [1]; here, they are extended in the Introduction and Chapters B and C.

2. To meet the experimental precision of the FCC-ee Tera-Z for electroweak precision observables (EWPOs), even three-loop EW calculations of the $\mathrm{Zf\bar{f}}$ vertex will be needed, comprising the loop orders $\mathcal{O}(\alpha\alpha_{\mathrm{s}}^2), \mathcal{O}(N_{\mathrm{f}}\alpha^2\alpha_{\mathrm{s}}), \mathcal{O}(N_{\mathrm{f}}^2\alpha^3)$, and also the corresponding QCD four-loop terms. This was mainly a subject of the 2018 report [1].

3. To decrease the α_{QED} uncertainty by a factor of five to ten, to the level $(3\text{–}5) \times 10^{-5}$, will require improvements in low-energy experiments. Alongside this, the perturbative QCD (pQCD) prediction of the Adler function must be improved by a factor of two, accomplished with better uncertainty estimates for m_{c} and m_{b}. The next mandatory improvements required are:
   (a) four-loop massive pQCD calculation of the Adler function;
   (b) improved α_{s} in the low Q^2 region above the τ mass;
   (c) a better control and understanding of $\Delta\alpha_{\mathrm{had}}^{(5)}(M_{\mathrm{Z}}^2)$, in terms of R data;
   (d) different methods for directly accessing $\alpha(M_{\mathrm{Z}}^2)$, e.g., the muon forward–backward asymmetry, or for calculating α_{QED}, either based on a radiative return experiment, e.g., at the FCC-ee Tera-Z, or using lattice QCD methods.

   This is discussed in Chapter B.

4. FCC-ee precision measurements require many improvements on the theoretical QCD side. These include: (i) higher-order pQCD fixed-order calculations; (ii) higher-order logarithmic resummations; (iii) per-mille-precision extractions of the α_{s} coupling; and (iv) an accurate control of non-perturbative QCD effects (such as, e.g., colour reconnection, hadronization), both analytically and as implemented in the Monte Carlo generators. These issues are discussed in Chapter B.

5. The reduction of the theoretical uncertainty of the total W pair production cross-section to the level of $\sim 0.01\%$ at the FCC-ee-W requires at least the calculation of $\mathcal{O}(\alpha^2)$ and dominant $\mathcal{O}(\alpha^3)$ corrections to double-resonant diagrams. Estimates within an effective field theory (EFT) approach show that the theory-induced systematic uncertainty of the mass measurement from a threshold scan can be at the level of $\Delta M_{\rm W} = (0.15-0.60)$ MeV. The lower value results from assuming that the non-resonant corrections are under control. In addition, it is also essential to reduce the uncertainty from initial-state radiation (ISR) corrections and QCD corrections for hadronic final states to the required accuracy. This is discussed in Chapter B.

6. Predictions for H decay widths and branching ratios are known with sufficient accuracy for the LHC. At the FCC-ee, the Higgs mass can be measured with a precision below 0.05 GeV. The dependence of EWPOs on $M_{\rm H}$ is mild, $\propto \alpha \log(M_{\rm H}/M_{\rm W})$, and an accuracy of 0.05 GeV of $M_{\rm H}$ will not affect their determination. The main improvements in Higgs boson studies will be connected with a better determination of branching ratios and self-couplings. More on related issues is discussed in the Introduction and in Chapter B.

7. The top pair line shape for centre-of-mass energies close to the $\rm t\bar{t}$ production threshold is highly sensitive to the mass of the top quark, allowing its determination with unprecedented precision. The statistical uncertainty of the measurement ($\sim$ 20 MeV) is projected to be significantly less than the current theoretical uncertainty. It is crucial to continuously improve the theoretical prediction. The most sensitive observable is the total production cross-section for $\rm b\bar{b}W^+W^-X$ final states near the top pair production threshold. A very precise knowledge of the strong coupling constant from other sources will be crucial in order to meaningfully constrain the top Yukawa coupling. These issues are discussed in Chapter B.

8. Proper truncation of the ultraviolet scale Λ depends on the experimental precision of the observables and Standard Model effective field theories (SMEFTs) must be adjusted to FCC-ee experimental conditions, e.g., in construction of appropriate complete operator bases and Wilson coefficients (WCs) for Beyond the Standard Model (BSM) theories. This issue is discussed in Chapter D.

9. The FCC-ee and the FCC-hh will both be sensitive to BSM physics and exotic massive states reaching tens of TeV or very weak couplings. It is proposed to use the SMEFT framework and constrain the Higgs triple coupling by analysing precision measurements. For these studies, but also exotic Higgs decays, it will be important to combine the LHC and HL-LHC data with an analysis at the FCC-ee.
   These issues are discussed in Chapter E.

Reference

[1] A. Blondel *et al.*, Standard Model theory for the FCC-ee Tera-Z stage, CERN (CERN Yellow Rep. Monogr. 3, Geneva, Switzerland),
`arXiv:1809.01830`, `doi:10.23731/CYRM-2019-003`