2 Precision quantum chromodynamics

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The unprecedentedly small experimental uncertainties expected in the electron–positron measurements at the FCC-ee, key to searches for physics beyond the SM up to $\Lambda \approx 50\,\mathrm{TeV}$, impose precise calculations for the corresponding theoretical observables. At the level of theoretical precision required to match that of the FCC-ee experimental measurements, the current relevant QCD uncertainties have to be reduced at at least four different levels.

1. Purely theoretical perturbative uncertainties from missing higher-order (HO) corrections in perturbative QCD (pQCD) calculations of $\mathrm{e}^+\mathrm{e}^-$ scattering amplitudes and decay processes involving multiple real emissions or virtual exchanges of quarks and gluons. Such fixed-order (FO) corrections include pure QCD and mixed QCD–QED or QCD–weak terms. Reducing such uncertainties requires pQCD calculations beyond the current state of the art, often given by next-to-next-to-leading-order (NNLO) accuracy, pure, or mixed with higher-order electroweak terms.

2. Theoretical uncertainties due to incomplete logarithmic resummations of different energy scales potentially appearing in the theoretical calculations. Examples include resummations of (i) soft and collinear logs in final states dominated by jets—either in analytical calculations or (only partially) incorporated into matched parton shower Monte Carlo generators—and (ii) logarithmic terms in the velocity of the produced top quarks in $\mathrm{e}^+\mathrm{e}^- \to \mathrm{t}\bar{\mathrm{t}}$ cross-sections. Reducing such uncertainties requires calculations beyond the current state of the art, often given by the next-to-next-to-leading-log (NNLL) accuracy.

3. Parametric uncertainties propagated into the final theoretical result, owing to the dependence of the calculation on the input values of (i) the QCD coupling at the Z pole scale, $\alpha_\mathrm{s}(m_\mathrm{Z})$, known today with a relatively poor $\pm 0.9\%$ precision, and (ii) the heavy quark (charm and bottom) masses m_c and m_b. Theoretical progress in lattice QCD determinations of α_s and $m_\mathrm{c,b}$ is needed, complemented with much more precise experimental measurements. A per-mille extraction of $\alpha_\mathrm{s}(m_\mathrm{Z})$ is thereby also a key axis of the FCC-ee physics programme [1].

4. Non-perturbative uncertainties from final-state hadronic effects linked to power-suppressed infrared phenomena, such as colour reconnection, hadronization, and multiparticle correlations (in spin, colour, space, momenta), that cannot be currently computed from first-principles QCD theory, and that often rely on phenomenological Monte Carlo models. The high-precision study of parton hadronization and other non-pQCD phenomena is also an intrinsic part of the FCC-ee physics programme [2].

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Examples of key observables where such four sources of QCD uncertainty will have an impact at the FCC-ee are numerous.

1. Uncertainties from missing HO terms are non-negligible in theoretical predictions for electroweak precision observables (EWPOs) at the Z pole, WW and $\mathrm{t\bar{t}}$ cross-sections, (N)MSSM Higgs cross-sections and decays, etc.

2. Uncertainties from missing soft and collinear log resummations, in analytical calculations or in parton shower MC generators, impact all $\mathrm{e^+e^-}$ final states with jets—e.g., the accurate extraction of forward–backward quark asymmetries at the Z pole—as well as precision flavour physics studies via B meson decays. Similarly, the size of the NNLL corrections (in the $\ln v$ top quark velocity) appears to be as large as that from the FO $\mathrm{N^3LO}$ terms in $\mathrm{e^+e^-} \to \mathrm{t\bar{t}}$ cross-section calculations.

3. The $\alpha_\mathrm{s}(m_\mathrm{Z})$ parametric uncertainty has a significant effect on the determination of all top properties (m_top, λ_top , Γ_top), all hadronic Higgs decay widths ($\mathrm{H} \to \mathrm{c\bar{c}}, \mathrm{b\bar{b}}, \mathrm{q\bar{q}}, \mathrm{g\,g}$) and associated Yukawa couplings, as well as on the extraction of other similarly crucial SM parameters ($m_\mathrm{c}, m_\mathrm{b}, \alpha_\mathrm{QED}$).

4. Non-perturbative uncertainties, in particular colour reconnection and hadronization effects, impact hadronic final states in $\mathrm{e^+e^-} \to \mathrm{WW}$ and $\mathrm{e^+e^-} \to \mathrm{t\bar{t}}$, and forward–backward angular asymmetries of quarks at the Z pole.

In the following sections, the current status and FCC-ee prospects for these four axes of QCD studies are summarised.

2.1 Higher fixed-order pQCD corrections

Computations of pQCD corrections beyond the $\mathrm{N^{2,3}LO}$ accuracy are required for many theoretical FCC-ee observables, in order to match their expected experimental precision. New analytical, algorithmic, and numerical concepts and tools are needed to be able to compute HO QCD and mixed QCD+electroweak multiloop, -legs, and -scales corrections for processes involving the heaviest SM particles (W, Z, H, t) to be carefully scrutinised at the FCC-ee. Concrete developments are covered in more detail in various other sections of this report, and are summarised here.

1. EWPOs: Mixed QCD-electroweak calculations of the $\mathrm{Zf\bar{f}}$ vertex will be needed at the FCC-ee at higher order than known today, including the $\mathcal{O}(\alpha\alpha_\mathrm{s}^2), \mathcal{O}(N_\mathrm{f}\alpha^2\alpha_\mathrm{s}), \mathcal{O}(N_\mathrm{f}^2\alpha^3)$ loop orders, where N_f^n denotes n or more closed internal fermion loops, plus the corresponding QCD four-loop terms [3]. The number of QCD diagrams for $\mathrm{Z} \to \mathrm{b\bar{b}}$ decays at two (three) loops is 98 (10 386) [3]. Section 9 provides, e.g., details on the extension of calculations beyond the two-loop QCD off-shell vertex functions, noting that for the triple-gluon vertex there are 2382 (63 992) three- (four-) loop graphs to evaluate. Including massive quarks in three- and four-point functions is a further requirement in order to reduce the FO theoretical uncertainties.

2. W bosons (Section 7): The resonant $\mathrm{e^+e^-} \to \mathrm{WW}$ cross-section contains soft corrections to the Coulomb function, analogous to ultrasoft ($m_\mathrm{top}v^2$) QCD corrections in $\mathrm{t\bar{t}}$ production [4]. For the W hadronic decay modes, QCD corrections to the partial decay

widths have to be included beyond NNLO to match the corresponding theoretical QED precision given by the counting $\mathcal{O}(\alpha_s^2) \sim \mathcal{O}(\alpha_{\text{QED}})$. QCD corrections to W self-energies and decay widths up to $\mathcal{O}(\alpha_{\text{QED}}\,\alpha_s^2)$ and $\mathcal{O}(\alpha_s^4)$ are required. Currently, $\mathcal{O}(\alpha_s^4)$ corrections for inclusive hadronic vector boson decays are known [5], while mixed QCD-EW corrections are known up to $\mathcal{O}(\alpha_{\text{QED}}\,\alpha_s)$ [6].

3. Higgs bosons (Section 12): The pure QCD corrections to Higgs boson decays into quarks, gluons, and photons are known up to N^{4}LO (no mass effects), N^{3}LO (heavy top limit), and NLO, respectively. Those translate into approximately 0.2%, 1%, and <3% scale uncertainties from missing HO corrections. In the case of the (N)MSSM Higgs sector (Section 3), HO pQCD corrections to the Higgs bosons decays are mostly known at NLO accuracy; thereby, their uncertainty is larger than for the SM Higgs case.

4. Top quarks (Section 11): The total cross-section for inclusive $e^+e^- \to b\bar{b}W^+W^-X$ production can be computed in a non-relativistic effective field theory with local effective vertices and matching corrections known up to N^{3}LO in pQCD [7]. Those translate into about 3% theoretical scale uncertainties of the threshold $t\bar{t}$ cross-sections that propagate into an uncertainty of ±60 MeV in the position of the resonant peak. Although the uncertainty has been reduced by a factor of two going from NNLO to N^{3}LO, perturbative progress is still needed, in particular in the threshold top mass definition translated into the $\overline{\text{MS}}$ scheme.

5. The extraction of α_{QED} from the R ratio requires the calculation of the four-loop massive pQCD calculation of the Adler function (together with better estimates of α_s in the low-Q^2 region above the τ mass, as well as of the m_c and m_b masses).

2.2 Higher-order logarithmic resummations

Improvements in the resummations of all-order logarithmic terms from different energy scales, appearing in the theoretical calculations for certain processes, are needed in various directions.

1. Soft and collinear parton radiation impacts many e^+e^- observables with jets in the final state. Such uncertainties enter through incomplete NNLL resummations in analytical calculations (e.g., based on soft-collinear effective theory, SCET), or through approximate models of the coherent branching implemented in the parton shower MC generators used to unfold and interpret the experimental data. Among those experimental observables, the measured forward–backward (FB) angular asymmetries of charm and bottom quarks in e^+e^- collisions around the Z pole, directly connected to the weak mixing angle, will need a careful study. The asymmetry value measured at LEP, $(A_{\text{FB}}^{0,b})_{\text{exp}} = 0.0992 \pm 0.0016$, remains today the electroweak precision observable with the largest disagreement (2.9σ) with respect to the SM prediction, $(A_{\text{FB}}^{0,b})_{\text{th}} = 0.1038$ [8,9]. Consequently, so also does the effective weak mixing angle derived from it, $\sin^2\theta_{\text{eff}}^{\text{f}} = 0.232\,21 \pm 0.000\,29$, compared with the $\sin^2\theta_{\text{eff}}^{\text{f}} = 0.231\,54 \pm 0.000\,03$ world-average [10]. The dominant systematic uncertainties on $(A_{\text{FB}}^{0,b})_{\text{exp}}$ arise from angular decorrelations induced in the thrust axis by soft and collinear parton radiation or parton-to-hadron b quark hadronization, and were estimated using MC simulations 20 years ago [11]. A recent reanalysis of the QCD corrections to $A_{\text{FB}}^{0,b}$ [12], with different modern parton shower models [13–15], indicates propagated uncertainties of about 1% (0.4%) for the lepton (jet) charge-based measurements, slightly

smaller but still consistent with the original measurements derived at LEP. The measurement of $A_{\mathrm{FB}}^{0,\mathrm{b}}$ at the FCC-ee will feature insignificant statistical uncertainties, and improvements in the modelling of parton radiation will be required for any high-precision extraction of the associated $\sin^2\theta_{\mathrm{eff}}^{\mathrm{f}}$ value.

2. Another field of $\mathrm{e}^+\mathrm{e}^-$ measurements where progress in logarithmic resummations is needed is in the studies of event shapes—such as the thrust T, C parameter, and jet broadening. All those observables are commonly used to extract the QCD coupling [1]. Theoretical studies of event shapes supplement FO perturbation theory with the resummation of enhanced logarithmic contributions, specifically accounting for terms ranging from $\alpha_{\mathrm{s}}^n \ln^{n+1}$ down to $\alpha_{\mathrm{s}}^n \ln^{n-2}$, i.e., N^{3}LL [16]. However, the $\alpha_{\mathrm{s}}(m_{\mathrm{Z}})$ values derived from the T and C measurements differ and their combination has thereby a final 2.9% systematic uncertainty [10]. This result points to limits in the resummation formalism that (i) hold only for $C, 1-T \ll 1$, where every emission is so soft and collinear that one can effectively neglect the kinematic cross-talk (e.g., energy–momentum conservation) that arises when there are a number of emissions, and (ii) use a power correction valid only in the two-jet limit, $1-T \ll 1$ [16].

3. High-precision studies of n-jet rates at the FCC-ee will also benefit from a reduction of resummation uncertainties. Jet rates in $\mathrm{e}^+\mathrm{e}^-$ rely on an algorithm to reconstruct them that comes with a parameter ($y_{\mathrm{cut}} = k_{\mathrm{T}}^2/s$, in the k_{T} Durham [17] and Cambridge [18] cases) to define how energetic the emission should be in order to be considered a jet. For $\ln y_{\mathrm{cut}} > -4$, the extracted α_{s} value from three-jet rates is fairly independent of y_{cut}, whereas the result depends substantially on the choice of y_{cut} below that [19]. This feature points to a breakdown of FO perturbation theory, owing to logarithmically enhanced $(\alpha_{\mathrm{s}} \ln^2 y_{\mathrm{cut}})^n$ terms. Jet rates at the one-in-a-million level in $\mathrm{e}^+\mathrm{e}^-$ at the Z pole will be available at the FCC-ee, including: four-jet events up to $k_{\mathrm{T}} \approx 30\,\mathrm{GeV}$ (corresponding to $|\ln y_{\mathrm{cut}}| \approx 2$), five-jet events at $k_{\mathrm{T}} \approx 20\,\mathrm{GeV}$ ($|\ln y_{\mathrm{cut}}| \approx 3$), six-jet events at $k_{\mathrm{T}} \approx 12\,\mathrm{GeV}$ ($|\ln y_{\mathrm{cut}}| \approx 4$), and seven-jet events at $k_{\mathrm{T}} \approx 7.5\,\mathrm{GeV}$ ($|\ln y_{\mathrm{cut}}| \approx 5$). Such results will be compared with theoretical calculations with an accuracy beyond the NNLO + NNLL provided today by the EERAD3 [20], MERCUTIO 2 [21], and CoLoRFulNNLO [22] (NNLO), and ARES [23] (NNLL) codes, thereby leading to α_{s} extractions with uncertainties well below the current few-percent level. In general, with the envisioned FCC-ee luminosities, jet measurements will extend along the six axes of higher accuracy, finer binning, higher jet resolution scales, larger numbers of resolved final-state objects, more differential distributions, and possibility placing stringent additional cuts to isolate specific interesting regions of the n-jet phase spaces not strongly constrained by LEP measurements [24].

4. In top physics studies, the size of the NNLL corrections (in top quark velocity, $\ln v$) in $\mathrm{e}^+\mathrm{e}^- \to \mathrm{t}\bar{\mathrm{t}}$ cross-section calculations appears to be as large as that from the FO N^{3}LO terms [7], calling for improved resummation studies for such an observable.

5. In the sector of flavour physics (Section 10), new tools based on SCET, developed to study processes with energetic quarks and gluons, can be applied after certain modifications to improve the accuracy of theoretical corrections for B-physics studies at the FCC-ee, in particular for regions of phase space where the perturbative approach breaks down, owing to the presence of large logarithmic enhancements, and where the next-to-soft effects become more important.

2.3 Per-mille-precision α_s extraction

The strong coupling, α_s, is one of the fundamental parameters of the Standard Model, and its value not only directly affects the stability of the electroweak vacuum [25] but it chiefly impacts all theoretical calculations of e^+e^- scattering and decay processes involving real or virtual quarks and gluons [1]. Known today with a 0.9% precision, making it the worst known of all fundamental interaction couplings in nature [10], the input value of $\alpha_s(m_Z)$ propagates as a parametric uncertainty into many of the FCC-ee physics observables, chiefly in the Z, Higgs, and top quark sectors.

1. The leading source of uncertainty in the calculation of crucial EWPOs at the Z pole, such as Γ_Z, $\sigma^0_{\rm had}$, and R_ℓ, is the propagated $\delta\alpha_s$ parametric source [3].
2. In the Higgs sector (Section 12), the current $\alpha_s(m_Z)$ parametric uncertainty (combined with uncertainties arising from our imperfect knowledge of m_c and m_b) propagates into total final uncertainties of ~2% for the BR(H→WW, ZZ) and BR($H \to \tau^+\tau^-, \mu^+\mu^-$) branching ratios, of ~6–7% for BR(H → gg) and BR($H \to c\bar{c}$), ~3% for BR($H \to \gamma\gamma$), and ~7% for BR($H \to Z\gamma$).
3. Precise studies of the $e^+e^- \to t\bar{t}$ cross-section (Section 11) indicate that it should be possible to extract the top quark width and mass with an uncertainty of around 50 MeV, provided that a precise independent extraction of the strong coupling is available. Such a requirement is, in particular, crucial to meaningfully constrain the top Yukawa coupling.

The current world-average value, $\alpha_s(m_Z) = 0.1181 \pm 0.0011$, is derived from a combination of six subclasses of approximately independent observables [10] measured in e^+e^- (hadronic Z boson and τ decays, and event shapes and jet rates), DIS (structure functions and global fits of parton distributions functions), and p–p collisions (top pair cross-sections), as well as from lattice QCD computations constrained by the empirical values of hadron masses and decay constants. To enter into the $\alpha_s(m_Z)$ world-average, the experimental (or lattice) results need to have a counterpart pQCD theoretical prediction at NNLO (or beyond) accuracy.

Of the current six $\alpha_s(m_Z)$ extractions entering in the PDG average, that derived from comparisons of NNLO pQCD predictions with lattice QCD results (Wilson loops, $q\bar{q}$ potentials, hadronic vacuum polarisation, QCD static energy) [26] today provides the most precise result: $\alpha_s(m_Z) = 0.1188 \pm 0.0011$. The current ~0.9% uncertainty is dominated by finite lattice spacing, truncations of the pQCD expansion up to NNLO, and hadron extrapolations. Over the next 10 years, reduction of the statistical uncertainties, at least by a factor of two, can be anticipated with increased computing power, while reaching the ~0.1% uncertainty level will also require the computation of fourth-order pQCD corrections [1].

After the lattice result, the most theoretically and experimentally 'clean' extractions of α_s are those based on the hadronic decays of the τ lepton, and W and Z bosons that will be measured with unparalleled accuracies at the FCC-ee. To derive $\alpha_s(m_Z)$, the experimental ratios of hadronic-to-leptonic decays are compared with the corresponding pQCD theoretical prediction, known today up to $\mathcal{O}(\alpha_s^4)$ [5,27]:

$$
\begin{aligned}
R_\ell^{\tau,W,Z}(Q = m_\tau, m_W, m_Z) &= \frac{\sigma(e^+e^- \to (\tau, W, Z) \to \text{hadrons})}{\sigma(e^+e^- \to (\tau, W, Z) \to \ell^+\ell^-)} \\
&= R_{\rm EW}(Q)\left(1 + \sum_{i=1}^{N=4} c_n(Q)\left(\frac{\alpha_s(Q)}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_m + \delta_{np}\right). \qquad (2.1)
\end{aligned}
$$

In this equation, Q is the typical momentum transfer in the process used for measuring R_ℓ, c_n are coefficients of the perturbative series that can, in practice, be calculated up to some finite order $n = N$, and the terms δ_m and δ_np correspond, respectively, to mixed QCD+EW higher-order and power-suppressed $\mathcal{O}(\Lambda^p/Q^p)$ non-perturbative corrections, which affect, differently, the tau lepton and electroweak boson decays. For $\alpha_\mathrm{s}(m_\mathrm{Z}) = 0.118$, the size of the QCD term in Eq. (2.1) amounts to a 4% effect, so at least per-mille measurement accuracies for the R_ℓ ratios are required for a competitive $\alpha_\mathrm{s}(m_\mathrm{Z})$ determination [8]. Such an experimental precision has been reached in measurements of τ and Z boson decays, but not for the W boson and that is why the latter still does not provide a precise α_s extraction [28]. Reaching per-mille uncertainties in α_s determinations based on Eq. (2.1) requires 100 times smaller uncertainties in the experimental τ, W, and Z measurements, a situation only reachable at the FCC-ee.

The ratio of hadronic to leptonic tau decays, known experimentally to within ±0.23% ($R_\ell^{\tau,\mathrm{exp}} = 3.4697 \pm 0.0080$), compared with next-to-NNLO (N^{3}LO) calculations, yields $\alpha_\mathrm{s}(m_\mathrm{Z}) = 0.1192 \pm 0.0018$, i.e., a 1.5% uncertainty, through a combination of results from different theoretical approaches (contour-improved perturbation theory (CIPT) and fixed-order perturbation theory (FOPT)) with different treatments of non-pQCD corrections [29,30]. The current α_s uncertainty is shared roughly equally between experimental and theoretical systematics. The latter are driven by differences in the CIPT and FOPT results, although the power-suppressed non-perturbative δ_np term in Eq. (2.1), which is of $\mathcal{O}(\Lambda^2/m_\tau^2) \approx 10^{-2}$, is not negligible for the tau, at variance with the much heavier W and Z bosons. High-statistics τ spectral functions (e.g., from B factories now, and the FCC-ee in the future), and solving CIPT–FOPT discrepancies (extending the calculations to N^{4}LO accuracy and controlling the non-pQCD uncertainties) are required to reduce the relative α_s uncertainty below the ∼1% level.

The current state-of-the-art N^{3}LO calculations of W boson decays [6] would allow a theoretical extraction of α_s with a ∼0.7% uncertainty, provided that one would have experimental measurements of sufficient precision. Unfortunately, the relevant LEP W$^+$W$^-$ data are poor, based on 5×10^4 W bosons alone, and result in a QCD coupling extraction, $\alpha_\mathrm{s}(m_\mathrm{Z}) = 0.117 \pm 0.040$, with a huge ∼37% uncertainty today [28]. A determination of α_s with per-mille uncertainty from W boson decays can only be achieved through the combination of two developments: (i) data samples commensurate with those expected at the FCC-ee (10^8 W bosons) and (ii) a significantly reduced uncertainty of the $\mathrm{V_{cs}}$ CKM element, which directly enters into the leading $R_\mathrm{EW}(Q)$ prefactor of Eq. (2.1) and propagates into a significant parametric uncertainty on the extracted α_s. Figure B.2.1 (left) shows the expected $\alpha_\mathrm{s}(m_\mathrm{Z})$ value derived from the R_ℓ^W ratio with 10^8 W bosons at the FCC-ee, assuming that $\mathrm{V_{cs}}$ has a negligible uncertainty (or, identical, assuming Cabibbo–Kobayashi–Maskawa (CKM) matrix unitarity). The extracted QCD coupling would have ∼0.2% propagated experimental uncertainties.

The current QCD coupling extraction based on Z boson hadron decays uses three closely related pseudo-observables measured at the LEP: $R_\ell^0 = \Gamma_\mathrm{had}/\Gamma_\ell$, $\sigma_\mathrm{had}^0 = 12\pi/m_Z \cdot \Gamma_e\Gamma_\mathrm{had}/\Gamma_\mathrm{Z}^2$, and Γ_Z, combined with N^{3}LO calculations, to give $\alpha_\mathrm{s}(m_\mathrm{Z}) = 0.1203 \pm 0.0028$ with a 2.5% uncertainty [10]. Alternatively, fixing all SM parameters to their measured values and letting free α_s in the electroweak fit yields $\alpha_\mathrm{s} = 0.1194 \pm 0.0029$ (∼2.4% uncertainty, shallow blue curve in Fig. B.2.1 (right)) [31]. At the FCC-ee, with 10^{12} Z bosons providing high-precision measurements with $\Delta m_\mathrm{Z} = 0.1$ MeV, and $\Delta\Gamma_Z = 0.1\,\mathrm{MeV}$, $\Delta R_\ell^0 = 10^{-3}$ (achievable thanks to the possibility of performing a threshold scan including energy self-calibration with resonant depolarisation) reduces the uncertainty on $\alpha_\mathrm{s}(m_\mathrm{Z})$ to ∼0.15%. Figure B.2.1 (right) shows the expected α_s extractions from R_ℓ^Z and Γ^Z at the FCC-ee (yellow band) with the experimental

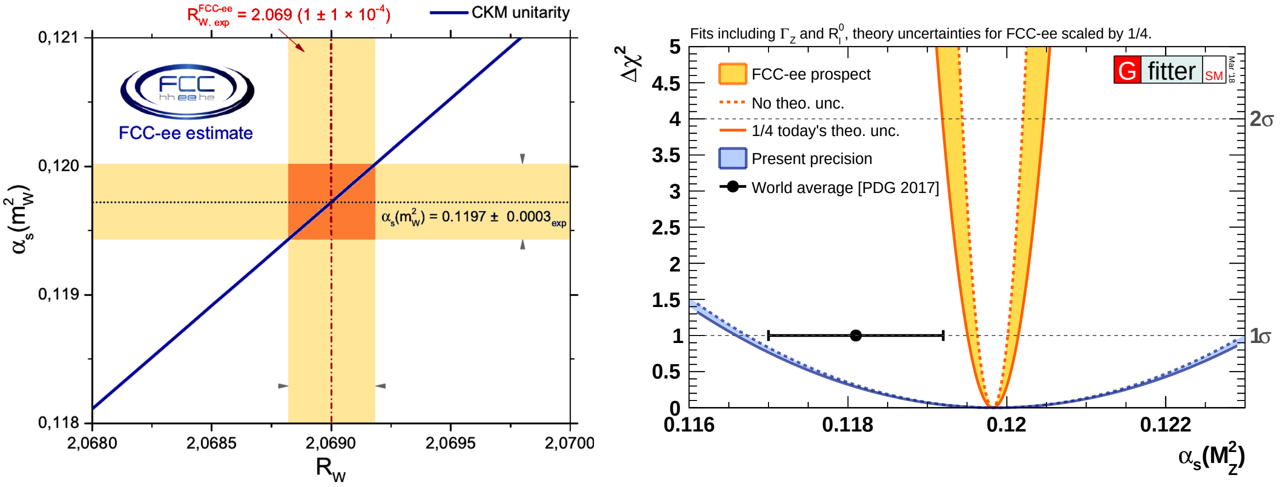

Fig. B.2.1: Left: Expected α_s determination from the W hadronic-to-leptonic decay ratio (R_ℓ^{W}) at the FCC-ee (the diagonal blue line assumes CKM matrix unitarity) [28]. Right: Precision on α_s derived from the electroweak fit today (blue band) [31] and expected at the FCC-ee (yellow band, without theoretical uncertainties and with the current theoretical uncertainties divided by a factor of four).

uncertainties listed in Table (A.1.2), without theoretical uncertainties (dotted red curve) and with the theoretical uncertainties reduced to one-quarter of their current values (solid red curve) [31].

The FCC-ee will not only provide an unprecedented amount of electroweak boson data, but also many orders of magnitude more jets than collected at LEP. The large and clean set of accurately reconstructed (and flavour-tagged) $\mathrm{e^+e^-}$ hadronic final states will provide additional high-precision α_s determinations from studies of event shapes, jet rates, and parton-to-hadron fragmentation functions (FFs) [1]. The existing measurements of $\mathrm{e^+e^-}$ event shapes (thrust T, C parameter) [23,32–34] and n-jet rates [19,35,36], analysed with $\mathrm{N^{2,3}LO}$ calculations matched, in some cases, to soft and collinear $\mathrm{N^{(2)}LL}$ resummations, yield $\alpha_s(m_Z) = 0.1169 \pm 0.0034$, with a 2.9% uncertainty [10]. This relatively large uncertainty is mostly driven by the span of individual extractions that use different (Monte Carlo or analytical) approaches to account for soft and collinear radiation as well as to correct for hadronization effects. Modern jet substructure techniques [37] can help mitigate the latter corrections. In terms of event shapes, the recent combination of the CoLoRFulNNLO subtraction method [38] with NNLL corrections in the back-to-back region [39] has led to a precise calculation of the energy–energy correlation (EEC) observable in electron–positron collisions, and thereby an accurate NNLO+NNLL extraction of $\alpha_s(m_Z) = 0.1175 \pm 0.0029$ (~2.5% uncertainty) [40], as discussed in detail in Section 4. Moreover, a very recent analysis of two-jet rates in $\mathrm{e^+e^-}$ collisions at $\mathrm{N^3LO}$+NNLL accuracy [41] has provided a new QCD coupling determination with ~1% uncertainty: $\alpha_s(m_Z) = 0.118\,81 \pm 0.001\,32$. In addition, other sets of observables computed today with a lower degree of accuracy (NLO, or approximately NNLO, bottom part of Fig. B.2.2), and thereby not now included in the PDG average, will provide additional constraints [1]. The energy dependence of the low-z FFs today provides $\alpha_s(m_Z) = 0.1205 \pm 0.0022$ (~2% uncertainty) at NNLO*+NNLL [42,43], whereas NLO scaling violations of the high-z FFs yield $\alpha_s(m_Z) = 0.1176 \pm 0.0055$ (~5% uncertainty, mostly of experimental origin) [44]. In addition, measurements of the photon structure function $\mathrm{F}_2^{\gamma}(x, Q^2)$, via $\mathrm{e^+e^-} \rightarrow \gamma\gamma \rightarrow$ hadrons, have been employed to derive $\alpha_s(m_Z) = 0.1198 \pm 0.0054$ (~4.5%

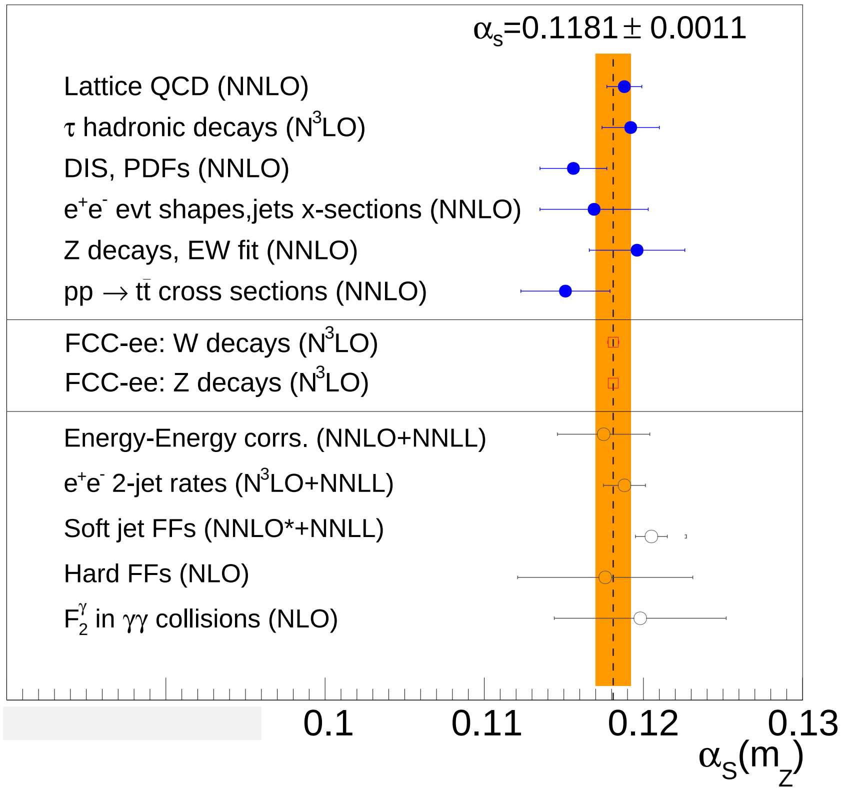

Fig. B.2.2: Summary of the $\alpha_\mathrm{s}(m_\mathrm{z})$ determinations discussed here. Top: Subclasses entering in the current PDG world-average (solid dots, orange band) whose numerical value is listed on top [10]. Middle: Expected FCC-ee values via W, Z hadronic decays (open squares). Bottom: Other methods based on $\mathrm{e^+e^-}$ data not (yet) in the $\alpha_\mathrm{s}(m_\mathrm{z})$ world-average: recent EEC [40] and two-jet rates [41], plus other extractions at a (currently) lower level of theoretical accuracy.

uncertainty) at NLO [45]. Extension to full-NNLO accuracy of the FFs and $\mathrm{F}_2^\gamma(x, Q^2)$ fits using the much larger $\mathrm{e^+e^-}$ datasets available at various centre-of-mass energies at the FCC-ee will enable subpercentage precision in $\alpha_\mathrm{s}(m_\mathrm{z})$ to be attained. Figure B.2.2 presents a comparison of the current $\alpha_\mathrm{s}(m_\mathrm{z})$ results (top), the expected FCC-ee extractions (middle), and the other aforementioned methods based on $\mathrm{e^+e^-}$ data not currently included in the world-average.

2.4 High-precision non-perturbative QCD

All $\mathrm{e^+e^-}$ processes with quarks and gluons in the final state have an intrinsic uncertainty linked to the final non-perturbative conversion of the partons, present in the last stage of the QCD shower, into hadrons. Such a process cannot be computed using first-principles QCD calculations and is described using phenomenological models, such as the Lund string [46], as implemented in the PYTHIA MC generator [13], or the cluster hadronization approach [47] typical of the HERWIG event generator [48]. The analysis and unfolding of any $\mathrm{e^+e^-}$ experimental measurement of hadronic final states relies on these very same Monte Carlo generators; therefore, the final results are sensitive to their particular implementation of soft and collinear parton radiation (whose MC modelling is equivalent to an approximate next-to-leading-log (NLL) accuracy [49]) and of the hadronization process. Examples of such propagated uncertainties have been discussed already in the context of α_s extractions from various experimental $\mathrm{e^+e^-}$ observables. An improved MC reproduction of the experimental hadron data can, e.g., help in enabling advanced light quark and gluon jet tagging in constraints of the Higgs Yukawa couplings to the first and second family of quarks. Controlling the uncertainties linked to hadronization and other final-state partonic effects, such as colour reconnection and multiparticle (spin, momenta, space, etc.) correlations, is, therefore, basic for many high-precision SM studies. Such effects are optimally studied in the clean environment provided by e^+e^- collisions, without coloured objects in the initial state. An FCC-ee goal, therefore, is to produce truly precise QCD measurements to constrain many aspects of non-perturbative dynamics to the 1% level or better, leaving an important legacy for MC generators for the FCC-eh and FCC-hh physics programme, much as those from LEP proved crucial for the parton shower models used today at the LHC [2]. In particular, the FCC-ee operating at different c.m. energies will enormously help to control resummation and hadronization effects in event shape distributions, reducing, in particular, non-perturbative uncertainties from a 9% effect at $\sqrt{s} = 91.2\,\mathrm{GeV}$ to a 2% at $400\,\mathrm{GeV}$ [2, 50].

The modelling of parton hadronization in the current MC event generators has achieved a moderate success, and the LHC data have only further complicated the situation. First, the production of baryons (in particular containing strange quarks) remains poorly understood and is hard to measure in the complicated hadron–hadron environment. Second, and most importantly, the LHC measurements have challenged the standard assumption of parton hadronization universality, i.e., that models developed from e^+e^- data can be directly applied to hadron–hadron collisions. Strong final-state effects, more commonly associated with heavy-ion physics and quark–gluon–plasma formation, such as the 'ridge' [51] or the increase of strangeness production in high-multiplicity pp events [52], cannot be accommodated within the standard MC generators. The large statistical samples available at the FCC-ee will allow parton hadronization to be controlled in the QCD vacuum with subpercentage uncertainties, and thereby provide a better understanding of any collective final-state effects present in hadron–hadron collisions, starting with multistrange baryons, whose total production rates could only be determined with 5–20% accuracy at the LEP [53, 54], and going further to excited [54, 55], exotic, or multiple heavy hadrons, with implications for more advanced fragmentation models. For Λ–Λ correlation distributions, where MC generator programs today fail to describe the LEP [56] and LHC data, the huge FCC-ee samples of hadronic Z decays will have statistical uncertainties matching the best LEP systematic uncertainties, corresponding to a total errors reduction by a factor of ten or more.

In $e^+e^- \to t\bar{t}$, when the top and antitop quarks decay and hadronize close to each other, interactions and interferences between them, the decay bottoms, and any radiated gluons affect the rearrangement of the colour flow and thereby the kinematic distributions of the final hadronic state. Whereas the perturbative radiation in the process can, in principle, be theoretically controlled, there is a 'cross-talk' among the produced hadronic strings, also known as colour reconnection (CR), that can only be modelled phenomenologically [57]. In the pp case, such CR effects can decrease the precision that can be achieved in the extraction of the top mass, and constitute 20–40% of its uncertainty [58]. Colour reconnection can also impact limits for CP-violation searches in $H \to W^+W^- \to q_1\bar{q}_2q_3\bar{q}_4$ decays [59]. Searches for such effects can be optimally studied in the process $e^+e^- \to W^+W^- \to q_1\bar{q}_2q_3\bar{q}_4$ [59], where CR could lead to the formation of alternative 'flipped' singlets $q_1\bar{q}_4$ and $q_3\bar{q}_2$, and correspondingly more complicated string topologies [60]. The combination of results from all four LEP collaborations excluded the no-CR null hypothesis at 99.5% CL [61], but the size of the WW data sample was too small for any quantitative studies. At the FCC-ee, with the W mass determined to better than $1\,\mathrm{MeV}$ by a threshold scan, the semileptonic WW measurements (unaffected by

CR) can be used to probe the impact of CR in the hadronic WW events [2,62]. Alternative CR constraints at the FCC-ee have been proposed through the study of event shape observables sensitive to string overlap, such as sphericity for different hadron flavours, as described in ‘rope hadronization’ approaches [63,64].

References

[1] D. d’Enterria and P.Z. Skands (Eds.), Proc. High-Precision α_s Measurements from LHC to FCC-ee, CERN (CERN, Geneva, 2015), `arXiv:1512.05194`,

[2] D. d’Enterria and P.Z. Skands (Eds.), Proc. Parton Radiation and Fragmentation from LHC to FCC-ee, 2017. `arXiv:1702.01329`

[3] A. Blondel *et al.*, Standard Model theory for the FCC-ee Tera-Z stage, CERN (CERN Yellow Rep. Monogr. 3, Geneva, Switzerland), `arXiv:1809.01830`, `doi:10.23731/CYRM-2019-003`

[4] M. Beneke and Y. Kiyo, *Phys. Lett.* **B668** (2008) 143. `arXiv:0804.4004`, `doi:10.1016/j.physletb.2008.08.031`

[5] P.A. Baikov *et al.*, *Phys. Rev. Lett.* **101** (2008) 012002. `arXiv:0801.1821`, `doi:10.1103/PhysRevLett.101.012002`

[6] D. Kara, *Nucl. Phys.* **B877** (2013) 683. `arXiv:1307.7190`, `doi:10.1016/j.nuclphysb.2013.10.024`

[7] M. Beneke *et al.*, *Phys. Rev. Lett.* **115** (2015) 192001. `arXiv:1506.06864`, `doi:10.1103/PhysRevLett.115.192001`

[8] S. Schael *et al.*, *Phys. Rep.* **427** (2006) 257. `arXiv:hep-ex/0509008`, `doi:10.1016/j.physrep.2005.12.006`

[9] LEP Electroweak Working Group, Precision electroweak measurements and constraints on the Standard Model, `arXiv:1012.2367`

[10] C. Patrignani *et al.*, *Chin. Phys.* **C40** (2016) 100001. `doi:10.1088/1674-1137/40/10/100001`

[11] D. Abbaneo *et al.*, *Eur. Phys. J.* **C4** (1998) 185. `doi:10.1007/s100520050196`

[12] D. d’Enterria and C. Yan, Forward–backward b-quark asymmetry at the Z pole: QCD uncertainties redux, 53rd Rencontres de Moriond on QCD and High Energy Interactions (Moriond QCD 2018) La Thuile, Italy, 2018, `arXiv:1806.00141`

[13] T. Sjöstrand *et al.*, *Comput. Phys. Commun.* **191** (2015) 159. `arXiv:1410.3012`, `doi:10.1016/j.cpc.2015.01.024`

[14] W.T. Giele *et al.*, *Phys. Rev.* **D84** (2011) 054003. `arXiv:1102.2126`, `doi:10.1103/PhysRevD.84.054003`

[15] N. Fischer *et al.*, *Eur. Phys. J.* **C76** (2016) 589. `arXiv:1605.06142`, `doi:10.1140/epjc/s10052-016-4429-6`

[16] G.P. Salam, in *From My Vast Repertoire. . . : Guido Altarelli’s Legacy*, Eds. S. Forte *et al.* (World Scientific, Singapore, 2019), Chap. 7, pp. 101–121. `arXiv:1712.05165`, `doi:10.1142/9789813238053_0007`

[17] S. Catani *et al.*, *Phys. Lett.* **B269** (1991) 432. `doi:10.1016/0370-2693(91)90196-W`

[18] Y. L. Dokshitzer *et al.*, *JHEP* **08** (1997) 001. arXiv:hep-ph/9707323, doi:10.1088/1126-6708/1997/08/001

[19] G. Dissertori *et al.*, *Phys. Rev. Lett.* **104** (2010) 072002. arXiv:0910.4283, doi:10.1103/PhysRevLett.104.072002

[20] A. Gehrmann-De Ridder *et al.*, *Comput. Phys. Commun.* **185** (2014) 3331. arXiv:1402.4140, doi:10.1016/j.cpc.2014.07.024

[21] S. Weinzierl, *Eur. Phys. J.* **C71** (2011) 1565 [Erratum: **C71** (2011) 1717]. arXiv:1011.6247, doi:10.1140/epjc/s10052-011-1565-x, doi:10.1140/epjc/s10052-011-1717-z

[22] V. Del Duca *et al.*, *Phys. Rev. Lett.* **117** (2016) 152004. arXiv:1603.08927, doi:10.1103/PhysRevLett.117.152004

[23] A. Banf *et al.*, *JHEP* **05** (2015) 102. arXiv:1412.2126, doi:10.1007/JHEP05(2015)102

[24] N. Fischer *et al.*, *Eur. Phys. J.* **C75** (2015) 571. arXiv:1505.01636, doi:10.1140/epjc/s10052-015-3766-1

[25] D. Buttazzo *et al.*, *JHEP* **12** (2013) 089. arXiv:1307.3536, doi:10.1007/JHEP12(2013)089

[26] S. Aoki *et al.*, FLAG review 2019, arXiv:1902.08191

[27] P.A. Baikov *et al.*, *Phys. Lett.* **B714** (2012) 62. arXiv:1206.1288, doi:10.1016/j.physletb.2012.06.052

[28] D. d'Enterria and M. Srebre, *Phys. Lett.* **B763** (2016) 465. arXiv:1603.06501, doi:10.1016/j.physletb.2016.10.012

[29] A. Pich and A. Rodríguez-Sánchez, *Phys. Rev.* **D94** (2016) 034027. arXiv:1605.06830, doi:10.1103/PhysRevD.94.034027

[30] D. Boito *et al.*, *Phys. Rev.* **D95** (2017) 034024. arXiv:1611.03457, doi:10.1103/PhysRevD.95.034024

[31] J. Haller *et al.*, *Eur. Phys. J.* **C78** (2018) 675. arXiv:1803.01853, doi:10.1140/epjc/s10052-018-6131-3

[32] S. Bethke *et al.*, *Eur. Phys. J.* **C64** (2009) 351. arXiv:0810.1389, doi:10.1140/epjc/s10052-009-1149-1

[33] R. Abbate *et al.*, *Phys. Rev.* **D86** (2012) 094002. arXiv:1204.5746, doi:10.1103/PhysRevD.86.094002

[34] A.H. Hoang *et al.*, *Phys. Rev.* **D91** (2015) 094018. arXiv:1501.04111, doi:10.1103/PhysRevD.91.094018

[35] S. Weinzierl, *Phys. Rev. Lett.* **101** (2008) 162001. arXiv:0807.3241, doi:10.1103/PhysRevLett.101.162001

[36] J. Schieck *et al.*, *Eur. Phys. J.* **C73** (2013) 2332. arXiv:1205.3714, doi:10.1140/epjc/s10052-013-2332-y

[37] Les Houches 2017: Physics at TeV colliders Standard Model working group report. arXiv:1803.07977.

[38] V. Del Duca *et al.*, *Phys. Rev.* **D94** (2016) 074019. arXiv:1606.03453, doi:10.1103/PhysRevD.94.074019

[39] D. de Florian and M. Grazzini, *Nucl. Phys.* **B704** (2005) 387. arXiv:hep-ph/0407241, doi:10.1016/j.nuclphysb.2004.10.051

[40] A. Kardos *et al.*, *Eur. Phys. J.* **C78** (2018) 498. arXiv:1804.09146, doi:10.1140/epjc/s10052-018-5963-1

[41] A. Verbytskyi *et al.*, *JHEP* **2019** (2019) 129. arXiv:1902.08158 doi:10.1007/JHEP08(2019)129

[42] R. Pérez-Ramos and D. d'Enterria, *JHEP* **08** (2014) 068. arXiv:1310.8534, doi:10.1007/JHEP08(2014)068

[43] D. d'Enterria and R. Pérez-Ramos, α_s determination at NNLO*+NNLL accuracy from the energy evolution of jet fragmentation functions at low z, Proc. 50th Rencontres de Moriond, QCD and high energy interactions: La Thuile, Italy (2015), p. 117. arXiv:1505.02624

[44] S. Albino *et al.*, *Nucl. Phys.* **B725** (2005) 181. arXiv:hep-ph/0502188, doi:10.1016/j.nuclphysb.2005.07.010

[45] S. Albino *et al.*, *Phys. Rev. Lett.* **89** (2002) 122004. arXiv:hep-ph/0205069, doi:10.1103/PhysRevLett.89.122004

[46] B. Andersson *et al.*, *Z. Phys.* **C20** (1983) 317. doi:10.1007/BF01407824

[47] B.R. Webber, *Nucl. Phys.* **B238** (1984) 492. doi:10.1016/0550-3213(84)90333-X

[48] G. Corcella *et al.*, *JHEP* **01** (2001) 010. arXiv:hep-ph/0011363, doi:10.1088/1126-6708/2001/01/010

[49] M. Dasgupta *et al.*, *JHEP* **09** (2018) 033. arXiv:1805.09327, doi:10.1007/JHEP09(2018)033

[50] G. Bell *et al.*, *JHEP* **01** (2019) 147. arXiv:1808.07867, doi:10.1007/JHEP01(2019)147

[51] V. Khachatryan *et al.*, *JHEP* **09** (2010) 091. arXiv:1009.4122, doi:10.1007/JHEP09(2010)091

[52] J. Adam *et al.*, *Nat. Phys.* **13** (2017) 535. arXiv:1606.07424, doi:10.1038/nphys4111

[53] P. Abreu *et al.*, *Z. Phys.* **C67** (1995) 543. doi:10.1007/BF01553980

[54] G. Alexander *et al.*, *Z. Phys.* **C73** (1997) 569. doi:10.1007/s002880050349

[55] R. Akers *et al.*, *Z. Phys.* **C68** (1995) 1. doi:10.1007/BF01579799

[56] G. Abbiendi *et al.*, *Eur. Phys. J.* **C13** (2000) 185. arXiv:hep-ex/9808031, doi:10.1007/s100520000207,10.1007/s100520050685

[57] V.A. Khoze and T. Sjöstrand, *Phys. Lett.* **B328** (1994) 466. arXiv:hep-ph/9403394, doi:10.1016/0370-2693(94)91506-7

[58] S. Argyropoulos and T. Sjöstrand, *JHEP* **11** (2014) 043. arXiv:1407.6653, doi:10.1007/JHEP11(2014)043

[59] J.R. Christiansen and T. Sjöstrand, *Eur. Phys. J.* **C75** (2015) 441. arXiv:1506.09085, doi:10.1140/epjc/s10052-015-3674-4

[60] T. Sjöstrand and V.A. Khoze, *Z. Phys.* **C62** (1994) 281. arXiv:hep-ph/9310242, doi:10.1007/BF01560244

[61] S. Schael *et al.*, *Phys. Rep.* **532** (2013) 119. arXiv:1302.3415, doi:10.1016/j.physrep.2013.07.004

[62] A. Abada *et al.*, *Eur. Phys. J.* **C79** (2019), 474, `doi:10.1140/epjc/s10052-019-6904-3`

[63] C. Bierlich *et al.* A shoving model for collectivity in hadronic collisions, `arXiv:1612.05132`

[64] C. Bierlich, *EPJ Web Conf.* **171** (2018) 14003. `arXiv:1710.04464`, `doi:10.1051/epjconf/201817114003`