10 Effective field theory approach to QED corrections in flavour physics

Contribution* by: M. Beneke, C. Bobeth, R. Szafron
Corresponding author: R. Szafron [robert.szafron@tum.de]

10.1 Introduction and motivation

Thanks to the accurate measurements performed at the low-energy facilities [1] and the LHC, flavour physics of light quarks, especially the bottom quark, emerged on the precision frontier for tests of the Standard Model (SM) and in searches for new physics effects. On the theoretical side, short-distance perturbative higher-order QCD and electroweak corrections are under good control for many processes. Moreover, tremendous progress in lattice computations [2] allows percentage to even subpercentage accuracy to be achieved for long-distance non-perturbative quantities. This allows for the prediction of some key observables with unprecedented accuracy and, in turn, the determination of short-distance parameters, such as the elements of the quark-mixing matrix (CKM) in the framework of the SM. Given these prospects, it is also desirable to improve the understanding and treatment of QED corrections, which are generally assumed to be small. Unfortunately, not much new development has taken place in the evaluation of such corrections.

For the future e^+e^- machines, the proper computation of QED corrections will be particularly important because the large data samples allow for precision measurements that require their inclusion in theoretical predictions. We would like to advocate a framework for a proper and systematic treatment of QED effects based on the effective field theory (EFT) approach, which exploits scale hierarchies present in processes involving mesons. In this spirit, QED corrections to $B_s \to \mu^+\mu^-$ have recently been analysed [3], revealing an unexpectedly large contribution owing to power enhancement. Such an effect cannot be found in the standard approach based on soft-photon approximation [4–6], as it requires a helicity flip induced by the photon. Further, the common assumption that hadrons are point-like objects neglects effects related to the structure of hadrons. It implies implicitly that the soft-photon approximation itself is performed in the framework of an EFT in which photons have virtuality below a typical hadronic binding scale $\Lambda_{\rm QCD} \sim \mathcal{O}(100\ \rm MeV)$ of partons in hadrons, below which they do not resolve the partonic structure of the hadrons. In consequence, this approach cannot address QED corrections, owing to virtualities above the scale $\Lambda_{\rm QCD}$. These observations are a motivation to scrutinise further QED corrections in flavour physics in the light of upcoming precise measurements and existing tensions in flavour measurements, in particular, related to tests of lepton flavour universality.

In addition to a systematic power counting, the EFT treatment offers the possibility of the all-order resummation of the corrections. This is particularly important for the mixed QCD–QED corrections, owing to the size of the QCD coupling constant and the presence of large logarithmic corrections. While the soft-exponentiation theorem allows resumming leading QED

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effects related to ultrasoft photons that do not resolve the partonic structure of hadrons, not much is known about the resummation of the subleading logarithms in QED for photons with larger virtuality. Standard factorisation theorems derived in QCD cannot be directly translated to QED, for, in the QCD case, the mass effects related to light degrees of freedom are typically neglected. This is not the case in QED, where the lepton mass provides a cut-off for collinear divergences. Moreover, the fact that in QCD one can observe only colour singlet states additionally simplifies the computations, while in QED, and more generally in the electroweak sector of the SM [7,8], it is necessary to account for charged particles in both the final and initial states. As a result, the QED factorisation theorems have not been explored intensively in the literature so far, but this gap should be filled before a precise e^+e^- collider becomes operational.

Power corrections to the standard soft approximation may also play an important role in certain processes. Studies of power corrections in the QCD case recently gained much attention [9–15]. New tools based on soft-collinear EFT (SCET) developed to study processes with energetic quarks and gluons can, after certain modifications, be applied to improve the accuracy of electroweak corrections in future lepton colliders. This is particularly important in collider physics for regions of phase space where the perturbative approach breaks down, owing to the presence of large logarithmic enhancements, and the next-to-soft effects become more important. Particularly interesting are mass-suppressed effects related to soft fermion exchange [16–18], whose consistent treatment in the SCET language is not yet fully known. Beyond applications to precision SM physics, the SCET framework may be necessary after possible discovery of new physics at the LHC [19,20].

10.2 QED corrections in $B_q \to \ell^+\ell^-$

The decay of a neutral meson $B_q \to \ell^+\ell^-$ ($\ell = e, \mu, \tau$) is the first step in an investigation of QED effects in QCD bound states. Its purely leptonic final state and neutral initial state keep complications related to the non-perturbative nature of QCD to the necessary minimum. Yet, as we shall see, even this simple example requires investigation of power corrections in SCET. The importance of this decay derives from the fact that it depends, at leading order (LO) in QED, only on the B_q meson decay constant, which can nowadays be calculated with subpercentage precision on the lattice [21], necessitating the inclusion of higher-order QED corrections from all scales at this level. This decay has been observed for $\ell = \mu$ by LHCb [22, 23], CMS [24], and ATLAS [25]. The currently measured branching fraction for B_s decays of about 3×10^{-9} is compatible with the latest SM predictions [3,26,27] and it is expected that the LHCb experiment will be able to measure the branching fraction with 5% accuracy with 50/fb (Run 4) around the year 2030 [28]. The FCC-ee running on the Z resonance is expected to provide, with about $\mathcal{O}(10^3)$ reconstructed events [29], an even higher event yield compared with LHCb Run 4. This, together with the cleaner hadronic environment at the FCC-ee, should allow better control of backgrounds and also systematic uncertainties, such that one can expect improved accuracy. However, the gain in accuracy cannot be quantified without a dedicated study.

On the theory side, electroweak and QCD corrections above the scale $\mu_b \sim 5$ GeV of the order of the b quark mass m_b are treated in the standard framework of weak EFT of the SM [30]. The effective Lagrangian is a sum of four-fermion and dipole operators

$$\mathcal{L}_{\Delta B=1} = \mathcal{N}_{\Delta B=1} \left[\sum_{i=1}^{10} C_i(\mu_b)\, Q_i \right] + \text{h.c.}\,, \tag{10.1}$$

with $\mathcal{N}_{\Delta B=1} \equiv 2\sqrt{2} G_F V_{tb} V_{tq}^*$ and covers, in principle, all weak decays of b hadrons. The pertinent operators relevant for $\mathrm{B_q} \to \ell^+\ell^-$ ($\mathrm{q} = \mathrm{d}, \mathrm{s}$) are

$$\begin{aligned} Q_7 &= \frac{e}{(4\pi)^2}\Big[\bar{q}\sigma^{\mu\nu}(m_\mathrm{b}P_R + m_\mathrm{q}P_L)b\Big]F_{\mu\nu}\,, \\ Q_9 &= \frac{\alpha_\mathrm{em}}{4\pi}\Big[\bar{q}\gamma^\mu P_L b\Big]\sum_\ell\Big[\bar{\ell}\gamma_\mu\ell\Big]\,, \\ Q_{10} &= \frac{\alpha_\mathrm{em}}{4\pi}\Big[\bar{q}\gamma^\mu P_L b\Big]\sum_\ell\Big[\bar{\ell}\gamma_\mu\gamma_5\ell\Big]\,. \end{aligned} \tag{10.2}$$

The matching $C_i(\mu_\mathrm{b})$ coefficients are computed at the electroweak scale $\mu_\mathrm{W} \sim \mathcal{O}(100\ \mathrm{GeV})$ and evolved to the scale of $\mu_\mathrm{b} \sim m_\mathrm{b}$ with the renormalization group equation of the weak EFT.

Because the neutral $\mathrm{B_q}$ meson is a pseudo-scalar and the SM interactions are mediated by axial and vector currents, the decay rate must vanish in the limit $m_\ell \to 0$, and therefore the decay amplitude is proportional to the lepton mass. The hadronic matrix element at LO in QED is parametrized by a single decay constant $f_{\mathrm{B_q}}$, defined by $\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_\mathrm{q}(p)\rangle = \mathrm{i}f_{\mathrm{B_q}}p^\mu$. The leading amplitude for $\mathrm{B_q} \to \ell^+\ell^-$ is

$$\mathrm{i}\mathcal{A} = m_\ell\, f_{\mathrm{B_q}}\,\mathcal{N}\,C_{10}(\mu_\mathrm{b})\,\Big[\bar{\ell}\gamma_5\ell\Big], \qquad \left(\mathcal{N} \equiv \mathcal{N}_{\Delta B=1}\frac{\alpha_\mathrm{em}}{4\pi}\right) \tag{10.3}$$

and the branching fraction is

$$\mathrm{Br}^{(0)}_{\mathrm{q}\ell} \equiv \mathrm{Br}^{(0)}\Big[\mathrm{B_q} \to \ell^+\ell^-\Big] = \frac{\tau_{\mathrm{B_q}} m^3_{\mathrm{B_q}} f^2_{\mathrm{B_q}}}{8\pi}\,|\mathcal{N}|^2\,\frac{m^2_\ell}{m^2_{\mathrm{B_q}}}\sqrt{1 - \frac{4m^2_\ell}{m^2_{B_\mathrm{q}}}}\,|C_{10}|^2\,, \tag{10.4}$$

with $m_{\mathrm{B_q}}$ denoting the mass of the meson and $\tau_{\mathrm{B_q}}$ its total lifetime. For neutral $\mathrm{B_s}$ mesons, the mixing needs to be accounted for [31], thereby allowing for the measurement of related CP asymmetries, to be discussed next. In this case, Eq. (10.4) refers to the 'instantaneous' branching fraction at time $t = 0$, which differs from the measured untagged time-integrated branching fraction by the factor $(1 - y_\mathrm{s}^2)/(1 + y_\mathrm{s}\mathcal{A}_{\Delta\Gamma})$, where $y_\mathrm{s} = \Delta\Gamma_\mathrm{s}/(2\Gamma_\mathrm{s})$ is related to the lifetime difference and $\mathcal{A}_{\Delta\Gamma}$ denotes the mass-eigenstate rate asymmetry. Concerning QED corrections, this branching fraction refers to the 'non-radiative' one prior to the inclusion of photon bremsstrahlung effects.

If one takes into account soft-photon radiation (both real and virtual) with energies smaller than the lepton mass, the decay amplitude is dressed by the standard Yennie–Frautschi–Suura exponent [4, 32]

$$\mathrm{Br}\Big[\mathrm{B_q} \to \ell^+\ell^- + n\gamma\Big] = \mathrm{Br}^{(0)}_{\mathrm{q}\ell} \times \left(\frac{2E_\mathrm{max}}{m_{\mathrm{B_q}}}\right)^{2\frac{\alpha_\mathrm{em}}{\pi}\left(\ln\frac{m^2_{\mathrm{B_q}}}{m^2_\ell} - 1\right) + \mathcal{O}(m_\ell)}\,. \tag{10.5}$$

This 'photon-inclusive' branching fraction is based on eikonal approximation, in the limit when the total energy carried away by the n photons, E_max, is much smaller than the lepton mass. The QED corrections in the initial state are entirely neglected and photons are assumed to couple to leptons through eikonal currents

$$J^\mu(q) = e\sum_i Q_i\eta_i\frac{p_i^\mu}{p_i\cdot q}, \tag{10.6}$$

where $\eta = -1$ for incoming particles and $\eta = +1$ for outgoing particles. The sum runs over all charged particles with momenta p_i and charges Q_i. Eikonal currents are spin-independent and thus they do not change the helicity of the leptons.

From this point, we focus only on the case of muons in the final state, $\ell = \mu$. In the experimental analysis [23–25], the signal is simulated fully inclusive of final-state radiation off the muons by applying PHOTOS [33] corresponding to a convolution of the $E_{\rm max}$-dependent exponential factor in the determination of the signal efficiency. Conversely, photon emission from the quarks (initial state) vanishes in the limit of small photon energies because it is infrared-safe, since the decaying meson is electrically neutral. Hence, it can be neglected as long as the signal window is sufficiently small, in practice of $\mathcal{O}(60\ \text{MeV})$ [34], and is effectively treated as negligible background on both experimental and theory sides. In consequence, the experimental analyses currently provide the non-radiative branching fraction relying on the simulation with PHOTOS.

The limitations of the conventional approximation had missed the important effect responsible for the power enhancement of QED corrections to the $\mathrm{B_s} \to \mu^+\mu^-$ decay. Indeed, even when the cut on the real photon emission is much smaller than the muon mass, virtual photons with virtualities of the order of the muon mass or larger can resolve the structure of the meson, whose typical size is of the order of $1/\Lambda_{\rm QCD}$. In this case, the meson cannot be treated as a point-like object. Moreover, the eikonal approximation is not suitable for such photons, as they can induce a helicity flip of the leptons. However, straightforward computation of the QED corrections is not possible, as it requires the evaluation of non-local time-ordered products of the $\mathcal{L}_{\Delta B=1}(0)$ Lagrangian with the electromagnetic current $j_{\rm QED} = Q_{\rm q}\bar{q}\gamma^\mu q$, such as

$$\langle 0| \int \mathrm{d}^4x\, T\{j_{\rm QED}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_{\rm q}\rangle. \tag{10.7}$$

Currently, this object is beyond the reach of lattice QCD, while the SCET approach allows one to systematically expand this matrix element and reduce the non-perturbative quantities to universal ones at leading order.

Let us consider Fig. B.10.1, where the photon is exchanged between the light quark and the lepton. There are two low-energy scales in the diagram, set by the external kinematics of the process $\mathrm{B_q} \to \mu^+\mu^-$. One is the muon mass m_μ, which is related to the collinear scale. We parametrize the lepton momentum in terms of the light-cone co-ordinates as $p_\ell = (n_+p_\ell,\, n_-p_\ell,\, p_\ell^\perp) \sim m_{\rm b}\,(1,\, \lambda_{\rm c}^2,\, \lambda_{\rm c})$, where we introduced the small counting parameter $\lambda_{\rm c} \sim m_\mu/m_{\rm b}$. The second low-energy scale is related to the typical size of the soft light quark momentum $l_{\rm q} \sim \Lambda_{\rm QCD}$ and for counting purposes we introduce $\lambda_{\rm s} \sim \Lambda_{\rm QCD}/m_{\rm b}$. In the case of muons, it happens that numerically $\lambda_{\rm c} \approx \lambda_{\rm s}$ and in the following we equate them and do not distinguish between them. It turns out that there also exists a hard-collinear invariant constructed from the lepton and quark momentum $p_\ell \cdot l_{\rm q} \sim \lambda m_{\rm b}^2$, thus in addition to the collinear and soft regions we must also consider a hard-collinear region, where momenta scale as $k \sim m_{\rm b}\left(1,\, \lambda,\, \lambda^{1/2}\right)$. This non-trivial hierarchy of intermediate scales must be properly accounted to evaluate the leading QED corrections, which can be done by subsequent matching on $\text{SCET}_{\rm I}$ and $\text{SCET}_{\rm II}$ [35] at the hard ($\sim m_{\rm b}$) and hard-collinear scales, respectively.

The power enhancement is directly related to the interplay of collinear and hard-collinear scales. When the hard-collinear or collinear photon interacts with the soft quark, momentum conservation forces the quark to become hard-collinear. These modes can be integrated out perturbatively with the help of the EFT methods. In this case, we must first match the operators

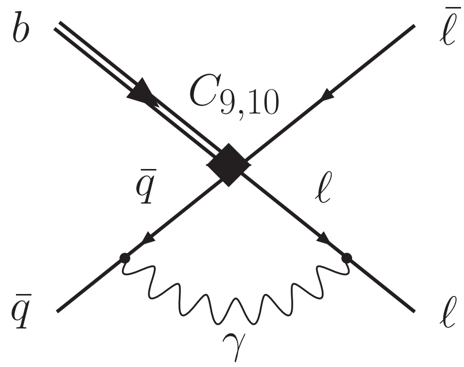

Fig. B.10.1: Example diagram that gives rise to the power-enhanced QED correction. A photon can be either collinear with virtuality $k^2 \sim m_\mu^2$ or hard-collinear, $k^2 \sim m_\mu m_b$.

in Eq. (10.2) on SCET_I currents [36]. In SCET_I, we retain soft, collinear, and hard-collinear modes; only the hard modes are integrated out. The leading SCET_I operator contains a hard-collinear quark field, which scales as $\lambda^{1/2}$ instead of the soft quark field with scaling $\lambda^{3/2}$. When we integrate out the hard-collinear modes, we must convert the hard-collinear quark field $\xi_\text{C}(x)$ to the soft quark field q_s. This is done with the help of power-suppressed Lagrangian [37]

$$\mathcal{L}^{(1)}_{\xi\text{q}} = \bar{q}_\text{s}(x_-)\, W^\dagger_{\xi\text{C}} \text{i}\not{D}_\perp \,\xi_\text{C}(x) - \bar{\xi}_\text{C}(x)\,\text{i}\overleftarrow{\not{D}}_\perp W_{\xi\text{C}}\, q_\text{s}(x_-),$$

where $W_{\xi\text{C}}$ is a collinear Wilson line carrying charge of the collinear field ξ_C. This Lagrangian insertion costs an additional power of $\lambda^{1/2}$, but the resulting SCET_II operators are still power-enhanced, as compared with the operators obtained without an intermediate hard-collinear scale. The power-enhanced correction to the amplitude is [3]

$$\begin{aligned} i\Delta\mathcal{A} = \frac{\alpha_\text{em}}{4\pi} Q_\ell Q_\text{q} m_\ell m_{\text{B}_\text{q}} f_{\text{B}_\text{q}} \mathcal{N}\left[\bar{\ell}(1+\gamma_5)\ell\right] \\ \times \Bigg\{ \int_0^1 \text{d}u (1-u) C_9^\text{eff}(u m_\text{b}^2) \int_0^\infty \frac{\text{d}\omega}{\omega} \phi_{\text{B}+}(\omega) \left[\ln \frac{m_\text{b}\omega}{m_\ell^2} + \ln \frac{u}{1-u}\right] \\ - Q_\ell C_7^\text{eff} \int_0^\infty \frac{\text{d}\omega}{\omega}\, \phi_{\text{B}+}(\omega) \left[\ln^2 \frac{m_\text{b}\omega}{m_\ell^2} - 2\ln \frac{m_\text{b}\omega}{m_\ell^2} + \frac{2\pi^2}{3}\right] \Bigg\}, \end{aligned} \tag{10.8}$$

where $\phi_{\text{B}+}(\omega)$ is the B_q meson light-cone distribution amplitude (LCDA), which contains information about the non-perturbative structure of the meson. This virtual correction is, by itself, infrared-finite, as it modifies the exclusive decay rate. The power enhancement manifests itself in Eq. (10.8) as the inverse power of the ω variable that results from the decoupling of the hard-collinear quark modes

$$m_{\text{B}_\text{q}} \int_0^\infty \frac{\text{d}\omega}{\omega}\, \phi_{\text{B}+}(\omega)\, \ln^k \omega \sim \frac{m_{\text{B}_\text{q}}}{\Lambda_\text{QCD}} \sim \frac{1}{\lambda}. \tag{10.9}$$

The ω may be interpreted as a momentum of the soft quark along the light-cone direction of the lepton, and thus $\omega \sim \Lambda_\text{QCD}$. The annihilation of the quark into leptons is a non-local process in the presence of the QED interactions and the virtual leptons with the wrong helicity can propagate over distances of the order of the meson size. Thus, the helicity flip costs a factor $m_\ell/\Lambda_{\rm QCD}$ instead of the typical suppression factor of $m_\ell/m_{\rm b}$ present in the leading-order amplitude.

The terms proportional to C_{10} cancel after the collinear and anticollinear contributions are added, such that only C_9 contributes out of the semileptonic operators. The term $\propto C_7$ requires separate treatment since the convolution integral containing the hard matching coefficient exhibits an endpoint singularity. In addition, the collinear contribution has a rapidity-type divergence. There exists an additional contribution related to the soft region, which, after a suitable rapidity regularisation, can be combined with the collinear contribution. When the convolution integral is performed in dimensional regularisation before taking the limit $d \to 4$, the total correction is finite and exhibits the double-logarithmic enhancement.

The numerical evaluation [3] of the power-enhanced correction (Eq. (10.8)) shows a partial cancellation of the terms $\propto C_9^{\rm eff}$ and $\propto C_7^{\rm eff}$. The final impact on the branching fraction $\mathrm{Br}^{(0)}_{\mathrm{q}\mu}$ is a decrease in the range (0.3–1.1)%, with a central value of 0.7%. Despite the cancellation, the overall correction is still sizeable compared with the natural size of a QED correction of $\alpha_{\rm em}/\pi \sim 0.3\%$. The large uncertainties of the power-enhanced QED correction are due to the poorly known inverse moment $\lambda_{\rm B}$ and almost unknown inverse-logarithmic moments σ_1 and σ_2 of the B meson LCDA.* The prediction for the muonic modes for the untagged time-integrated branching fractions for $B_{\rm s} \to \mu^+\mu^-$ and $B_{\rm d} \to \mu^+\mu^-$ are

$$\overline{\mathrm{Br}}^{(0)}_{\mathrm{s}\mu} = \begin{pmatrix} 3.59 \\ 3.65 \end{pmatrix} \left[1 \pm \begin{pmatrix} 0.032 \\ 0.011 \end{pmatrix}_{f_{\mathrm{B_s}}} \pm 0.031|_{\rm CKM} \pm 0.011|_{m_{\rm t}} \pm 0.012|_{\text{non-pmr}} \pm 0.006|_{\rm pmr} \pm {}^{+0.003}_{-0.005}|_{\rm QED} \right] \times 10^{-9}, \tag{10.10}$$

$$\overline{\mathrm{Br}}^{(0)}_{\mathrm{d}\mu} = \begin{pmatrix} 1.05 \\ 1.02 \end{pmatrix} \left[1 \pm \begin{pmatrix} 0.045 \\ 0.014 \end{pmatrix}_{f_{\mathrm{B_d}}} \pm 0.046|_{\rm CKM} \pm 0.011|_{m_{\rm t}} \pm 0.012|_{\text{non-pmr}} \pm 0.003|_{\rm pmr} \pm {}^{+0.003}_{-0.005}|_{\rm QED} \right] \times 10^{-10}, \tag{10.11}$$

where we group uncertainties: (i) main parametric long-distance ($f_{\mathrm{B_q}}$) and short-distance (CKM and $m_{\rm t}$), (ii) remaining non-QED parametric ($\tau_{\mathrm{B_q}}$, $\alpha_{\rm s}$) and non-QED non-parametric ($\mu_{\rm W}$, $\mu_{\rm b}$, higher order, see Ref. [26]), and (iii) from the QED correction ($\lambda_{\rm B}$ and $\sigma_{1,2}$, see Ref. [3]). We provide here two values, depending on the choice of the lattice calculation of $f_{\mathrm{B_q}}$ for $N_f = 2+1$ (upper) and $N_f = 2+1+1$ (lower), with averages from FLAG 2019 [2]. Note that the small uncertainties of the $N_f = 2+1+1$ results are currently dominated by a single group [21] and confirmation by other lattice groups in the future is desirable. It can be observed that, in this case, the largest uncertainties are due to CKM parameters, such that they can be determined provided the accuracy of the measurements at the FCC-ee is at the 1% level. Still fairly large errors are due to the top quark mass $m_{\rm t} = (173.1 \pm 0.6)$ GeV, here assumed to be in the pole scheme, where an additional non-parametric uncertainty of 0.2% is included (in 'non-pmr') for the conversion to the $\overline{\rm MS}$ scheme. Further 'non-pmr' contains a 0.4% uncertainty from $\mu_{\rm W}$ variation and 0.5% further higher-order uncertainty, all linearly added. For the CKM input, we use Refs. [3, 27].

As mentioned, for the $B_{\rm s}$ meson, the mixing provides the opportunity to measure CP asymmetries in a time-dependent analysis

$$\frac{\Gamma[\mathrm{B_s}(t) \to \mu^+_\lambda \mu^-_\lambda] - \Gamma[\bar{\mathrm{B}}_{\rm s}(t) \to \mu^+_\lambda \mu^-_\lambda]}{\Gamma[\mathrm{B_s}(t) \to \mu^+_\lambda \mu^-_\lambda] + \Gamma[\bar{\mathrm{B}}_{\rm s}(t) \to \mu^+_\lambda \mu^-_\lambda]} = \frac{C_\lambda \cos(\Delta m_{\mathrm{B_s}} t) + S_\lambda \sin(\Delta m_{\mathrm{B_s}} t)}{\cosh(y_{\rm s} t/\tau_{\mathrm{B_s}}) + \mathcal{A}^\lambda_{\Delta\Gamma} \sinh(y_{\rm s} t/\tau_{\mathrm{B_s}})}\,, \tag{10.12}$$

*Throughout, the same numerical values as in Ref. [3] are used for $B_{\rm s}$ and $B_{\rm d}$, neglecting $SU(3)$-flavour breaking effects.

where all quantities are defined in Ref. [31] and $|\mathcal{A}^\lambda_{\Delta\Gamma}|^2 + |C_\lambda|^2 + |S_\lambda|^2 = 1$ holds. For example, the mass-eigenstate rate asymmetry $\mathcal{A}_{\Delta\Gamma} = +1$ in the SM exactly, if only a pseudo-scalar amplitude exists, and is therefore assumed to be very sensitive to possible new flavour-changing interactions, with essentially no uncertainty from SM background. We now see that the QED correction of the SM itself generates small 'contamination' of the observable, given by Ref. [3]

$$\mathcal{A}^\lambda_{\Delta\Gamma} \approx 1 - 1.0 \cdot 10^{-5}\,, \qquad S_\lambda \approx -0.1\%\,, \qquad C_\lambda \approx \eta_\lambda\, 0.6\%\,, \tag{10.13}$$

where $\eta_{L/R} = \pm 1$. Present measurements [23] set only very weak constraints on the deviations of $A^\lambda_{\Delta\Gamma}$ from unity, and C_λ, S_λ have not yet been measured,† but the uncertainty in the B meson LCDA is, in principle, a limiting factor for the precision with which new physics can be constrained from these observables. Also, S_λ and C_λ deviate marginally from the leading-order SM prediction of zero, but signals from new physics should be substantially larger to distinguish them from the SM QED correction.

A similar framework can be used to analyse QED corrections to $\mathrm{B}^\pm \to \ell^\pm \nu_\ell$. In this case, power enhancement does not arise, owing to the different chirality structure of the current and the presence of only one charged lepton in the final state [3]. QED corrections that depend on the meson structure are subleading in this case. The leading QED corrections for this process can be obtained from the usual soft-photon approximation, where the charged meson is treated as a point-like charge.

10.3 Summary and outlook

The proper treatment of QED corrections in theoretical predictions is essential to the success of future $\mathrm{e}^+\mathrm{e}^-$ colliders. We have shown how this goal could be achieved in flavour physics for the example of a power-enhanced leading QED correction to the leptonic decays $\mathrm{B_q} \to \mu^+\mu^-$ with $\mathrm{q} = \mathrm{d,s}$ [3] and provided updated predictions. A systematic expansion based on the appropriate EFTs must be implemented to cover dynamics from the hard scale $\mu_\mathrm{b} \sim 5$ GeV over hard-collinear ($\mathrm{SCET_I}$) and collinear scales ($\mathrm{SCET_{II}}$) down to the ultrasoft scales $\mathcal{O}(10\,\mathrm{MeV})$. Further, the EFTs allow for a systematic resummation of the leading logarithmic corrections and they provide a field-theoretical definition of non-perturbative objects in the presence of QED, as, for example, generalised light-cone distribution amplitudes of the B meson dressed by process-dependent Wilson lines [36]. The consistent evaluation of the QED corrections is thus a challenging task, but it can be accomplished with the help of effective field theory.

In the example at hand, the special numerical value of the muon mass and its proximity to the typical size of hadronic binding energies Λ_QCD gave rise to a special tower of EFTs. The application to the cases of electrons and taus requires additional considerations. Full theoretical control of QED corrections is also desirable for other decays that will allow future precision determination of short-distance parameters. For example, an important class is that of exclusive $\mathrm{b} \to \mathrm{u}\ell\bar{\nu}_\ell$ and $\mathrm{b} \to \mathrm{c}\ell\bar{\nu}_\ell$ decays for the determination of CKM elements V_ub and V_cb, respectively. Owing to the absence of resonant hadronic contributions, the only hadronic uncertainties from $\mathrm{B} \to \mathrm{M}$ form factors could become controllable with high accuracy in lattice calculations for large dilepton invariant masses, i.e., energetic leptons, which is also the preferred kinematic region for the tower of EFTs discussed here. Other interesting applications are observables that are predicted in the SM to vanish when restricting to the leading order in the weak

†Note that C^λ requires the measurement of the muon helicity, whereas $\mathcal{A}^\lambda_{\Delta\Gamma}$ and S_λ can also be determined as averages over the muon helicity; furthermore, $\mathcal{A}^\lambda_{\Delta\Gamma}$ can be measured without flavour-tagging, whereas it is required for S_λ and C_λ.

operator product expansion but might be sensitive to non-standard interactions. Then the QED corrections in the SM provide a background to the new physics searches, as in the example of $\mathcal{A}_{\Delta\Gamma}$ in $\mathrm{B_s} \to \mu^+\mu^-$ given here. This concerns observables in the angular distributions of $\mathrm{B} \to \mathrm{K}^{(*)}\ell^+\ell^-$ as, for example, discussed in Refs. [38, 39].

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