3 Precision predictions for Higgs decays in the (N)MSSM

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3.1 Introduction

The signal that was discovered in the Higgs searches at ATLAS and CMS at a mass of ∼125 GeV [1–3] is, within current theoretical and experimental uncertainties, compatible with the properties of the Higgs boson predicted within the Standard Model (SM) of particle physics. No conclusive signs of physics beyond the SM have been reported so far. However, the measurements of Higgs signal strengths for the various channels leave considerable room for Beyond Standard Model (BSM) interpretations. Consequently, the investigation of the precise properties of the discovered Higgs boson will be one of the prime goals at the LHC and beyond. While the mass of the observed particle is already known with excellent accuracy [4, 5], significant improvements of the information about the couplings of the observed state are expected from the upcoming runs of the LHC [3, 6–9] and even more so from the high-precision measurements at a future e^+e^- collider [10–18]. For the accurate study of the properties of the Higgs boson, precise predictions for the various partial decay widths, the branching ratios (BRs), and the Higgs boson production cross-sections, along with their theoretical uncertainties, are indispensable.

Motivated by the 'hierarchy problem', supersymmetry (SUSY) inspired extensions of the SM play a prominent role in the investigations of possible new physics. As such, the minimal supersymmetric Standard Model (MSSM) [19, 20] or its singlet extension, the next-to-MSSM (NMSSM) [21, 22], have been the object of many studies in the last decades. Despite this attention, these models are not yet prepared for an era of precision tests, as the uncertainties at the level of the Higgs mass calculation [23–25] are about one order of magnitude larger than the experimental uncertainty. At the level of the decays, the theoretical uncertainty arising from unknown higher-order corrections has been estimated for the case of the Higgs boson of the SM (where the Higgs mass is treated as a free input parameter) in Refs. [26, 27] and updated in Ref. [28]: depending on the channel and the Higgs mass, it typically falls in the range of ∼0.5–5%. To our knowledge, no similar analysis has been performed in SUSY-inspired models (or other BSM models), but one can expect the uncertainties from missing higher-order corrections to be larger in general—with many nuances, depending on the characteristics of the Higgs state and the considered point in parameter space: we provide some discussion of this issue at the end of this section. In addition, parametric uncertainties that are induced by the experimental errors of the input parameters should be taken into account. For the case of the SM decays, those parametric uncertainties have been discussed in the references cited. In the SUSY case, the parametric uncertainties induced by the (known) SM input parameters can be determined in the same way as for the SM, while the dependence on unknown SUSY parameters can be utilised in setting constraints on those parameters. While still competitive

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today, the level of accuracy of the theoretical predictions of Higgs boson decays in SUSY models should soon become outclassed by the achieved experimental precision (in particular at future e^+e^- colliders) on the decays of the observed Higgs signal. Without comparable accuracy of the theoretical predictions, the impact of the exploitation of the precision data will be diminished—either in terms of further constraining the parameter space or of interpreting deviations from the SM results. Further efforts towards improving the theoretical accuracy are therefore necessary in order to enable a thorough investigation of the phenomenology of these models. Besides the decays of the SM-like state at 125 GeV of a SUSY model—where the goal is clearly to reach an accuracy that is comparable to the case of the SM—it is also of interest to obtain reliable and accurate predictions for the decays of the other Higgs bosons in the spectrum. The decays of the non-SM-like Higgs bosons can be affected by large higher-order corrections as a consequence of either large enhancement factors or a suppression of the lowest-order contribution. Confronting accurate predictions with the available search limits yields important constraints on the parameter space. Here, we review the evaluation of the decays of the neutral Higgs bosons of the $\mathbb{Z}_3$-conserving NMSSM into SM particles, as presented in Ref. [29].

Current work focusing on NMSSM Higgs decays is part of the effort for developing a version of `FeynHiggs` [23, 30–37] dedicated to the NMSSM [38, 39]. The general methodology relies on a Feynman-diagrammatic calculation of radiative corrections, which employs `FeynArts` [40,41], `FormCalc` [42], and `LoopTools` , [42]. The renormalization scheme has been implemented within the NMSSM [39] in such a way that the result in the MSSM limit of the NMSSM exactly coincides with the MSSM result obtained from `FeynHiggs` without any further adjustments of parameters.

3.2 Higgs decays to SM particles in the $\mathcal{CP}$-violating NMSSM

In this section, we review the technical aspects of our calculation of the Higgs decays. Our notation and the renormalization scheme that we employ for the $\mathbb{Z}_3$-conserving NMSSM in the general case of complex parameters are presented in Section 2 of Ref. [39], and we refer the reader to that article for further details.

3.2.1 Decay amplitudes for a physical (on-shell) Higgs state—generalities

3.2.1.1 On-shell external Higgs leg

In this section, we consider the decays of a physical Higgs state, i.e., an eigenstate of the inverse propagator matrix for the Higgs fields, evaluated at the corresponding pole eigenvalue. The connection between such a physical state and the tree-level Higgs fields entering the Feynman diagrams is non-trivial in general since the higher-order contributions induce mixing among the Higgs states and between the Higgs states and the gauge bosons (as well as the associated Goldstone bosons). The LSZ reduction fully determines the (non-unitary) transition matrix $\mathbf{Z}^{\text{mix}}$ between the loop-corrected mass eigenstates and the lowest-order states. Then, the amplitude describing the decay of the physical state h_i^{phys} (we shall omit the superscript 'phys' later on), into e.g., a fermion pair $\text{f}\bar{\text{f}}$, relates to the amplitudes in terms of the tree-level states h_j^0 according to (see the following for the mixing with gauge bosons and Goldstone bosons):

$$\mathcal{A}[h_i^{\text{phys}} \to \text{f}\bar{\text{f}}\,] = Z_{ij}^{\text{mix}}\, \mathcal{A}[h_j^0 \to \text{f}\bar{\text{f}}\,]\,. \tag{3.1}$$

Here, we characterize the physical Higgs states according to the procedure outlined in Ref. [39] (see also Refs. [32, 43, 44]).

1. The Higgs self-energies include full one-loop and leading $\mathcal{O}(\alpha_t\alpha_s, \alpha_t^2)$ two-loop corrections (with two-loop effects obtained in the MSSM approximation via the publicly available code `FeynHiggs`[†]).

2. The pole masses correspond to the zeros of the determinant of the inverse propagator matrix.

3. The (5×5) matrix $\mathbf{Z}^{\text{mix}}$ is obtained in terms of the solutions of the eigenvector equation for the effective mass matrix evaluated at the poles, and satisfying the appropriate normalization conditions (see Section 2.6 of Ref., [39]).

In correcting the external Higgs legs by the full matrix $\mathbf{Z}^{\text{mix}}$—instead of employing a simple diagrammatic expansion—we resum contributions to the transition amplitudes that are formally of higher-loop order. This resummation is convenient for taking into account numerically relevant leading higher-order contributions. It can, in fact, be crucial for the frequent case where radiative corrections mix states that are almost mass-degenerate in order to properly describe the resonance-type effects that are induced by the mixing. Conversely, care needs to be taken to avoid the occurrence of non-decoupling terms when Higgs states are well-separated in mass, since higher-order effects can spoil the order-by-order cancellations with vertex corrections.

We stress that all public tools, with the exception of `FeynHiggs`, neglect the full effect of the transition to the physical Higgs states encoded within $\mathbf{Z}^{\text{mix}}$, and instead employ the unitary approximation $\mathbf{U}^0$ neglecting external momenta (which is in accordance with leading-order or QCD-improved leading-order predictions). We refer the reader to Refs. [32,39,44] for the details of the definition of $\mathbf{U}^0$ or $\mathbf{U}^m$ (another unitary approximation), as well as a discussion of their impact at the level of Higgs decay widths.

3.2.1.2 Higgs–electroweak mixing

For the mass determination, we do not take into account contributions arising from the mixing of the Higgs fields with the neutral Goldstone or Z bosons, since these corrections enter at the subdominant two-loop level (contributions of this kind can also be compensated by appropriate field-renormalization conditions [47]). We note that, in the $\mathcal{CP}$-conserving case, only external $\mathcal{CP}$-odd Higgs components are affected by such a mixing. Yet, at the level of the decay amplitudes, the Higgs mixing with the Goldstone and Z bosons already enters at the one-loop order (even if the corresponding self-energies are cancelled by an appropriate field-renormalization condition, this procedure will still provide a contribution to the $h_i f\bar{f}$ counterterm). Therefore, for a complete one-loop result of the decay amplitudes, it is, in general, necessary to incorporate Higgs–Goldstone and Higgs–Z self-energy transition diagrams [43, 48, 49]. In the following, we evaluate such contributions to the decay amplitudes in the usual diagrammatic fashion (as prescribed by the LSZ reduction), with the help of the `FeynArts` model file for the $\mathcal{CP}$-violating NMSSM [39]. The corresponding one-loop amplitudes (including the associated counterterms) will be symbolically denoted as $\mathcal{A}^{1L}_{G/Z}$. These amplitudes can be written in terms of the self-energies $\Sigma_{h_i G/Z}$ with Higgs and Goldstone or Z bosons in the external legs. In turn, these self-energies are connected by a Slavnov–Taylor identity (see e.g., Appendix A of

[†]The Higgs masses in `FeynHiggs` could be computed with additional improvements, such as additional fixed-order results [45,46] or the resummation of large logarithms for very heavy SUSY particles [33–35]; for simplicity, we do not take such refinements into account in this section.

Ref. [50]):[‡]

$$\begin{aligned} 0 &= M_{\mathrm{Z}}\,\Sigma_{h_i\mathrm{G}}\left(p^2\right) + \mathrm{i}\,p^2\,\Sigma_{h_i\mathrm{Z}}\left(p^2\right) + M_{\mathrm{Z}}\left(p^2 - m_{h_i}^2\right) f\left(p^2\right) \\ &\quad - \frac{e}{2\,s_{\mathrm{w}}\,c_{\mathrm{w}}} \sum_j \left[(U_n)_{i1}(U_n)_{j4} - (U_n)_{i2}(U_n)_{j5} - (U_n)_{j1}(U_n)_{i4} + (U_n)_{j2}(U_n)_{i5}\right] T_{h_j}\,, \end{aligned} \tag{3.2a}$$

$$\begin{aligned} f\left(p^2\right) \equiv -\frac{\alpha}{16\,\pi\,s_{\mathrm{w}}\,c_{\mathrm{w}}} \sum_j &\left[(U_n)_{i1}(U_n)_{j4} - (U_n)_{i2}(U_n)_{j5} - (U_n)_{j1}(U_n)_{i4} + (U_n)_{j2}(U_n)_{i5}\right] \\ &\times \left[c_\beta\,(U_n)_{j1} + s_\beta\,(U_n)_{j2}\right] B_0\left(p^2, m_{h_j}^2, M_Z^2\right), \end{aligned} \tag{3.2b}$$

where the T_{h_i} correspond to the tadpole terms of the Higgs potential and $(U_n)_{ij}$ are the elements of the transition matrix between the gauge- and tree-level mass-eigenstate bases of the Higgs bosons—the notation is introduced in Section 2.1 of Ref. [39]. Similar relations in the MSSM are also provided in Eq. (127) of Ref. [43]. We checked this identity at the numerical level.

3.2.1.3 Inclusion of one-loop contributions

The wave function normalization factors contained in $\mathbf{Z}^{\mathrm{mix}}$, together with the described treatment of the mixing with the Goldstone and Z bosons, ensure the correct on-shell properties of the external Higgs leg in the decay amplitude, so that no further diagrams correcting this external leg are needed. Moreover, the SM fermions and gauge bosons are also treated as on-shell particles in our renormalization scheme. Beyond the transition to the loop-corrected states incorporated by $\mathbf{Z}^{\mathrm{mix}}$, we thus compute the decay amplitudes at the one-loop order as the sum of the tree-level contribution $\mathcal{A}^{\mathrm{tree}}$ (possibly equal to zero), the Higgs–electroweak one-loop mixing $\mathcal{A}^{\mathrm{1L}}_{\mathrm{G/Z}}$ and the (renormalised) one-loop vertex corrections $\mathcal{A}^{\mathrm{1L}}_{\mathrm{vert}}$ (including counterterm contributions)—we note that each of these pieces of the full amplitude is separately ultraviolet-finite. In the example of the $\mathrm{f\bar{f}}$ decay, the amplitudes with a tree-level external Higgs field h_j^0—on the right-hand side of Eq. (3.1)—thus symbolically read

$$\mathcal{A}[h_j^0 \to \mathrm{f\bar{f}}\,] = \mathcal{A}^{\mathrm{tree}}[h_j^0 \to \mathrm{f\bar{f}}\,] + \mathcal{A}^{\mathrm{1L}}_{\mathrm{G/Z}}[h_j^0 \to \mathrm{f\bar{f}}\,] + \mathcal{A}^{\mathrm{1L}}_{\mathrm{vert}}[h_j^0 \to \mathrm{f\bar{f}}\,]\,. \tag{3.3}$$

All the pieces on the right-hand side of this equation are computed with the help of `FeynArts` [40,41], `FormCalc` [42], and `LoopTools` [42], according to the prescriptions that are encoded in the model file for the $\mathcal{CP}$-violating NMSSM. However, we use a specific treatment for some of the contributions, such as QED and QCD one-loop corrections to Higgs decays into final-state particles that are electrically or colour charged, or include certain higher-order corrections. We describe these channel-specific modifications in the following subsections.

3.2.1.4 Goldstone-boson couplings

The cubic Higgs–Goldstone-boson vertices can be expressed as

$$\begin{aligned} \mathcal{L} \ni -\frac{1}{\sqrt{2}\,v} \Bigg\{ &\sum_j m_{h_j}^2 \left[\cos\beta\,(U_n)_{j1} + \sin\beta\,(U_n)_{j2}\right] h_j^0 \left[G^+G^- + \tfrac{1}{2}\left(G^0\right)^2\right] \\ &+ \Bigg[\sum_j \left(m_{\mathrm{H}^\pm}^2 - m_{h_j}^2\right) \left(\sin\beta\left[(U_n)_{j1} + \mathrm{i}\,(U_n)_{j4}\right] - \cos\beta\left[(U_n)_{j2} - \mathrm{i}\,(U_n)_{j5}\right]\right) h_j^0 H^+G^- + \mathrm{h.\,c.}\Bigg] \end{aligned}$$

[‡]We denote the imaginary unit by i.

$$+\frac{1}{2}\sum_{j,k}\left(m_{h_k}^2 - m_{h_j}^2\right)\left[(U_n)_{j1}(U_n)_{k4} - (U_n)_{j2}(U_n)_{k5} - (j \leftrightarrow k)\right] h_j^0 h_k^0 G^0 \Bigg\} . \quad (3.4)$$

The doublet vacuum expectation value (VEV), $v = M_{\mathrm{W}}\, s_{\mathrm{w}}/\sqrt{2\,\pi\,\alpha}$, is expressed in terms of the gauge-boson masses M_{W} and M_{Z} $\left(s_{\mathrm{w}} = \sqrt{1 - M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2}\right)$, as well as the electromagnetic coupling α. The symbol $m_{h_j}^2$, $(j = 1, \ldots, 5)$, represents the tree-level mass squared of the neutral Higgs state h_j^0, and $m_{\mathrm{H}^\pm}^2$ represents the mass squared of the charged Higgs state.

The use of the tree-level couplings of Eq. (3.4), together with a physical (loop-corrected) external Higgs leg $h_i = \sum_j Z_{ij}^{\mathrm{mix}}\, h_j^0$, is potentially problematic regarding the gauge properties of the matrix elements. The structure of the gauge theory and its renormalization indeed guarantee that the gauge identities are observed at the order of the calculation (one loop). However, the evaluation of Feynman amplitudes is not protected against a violation of the gauge identities at the (incomplete) two-loop order. We detected such gauge-violating effects of two-loop order at several points in our calculation of the neutral Higgs decays.

1. The Ward identity in $h_i \to \gamma\gamma$ is not satisfied (see also Ref. [51]).

2. Infrared (IR) divergences of the virtual corrections in $h_i \to \mathrm{W}^+\mathrm{W}^-$ do not cancel their counterparts in the bremsstrahlung process $h_i \to \mathrm{W}^+\mathrm{W}^-\gamma$ (see also Ref. [52]).

3. Computing $h_i \to \mathrm{f}\bar{\mathrm{f}}$ in an R_ξ gauge entails non-vanishing dependence of the amplitudes on the electroweak gauge-fixing parameters ξ_{Z} and ξ_{W}.

As these gauge-breaking effects could intervene with sizeable and uncontrolled numerical impact, it is desirable to add two-loop order terms, restoring the gauge identities at the level of the matrix elements. Technically, there are different possible procedures to achieve this: one would amount to replacing the kinematic Higgs masses that appear in Higgs–gauge-boson couplings with tree-level Higgs masses; we prefer the alternative procedure, which involves changing the Higgs–Goldstone-boson couplings of Eq. (3.4): for the Higgs mass associated to the external Higgs leg, the loop-corrected Higgs mass M_{h_i} is used instead of the tree-level one. This is actually the form of the Higgs–Goldstone-boson coupling that would be expected in an effective field theory of the physical Higgs boson h_i. Using the definition of Z_{ij}^{mix} as an eigenvector of the loop-corrected mass matrix for the eigenvalue $M_{h_i}^2$—see Section 2.6 of Ref. [39]—one can verify that the effective Higgs–Goldstone-boson vertices employing the physical Higgs mass differ from their tree-level counterparts by a term of one-loop order (proportional to the Higgs self-energies) so that the alteration of the one-loop amplitudes is indeed of two-loop order. Employing this shift of the Higgs–Goldstone couplings cures the gauge-related issues that we mentioned earlier.

Another issue with gauge invariance appears in connection with the amplitudes $\mathcal{A}_{\mathrm{G/Z}}^{1\mathrm{L}}$. The Goldstone and Z boson propagators generate denominators with pole M_{Z}^2 (or $\xi_{\mathrm{Z}}\, M_{\mathrm{Z}}^2$ in an R_ξ gauge): in virtue of the Slavnov–Taylor identity of Eq. (3.2a), these terms should cancel one another in the total amplitude at the one-loop order—we refer the reader to Section 4.3 of Ref. [43] for a detailed discussion. However, the term $(p^2 - M_{\mathrm{Z}}^2)^{-1}$ multiplying $f(p^2)$ of Eq. (3.2a) only vanishes if $p^2 = m_{h_i}^2$: if we employ $p^2 = M_{h_i}^2$ (the loop-corrected Higgs mass), the cancellation is spoilt by a term of two-loop order. To address this problem, we redefine

$\mathcal{A}^{1L}_{G/Z}$ by adding a two-loop term:

$$\tilde{\mathcal{A}}^{1L}_{G/Z}[h_i \to f\bar{f}\,] \equiv Z^{mix}_{ij} \cdot \mathcal{A}^{1L}_{G/Z}[h^0_j \to f\bar{f}\,] + \frac{\Gamma^{tree}_{Gf\bar{f}}}{M^2_{h_i}} \sum_{j,k} \hat{\Sigma}_{h_j h_k}\left(M^2_{h_i}\right) \cdot Z^{mix}_{ik} \frac{f\left(M^2_{h_i}\right) \xi_Z M^2_Z}{M^2_{h_i} - \xi_Z M^2_Z}, \tag{3.5}$$

where $\Gamma^{tree}_{Gf\bar{f}}$ represents the tree-level vertex of the neutral Goldstone boson with the fermion f (in the particular example of a Higgs decay into $f\bar{f}$). Then it is straightforward to check that $\tilde{\mathcal{A}}^{1L}_{G/Z}$ is gauge-invariant. The transformation of Eq. (3.5) can also be interpreted as a two-loop shift redefining $\Sigma_{h_i Z}$, so that it satisfies a generalised Slavnov–Taylor identity of the form of Eq. (3.2a), but applied to a physical (loop-corrected) Higgs field, with the term $\left(p^2 - m^2_{h_i}\right) f(p^2)$ of Eq. (3.2a) replaced with $\left(p^2 - M^2_{h_i}\right) f(p^2)$.

3.2.1.5 Numerical input in the one-loop corrections

As usual, the numerical values of the input parameters need to reflect the adopted renormalization scheme, and the input parameters corresponding to different schemes differ from each other by shifts of the appropriate loop order (at the loop level, there exists some freedom to use a numerical value of an input parameter that differs from the tree-level value by a one-loop shift, since the difference induced in this way is of higher order). Concerning the input values of the relevant light quark masses, we follow in our evaluation the choice of `FeynHiggs` and employ $\overline{\text{MS}}$ quark masses with three-loop QCD corrections evaluated at the scale of the mass of the decaying Higgs, $m^{\overline{\text{MS}}}_q(M_{h_i})$, in the loop functions and the definition of the Yukawa couplings. In addition, the input value for the pole top mass is converted to $m^{\overline{\text{MS}}}_t(m_t)$ using up to two-loop QCD and one-loop top Yukawa or electroweak corrections (corresponding to the higher-order corrections included in the Higgs boson mass calculation). Furthermore, the $\tan\beta$-enhanced contributions are always included in the defining relation between the bottom Yukawa coupling and the bottom mass (and similarly for all other down-type quarks). Concerning the Higgs VEV appearing in the relation between the Yukawa couplings and the fermion masses, we parametrize it in terms of $\alpha(M_Z)$. Finally, the strong coupling constant employed in SUSY-QCD diagrams is set to the scale of the supersymmetric particles entering the loop. We will comment on deviations from these settings if needed.[§]

3.2.2 Higgs decays into SM fermions

Our calculation of the Higgs decay amplitudes into SM fermions closely follows the procedure outlined in the previous subsection. However, we include the QCD and QED corrections separately, making use of analytical formulae that are well-documented in the literature [54,55]. We also employ an effective description of the Higgs–$b\bar{b}$ interactions in order to resum potentially large effects for large values of $\tan\beta$. Next, we comment on these two issues and discuss further the derivation of the decay widths for this class of channel.

[§]Possibly large contributions by electroweak double-logarithms of the Sudakov type as well as the corresponding counterparts in fermionic Higgs decays with additional real radiation of gauge bosons are investigated in a separate article [53].

3.2.2.1 *Tree-level amplitude*

At the tree level, the decay $h_j^0 \to \mathrm{f}\bar{\mathrm{f}}$ is determined by the Yukawa coupling Y_f and the decomposition of the tree-level state h_j^0 in terms of the Higgs-doublet components:

$$\mathcal{A}^{\mathrm{tree}}[h_j^0 \to \mathrm{f}\bar{\mathrm{f}}] = -\mathrm{i}\frac{Y_\mathrm{f}}{\sqrt{2}}\bar{u}_\mathrm{f}(p_\mathrm{f})\left\{\delta_{\mathrm{f},d_k/e_k}(U_n)_{j1} + \delta_{\mathrm{f},u_k}(U_n)_{j2} - \mathrm{i}\gamma_5\left[\delta_{\mathrm{f},d_k/e_k}(U_n)_{j4} + \delta_{\mathrm{f},u_k}(U_n)_{j5}\right]\right\} v_\mathrm{f}(p_{\bar{\mathrm{f}}}) \tag{3.6}$$

$$\equiv -\mathrm{i}\bar{u}_\mathrm{f}(p_\mathrm{f})\left\{g^S_{h_j\mathrm{ff}} - \gamma_5 g^P_{h_j\mathrm{ff}}\right\} v_\mathrm{f}(p_{\bar{\mathrm{f}}})\,. \tag{3.7}$$

The δs are Kronecker symbols selecting the appropriate Higgs matrix element for the fermionic final state, $u_k = \mathrm{u,c,t}$, $d_k = \mathrm{d,s,b}$, or $e_k = \mathrm{e},\mu,\tau$. We have written the amplitude in the Dirac-fermion convention, separating the scalar part $g^S_{h_j\mathrm{ff}}$ (first two terms between curly brackets in the first line) from the pseudo-scalar one $g^P_{h_j\mathrm{ff}}$ (last two terms). The fermion and antifermion spinors are denoted $\bar{u}_\mathrm{f}(p_\mathrm{f})$ and $v_\mathrm{f}(p_{\bar{\mathrm{f}}})$, respectively.

3.2.2.2 *Case of the* $\mathrm{b}\bar{\mathrm{b}}$ *final state:* $\tan\beta$*-enhanced corrections*

In the case of a decay to $\mathrm{b}\bar{\mathrm{b}}$ (and analogously for down-type quarks of first and second generation, but with smaller numerical impact), the loop contributions that receive a $\tan\beta$ enhancement may have a sizeable impact, thus justifying an effective description of the Higgs–$\mathrm{b}\bar{\mathrm{b}}$ vertex that provides a resummation of large contributions [43, 56–62]. We denote the neutral components of $\mathcal{H}_1$ and $\mathcal{H}_2$ from Eq. (2.2) of Ref. [39] by H^0_d and H^0_u, respectively. The large $\tan\beta$-enhanced effects arise from contributions to the $(H^0_u)^*\,\bar{b}\,P_\mathrm{L}\,b$ operator—$P_\mathrm{L,R}$ are the left- and right-handed projectors in the Dirac description of the b spinors—and can be parametrized in the following fashion:

$$\mathcal{L}^{\mathrm{eff}} = -Y_\mathrm{b}\,\bar{\mathrm{b}}\left[H^0_\mathrm{d} + \frac{\Delta_\mathrm{b}}{\tan\beta}\left(\frac{\lambda}{\mu_\mathrm{eff}}\,S\,H^0_\mathrm{u}\right)^*\right]P_\mathrm{L}\,b + \mathrm{h.c.} \equiv -\sum_j g^{L\,\mathrm{eff}}_{h_j\mathrm{bb}}\,h^0_j\,\bar{b}\,P_\mathrm{L}\,b + \mathrm{h.\,c.} \tag{3.8}$$

Here, Δ_b is a coefficient that is determined via the calculation of the relevant ($\tan\beta$-enhanced) one-loop diagrams to the Higgs–$\mathrm{b}\bar{\mathrm{b}}$ vertex, involving gluino–sbottom, chargino–stop, and neutralino–sbottom loops.[¶] The symbol μ_eff represents the effective μ term that is generated when the singlet field acquires a VEV. The specific form of the operator, $(S\,H^0_\mathrm{u})^*\,\bar{b}\,P_\mathrm{L}\,b$, is designed so as to preserve the $\mathbb{Z}_3$ symmetry, and it can be shown that this operator is the one that gives rise to leading contributions to the $\tan\beta$-enhanced effects. We evaluate Δ_b at a scale corresponding to the arithmetic mean of the masses of the contributing SUSY particles: this choice is consistent with the definition of Δ_b employed for the Higgs mass calculation.

From the parametrization of Eq. (3.8), one can derive the non-trivial relation between the 'genuine' Yukawa coupling Y_b and the effective bottom mass m_b: $Y_\mathrm{b} = m_\mathrm{b}/(v_1\,(1+\Delta_\mathrm{b}))$. Then, the effective couplings of the neutral Higgs fields to $\mathrm{b}\bar{\mathrm{b}}$ read:

$$g^{L\,\mathrm{eff}}_{h_j\mathrm{bb}} = \frac{m_\mathrm{b}}{\sqrt{2}\,v_1\,(1+\Delta_b)}\left\{(U_n)_{j1} + \mathrm{i}\,(U_n)_{j4} + \frac{\Delta_\mathrm{b}}{\tan\beta}\left((U_n)_{j2} - \mathrm{i}\,(U_n)_{j5} + \tfrac{\lambda^* v_2}{\mu^*_\mathrm{eff}}\left[(U_n)_{j3} - \mathrm{i}\,(U_n)_{j6}\right]\right)\right\}. \tag{3.9}$$

This can be used to substitute $\mathcal{A}^{\mathrm{tree}}[h_j^0 \to \mathrm{b}\bar{\mathrm{b}}]$ in Eq. (3.3) for:

$$\mathcal{A}^{\mathrm{eff}}[h_j^0 \to \mathrm{b}\bar{\mathrm{b}}] = -\mathrm{i}\,\bar{u}_\mathrm{b}(p_\mathrm{b})\left[g^{L\,\mathrm{eff}}_{h_j\mathrm{bb}}\,P_\mathrm{L} + g^{L\,\mathrm{eff}\,*}_{h_j\mathrm{bb}}\,P_\mathrm{R}\right]v_\mathrm{b}(p_{\bar{\mathrm{b}}})\,, \tag{3.10}$$

[¶] Two-loop corrections to Δ_b have also been reported in Refs. [63, 64].

where this expression resums the effect of $\tan\beta$-enhanced corrections to the $h_j^0 \mathrm{b}\bar{\mathrm{b}}$ vertex. However, if one now adds the one-loop amplitude $\mathcal{A}^{\mathrm{1L}}_{\mathrm{vert}}$, the one-loop effects associated with the $\tan\beta$-enhanced contributions would be included twice. To avoid this double counting, the terms that are linear in Δ_{b} in Eq. (3.9) need to be subtracted. Employing the 'subtraction' couplings

$$g^{L\,\mathrm{sub}}_{h_j \mathrm{bb}} = \frac{m_{\mathrm{b}}\,\Delta_{\mathrm{b}}}{\sqrt{2}\,v_1}\left\{(U_n)_{j1} + \mathrm{i}\,(U_n)_{j4} - \frac{1}{\tan\beta}\left((U_n)_{j2} - \mathrm{i}\,(U_n)_{j5} + \frac{\lambda^*\,v_{\mathrm{u}}}{\mu^*_{\mathrm{eff}}}\left[(U_n)_{j3} - \mathrm{i}\,(U_n)_{j6}\right]\right)\right\}, \tag{3.11}$$

we define the following 'tree-level' amplitude for the Higgs decays into bottom quarks:

$$\mathcal{A}^{\mathrm{tree}}[h_j^0 \to \mathrm{b}\bar{\mathrm{b}}] = \mathcal{A}^{\mathrm{eff}}[h_j^0 \to \mathrm{b}\bar{\mathrm{b}}] + \mathcal{A}^{\mathrm{sub}}[h_j^0 \to \mathrm{b}\bar{\mathrm{b}}], \tag{3.12a}$$

$$\mathcal{A}^{\mathrm{sub}}[h_j^0 \to \mathrm{b}\bar{\mathrm{b}}] \equiv -\mathrm{i}\,\bar{u}_{\mathrm{b}}(p_{\mathrm{b}})\left[g^{L\,\mathrm{sub}}_{h_j \mathrm{bb}}\,P_{\mathrm{L}} + g^{L\,\mathrm{sub}\,*}_{h_j \mathrm{bb}}\,P_{\mathrm{R}}\right] v_{\mathrm{b}}(p_{\bar{\mathrm{b}}}). \tag{3.12b}$$

3.2.2.3 QCD and QED corrections

The inclusion of QCD and QED corrections requires a proper treatment of IR effects in the decay amplitudes. The IR-divergent parts of the virtual contributions by gluons or photons in $\mathcal{A}^{\mathrm{1L}}_{\mathrm{vert}}$ are cancelled by their counterparts in processes with radiated photons or gluons. We directly employ the QCD and QED correction factors that are well-known analytically (see next) and therefore omit the Feynman diagrams involving a photon or gluon propagator when computing, with `FeynArts` and `FormCalc`, the one-loop corrections to the $h_j^0 \mathrm{f}\bar{\mathrm{f}}$ vertex and to the fermion mass and wave function counterterms. The QCD and QED correction factors applying to the fermionic decays of a $\mathcal{CP}$-even Higgs state are given in Ref. [54]. The $\mathcal{CP}$-odd case was addressed later in Ref. [55]. In the $\mathcal{CP}$-violating case, it is useful to observe that the $h_j \mathrm{f}\bar{\mathrm{f}}$ scalar and pseudo-scalar operators do not interfere, so that the $\mathcal{CP}$-even and $\mathcal{CP}$-odd correction factors can be applied directly at the level of the amplitudes—although they were obtained at the level of the squared amplitudes:

$$\mathcal{A}^{\mathrm{tree+QCD/QED}}[h_j^0 \to \mathrm{f}\bar{\mathrm{f}}] = -\mathrm{i}\,\frac{m_{\mathrm{f}}^{\overline{\mathrm{MS}}}(M_{h_i})}{m_{\mathrm{f}}}\,\bar{u}_{\mathrm{f}}(p_{\mathrm{f}})\left\{g^{S}_{h_j \mathrm{ff}}\,c_S - \gamma_5\,g^{P}_{h_j \mathrm{ff}}\,c_P\right\} v_{\mathrm{f}}(p_{\bar{\mathrm{f}}}), \tag{3.13a}$$

$$c_{S,P} = \sqrt{1 + c^{\mathrm{QED}}_{S,P} + c^{\mathrm{QCD}}_{S,P}}, \tag{3.13b}$$

$$c^{\mathrm{QED}}_{S,P} \equiv \frac{\alpha}{\pi}\,Q_{\mathrm{f}}^2\,\Delta_{S,P}\left(\sqrt{1 - \frac{4\,m_{\mathrm{f}}^2}{M_{h_i}^2}}\right), \tag{3.13c}$$

$$c^{\mathrm{QCD}}_{S,P} \equiv \frac{\alpha_s(M_{h_i})}{\pi}\,C_2(f)\left[\Delta_{S,P}\left(\sqrt{1 - \frac{4\,m_{\mathrm{f}}^2}{M_{h_i}^2}}\right) + 2 + 3\log\left(\frac{M_{h_i}}{m_{\mathrm{f}}}\right)\right]. \tag{3.13d}$$

Here, Q_{f} is the electric charge of the fermion f, $C_2(f)$ is equal to 4/3 for quarks and equal to 0 for leptons, M_{h_i} corresponds to the kinematic (pole) mass in the Higgs decay under consideration and the functions $\Delta_{S,P}$ are explicated in e.g., Section 4 of Ref. [65]. In the limit of $M_{h_i} \gg m_{\mathrm{f}}$, both $\Delta_{S,P}$ reduce to $\left[-3\log\left(M_{h_i}/m_{\mathrm{f}}\right) + \frac{9}{4}\right]$. As noted in Ref. [54], the leading logarithm in the QCD correction factor can be absorbed by the introduction of a running $\overline{\mathrm{MS}}$ fermion mass in the definition of the Yukawa coupling Y_{f}. Therefore, it is motivated to factorise $m_{\mathrm{f}}^{\overline{\mathrm{MS}}}(M_{h_i})$, with higher orders included in the definition of the QCD beta function.

The QCD (and QED) correction factors generally induce a sizeable shift of the tree-level width of as much as ∼50%. While these effects were formally derived at the one-loop order, we apply them over the full amplitudes (without the QCD and QED corrections), i.e., we include

the one-loop vertex amplitude without QCD/QED corrections $\mathcal{A}_{\text{vert}}^{\text{1Lwo. QCD/QED}}$ and $\mathcal{A}_{\text{G/Z}}^{\text{1L}}$ in the definitions of the couplings $g_{h_j\text{ff}}^{S,P}$ that are employed in Eq. (3.13)—we will use the notation $g_{h_j\text{ff}}^{S,P\,\text{1L}}$ in the following. The adopted factorisation corresponds to a particular choice of the higher-order contributions beyond the ones that have been explicitly calculated.

3.2.2.4 Decay width

Putting together the various pieces discussed before, we can express the decay amplitude at the one-loop order as

$$\mathcal{A}[h_i \to \text{f}\bar{\text{f}}\,] = -\text{i}\,\frac{m_{\text{f}}^{\overline{\text{MS}}}(M_{h_i})}{m_{\text{f}}}\,Z_{ij}^{\text{mix}}\,\bar{u}_{\text{f}}(p_{\text{f}})\left\{g_{h_j\text{ff}}^{S\,\text{1L}}\,c_S - \gamma_5\,g_{h_j\text{ff}}^{P\,\text{1L}}\,c_P\right\}v_{\text{f}}(p_{\bar{\text{f}}})\,, \tag{3.14a}$$

$$-\text{i}\,\bar{u}_{\text{f}}(p_{\text{f}})\left\{g_{h_j\text{ff}}^{S\,\text{1L}} - \gamma_5\,g_{h_j\text{ff}}^{P\,\text{1L}}\right\}v_{\text{f}}(p_{\bar{\text{f}}}) \equiv \Big(\mathcal{A}^{\text{tree}} + \mathcal{A}_{\text{vert}}^{\text{1Lwo. QCD/QED}} + \mathcal{A}_{\text{G/Z}}^{\text{1L}}\Big)\Big[h_j \to \text{f}\bar{\text{f}}\,\Big]\,. \tag{3.14b}$$

Summing over spinor and colour degrees of freedom, the decay width is then obtained as

$$\Gamma\Big[h_i \to \text{f}\bar{\text{f}}\,\Big] = \frac{1}{16\,\pi\,M_{h_i}}\sqrt{1-\frac{4m_{\text{f}}^2}{M_{h_i}^2}}\sum_{\substack{\text{polarisation,}\\ \text{colour}}}\Big|\mathcal{A}[h_i^{\text{phys.}} \to \text{f}\bar{\text{f}}\,]\Big|^2\,. \tag{3.15}$$

At the considered order, we could dismiss the one-loop squared terms in $|\mathcal{A}[h_i \to \text{f}\bar{\text{f}}\,]|^2$. However, to tackle the case where the contributions from irreducible one-loop diagrams are numerically larger than the tree-level amplitude, we keep the corresponding squared terms in the expression (it should be noted that the QCD and QED corrections have been stripped off from the one-loop amplitude, which gets squared). The approach of incorporating the squared terms should give a reliable result in a situation where the tree-level result is significantly suppressed, since the other missing contribution at this order, consisting of the tree-level amplitude times the two-loop amplitude, would be suppressed, owing to the small tree-level result. In such a case, however, the higher-order uncertainties are expected to be comparatively larger than in the case where one-loop effects are subdominant to the tree level.

The kinematic masses of the fermions are easily identified in the leptonic case. For decays into top quarks, the 'pole' mass m_{t} is used, while for all other decays into quarks we employ the $\overline{\text{MS}}$ masses evaluated at the scale of the Higgs mass $m_{\text{q}}^{\overline{\text{MS}}}(M_{h_i})$. We note that these kinematic masses have little impact on the decay widths, as long as the Higgs state is much heavier. In the NMSSM, however, singlet-like Higgs states can be very light, in which case the choice of an $\overline{\text{MS}}$ mass is problematic. Yet, in this case, the Higgs state is typically near threshold so that the free-parton approximation in the final state is not expected to be reliable. Our current code is not properly equipped to address decays directly at threshold independently of the issue of running kinematic masses. Improved descriptions of the hadronic decays of Higgs states close to the $\text{b}\bar{\text{b}}$ threshold or in the chiral limit have been presented in, e.g., Refs. [66–71].

3.2.3 Decays into SM gauge bosons

Now we consider Higgs decays into the gauge bosons of the SM. Almost each of these channels requires a specific processing in order to include higher-order corrections consistently or to deal with off-shell effects.

3.2.3.1 Decays into electroweak gauge bosons

Higgs decays into on-shell Ws and Zs can easily be included at the one-loop order in comparable fashion to the fermionic decays. However, the notion of WW or ZZ final states usually includes contributions from off-shell gauge bosons as well, encompassing a wide range of four-fermion final states. Such off-shell effects mostly impact the decays of Higgs bosons with a mass below the WW or ZZ thresholds. Instead of a full processing of the off-shell decays at one-loop order, we pursue two distinct evaluations of the decay widths in these channels.

Our first approach is that already employed in `FeynHiggs` for the corresponding decays in the MSSM. It involves exploiting the precise one-loop results of `Prophecy4f` for the SM Higgs decays into four fermions [72–74]. For an (N)MSSM Higgs boson h_i, the SM decay width is thus evaluated at the mass M_{h_i} and then rescaled by the squared ratio of the tree-level couplings to gauge bosons for h_i and an SM Higgs boson $H_{\rm SM}$ ($V = \mathrm{W}, \mathrm{Z}$):

$$\Gamma[h_i \to VV] = \Gamma^{\rm SM}[H_{\rm SM}(M_{h_i}) \to VV] \left| \mathcal{R}_{ij} \cdot \frac{g^{\rm NMSSM}_{h_j VV}}{g^{\rm SM}_{\rm HVV}} \right|^2 , \tag{3.16a}$$

$$\frac{g^{\rm NMSSM}_{h_j VV}}{g^{\rm SM}_{\rm HVV}} \equiv \cos\beta\,(U_n)_{j1} + \sin\beta\,(U_n)_{j2}\,, \tag{3.16b}$$

where $\Gamma[h_i \to VV]$ represents the decay width of the physical Higgs state h_i in the NMSSM, while $\Gamma^{\rm SM}[H_{\rm SM}(M_{h_i}) \to VV]$ denotes the decay width of an SM Higgs boson with mass M_{h_i}. The matrix elements $\mathcal{R}_{ij}$ reflect the connection between the tree-level Higgs states and the physical states. This role is similar to $\mathbf{Z}^{\rm mix}$. However, decoupling in the SM limit of the model yields the additional condition that the ratio in Eq. (3.16a) reduces to 1 in this limit for the SM-like Higgs boson of the NMSSM. For this reason, `FeynHiggs` employs the matrix $\mathbf{U}^m$ (or $\mathbf{U}^0$) as a unitary approximation of $\mathbf{Z}^{\rm mix}$—see Section 2.6 of Ref. [39]. An alternative choice involves using $X_{ij} \equiv Z^{\rm mix}_{ij} \big/ \sqrt{\sum_k |Z^{\rm mix}_{ik}|^2}$. However, the difference of the widths when employing $\mathbf{U}^0$, $\mathbf{U}^m$, $\mathbf{Z}^{\rm mix}$, or $\mathbf{X} \equiv (X_{ij})$ corresponds to effects of higher order, which should be regarded as part of the higher-order uncertainty. The rescaling of the one-loop SM width should only be applied for the SM-like Higgs of the NMSSM, where this implementation of the $h_i \to VV$ widths is expected to provide an approximation that is relatively close to a full one-loop result incorporating all NMSSM contributions. However, for the other Higgs states of the NMSSM, one-loop contributions beyond the SM may well be dominant. Actually, the farther the quantity $[\mathcal{R}_{ij} \cdot (U_n)_{j2}] \big/ [\mathcal{R}_{ij} \cdot (U_n)_{j1}]$ departs from $\tan\beta$, the more inaccurate the prediction based on SM-like radiative corrections becomes.

Our second approach involves a one-loop calculation of the Higgs decay widths into on-shell gauge bosons (see Ref. [52] for the MSSM case), including tree-level off-shell effects. This evaluation is meant to address the case of heavy Higgs bosons at the full one-loop order. The restriction to on-shell kinematics is justified above the threshold for electroweak gauge-boson production (off-shell effects at the one-loop level could be included via a numerical integration over the squared momenta of the gauge bosons in the final state—see Refs. [75,76] for a discussion in the MSSM). For details of our implementation, see Ref. [29], with the noteworthy feature that contributions from Higgs–electroweak mixing $\mathcal{A}^{\rm 1L}_{\rm G/Z}$ vanish. In the case of the $\mathrm{W^+W^-}$ final state, the QED IR divergences are regularised with a photon mass and cancel with bremsstrahlung corrections: soft and hard bremsstrahlung are included according to Refs. [77, 78] (see also Ref. [52]). We stress that the exact cancellation of the IR divergences is only achieved through the replacement of the $h_i\mathrm{G^+G^-}$ coupling with the expression in terms of the kinematic Higgs

mass (see Ref. [29] for more details). This fact had already been observed in Ref. [52]. To extend the validity of the calculation below the threshold, we process the Born-order term separately, applying an off-shell kinematic integration over the squared external momentum of the gauge bosons—see, e.g., Eq. (37) in Ref. [79]. Thus, this evaluation is performed at tree level below threshold and at full one-loop order (for the on-shell case) above threshold. The vanishing on-shell kinematic factor multiplying the contributions of one-loop order ensures the continuity of the prediction at threshold. Finally, we include the one-loop squared term in the calculation. Indeed, as we will discuss later, the tree-level contribution vanishes for a decoupling doublet, meaning that the Higgs decays to WW/ZZ can be dominated by one-loop effects. To this end, the infrared divergences of two-loop order are regularised in an ad-hoc fashion—which appears compulsory as long as the two-loop order is incomplete—making use of the one-loop real radiation and estimating the logarithmic term in the imaginary part of the one-loop amplitude.

3.2.3.2 Radiative decays into gauge bosons

Higgs decays into photon pairs, gluon pairs, or γZ appear at the one-loop level—i.e., $\mathcal{A}^{\text{tree}} = 0$ for all these channels. We compute the one-loop order using the `FeynArts` model file, although the results are well-known analytically in the literature—see, e.g., Ref. [51] or Section III of Ref. [80] (Ref. [79] for the MSSM). The electromagnetic coupling in these channels is set to the value $\alpha(0)$, corresponding to the Thomson limit.

The use of tree-level Higgs–Goldstone couplings together with loop-corrected kinematic Higgs masses M_{h_i} in our calculation would induce an effective violation of Ward identities by two-loop order terms in the amplitude: we choose to restore the proper gauge structure by redefining the Higgs–Goldstone couplings in terms of the kinematic Higgs mass M_{h_i} (see Ref. [29] for more details). Since our calculation is restricted to the leading—here, one-loop—order, the transition of the amplitude from tree-level to physical Higgs states is performed via $\mathbf{U}^m$ or $\mathbf{X}$ instead of $\mathbf{Z}^{\text{mix}}$ in order to ensure the appropriate behaviour in the decoupling limit.

Leading QCD corrections to the diphoton Higgs decays have received substantial attention in the literature. A frequently used approximation for this channel involves multiplying the amplitudes driven by quark and squark loops by the factors $[1 - \alpha_{\text{s}}(M_{h_i})/\pi]$ and $[1 + 8\,\alpha_{\text{s}}(M_{h_i})/(3\,\pi)]$, respectively—see, e.g., Ref. [81]. However, these simple factors are only valid in the limit of heavy quarks and squarks (compared with the mass of the decaying Higgs boson). More general analytical expressions can be found in, e.g., Ref. [82]. In our calculation, we apply the correction factors $[1 + C^S(\tau_{\text{q}})\,\alpha_{\text{s}}(M_{h_i})/\pi]$ and $[1 + C^P(\tau_{\text{q}})\,\alpha_{\text{s}}(M_{h_i})/\pi]$ to the contributions of the quark q to the $\mathcal{CP}$-even and the $\mathcal{CP}$-odd $h_i\gamma\gamma$ operators, respectively, and $[1 + C(\tau_{\tilde{Q}})\,\alpha_{\text{s}}(M_{h_i})/\pi]$ to the contributions of the squark $\tilde{Q}$ (to the $\mathcal{CP}$-even operator). Here, τ_X denotes the ratio $\left[4\,m_X^2(M_{h_i}/2)/M_{h_i}^2\right]$. The coefficients $C^{S,P}$ and C are extracted from Refs. [83] and [84]. To obtain a consistent inclusion of the $\mathcal{O}(\alpha_{\text{s}})$ corrections, the quark and squark masses m_X entering the one-loop amplitudes or the correction factors are chosen as defined in Eq. (5) of Ref. [83] and Eq. (12) of Ref. [84] (rather than $\overline{\text{MS}}$ running masses).

The QCD corrections to the digluon decays include virtual corrections but also gluon and light quark radiation. They are thus technically defined at the level of the squared amplitudes. In the limit of heavy quarks and squarks, the corrections are known beyond NLO—see the discussion in Ref. [79] for a list of references. The full dependence in mass was derived at NLO in Refs. [83, 84], for both quark and squark loops. In our implementation, we follow the prescriptions of Eqs. (51), (63), and (67) of Ref. [79] in the limit of light radiated quarks

and heavy particles in the loop. For consistency, the masses of the particles in the one-loop amplitude are taken as pole masses. Effects beyond this approximation can be sizeable, as evidenced by Fig. 20 of Ref. [83] and Fig. 12 of Ref. [84]. As the $\mathcal{CP}$-even and $\mathcal{CP}$-odd Higgs–gg operators do not interfere, it is straightforward to include both correction factors in the $\mathcal{CP}$-violating case. Finally, we note that parts of the leading QCD corrections to $h_i \to$ gg are induced by the real radiation of quark–antiquark pairs. In the case of the heavier quark flavours (top, bottom, and possibly charm), the channels are experimentally easily distinguishable from gluonic decays. Therefore, the partial widths related to these corrections could be attached to the Higgs decays into quarks instead [85]. The resolution of this ambiguity would involve a dedicated experimental analysis of the kinematics of the gluon radiation in $h_i \to \mathrm{gq\bar{q}}$ (collinear or back-to-back emission).

The QCD corrections to the quark loops of an SM Higgs decay into γZ have been studied in Refs. [86–88], but we do not consider them here.

3.3 Discussion concerning the remaining theoretical uncertainties

Next, we provide a summary of the main sources of theoretical uncertainties from unknown higher-order corrections applying to our calculation of the NMSSM Higgs decays. We do not discuss here the parametric theoretical uncertainties arising from the experimental errors of the input parameters. For the experimentally known SM-type parameters, the induced uncertainties can be determined in the same way as for the SM case (see, e.g., Ref. [26]). The dependence on the unknown SUSY parameters, however, is usually not treated as a theoretical uncertainty but rather exploited for setting indirect constraints on those parameters.

3.3.1 Higgs decays into quarks ($h_i \to \mathrm{q\bar{q}}$, $\mathrm{q} = \mathrm{c, b, t}$)

In our evaluation, these decays have been implemented at full one-loop order, i.e., at QCD, electroweak, and SUSY next-to-leading order (NLO). In addition, leading QCD logarithmic effects have been resummed within the parametrization of the Yukawa couplings in terms of a running quark mass at the scale of the Higgs mass. The Higgs propagator-type corrections determining the mass of the considered Higgs particle, as well as the wave function normalization at the external Higgs leg of the process, contain full one-loop and dominant two-loop contributions.

For an estimate of the remaining theoretical uncertainties, several higher-order effects should be taken into account.

1. First, we should assess the magnitude of the missing QCD NNLO (two-loop) effects. We stress that there should be no large logarithms associated with these corrections, since these are already resummed through the choice of running parameters and the renormalization scale. For the remaining QCD pieces, we can directly consider the situation in the SM. In the case of the light quarks, the QCD contributions of higher order have been evaluated and amount to ∼4% at $m_{\mathrm{H}} = 120\,\mathrm{GeV}$ (see, e.g., Ref. [89]). For the top quark, the uncertainty due to missing QCD NNLO effects was estimated at 5% [26].

2. Concerning the electroweak corrections, the numerical analysis in Ref. [29] suggests that the one-loop contribution is small—at the percentage level—for an SM-like Higgs, which is consistent with earlier estimates in the SM [26]. For the heavy Higgs states, the numerical analysis in Ref. [29] indicates a larger impact of such effects—at the level of ∼10% in the

considered scenario. Assuming that the electroweak NNLO corrections are comparable to the squared one-loop effects, our estimate for pure electroweak higher orders in decays of heavy Higgs states reaches the percentage level. In fact, for multiteraelectronvolt Higgs bosons, the electroweak Sudakov logarithms may require a resummation (see Ref. [53]). Furthermore, mixed electroweak–QCD contributions are expected to be larger than the pure electroweak NNLO corrections, adding a few more percent to the uncertainty budget. For light Higgs states, the electroweak effects are much smaller, since the Sudakov logarithms remain of comparatively modest size.

3. Finally, the variations with the squark masses in the numerical analysis in Ref. [29] for the heavy doublet states show that the one-loop SUSY effects could amount to 5–10% for a subteraelectronvolt stop or sbottom spectrum. In such a case, the two-loop SUSY and the mixed QCD or electroweak–SUSY corrections may reach the percentage level. Conversely, for very heavy squark spectra, we expect to recover an effective singlet-extended two-Higgs-doublet model (an effective SM if the heavy doublet and singlet states also decouple) at low energy. However, all the parameters of this low-energy effective field theory implicitly depend on the SUSY radiative effects, since unsuppressed logarithms of SUSY origin generate terms of dimension ≤ 4—e.g., in the Higgs potential or the Higgs couplings to SM fermions. Conversely, the explicit dependence of the Higgs decay widths on SUSY higher-order corrections is suppressed for a large SUSY scale. In this case, the uncertainty from SUSY corrections reduces to a parametric effect, that of the matching between the NMSSM and the low-energy Lagrangian—e.g., in the SM limit, the uncertainty on the mass prediction for the SM-like Higgs continues to depend on SUSY logarithms and would indirectly affect the uncertainty on the decay widths.

Considering all these higher-order effects together, we conclude that the decay widths of the SM-like Higgs should be relatively well-controlled (up to $\sim$5%), while those of a heavy Higgs state could receive sizeable higher-order contributions, possibly adding up to the level of $\sim$10%.

3.3.2 Higgs decays into leptons

Here, QCD corrections appear only at two-loop order in the Higgs propagator-type corrections, as well as in the counterterms of the electroweak parameters, and only from three-loop order onwards in the genuine vertex corrections. Thus, the theory uncertainty is expected to be substantially smaller than in the case of quark final states. For an SM-like Higgs, associated uncertainties were estimated to be below the percentage level [28]. For heavy Higgs states, however, electroweak one-loop corrections are enhanced by Sudakov logarithms (see Ref. [53]) and reach the $\sim$10% level for Higgs masses of the order of 1 TeV, so that the two-loop effects could amount to a few percent. In addition, light status may generate a sizeable contribution of SUSY origin, where the unknown corrections are of two-loop electroweak order.

3.3.3 Higgs decays into WW/ZZ

The complexity of these channels is illustrated by our presentation of two separate estimates, expected to perform differently in various regimes.

1. In the SM, the uncertainty of `Prophecy4f` in the evaluation of these channels was assessed at the subpercentage level below 500 GeV, but up to $\sim$15% at 1 TeV [26]. For an SM-like Higgs, our numerical analysis in Ref. [29] shows that the one-loop electroweak

corrections are somewhat below 10%, making plausible a subpercentage uncertainty in the results employing `Prophecy4f`. Conversely, the assumption that the decay widths for an NMSSM Higgs boson can be obtained through a simple rescaling of the result for the width in the SM by tree-level couplings is, in itself, a source of uncertainties. We expect this approximation to be accurate only in the limit of a decoupling SM-like composition of the NMSSM Higgs boson. If these SM-like characteristics are altered through radiative corrections of SUSY origins or NMSSM Higgs mixing effects—both of which may still reach the level of several percentage in a phenomenologically realistic set-up—the uncertainty in the rescaling procedure for the decay widths should be of corresponding magnitude.

2. In the case of heavier states, our numerical analysis in Ref. [29] indicates that the previous procedure is unreliable in the mass range $\gtrsim$500 GeV. In particular, for heavy doublets in the decoupling limit, radiative corrections dominate over the—then vanishing—tree-level amplitude, shifting the widths by orders of magnitude. In such a case, our one-loop calculation captures only the leading order and one can expect sizeable contributions at the two-loop level: as discussed in the numerical analysis in Ref. [29], shifting the quark masses between pole and $\overline{\text{MS}}$ values—two legitimate choices at the one-loop order that differ in the treatment of QCD two-loop contributions—results in modifications of the widths of order $\sim$50%. Conversely, one expects the decays of a decoupling heavy doublet into electroweak gauge bosons to remain a subdominant channel, so that a less accurate prediction may be tolerable. It should be noted, however, that the magnitude of the corresponding widths is sizeably enhanced by the effects of one-loop order; this may be of interest regarding their phenomenological impact.

3.3.4 Radiative decays into gauge bosons

As these channels appear at the one-loop order, our (QCD-corrected) results represent (only) an improved leading-order evaluation. Yet the situation is contrasted.

1. In the SM, the uncertainty for a Higgs decay into $\gamma\gamma$ was estimated at the level of 1% in Ref. [26]; however, the corresponding calculation includes both QCD NLO and electroweak NLO corrections. In our case, only QCD NLO corrections (with full mass dependence) are taken into account. The comparison with `NMSSMCALC` in Ref. [29] provides us with a lower bound on the magnitude of electroweak NLO and QCD NNLO effects: both evaluations are of the same order but differ by a few percent. The uncertainty in the SUSY contribution should be considered separately, as light charginos or sfermions could have a sizeable impact. In any case, we expect the accuracy of our calculation to perform at the level of $\gtrsim$4%.

2. In the case of the Higgs decays into gluons, for the SM prediction—including QCD corrections with full mass dependence and electroweak two-loop effects—an uncertainty of 3% from QCD effects and 1% from electroweak effects was estimated in Ref. [26]. In our case, the QCD corrections are only included in the heavy-loop approximation, and NLO electroweak contributions have not been considered. Consequently, the uncertainty budget should settle above the corresponding estimate for the SM quoted here. In the case of heavy Higgs bosons, the squark spectrum could have a significant impact on the QCD two-loop corrections, as exemplified in Fig. 5 of Ref. [84].

3. For $h_i \to \gamma Z$, QCD corrections are not yet available, so the uncertainty should be above the $\sim$5% estimated in the SM [26].

3.3.5 Additional sources of uncertainty from higher orders

For an uncertainty estimate, the following effects apply to essentially all channels and should be considered as well.

1. The mixing in the Higgs sector plays a central role in the determination of the decay widths. Following the treatment in `FeynHiggs`, we have considered $\mathbf{Z}^{\rm mix}$ in all our one-loop evaluations, as prescribed by the LSZ reduction. Most public codes consider a unitary approximation in the limit of the effective scalar potential ($\mathbf{U}^0$, in our notation). The analysis of Ref. [39] and our most recent analysis in Ref. [29]—employing $\mathbf{U}^m$, a more reliable unitary approximation than $\mathbf{U}^0$—indicate that the different choices of mixing matrices may affect the Higgs decays by a few percent (and far more in contrived cases). However, even the use of $\mathbf{Z}^{\rm mix}$ is, of course, subject to uncertainties from unknown higher-order corrections. While the Higgs propagator-type corrections determining the mass of the considered Higgs boson and the wave function normalization contain corrections up to the two-loop order, the corresponding prediction for the mass of the SM-like Higgs still has an uncertainty at the level of about 2%, depending on the SUSY spectrum.

2. In this section, we confined ourselves to the evaluation of the Higgs decay widths into SM particles and did not consider the branching ratios. For the latter, an implementation at the full one-loop order of many other two-body decays, relevant, in particular, for the heavy Higgs states, would be desirable, but goes beyond the scope of the present analysis. Furthermore, to consider the Higgs branching ratios at the one-loop order, we would have to consider three-body widths at the tree level, for instance $h_i \to b\bar{b}Z$, since these are formally of the same magnitude as the one-loop effects for two-body decays [53]. In addition, these three-body decays—typically real radiation of electroweak and Higgs bosons—exhibit Sudakov logarithms that would require resummation in the limit of heavy Higgs states [53].

3. At decay thresholds, the approximation of free particles in the final state is not sufficient, and a more accurate treatment would require the evaluation of final-state interactions. Several cases have been discussed in, e.g., Refs. [69, 71, 90].

In this discussion, we did not attempt to provide a quantitative estimate of the remaining theoretical uncertainties from unknown higher-order corrections, as such an estimate would, in any case, sensitively depend on the considered region in parameter space. Instead, we have pointed out the various sources of higher-order uncertainties remaining at the level of our state-of-the-art evaluation of the Higgs decays into SM particles in the NMSSM. For a decoupling SM-like Higgs boson, one would ideally expect that the level of accuracy of the predictions approaches that achieved in the SM. However, even in this limit, missing NNLO pieces—which are known for the SM, but not for the NMSSM—give rise to a somewhat larger theoretical uncertainty in the NMSSM. Furthermore, uncertainties of parametric nature (for instance, from the theoretical prediction of the Higgs boson mass) need to be taken into account as well. For heavy Higgs states, the impact of electroweak Sudakov logarithms and SUSY corrections add to the theoretical uncertainty to an extent that is strongly dependent on the details of the

spectrum and the characteristics of the Higgs state (see Ref. [53]). For a decoupling doublet at ∼1 TeV, an uncertainty of ∼5–15% may be used as a guideline for the fermionic and radiative decays, while the uncertainty may be as large as ∼50% in $h_i \to$ WW/ZZ.

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