Quantum Field Theory and the Electroweak Standard Model

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Abstract

Lecture notes with a brief introduction to Quantum field theory and the Standard Model are presented. The lectures were given at the 2015 European School of High-Energy Physics. The main features, the present status, and problems of the Standard Model are discussed.

Keywords

Lectures; ESHEP; quantum field theory; Standard Model.

1 Introduction

The lecture course consists of four main parts. In the Introduction, we will discuss what is the Standard Model [1–3], its particle content, and the main principles of its construction. The second Section contains brief notes on Quantum Field Theory (QFT), where we remind the main objects and rules required further for construction of the SM. Sect. 3 describes some steps of the SM development. The Lagrangian of the model is derived and discussed. Phenomenology and high-precision tests of the model are overviewed in Sect. 4. The present status, problems, and prospects of the SM are summarized in Conclusions. Some simple useful exercises and questions are given for students in each Section. These lectures give only an overview of the subject while for details one should look in textbooks, e.g., [4–6], and modern scientific papers.

1.1 What is the Standard Model?

Let us start with the definition of the subject of the lecture course. It is the so-called *Standard Model* (SM). This name is quite widely accepted and commonly used to define a certain theoretical model in high energy physics. This model is suited to describe properties and interactions of *elementary* particles. One can say that at the present moment, the Standard Model is the most successful physical model ever. In fact it describes with a high precision hundreds and hundreds independent observables. The model made also a lot of predictions which have been verified later experimentally. Among other physical models pretending to describe fundamental properties of Nature, the SM has the highest predictive power. Moreover, the model is minimal: it is constructed using only fields, interactions and parameters which are necessary for consistency and/or observed experimentally. The minimality and in general the success of the model is provided to a great extent by application of symmetry principles.

In spite of the nice theoretical features and successful experimental verification of the SM, we hardly can believe that it is the true fundamental theory of Nature. First of all, it is only one of an infinite number of possible models within Quantum field theory. So it has well defined grounds but its uniqueness is questionable. Second, we will see that the SM and QFT itself do not seem to be the most adequate (mathematical) language to describe Nature. One can also remind that gravity is not (yet) joined uniformly with other interactions.

But in any case, the SM is presently the main theoretical tool in high-energy physics. Most likely this status will be preserved even if some new more fundamental physical model would be accepted by the community. In this case the SM can be treated as an approximation (a low-energy limit) of a more general theory.
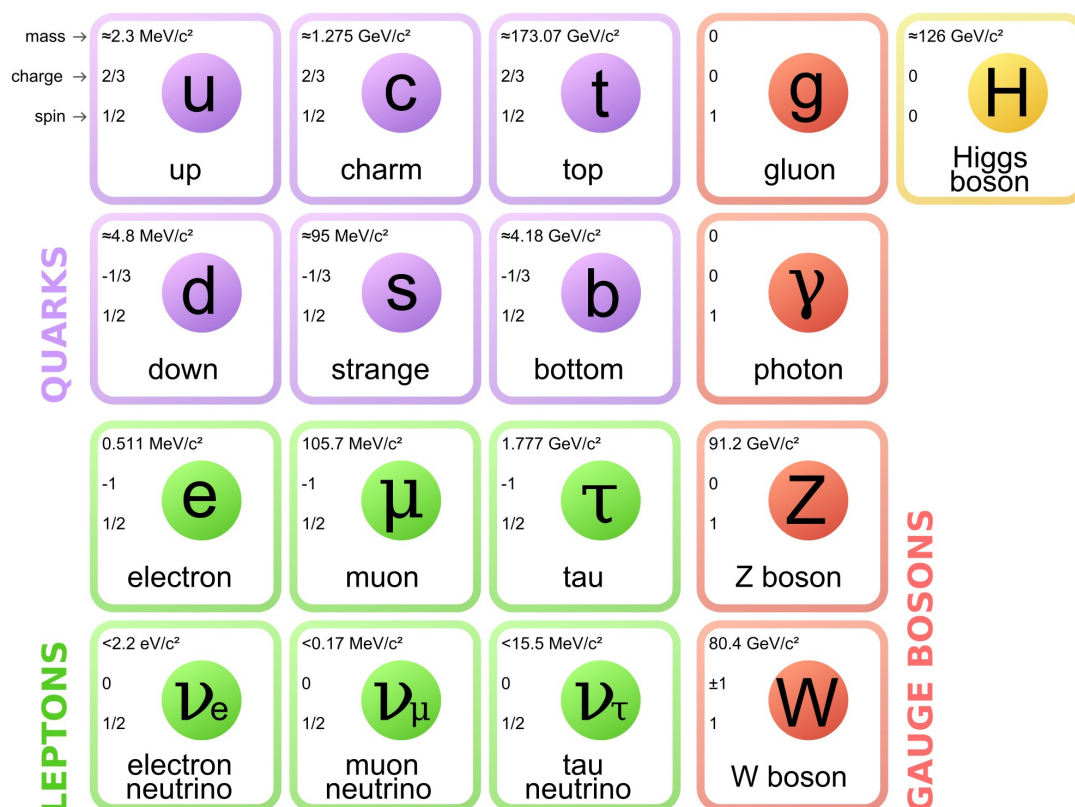
Fig. 1: Particle content of the Standard Model. Courtesy to Wikipedia: 'Standard Model of Elementary Particles' by MissMJ — Own work by uploader, PBS NOVA, Fermilab, Office of Science, United States Department of Energy, Particle Data Group.

1.2 Particle content of the Standard Model

Before construction of the SM, let us defined its content in the sense of fields and particles.

We would like to underline that the discovery of the Higgs boson at LHC in 2012 [7, 8] just finalized the list of SM particles from the experimental point of view. Meanwhile the Higgs boson is one of the key ingredients of the SM, so it was always in the list even so that its mass was unknown.

The particle content of the SM is given on Fig. 1. It consists of 12 fermions (spin = 1/2), 4 vector gauge bosons (spin = 1), and one scalar Higgs boson (spin = 0). For each particle the chart contains information about its mass, electric charge, and spin. One can see that the data on neutrino masses is represented in the form of upper limits, since they have not been yet measured. Strictly speaking the information about neutrino masses should be treated with care. According to the present knowledge, as discussed in the course of lectures given by S. Petcov, a neutrino particle of a given lepton flavor e.g., ν_τ, is not a mass eigenstate but a superposition of (at least) three states with different masses.

Fermions are of two types: *leptons* and *quarks*. They are:
— 3 charged leptons (e, μ, τ);
— 3 neutrinos ν_e, ν_μ, ν_τ (or ν_1, ν_2, ν_3, see lectures by S. Petcov);
— 6 quarks of different *flavors*, see lectures by S. Gori.

Each quark can have one of three *colours*, see lectures by A. Mitov. Each fermion has 2 degrees of freedom e.g., can have spin up or down, or can be either *left* or *right*. Each fermion particle in the SM has an *anti-particle*, $f \neq \bar{f}$. The later statement is not yet verified for neutrinos, they might be Majorana particles.

Traditionally fermions are called *matter fields*, contrary the the so-called *force fields*, i.e., intermediate vector bosons which are mediate interactions. Please keep in mind that this notion doesn't correspond to the common sense directly. In fact most of fermions are unstable and do not form the 'ordinary matter' around us, while e.g., mass of nuclear matter is provided to a large extent by gluons. Moreover, looking at various Feynman diagrams we can see that fermions can serve as intermediate particles in interaction processes.

In the SM we have the following boson fields:
— 8 vector (spin=1) *gluons*;
— 4 vector (spin=1) *electroweak bosons*: γ, Z, W^+, W^-;
— 1 scalar (spin=0) *Higgs boson*.

Gluons and photon are *massless* and have 2 degrees of freedom (polarizations), Z and W bosons are *massive* and have 3 degrees of freedom (polarizations). By saying massless or massive we mean the absence or presence of the corresponding terms in the Lagrangian of the SM. This is not always related to observables in a straightforward way: e.g., gluons are not observed as free asymptotic states, and masses of unstable W and Z bosons are defined indirectly from kinematics of their decay products.

Gluons and Electroweak (EW) bosons are *gauge bosons*, their interactions with fermions are fixed by certain symmetries of the Lagrangian. Note that electrically neutral bosons (H, γ, Z, and gluons) coincide with their anti-particles e.g., $\gamma \equiv \bar{\gamma}$.

Besides the particle content, we have to list the interactions which are described by the Standard Model. Our final goal would be to answer the question *"How many fundamental interactions are there in Nature?"* But we should understand that it is only a dream, a primary motivation of our studies. Being scientists we might be always unsure about the true answer to this question. On the other hand, we can certainly say, how many different interactions is there in a given model, for example the SM one. To answer this question we have to look at the Lagrangian of the model, see e.g., book [9]. For the SM it looks very long and cumbersome. The SM Lagrangian contains kinetic terms for all listed above fields and dozens of terms that describe interactions between them. Before trying to count the number of interactions we should understand the structure and symmetries of the Lagrangian.

1.3   Principles of the Standard Model

We are going to construct the SM Lagrangian. For this purpose, we have to define first the guiding principles. That is important for optimization of the procedure. The same principles might be used further in construction of other models.

First of all, we have to keep in mind that the SM is a model that is built within the local Quantum field theory. From the beginning this condition strongly limits the types of terms that can appear in the Lagrangian because of the Lorentz invariance, the Hermitian condition, the locality etc. One can make a long list of various conditions. Here I list only the main principles which will be exploited in our way of the SM construction:

– the *generalized correspondence* to various existing theories and models like Quantum Mechanics, QED, the Fermi model etc.;

– the *minimality*, i.e., only observed and/or unavoidable objects (fields and interactions) are involved;

– the *unitarity* which is a general condition for cross sections and various transformations of fields related to the fact that any probability limited from above by unity;

– the *renormalizability* is necessary for derivation of finite prediction of observable quantities at the quantum level;

– the *gauge* principle for introduction of interactions.

The main guiding principle is the *symmetry* one. The SM possesses several different symmetries:

— the Lorentz (and Poincaré) symmetry,
— the CPT symmetry,
— three gauge symmetries $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$,
— the global $SU(2)_L \times SU(2)_R$ symmetry in the Higgs sector (it is broken spontaneously);
— some other symmetries, like the one between three generation of fermions, the one that provides cancellation of anomalies etc.

In this context, one can mention also the conformal symmetry which is obviously broken in the SM, but the mechanism of its breaking and the consequences are very important for the model.

2   Brief notes on Quantum field theory

The Standard Model is a model constructed within the local relativistic Quantum field theory. It means that the SM obeys the general QFT rules. We should keep in mind that there are many other possible QFT models, and the SM is distinguished between them mostly because of its successful experimental verifications but also because of a number of its features like renormizablity, unitarity, and cancellation of axial anomalies. I assume that all students of the ESHEP school had courses on Quantum field theory. Here we will just remind several features of QFT which are important for further construction of the SM Lagrangian.

As it was already mentioned, we are going to preserve the correspondence to Quantum Mechanics (QM). Historically, QFT was developed on the base of QM, in particular using the quantum oscillator ansatz. But by itself QFT can be considered as a more profound fundamental construction, so one should be able to define this theory without referring to QM. In fact, QFT can be formulated starting from the basic classification of fields as unitary irreducible representations of the Lorentz group.

Let us first of all fix the notation. We will work in the natural system of units where the speed of light $c = 1$ and the reduced Planck constant $\hbar = 1$. The Lorentz indexes will be denoted by Greek letters, like $\mu = 0, 1, 2, 3$; p_μ is a four-momentum of a particle, $\mathbf{p} = (p_1, p_2, p_3)$ is a three-momentum, $p_0 = E$ is the particle energy.

The metric tensor of the Minkowsky space is chosen in the form

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad g_{\mu\nu}p_\nu = p_\mu, \quad g_{\mu\mu} = 4. \tag{1}$$

We will always assume summation over a Lorentz index if it is repeated twice: $A_\mu B_\mu \equiv A_0 B_0 - A_1 B_1 - A_2 B_2 - A_3 B_3$, where the metric tensor is used. In particular, the scalar product of two four-vectors is defined as $pq = p_\mu q_\mu = p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3$. It is a relativistic *invariant*.

We will assume that there exist so-called asymptotic free final states for particle-like excitation of quantum fields. These asymptotic states will be associated with initial or final state (elementary) particles which fly in a free space without interactions. For such states we apply the *on-mass-shell condition* $p^2 = pp = p_0^2 - \mathbf{p}^2 = E^2 - \mathbf{p}^2 = m^2$ where m is the mass of the particle.

Now we will postulate the properties of the fields that are required for the construction of the SM. A neutral scalar field can be defined as

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}\mathbf{p}}{\sqrt{2p_0}} \left(e^{-ipx} a^-(\mathbf{p}) + e^{+ipx} a^+(\mathbf{p}) \right), \tag{2}$$

where $a^\pm(\mathbf{p})$ are *creation* and *annihilation* operators. Their commutation relations read

$$[a^-(\mathbf{p}), a^+(\mathbf{p}')] \equiv a^-(\mathbf{p})a^+(\mathbf{p}') - a^+(\mathbf{p}')a^-(\mathbf{p}) = \delta(\mathbf{p} - \mathbf{p}'),$$
$$[a^-(\mathbf{p}), a^-(\mathbf{p}')] = [a^+(\mathbf{p}), a^+(\mathbf{p}')] = 0. \tag{3}$$

The field is a function of four-coordinate x in the Minkowsky space. It behaves as a plane wave in the whole space. The Lagrangian[1] for the neutral scalar field can be chosen in the form

$$\mathcal{L}(x) = \frac{1}{2}(\partial_\mu\varphi\partial_\mu\varphi - m^2\varphi^2). \tag{4}$$

Note that it depends only on the field and its first derivative. Variation of the action $A \equiv \int \mathrm{d}^4\mathcal{L}(x)$ with respect to variations of the field $\varphi \to \varphi + \delta\varphi$ according to the *least action principle* gives

$$\delta \int \mathrm{d}x\mathcal{L}(x) = \int \mathrm{d}x\left(\frac{\partial\mathcal{L}}{\partial\varphi}\delta\varphi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\delta(\partial_\mu\varphi)\right) = 0. \tag{5}$$

Here we apply quite natural for QFT zero boundary conditions for the field and its derivative at infinity and get the well-known Klein–Gordon equation of motion

$$(\partial_\mu^2 + m^2)\varphi(x) = 0. \tag{6}$$

EXERCISE: Check that the postulated above field $\varphi(x)$ satisfies the equation.

Creation and annihilation operators act in the Fock space which consists of *vacuum* ground state denoted as $|0\rangle$ and *excitations* over it. For the vacuum state we postulate

$$a^-(\mathbf{p})|0\rangle = 0, \quad \langle 0|a^+(\mathbf{p}) = 0, \quad \langle 0|0\rangle = 1. \tag{7}$$

Actually, $a^-(\mathbf{p})|0\rangle = 0 \cdot |0\rangle$ but the vacuum state can be dropped since finally all observable quantities are proportional to $\langle 0|0\rangle$. The field excitations are states of the form

$$|f\rangle = \int \mathrm{d}\mathbf{p}\, f(\mathbf{p})a^+(\mathbf{p})|0\rangle, \quad |g\rangle = \int \mathrm{d}\mathbf{p}\mathrm{d}\mathbf{q}\, g(\mathbf{p},\mathbf{q})a^+(\mathbf{p})a^+(\mathbf{q})|0\rangle, \quad \ldots \tag{8}$$

The most simple excitation $a^+(\mathbf{p})|0\rangle \equiv |p\rangle$ is used to describe a single on-mass-shell particle with momentum $\mathbf{p}$. Then $a^+(\mathbf{p})a^+(\mathbf{q})|0\rangle$ is a two-particle state and so on. Because of the presence of modulating functions like $f(\mathbf{p})$ and $g(\mathbf{p},\mathbf{q})$, the Fock space is infinite-dimensional.

EXERCISES: 1) Find the norm $\langle p|p\rangle$; 2) check that operator $\hat{N} = \int \mathrm{d}\mathbf{p}\, a^+(\mathbf{p})a^-(\mathbf{p})$ acts as a particle number operator.

A charged scalar field is defined as

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}\mathbf{p}}{\sqrt{2p_0}} \left(e^{-ipx}a^-(\mathbf{p}) + e^{+ipx}b^+(\mathbf{p})\right),$$

$$\varphi^*(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}\mathbf{p}}{\sqrt{2p_0}} \left(e^{-ipx}b^-(\mathbf{p}) + e^{+ipx}a^+(\mathbf{p})\right),$$

$$[a^-(\mathbf{p}), a^+(\mathbf{p}')] = [b^-(\mathbf{p}), b^+(\mathbf{p}')] = \delta(\mathbf{p} - \mathbf{p}'), \quad [a^\pm, b^\pm] = 0,$$

where operators $a^\pm(\mathbf{p})$ create and annihilate particles, while operators $b^\pm(\mathbf{p})$ are used for the same purpose for *anti-particles*. The corresponding Lagrangian reads

$$\mathcal{L}(\phi, \phi^*) = \partial_\mu\varphi^*\partial_\mu\varphi - m^2\varphi^*\varphi. \tag{9}$$

Note that φ and φ^* are related by a generalized conjugation which involves operator transformations: $(a^\pm)^* = a^\mp$ and $(b^\pm)^* = b^\mp$. It is worth no to note also that φ and φ^* are not "a particle and an anti-particle".

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[1]Actually it is a Lagrangian density.

A massive charged vector field (remind W bosons) is defined as

$$U_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{p}}{\sqrt{2p_0}} \sum_{n=1,2,3} e_\mu^n(\mathbf{p}) \left(e^{-ipx}a_n^-(\mathbf{p}) + e^{+ipx}b_n^+(\mathbf{p})\right),$$

$$U_\mu^*(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{p}}{\sqrt{2p_0}} \sum_{n=1,2,3} e_\mu^n(\mathbf{p}) \left(e^{-ipx}b_n^-(\mathbf{p}) + e^{+ipx}a_n^+(\mathbf{p})\right),$$

$$[a_n^-(\mathbf{p}), a_l^+(\mathbf{p}')] = [b_n^-(\mathbf{p}), b_l^+(\mathbf{p}')] = \delta_{nl}\delta(\mathbf{p} - \mathbf{p}'), \quad [a^\pm, b^\pm] = 0.$$

For *polarization vectors* $e_\mu^n(\mathbf{p})$ the following conditions are applied:

$$e_\mu^n(\mathbf{p})e_\mu^l(\mathbf{p}) = -\delta_{nl}, \qquad p_\mu e_\mu^n(\mathbf{p}) = 0. \tag{10}$$

EXERCISE: Using the above orthogonality conditions, show that

$$\sum_{n=1,2,3} e_\mu^n(\mathbf{p})e_\nu^n(\mathbf{p}) = -\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right). \tag{11}$$

The Lagrangian for a massive charged vector field takes the form

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu U_\nu^* - \partial_\nu U_\mu^*\right)\left(\partial_\mu U_\nu - \partial_\nu U_\mu\right) + m^2 U_\mu^* U_\mu. \tag{12}$$

The corresponding Euler-Lagrange equation reads

$$-\partial_\mu(\partial_\mu U_\nu - \partial_\nu U_\mu) - m^2 U_\mu = 0.$$

EXERCISE: Using the above equation, show that $\partial_\nu U_\nu(x) = 0$, *i.e., derive the* Lorentz condition. Note that the Lorentz condition removes from the field one of four independent degrees of freedom (components).

A massless neutral vector field (a photon) is defined as

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{p}}{\sqrt{2p_0}} e_\mu^\lambda(\mathbf{p}) \left(e^{-ipx}a_\lambda^-(\mathbf{p}) + e^{+ipx}a_\lambda^+(\mathbf{p})\right), \tag{13}$$

$$[a_\lambda^-(\mathbf{p}), a_\nu^+(\mathbf{p}')] = -g_{\lambda\nu}\delta(\mathbf{p} - \mathbf{p}') \qquad e_\mu^\lambda(\mathbf{p})e_\nu^\lambda(\mathbf{p}) = g_{\mu\nu}, \qquad e_\mu^\lambda(\mathbf{p})e_\mu^\nu(\mathbf{p}) = g_{\lambda\nu}.$$

Formally this field has four polarizations, but only two of them correspond to physical degrees of freedom. The corresponding Lagrangian reads

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu}, \qquad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{14}$$

A Dirac fermion field is defined as

$$\Psi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{p}}{\sqrt{2p_0}} \sum_{r=1,2} \left(e^{-ipx}a_r^-(\mathbf{p})u_r(\mathbf{p}) + e^{+ipx}b_r^+(\mathbf{p})v_r(\mathbf{p})\right), \tag{15}$$

$$\overline{\Psi}(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{p}}{\sqrt{2p_0}} \sum_{r=1,2} \left(e^{-ipx}b_r^-(\mathbf{p})\bar{v}_r(\mathbf{p}) + e^{+ipx}a_r^+(\mathbf{p})\bar{u}_r(\mathbf{p})\right),$$

$$[a_r^-(\mathbf{p}), a_s^+(\mathbf{p}')]_+ = [b_r^-(\mathbf{p}), b_s^+(\mathbf{p}')]_+ = \delta_{rs}\delta(\mathbf{p} - \mathbf{p}'),$$

$$[a_r^+(\mathbf{p}), a_s^+(\mathbf{p}')]_+ = [a_r^-(\mathbf{p}), b_s^+(\mathbf{p}')]_+ = \ldots = 0.$$

EXERCISE: Show that $a_r^+(\mathbf{p})a_r^+(\mathbf{p}) = 0$, *i.e., verify the* Pauli principle.

Here, u_r, u_r, $\bar{u}_r$, and $\bar{v}_r$ are four-component spinors, so $\Psi(x) \equiv \{\Psi_\alpha(x)\}$ is a four-vector column, $\alpha = 1, 2, 3, 4$, and $\overline{\Psi}(x)$ is a four-vector row,

$$\bar{u}u = \sum_{\alpha=1}^{4} \bar{u}_\alpha u_\alpha = \sum_{\alpha=1}^{4} u_\alpha \bar{u}_\alpha = \mathrm{Tr}(u\bar{u}).$$

Spinors are solutions of the (Dirac) equations:

$$\begin{aligned}
(\hat{p} - m)u_r(\mathbf{p}) &= 0, & \bar{u}_r(\mathbf{p})(\hat{p} - m) &= 0, \\
(\hat{p} + m)v_r(\mathbf{p}) &= 0, & \bar{v}_r(\mathbf{p})(\hat{p} + m) &= 0, \\
\hat{p} \equiv p_\mu \gamma_\mu = p_0\gamma_0 - p_1\gamma_1 - p_2\gamma_2 - p_3\gamma_3, & & m &\equiv m\mathbf{1},
\end{aligned} \tag{16}$$

where $\mathbf{1}$ is the unit four-by-four matrix. For the solutions of the above equations we impose the normalization conditions

$$\bar{u}_r(\mathbf{p})u_s(\mathbf{p}) = -\bar{v}_r(\mathbf{p})v_s(\mathbf{p}) = 2m\delta_{rs}.$$

The gamma matrixes (should) satisfy the commutation condition

$$[\gamma_\mu, \gamma_\nu]_+ = 2g_{\mu\nu}\mathbf{1} \quad \Rightarrow \quad \gamma_0^2 = \mathbf{1}, \quad \gamma_1^2 = \gamma_2^2 = \gamma_3^2 = -\mathbf{1}$$

and the condition of Hermitian conjugation

$$\gamma_\mu^\dagger = \gamma_0 \gamma_\mu \gamma_0.$$

The latter leads to the rule of the *Dirac conjugation*:

$$\overline{\Psi} = \Psi^\dagger \gamma_0, \quad \bar{u} = u^\dagger \gamma_0, \quad \bar{v} = v^\dagger \gamma_0. \tag{17}$$

EXERCISE: Show that the Dirac conjugation rule is consistent with the set of Dirac equations (16).

Note that explicit expressions for gamma matrixes are not unique, but they are not necessary for construction of observables, *QUESTION: Why is that so?* The most common representations are the so-called Dirac's (standard) and Weyl's (spinor) ones.

Two values of index r in Eq. (15) correspond to two independent degrees of freedom for each spinor in other words to to independent solutions of the Dirac equations. These two degrees of freedom can be treated as two polarization states like 'spin up' and 'spin down'. But in the Standard Model, there is one special choice of the basis for spinors, namely we will distinguish *Left* (L) and *Right* (R) spinors. By definition,

$$\Psi_L \equiv P_L\Psi, \quad \Psi_R \equiv P_R\Psi, \quad P_{L,R} \equiv \frac{1 -, + \gamma_5}{2}, \quad \Psi = \Psi_L + \Psi_R. \tag{18}$$

Here $\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3$, this gamma-matrix has the properties

$$[\gamma_\mu, \gamma_5]_+ = 0, \quad \gamma_5^2 = \mathbf{1}, \quad \gamma_5^\dagger = \gamma_5. \tag{19}$$

As can be seen from Eq. (18), $P_{L,R}$ form a complete set of orthogonal projection operators,

$$P_{L,R}^2 = P_{L,R} \quad P_L P_R = P_R P_L = 0, \quad P_L + P_R = 1. \tag{20}$$

The sign before γ_5 in the definition of the projection operators in Eq. (18) corresponds to the standard representation of gamma matrixes[2]. The Dirac conjugation (17) of left and right spinors gives

$$\overline{\Psi}_L \equiv \overline{\Psi}\frac{1 + \gamma_5}{2}, \qquad \overline{\Psi}_R \equiv \overline{\Psi}\frac{1 - \gamma_5}{2}.$$

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[2]In the spinor representation the sign is opposite: $P_L \equiv (1 + \gamma_5)/2$ and $P_R \equiv (1 - \gamma_5)/2$.

Remind some properties of gamma matrixes

$$\mathrm{Tr}\gamma_\mu = \mathrm{Tr}\gamma_5 = 0, \quad \mathrm{Tr}\gamma_\mu\gamma_\nu = 4g_{\mu\nu}, \quad \mathrm{Tr}\gamma_5\gamma_\mu\gamma_\nu = 0,$$
$$\mathrm{Tr}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta = 4(g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}), \qquad \mathrm{Tr}\gamma_5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta = -4i\varepsilon_{\mu\nu\alpha\beta}.$$

The equations for u and v are chosen so that we get the conventional *Dirac equations*

$$(i\gamma_\mu\partial_\mu - m)\Psi(x) = 0, \qquad i\partial_\mu\overline{\Psi}(x)\gamma_\mu + m\overline{\Psi}(x) = 0.$$

These equations follow also from the Lagrangian

$$\mathcal{L} = \frac{i}{2}\left[\overline{\Psi}\gamma_\mu(\partial_\mu\Psi) - (\partial_\mu\overline{\Psi})\gamma_\mu\Psi\right] - m\overline{\Psi}\Psi \equiv i\overline{\Psi}\gamma_\mu\partial_\mu\Psi - m\overline{\Psi}\Psi.$$

Note that the right-hand side is a short notation for the true Lagrangian which is given in the middle.

In QFT Lagrangians (Hamiltonians) should be Hermitian: $\mathcal{L}^\dagger = \mathcal{L}$. *QUESTION: What kind of problems one can have with a non-Hermitian Hamiltonian?*

Up to now we considered only *free* non-interacting fields. Studies of transitions between free states is the main task of QFT[3].

Let us postulate the transition *amplitude* (matrix element) $\mathcal{M}$ of a physical process:

$$\mathcal{M} \equiv \langle out|S|in\rangle, \qquad S \equiv T\exp\left(i\int \mathrm{d}x\mathcal{L}_I(\varphi(x))\right). \tag{21}$$

Here S is the so-called S-matrix which is the general evolution operator of quantum states. Letter T means the *time ordering* operator, it will be discussed a bit later. The initial and final states are

$$|in\rangle = a^+(\mathbf{p}_1)\ldots a^+(\mathbf{p}_s)|0\rangle, \qquad |out\rangle = a^+(\mathbf{p}'_1)\ldots a^+(\mathbf{p}'_r)|0\rangle. \tag{22}$$

The differential probability to evolve from $|in\rangle$ to $|out\rangle$ is

$$\mathrm{d}w = (2\pi)^4\delta(\sum p'_i)\frac{n_1\ldots n_s}{2E_1\ldots E_s}|\mathcal{M}|^2\prod_{j=1}^r\frac{\mathrm{d}\mathbf{p}'_j}{(2\pi)^3 2E'_j}.$$

Here n_i is the particle number density of ith particle beam.

Nontrivial transitions happen due to interactions of fields. QFT prefers dealing with *local* interactions $\Rightarrow$ $\mathcal{L}_I = \mathcal{L}_I(\varphi(x))$. By 'local' we mean that all interaction terms in the Lagrangian are constructed as products of fields (or their first derivatives) taken the same space-time coordinate.

Examples of interaction Lagrangians:

$$g\varphi^3(x), \qquad h\varphi^4(x), \qquad y\varphi(x)\overline{\Psi}(x)\Psi(x),$$
$$e\overline{\Psi}(x)\gamma_\mu\Psi(x)A_\mu(x), \qquad G\overline{\Psi}_1(x)\gamma_\mu\Psi_1(x)\cdot\overline{\Psi}_2(x)\gamma_\mu\Psi_2(x).$$

IMPORTANT: Always keep in mind the dimension *of your objects!* The reference unit is the dimension of energy (mass):

$$[E] = [m] = 1 \quad \Rightarrow \quad [p] = 1, \qquad [x] = -1. \tag{23}$$

An action should be dimensionless

$$\left[\int \mathrm{d}x\mathcal{L}(x)\right] = 0 \Rightarrow [\mathcal{L}] = 4. \tag{24}$$

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[3]Collective, nonperturbative effects, bound states etc. are also of interest, but that goes beyond the scope of these lectures.

EXERCISE: Show that $[\varphi] = [A_\mu] = 1$ and $[\Psi] = 3/2$. Find the dimensions of the coupling constants g, h, y, e, and G in the examples above.

By definition the time ordering operator acts as follows:

$$T\, A_1(x_1)\dots A_n(x_n) = (-1)^l A_{i_1}(x_{i_1})\dots A_{i_n}(x_{i_n}) \text{ with } x_{i_1}^0 > \dots > x_{i_n}^0, \tag{25}$$

where l is the number of fermion field permutations.

The perturbative expansion of the S matrix exponent (21) leads to terms like

$$\frac{i^n g^n}{n!}\langle 0|a^-(\mathbf{p}'_1)\dots a^-(\mathbf{p}'_r)\int \mathrm{d}x_1\dots \mathrm{d}x_n T\varphi^3(x_1)\dots\varphi^3(x_n)a^+(\mathbf{p}_1)\dots a^+(\mathbf{p}_s)|0\rangle.$$

Remind that fields φ also contain creation and annihilation operators. By permutation of operators $a^-(\mathbf{p})a^+(\mathbf{p}') = a^+(\mathbf{p}')a^-(\mathbf{p}) + \delta(\mathbf{p} - \mathbf{p}')$ we move a^- to the right and a^+ to the left. At the end we get either 0 because $a^-|0\rangle >= 0$ or some terms proportional to $\langle 0|0\rangle = 1$.

EXERCISE: Show that $[a^-(\mathbf{p}), \varphi(x)] = \frac{e^{ipx}}{(2\pi)^{3/2}\sqrt{2p_0}}$ and $[a_r^-(\mathbf{p}), \overline{\Psi}(x)]_+ = \frac{e^{ipx}\bar{u}_r(\mathbf{p})}{(2\pi)^{3/2}\sqrt{2p_0}}.$

By definition the *causal Green function* is given by

$$\langle 0|T\varphi(x)\varphi(y)|0\rangle \equiv -iD^c(x-y). \tag{26}$$

It is a building block for construction of amplitudes. One can show (see textbooks) that

$$(\partial^2 + m^2)D^c(x) = \delta(x), \tag{27}$$

so that D^c is the Green function of the Klein-Gordon operator,

$$D^c(x) = \frac{-1}{(2\pi)^4}\int \frac{\mathrm{d}p\, e^{-ipx}}{p^2 - m^2 + i0}, \tag{28}$$

where $+i0$ is an infinitesimally small quantity imaginary quantity which shifts the poles of the Green function from the real axis. The sign of this quantity is chosen to fulfil the requirement of the time ordering operation in Eq. (26).

For other fields we have

$$\langle 0|T\,\Psi(x)\overline{\Psi}(y)|0\rangle = \frac{i}{(2\pi)^4}\int \frac{\mathrm{d}p\, e^{-ip(x-y)}(\hat{p}+m)}{p^2 - m^2 + i0},$$

$$\langle 0|T\,U_\mu(x)U_\nu^*(y)|0\rangle = \frac{-i}{(2\pi)^4}\int \frac{\mathrm{d}p\, e^{-ip(x-y)}(g_{\mu\nu} - p_\mu p_\nu/m^2)}{p^2 - m^2 + i0}, \tag{29}$$

$$\langle 0|T\,A_\mu(x)A_\nu(y)|0\rangle = \frac{-i}{(2\pi)^4}\int \frac{\mathrm{d}p\, e^{-ip(x-y)}g_{\mu\nu}}{p^2 + i0}.$$

The Wick theorem states that for any combinations of fields

$$T\,A_1\dots A_n \equiv \sum(-1)^l\langle 0|TA_{i_1}A_{i_2}|0\rangle\dots\langle 0|TA_{i_{k-1}}A_{i_k}|0\rangle\, :A_{i_k}\dots A_{i_n}: \tag{30}$$

The sum is taken over all possible ways to pair the fields.

The *normal ordering* operation acts as

$$:a_1^- a_2^+ a_3^- a_4^- a_5^+ a_6^- a_7^+: = (-1)^l a_2^+ a_5^+ a_7^+ a_1^- a_3^- a_4^- a_6^- \tag{31}$$

so that all annihilation operators go to the left and all creation operators go to the right. The number of fermion operator permutations l provides the factor $(-1)^l$.

Using the Wick theorem we construct the *Feynman rules* for simple $g\phi^3$ and $h\varphi^4$ interactions. But for the case of gauge interactions we need something more as we will see below.

It appears that symmetries play the crucial role in QFT. There are two major types of symmetries in the SM: *global* and *local* ones. By a global symmetry we mean invariance of a Lagrangian and observables with respect to certain transformations of coordinates and/or fields if the transformations are the same in each space-time point. If the transformations do depend on coordinates, the corresponding symmetry is called local.

The **1st Noether (Nöther) theorem**:
If an action is invariant with respect to transformations of a global Lie group G_r with r parameters, then there are r linearly independent combinations of Lagrange derivatives which become complete divergences; and vice versa.

If the field satisfies the Euler–Lagrange equations, then $\mathrm{div} J = \nabla J = 0$, i.e., the *Noether currents* are conserved. Integration of those divergences over a 3-dimensional volume (with certain boundary conditions) leads to r *conserved charges*. Remind that conservation of the electric charge in QED is related to the global $U(1)$ symmetry of this model, and that Poincaré symmetries lead to conservation of energy, momentum, and angular momentum.

Much more involved and actually important for us is the **2nd Noether theorem**:
If the action is invariant with respect to the infinite-dimensional r-parametric group $G_{\infty,r}$ with derivatives up to the kth order, then there are r independent relations between Lagrange derivatives and derivatives of them up to the kth order; and vice versa.

The importance of the second theorem is justified by the fact that gauge groups (and also the general coordinate transformation in Einstein's gravitational theory) are infinite-dimensional groups. The 2nd Noether theorem provides r conditions on the fields which are additional to the standard Euler–Lagrange equations. There conditions should be used to exclude *double counting* of physically *equivalent* field configurations.

2.1 Gauge symmetries

Let us start the discussion of local gauge symmetries with Quantum electrodynamics (QED). The free Lagrangians for electrons and photons

$$\mathcal{L}_0(\Psi) = i\overline{\Psi}\gamma_\mu\partial_\mu\Psi - m\overline{\Psi}\Psi, \quad \mathcal{L}_0(A) = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{32}$$

are invariant with respect to the *global $U(1)$* transformations

$$\Psi(x) \to \exp(ie\theta)\Psi(x), \quad \overline{\Psi}(x) \to \exp(-ie\theta)\overline{\Psi}(x), \quad A_\mu(x) \to A_\mu(x). \tag{33}$$

One can note that $F_{\mu\nu}$ is invariant also with respect to *local* transformations $A_\mu(x) \to A_\mu(x) + \partial_\mu\omega(x)$. For fermions the corresponding transformations are

$$\Psi(x) \to \exp(ie\omega(x))\Psi(x), \quad \overline{\Psi}(x) \to \exp(-ie\omega(x))\overline{\Psi}(x), \tag{34}$$

i.e., the global constant angle θ in Eq. (33) is substituted by a local function $\omega(x)$ which varies from one space-time point to another.

The question is how to make the fermion Lagrangian being also invariant? The answer is to introduce the so-called *covariant derivative:*

$$\partial_\mu \to D_\mu, \qquad D_\mu\Psi \equiv (\partial_\mu - ieA_\mu)\Psi, \quad D_\mu\overline{\Psi} \equiv (\partial_\mu + ieA_\mu)\overline{\Psi}. \tag{35}$$

Then we get the QED Lagrangian:

$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + i\overline{\Psi}\gamma_\mu D_\mu\Psi - m\overline{\Psi}\Psi$$

$$= -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + i\overline{\Psi}\gamma_\mu\partial_\mu\Psi - m\overline{\Psi}\Psi + e\overline{\Psi}\gamma_\mu\Psi A_\mu,$$

where the last term describes interaction of electrons and positrons with photons. The most important point here is that the structure of the interaction term is completely fixed by the gauge symmetry. Nevertheless, there is one specific feature of the abelian $U(1)$ case, namely the values of electric charges (coupling constants) can be different e.g., for up and down quarks.

One can note that local transformations are more natural because of causality of Special relativity.

EXERCISES: 1) Check the covariance: $D_\mu\Psi \to e^{ie\omega(x)}(D_\mu\Psi)$; *2) construct the Lagrangian of scalar QED (use Eqs. (9) and (14)).*

Let's look now again at the free photon Lagrangian

$$\mathcal{L}_0(A) = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 = -\frac{1}{2}A_\nu K_{\mu\nu}A_\nu,$$

$$K_{\mu\nu} = g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu \quad \Rightarrow K_{\mu\nu}(p) = p_\mu p_\nu - g_{\mu\nu}p^2.$$

Operator $K_{\mu\nu}(p)$ has zero modes (since $p_\mu K_{\mu\nu} = 0$), so it is not invertable. Definition of the photon propagator within the *functional integral formalism* becomes impossible. The reason is the unresolved symmetry. The solution is to introduce a *gauge fixing term* into the Lagrangian:

$$\mathcal{L}_0(A) = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A_\mu)^2 \quad \Rightarrow$$

$$\langle 0|T\, A_\mu(x)A_\nu(y)|0\rangle = \frac{-i}{(2\pi)^4}\int \mathrm{d}p\, e^{-ip(x-y)}\frac{g_{\mu\nu} + (\alpha-1)p_\mu p_\nu/p^2}{p^2 + i0}.$$

It is very important that physical quantities do not depend on the value of α.

Let us briefly discuss the features of non-abelian Gauge symmetries which will be also used in the construction of the SM. Transformations for a *non-abelian* case read

$$\Psi_i \to \exp ig\omega^a t^a_{ij}\Psi_j, \qquad [t^a, t^b] = if^{abc}t^c,$$

$$B^a_\mu \to B^a_\mu + \partial_\mu\omega^a + gf^{abc}B^b_\mu\omega^c, \quad F_{\mu\nu} \equiv \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + gf^{abc}B^b_\mu B^c_\nu,$$

where t^a are the group generators, f^{abc} are the structure constants (see details in the lectures by A. Mitov).

We introduce the covariant derivative

$$\partial_\mu\Psi \to D_\mu\Psi \equiv (\partial_\mu - igB^a_\mu t^a)\Psi$$

and get

$$\mathcal{L}(\Psi, B) = i\overline{\Psi}\gamma_\mu D_\mu\Psi + \mathcal{L}(B),$$

$$\mathcal{L}(B) = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu B^a_\mu)^2 = -\frac{1}{4}\left(\partial_\mu B^a_\nu - \partial_\nu B^a_\mu\right)^2 - \frac{1}{2\alpha}(\partial_\mu B^a_\mu)^2$$

$$- \frac{g}{2}f^{abc}\left(\partial_\mu B^a_\nu - \partial_\nu B^a_\mu\right)B^b_\mu B^c_\nu - \frac{g^2}{4}f^{abc}f^{ade}B^b_\mu B^c_\nu B^d_\mu B^e_\nu.$$

Note that $\mathcal{L}(B)$ contains self-interactions and can not be treated as a 'free Lagrangian'. There is no any mass term for the gauge field in the Lagrangian, $m_B \equiv 0$, because such a mass term would be not gauge-invariant. It is worth to note that the non-abelian charge g is *universal*, i.e., it is the same for all fields which are transformed by the given group.

Exclusion of double-counting due to the physical equivalence of the field configurations related to each other by non-abelian gauge transformations is nontrivial. Functional integration over those identical

configurations (or application of the BRST method) leads to the appearance of the so-called *Faddeev–Popov ghosts*:

$$\mathcal{L}(\Psi, B) \to \mathcal{L}(\Psi, B) + \mathcal{L}_{gh},$$
$$\mathcal{L}_{gh} = -\partial_\mu \bar{c}^a \partial_\mu c^a + g f^{acb} \bar{c}^a B_\mu^c \partial_\mu c^a = -\partial_\mu \bar{c}^a \partial_\mu c^a - g f^{acb} \partial_\mu \bar{c}^a B_\mu^c c^a, \qquad (36)$$

where c and $\bar{c}$ are ghost fields, they are fermions with a boson-like kinetic term. Keep in mind that Faddeev–Popov ghosts are *fictitious* particles. In the Feynman rules they (should) appear only as virtual states in propagators but not in the initial and final states. Formally, ghosts can be found also in QED, but they are non-interacting since $f^{abc} = 0$ there and can be totally omitted.

2.2 Regularization and renormalization

Higher-order terms in the perturbative series contain loop integrals e.g.,

$$I_2 \equiv \int \frac{\mathrm{d}^4 p}{(p^2 + i0)((k - p)^2 + i0)} \sim \int \frac{|p|^3 \, \mathrm{d}|p|}{|p|^4} \sim \ln \infty. \qquad (37)$$

Introduction of a cut-off M leads to a finite, i.e., *regularized* value of the integral:

$$I_2^{\text{cut-off}} = i\pi^2 \left(\ln \frac{M^2}{k^2} + 1 \right) + \mathcal{O}\left(\frac{k^2}{M^2} \right) = i\pi^2 \left(\ln \frac{M^2}{\mu^2} - \ln \frac{k^2}{\mu^2} + 1 \right) + \mathcal{O}\left(\frac{k^2}{M^2} \right). \qquad (38)$$

Another possibility is the *dimensional regularization* where $dim = 4 \to dim = 4 - 2\varepsilon$

$$I_2^{\text{dim.reg.}} = \mu^{2\varepsilon} \int \frac{\mathrm{d}^{4-2\varepsilon} p}{(p^2 + i0)((k - p)^2 + i0)} = i\pi^2 \left(\frac{1}{\varepsilon} - \ln \frac{k^2}{\mu^2} + 2 \right) + \mathcal{O}(\varepsilon). \qquad (39)$$

Here the divergence is parameterized by the ε^{-1} term. The origin of UV divergences is the *locality* of interactions in QFT.

Let's consider a three-point (vertex) function in the $g\phi^3$ model, it looks like

$$G = \int \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \, \varphi(x)\varphi(y)\varphi(z)F(x, y, z),$$

$$F^{\text{dim.reg.}} = \frac{A}{\varepsilon}\delta(y - x)\delta(z - x) + \dots$$

IMPORTANT: Divergent terms are *local* because of the delta-functions.

*A QFT model is called **renormalizable** if all UV-divergent terms are of the type of the ones existing in the (semi)classical Lagrangian. Otherwise the model is **non-renormalizable**.*

EXAMPLES:
a) renormalizable models: QED, QCD, the SM [proved by 't Hooft & Veltman], $h\varphi^4$, $g\varphi^3$;
b) non-renormalizable models: the Fermi model with $\mathcal{L} \sim G(\overline{\Psi}\gamma_\mu \Psi)^2$ and General Relativity.

It can be shown that models with dimensionful ($[G] < 0$) coupling constants are non-renormalizable.

In renormalizable models all UV divergences can be *subtracted* from amplitudes and shifted into *counter terms* in $\mathcal{L}$. Each term in $\mathcal{L}$ gets a *renormalization constant*:

$$\mathcal{L} = \frac{Z_2}{2}(\partial\varphi)^2 - \frac{Z_m m^2}{2}\varphi^2 + Z_4 h\varphi^4 = \frac{1}{2}(\partial\varphi_B)^2 - \frac{m_B^2}{2}\varphi^2 + h_B\varphi^4,$$

where $\varphi_B = \sqrt{Z_2}\varphi$, $m_B^2 = m^2 Z_M Z_2^{-1}$, $h_B = h Z_4 Z_2^{-2}$ are *bare* field, mass, and charge,

$$Z_i(h, \varepsilon) = 1 + \frac{Ah}{\varepsilon} + \frac{Bh^2}{\varepsilon^2} + \frac{Ch^2}{\varepsilon} + \mathcal{O}(h^3).$$
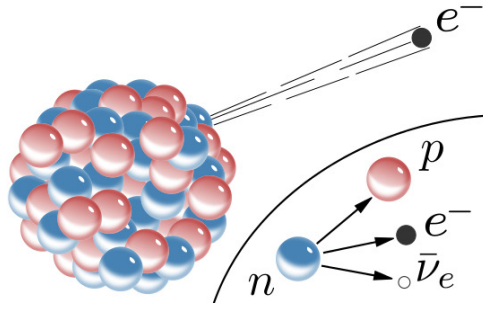
Fig. 2: Beta decay.

Remonrmalization constants are chosen in such a way that divergences in amplitudes are *cancelled out* with divergences in Z_i. The happens order by order.

R. Feynman said once: *"I think that the renormalization theory is simply a way to sweep the difficulties of the divergences of electrodynamics under the rug."* Physicists are still not fully satisfied by the renormalization procedure, but the method have be verified in many models. Moreover, renormalizable models including the SM appear to be the most successful ones in description of phenomenology.

Physical results should not depend on the auxiliary scale μ:

$$F(k, g, m) \xrightarrow{\infty} F_{\text{reg}}(k, M, g, m) \xrightarrow{M \to \infty} F_{\text{ren}}(k, \mu, g, m) \xrightarrow{RG} F_{\text{phys}}(k, \Lambda, m),$$

where Λ is a dimensionful scale.

Charge (and mass) become *running*, i.e., energy-dependent:

$$g \to g\left(g, \frac{\mu'}{\mu}\right), \qquad \beta(g) \equiv \left.\frac{\mathrm{d}g}{\mathrm{d}\ln\mu}\right|_{g_B = Const}. \tag{40}$$

Note that the renormalization scale μ unavoidably appears in any scheme. Scheme and scale dependencies are reduced after including higher and higher orders of the perturbation theory.

At this point we stop the brief introduction to Quantum field theory, comprehensive details can be found in textbooks, e.g., Refs. [4, 6].

3 Construction of the Standard Model

3.1 The Fermi model and Cabibbo–Kobayashi–Maskawa mixing matrix

To describe the β decay $n \to p + e^- + \nu_e$ in 1933, see Fig. 2, Enrico Fermi suggested a simple model:

$$\mathcal{L}_{int} = G \underbrace{\overline{\Psi}_n \gamma_\rho \Psi_p}_{J_\rho^{(N)}} \cdot \underbrace{\overline{\Psi}_\nu \gamma_\rho \Psi_e}_{J_\rho^{(l)\dagger}} + h.c.$$

with interactions in the form of a product of two vector currents. This model was inspired by QED where similar vector currents appear.

In 1957 R. Marshak & G. Sudarshan; and R. Feynman & M. Gell-Mann modified the model:

$$\mathcal{L}_{\text{Fermi}} = \frac{G_{\text{Fermi}}}{\sqrt{2}} J_\mu J_\mu^\dagger,$$

$$J_\mu = \overline{\Psi}_e \gamma_\rho \frac{1 - \gamma_5}{2} \Psi_{\nu_e} + \overline{\Psi}_\mu \gamma_\rho \frac{1 - \gamma_5}{2} \Psi_{\nu_\mu} + (V - A)_{\text{nucleons}} + h.c. \tag{41}$$

Explicit *V-A* (Vector minus Axial–vector) form of weak interactions means the 100% violation of parity. In fact, it appears that only left fermions participate in weak interactions, while right fermions don't. The modification of the model was required to describe differential distributions of beta decays. Note that the CP symmetry in Lagrangian (41) is still preserved.

The modern form of the Fermi Lagrangian includes 3 fermion generations:

$$\mathcal{L}_{\text{Fermi}} = \frac{G_{\text{Fermi}}}{\sqrt{2}} (\bar{e}_L \ \bar{\mu}_L \ \bar{\tau}_L) \gamma_\rho \begin{pmatrix} \nu_{e,L} \\ \nu_{\mu,L} \\ \nu_{\tau,L} \end{pmatrix} \cdot (\bar{u}'_L \ \bar{c}'_L \ \bar{t}'_L) V_u^\dagger \gamma_\rho V_d \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix} + \dots$$

Quarks $\{q'\}$ are the *eigenstates* of strong interactions, and $\{q\}$ are the eigenstates of the weak ones.

Matrixes V_d and V_u describe quark mixing (see details in lectures by S. Gori):

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_d \times \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}, \qquad V_u^\dagger V_d \equiv V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

By construction, in this model (and further in the SM) the mixing matrixes are unitary: $V_i^\dagger V_i = \mathbf{1}$. In a sense, this property just keeps the number of quarks to be conserved. V_{CKM} contains 4 independent parameters: 3 angles and 1 phase.

QUESTION: What is mixed by V_{CKM}? E.g., what is mixed by the V_{ud} element of V_{CKM}?

The Fermi model describes β-decays and the muon decay $\mu \to e + \bar{\nu}_e + \nu_\mu$ with a very high precision. Nevertheless, there are two critical problems:
1. The model is non-renormalizable, remind that the dimension of the Fermi coupling constant $[G_{\text{Fermi}}] = -2$.
2. Unitarity in this model is violated: consider e.g., electron-neutrino scattering

$$\sigma_{\text{total}}(e\nu_e \to e\nu_e) \sim \frac{G_{\text{Fermi}}^2}{\pi} s, \qquad s = (p_e + p_{\nu_e})^2. \tag{42}$$

This cross section obviously growth with energy. Meanwhile the unitarity condition for l^{th} partial wave in the scattering theory requires that $\sigma_l < \frac{4\pi(2l+1)}{s}$. For $l = 1$ we reach the *unitarity limit* at $s_0 = 2\pi\sqrt{3}/G_{\text{Fermi}} \approx 0.9 \cdot 10^6 \text{ GeV}^2$. So at energies above $\sim 10^3$ GeV the Fermi model is completely senseless.

3.2 (Electro)Weak interactions in SM

The modern point of view is: *a renormalizable model which preserves unitarity is a Yang–Mills (non-abelian) gauge model.* So we have to try to construct an interaction Lagrangian using the principle of gauge symmetry.

Let's try to do that for description of weak interactions The 1st step: we introduce a massive vector W boson

$$\mathcal{L}_{\text{int}} = -g_w(J_\alpha W_\alpha + J_\alpha^\dagger W_\alpha^\dagger). \tag{43}$$

Then the scattering amplitude, see Fig. 3, takes the form

$$T = i(2\pi)^4 g_w^2 J_\alpha \frac{g_{\alpha\beta} - k_\alpha k_\beta / M_W^2}{k^2 - M_W^2} J_\beta^\dagger, \tag{44}$$

where k is the W boson momentum. If $|k| \ll M_W$ we reproduce the Fermi model with

$$\frac{G_{\text{Fermi}}}{\sqrt{2}} = \frac{g_w^2}{M_W^2}.$$
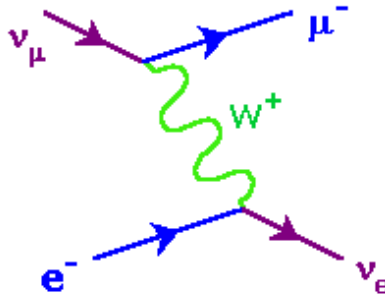
Fig. 3: Feynman diagram for electron-neutrino scattering with W boson exchange.

However such a way to introduce interactions again leads to a non-renormalizable model. The problem appears due to the specific momentum dependence in the propagator of a massive vector particle, see Eq. (29).

The minimal way to introduce electromagnetic and weak interactions as gauge ones is to take the group $SU(2) \otimes U(1)$. The abelian group $U(1)$ is the same as gives conservation of charge in QED. Instead of electric charge Q we introduce now the *hypercharge* Y. $U(1)$ gauge symmetry provides interactions of fermions with a massless vector (photon-like) field B_μ. The non-abelian group $SU(2)$ is the same as used for description of spinors in Quantum mechanics. Instead of spin we use here the *weak isospin* I. There are three massless vector Yang–Mills bosons in the adjoint representation of this group: W_μ^a, $a = 1, 2, 3$. Two of them can be charge and the third one should be neutral. Introduction of the third (electro)weak boson is unavoidable, even so that we did not have experimental evidences of weak neutral currents at the times of the SM invention.

QUESTION: Why weak interactions in the charged current (like muon and beta decays) were discovered experimentally much earlier than the neutral current ones?

One can show that the model built for gauge $SU(2) \otimes U(1)$ interactions of fermions and vector bosons is renormalizable and unitary. But this model doesn't describe the reality since all gauge bosons should be massless because of the symmetry condition. To resolve this problem we need a mechanism that will provide masses for some vector bosons without an explicit breaking of the gauge symmetry.

3.3 The Brout-Englert-Higgs mechanism

Let's consider the simple abelian $U(1)$ symmetry for interaction of a charged scalar field φ with a vector field A_μ:

$$\mathcal{L} = \partial_\mu \varphi^* \partial_\mu \varphi - V(\varphi) - \frac{1}{4} F_{\mu\nu}^2 + ie(\varphi^* \partial_\mu \varphi - \partial_\mu \varphi^* \varphi) A_\mu + e^2 A_\mu A_\mu \varphi^* \varphi.$$

If $V(\varphi) \equiv V(\varphi^* \cdot \varphi)$, $\mathcal{L}$ is invariant with respect to local $U(1)$ gauge transformations

$$\varphi \to e^{ie\omega(x)} \varphi, \quad \varphi^* \to e^{-ie\omega(x)} \varphi^*, \quad A_\mu \to A_\mu + \partial_\mu \omega(x). \tag{45}$$

In polar coordinates $\varphi \equiv \sigma(x) e^{i\theta(x)}$ and $\varphi^* \equiv \sigma(x) e^{-i\theta(x)}$ and the Lagrangian takes the form

$$\mathcal{L} = \partial_\mu \sigma \partial_\mu \sigma + e^2 \sigma^2 \underbrace{\left(A_\mu + \frac{1}{e} \partial_\mu \theta\right)}_{\equiv B_\mu} \underbrace{\left(A_\mu + \frac{1}{e} \partial_\mu \theta\right)}_{\equiv B_\mu} - V(\varphi^* \varphi) - \frac{1}{4} F_{\mu\nu}^2. \tag{46}$$

Note that after the change of variables $A_\mu + \frac{1}{e} \partial_\mu \theta \to B_\mu$, we have $F_{\mu\nu}(A) = F_{\mu\nu}(B)$.
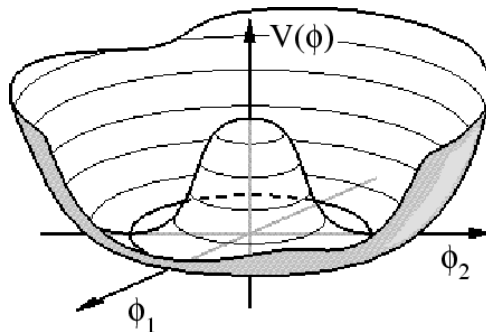
Fig. 4: The Higgs field potential. Picture courtesy: E.P.S. Shellard, DAMTP, Cambridge. From http://www.geocities.com/CapeCanaveral/ 2123/breaking.htm.

We see that $\theta(x)$ is completely swallowed by the field $B_\mu(x)$. So we made a change of variables. But which set of variables is the true physical one? This question is related to the choice of variables in which the secondary quantization should be performed. And the answer can be given by measurements. In fact, according to Quantum mechanics only quantum eigenstates can be observed, so we have a reference point.

R. Brout & F. Englert [10], and P. Higgs [11], see also a brief review in the Scientific Background on the Nobel Prize in Physics 2013 [12], suggested to take the scalar potential in the form

$$V(\varphi^*\varphi) = \lambda(\varphi^*\varphi)^2 + m^2\varphi^*\varphi. \tag{47}$$

For $\lambda > 0$ and $m^2 < 0$ we get the shape of a *Mexican hat*, see Fig. 4. We have chosen a potential for which $V(\varphi^*\varphi) = V(\sigma^2)$, while $\theta(x)$ corresponds to the rotational symmetry of the potential.

By looking at the derivative of the potential $\frac{\mathrm{d}V(\sigma)}{\mathrm{d}\sigma} = 0$, we find two critical points: $\sigma = 0$ is the local maximum and $\sigma_0 = \sqrt{-\frac{m^2}{2\lambda}}$ is the global minimum. We have to shift to the global minimum: $\sigma(x) \to h(x) + \sigma_0$. So we get

$$\mathcal{L} = \partial_\mu h \partial_\mu h + e^2 h^2 B_\mu B_\mu + 2e^2\sigma_0 h B_\mu B_\mu + e^2\sigma_0^2 B_\mu B_\mu - V(h) - \frac{1}{4}F_{\mu\nu}^2. \tag{48}$$

We see that field B_μ got the mass:

$$m_B^2 = 2e^2\sigma_0^2 = -\frac{e^2 m^2}{\lambda} > 0. \tag{49}$$

So, we generated a mass term for the vector field without putting it into the Lagrangian by hand. That is the core of the Brout–Englert–Higgs mechanism.

The quantity $\sigma_0 \equiv v$ is the *vacuum expectation value* (vev) of $\sigma(x)$,

$$v \equiv \langle 0|\sigma|0\rangle, \qquad v = \frac{1}{V_0}\int_{V_0} \mathrm{d}^3x\, \sigma(x). \tag{50}$$

Look now at the potential (keep in mind $m^2 = -2\lambda v^2$)

$$\begin{aligned} V(h) &= \lambda(h+v)^4 + m^2(h+v)^2 \\ &= \lambda h^4 + 4\lambda v h^3 + h^2\underbrace{(6\lambda v^2 + m^2)}_{2m_h^2 = 4\lambda v^2} + h\underbrace{(4\lambda v^3 + 2m^2 v)}_{=0} + \lambda v^4 + m^2 v^2. \end{aligned}$$

So the scalar field h has a normal (not tachyon-like) ($m_h^2 > 0$) mass term. One can see that the initial tachyons φ are not observable.

It is worth to note that even so the field content of the Lagrangian is changed, but the number of degrees of freedom is conserved. In fact initially we had two components of the scalar field and two components of the massless vector field, and after the change of variables we have a single scalar field plus a massive vector field with 3 independent components: $2 + 2 = 1 + 3$.

The field $\theta(x)$ is a *Nambu–Goldstone boson*. It is massless, $m_\theta = 0$, and corresponds to effortless rotations around the symmetry axis of the potential. In general, the Goldstone theorem claims that in a model with spontaneous breaking of a continuous global symmetry G_n (remind the first Noether theorem) there exist as many massless modes, as there are group generators which do not preserve the vacuum invariance.

The constant term $\lambda v^4 + m^2 v^2$ obviously doesn't affect equations of motion, but it contributes to the Universe energy density (too much, actually). That makes a problem for Cosmology. Formally, one can make a shift of the initial Lagrangian just by this term and avoid the problem.

Now let us return to the case of the Standard Model. To generate masses for 3 vector bosons we need at least 3 goldstones. The minimal possibility is to introduce one complex scalar doublet field:

$$\Phi \equiv \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \qquad \Phi^\dagger = (\Phi_1^* \ \Phi_2^*). \tag{51}$$

Then the following Lagrangian is $SU(2) \otimes U(1)$ invariant

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D_\mu \Phi) - m^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 - \frac{1}{4} W_{\mu\nu}^a W_{\mu\nu}^a - \frac{1}{4} B_{\mu\nu} B_{\mu\nu},$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu, \qquad W_{\mu\nu}^a \equiv \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\varepsilon^{abc} W_\mu^b W_\nu^c,$$

$$D_\mu \Phi \equiv \partial_\mu \Phi + ig W_\mu^a \frac{\tau^a}{2} \Phi + \frac{i}{2} g' B_\mu \Phi. \tag{52}$$

Again for $m^2 < 0$ there is a non-trivial minimum of the Higgs potential and a non-zero vev of a component: $\langle 0|\Phi_2|0\rangle = \eta/\sqrt{2}$. In accord with the Goldstone theorem, three massless bosons appear. The global $SU(2) \times SU(2)$ symmetry of the Higgs sector is reduced to the *custodial SU(2)* symmetry.

3.4 Electroweak bosons

The gauge bosons of the $SU(2) \otimes U(1)$ group can be represented as

$$W_\mu^+ = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}, \qquad W_\mu^- = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}, \qquad W_\mu^0 = W_\mu^3, \qquad B_\mu. \tag{53}$$

W_μ^0 and B_μ are both neutral and have the same quantum numbers, so they can mix. In a quantum world, 'can' means 'do':

$$W_\mu^0 = \cos\theta_w Z_\mu + \sin\theta_w A_\mu,$$
$$B_\mu = -\sin\theta_w Z_\mu + \cos\theta_w A_\mu, \tag{54}$$

where θ_w is the *weak mixing angle*, introduced first by S. Glashow, θ_w is known also the Weinberg angle. Remind that we have to choose variables which correspond to observables. Vector bosons Z_μ and A_μ are linear combinations of the primary fields W_μ^0 and B_μ.

It is interesting to note that Sheldon Glashow, Abdus Salam, and Steven Weinberg got the Nobel Prize in 1979, *before* the discovery of Z and W bosons in 1983, and even longer before the discovery of the Higgs boson. So the Standard Model has been distinguished before experimental confirmation of its key components.

Look now at the scalar fields:

$$\Phi \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \Psi_2(x) + i\Psi_1(x) \\ \eta + \sigma(x) + i\xi(x) \end{pmatrix}, \qquad \Phi^\dagger = \ldots$$

Fields $\Psi_{1,2}$ and ξ become massless Goldstone bosons. We *hide* them into the vector fields:

$$W_\mu^i \to W_\mu^i + \frac{2}{g\eta}\partial_\mu \Psi_i \;\Rightarrow\; M_W = \frac{g\eta}{2},$$

$$Z_\mu = \frac{g}{\sqrt{g^2 + g'^2}}W_\mu^0 - \frac{g'}{\sqrt{g^2 + g'^2}}B_\mu - \frac{2}{\eta\sqrt{g^2 + g'^2}}\partial_\mu \xi \;\Rightarrow\; M_Z = \frac{\eta\sqrt{g^2 + g'^2}}{2}. \quad (55)$$

The photon field appears massless by construction. Looking at the mixing we get

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}} = \frac{M_W}{M_Z}.$$

The non-abelian tensor

$$W_{\mu\nu}^a \equiv \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\varepsilon^{abc}W_\mu^b W_\nu^c$$

leads to triple and quartic self-interactions of the *primary* W_μ^a bosons, since

$$\mathcal{L} = -\frac{1}{4}W_{\mu\nu}^a W_{\mu\nu}^a + \dots \quad (56)$$

Fields B_μ and W_μ^a were not interacting between each other. But after the spontaneous breaking of the global symmetry in the Higgs sector, and the consequent change of the basis $\{W_\mu^0, B_\mu\} \to \{Z_\mu, A_\mu\}$, we get interactions of charged $W_\mu^\pm$ bosons with photons. That allows to fix the relation between the constants:

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} = g\sin\theta_w. \quad (57)$$

The value of the W boson charge ($\pm e$) is known from β decays. The very construction of the SM requires phenomenological input. So on the way of the SM building, not everything comes out automatically from symmetry principles etc.

3.5   EW interactions of fermions

We have chosen the $SU(2) \otimes U(1)$ symmetry group. To account for parity violation in weak decays, we assume different behavior of left and right fermions under $SU(2)_L$ transformations:

$$\text{left doublets} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \; \begin{pmatrix} u \\ d \end{pmatrix}_L \quad + 2 \text{ generations,}$$
$$\text{right singlets} \quad e_R, \; u_R, \; d_R, \; (\nu_{e,R}) \quad + 2 \text{ generations.}$$

The fermion Lagrangian is constructed with the help of covariant derivatives:

$$\mathcal{L}(\Psi) = \sum_{\Psi_i}\left[\frac{i}{2}\left(\overline{\Psi}_L\gamma_\alpha D_\alpha \Psi_L - D_\alpha\overline{\Psi}_L\gamma_\alpha\Psi_L\right) + \frac{i}{2}\left(\overline{\Psi}_R\gamma_\alpha D_\alpha \Psi_R - D_\alpha\overline{\Psi}_R\gamma_\alpha\Psi_R\right)\right],$$

$$D_\alpha\Psi_L \equiv \partial_\alpha\Psi_L + \frac{ig\tau^b}{2}W_\alpha^b\Psi_L - ig_1 B_\alpha\Psi_L, \qquad D_\alpha\Psi_R \equiv \partial_\alpha\Psi_L - ig_2 B_\alpha\Psi_L.$$

All interactions of the SM fermions with the vector bosons are here. But coupling constants $g_{1,2}$ still have to be fixed.

Fermions have weak isospins and hypercharges:

$$\Psi_L: \quad \left(\frac{1}{2}, -\frac{2g_1}{g'}\right), \qquad \Psi_R: \quad \left(0, -\frac{2g_2}{g'}\right). \quad (58)$$

Looking at interactions of e with A_μ in $\mathcal{L}(\Psi)$ we fix its hypercharges:

$$e_L: \quad \left(-\frac{1}{2}, \ -1\right), \qquad e_R: \quad \left(0, \ -2\right). \tag{59}$$

The *Gell-Mann—Nishijima formula* works for all fermions:

$$Q = I_3 + \frac{Y}{2}, \tag{60}$$

where Q is the electric charge, I_3 is the weak isospin projection, and Y is the hypercharge.

Interactions of leptons with $W^\pm$ and Z bosons come out in the form

$$\mathcal{L}_I = -\frac{g}{\sqrt{2}}\bar{e}_L\gamma_\mu\nu_{e,L}W^-_\mu + h.c. - \frac{gZ_\mu}{2\cos\theta_w}\left[\bar{\nu}_{e,L}\gamma_\mu\nu_{e,L}\right.$$

$$\left. +\bar{e}\gamma_\mu\left(-(1-2\sin^2\theta_w)\frac{1-\gamma_5}{2} + 2\sin^2\theta_w\frac{1+\gamma_5}{2}\right)e\right]$$

$$\Rightarrow g_w = \frac{g}{2\sqrt{2}}, \quad M_W^2 = \frac{g^2\sqrt{2}}{8G_{\text{Fermi}}} = \frac{e^2\sqrt{2}}{8G_{\text{Fermi}}\sin^2\theta_w} = \frac{\pi\alpha}{\sqrt{2}G_{\text{Fermi}}\sin^2\theta_w}.$$

That gives $M_W = \frac{38.5}{\sin\theta_w}$ GeV, remind $M_Z = \frac{M_W}{\cos\theta_w}$.

We can see that the Higgs boson vev is directly related to the Fermi coupling constant:

$$v = (\sqrt{2}G_{\text{Fermi}})^{-1/2} \approx 246.22 \text{ GeV}. \tag{61}$$

So this quantity has been know with a high precision long before the discovery of the Higgs boson and measurement of its mass.

QUESTION: Why neutral weak currents in the SM do not change flavour?

3.6 Self-interactions of EW bosons and Faddeev–Popov ghosts

Because of the non-abelian $SU(2)_L$ group and mixing of the neutral vector bosons, we have a rather reach structure of EW boson self-interactions, see Fig. 5. The corresponding contributions to the SM Lagrangian look as follows:

$$\mathcal{L}_3 \sim ie\frac{\cos\theta_w}{\sin\theta_w}\left[(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)W^+_\mu Z_\nu - (\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)W^-_\mu Z_\nu\right.$$

$$\left. +W^-_\mu W^+_\nu(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\right]$$

$$\mathcal{L}_4 \sim -\frac{e^2}{2\sin^2\theta_w}\left[(W^+_\mu W^-_\mu)^2 - W^+_\mu W^+_\mu W^-_\nu W^-_\nu\right],$$

$$-\frac{e^2\cos^2\theta_w}{\sin^2\theta_w}\left[W^+_\mu W^-_\mu Z_\nu Z_\nu - W^+_\mu Z_\mu W^-_\nu Z_\nu\right]$$

$$-\frac{e^2\cos^2\theta_w}{\sin^2\theta_w}\left[2W^+_\mu W^-_\mu Z_\nu A_\nu - W^+_\mu Z_\mu W^-_\nu A_\nu - W^+_\mu A_\mu W^-_\nu Z_\nu\right]$$

$$-e^2\left[W^+_\mu W^-_\mu A_\nu A_\nu - W^+_\mu A_\mu W^-_\nu A_\nu\right].$$

As we discussed earlier, an accurate treatment of non-abelian gauge symmetries leads to introduction of Faddeev–Popov ghosts. For the $SU(2)$ case we obtain 3 ghosts: $c_a(x)$, $a = 1, 2, 3$,

$$c_1 = \frac{X^+ + X^-}{\sqrt{2}}, \quad c_2 = \frac{X^+ - X^-}{\sqrt{2}}, \quad c_3 = Y_Z\cos\theta_w - Y_A\sin\theta_w,$$
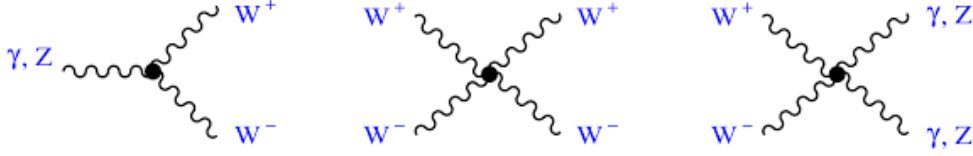
Fig. 5: Vertexes of EW boson self-interactions.

$$\mathcal{L}_{gh} = \underbrace{\partial_\mu \bar{c}_i (\partial_\mu c_i - g\varepsilon_{ijk} c_j W_\mu^k)}_{\text{kinetic + int. with } W^a} + \underbrace{\text{int. with } \Phi}_{M_{gh},\ \text{int. with } H} \; .$$

Propagators of the ghost fields read

$$D_{Y_\gamma}(k) = \frac{i}{k^2 + i0}, \qquad D_{Y_Z}(k) = \frac{i}{k^2 - \xi_Z M_Z^2 + i0}, \qquad D_X(k) = \frac{i}{k^2 - \xi_W M_W^2 + i0},$$

where ξ_i are the gauge parameters. Note that masses of the ghosts Y_γ, Y_Z, and $X^\pm$ coincide with the ones of photon, Z, and $W^\pm$, respectively. That is important for *gauge invariance* of total amplitudes. The ghosts appear only in propagators, but not in the final or initial states.

3.7 Generation of fermion masses

We observe massive fermions, but the $SU(2)_L$ gauge symmetry forbids fermion mass terms, since

$$m\overline{\Psi}\Psi = m\left(\overline{\Psi}\frac{1+\gamma_5}{2} + \overline{\Psi}\frac{1-\gamma_5}{2}\right)\left(\frac{1+\gamma_5}{2}\Psi + \frac{1-\gamma_5}{2}\Psi\right) = m(\overline{\Psi}_L\Psi_R + \overline{\Psi}_R\Psi_L) \tag{62}$$

while Ψ_L and Ψ_R are transformed in different ways under $SU(2)_L$. The SM solution is to introduce Yukawa interactions:

$$\begin{aligned}
\mathcal{L}_Y &= -y_d(\bar{u}_L\bar{d}_L)\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R - y_u(\bar{u}_L\bar{d}_L)\begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} u_R \\
&\quad - y_l(\bar{\nu}_L\bar{l}_L)\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} l_R - y_\nu(\bar{\nu}_L\bar{l}_L)\begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \nu_R + h.c.
\end{aligned}$$

This form of this Lagrangian is fixed by the condition of the $SU(2)_L$ gauge invariance. It is worth to note that neutrino masses can be generated in the same way as the up-quark ones. Of course, that requires introduction of additional Yukawa constants y_ν.

QUESTION: Why do we need 'h.c.' in $\mathcal{L}_Y$?

Spontaneous breaking of the global symmetry in the Higgs sector provides in $\mathcal{L}$ mass terms for fermions and Yukawa interactions of fermions with the Higgs boson:

$$\mathcal{L}_Y = -\frac{v+H}{\sqrt{2}}\left[y_d\bar{d}d + y_u\bar{u}u + y_l\bar{l}l + y_\nu\bar{\nu}\nu\right] \quad \Rightarrow \quad m_f = \frac{y_f}{\sqrt{2}}v.$$

By construction, the coupling of the Higgs boson to a fermion is proportional its mass m_f. It is interesting to note that the top quark Yukawa coupling is very close to 1. And there is a very strong hierarchy of fermion masses:

$$y_t \approx 0.99 \; \gg \; y_e \approx 3\cdot 10^{-6} \; \gg \; y_\nu \approx?$$

The question mark in the last case is given not only because we do not know neutrino masses, but also since we are not sure the they are generated by the same mechanism.

Quarks can mix and Yukawa interactions are not necessarily diagonal neither in the basis of weak interaction eigenstates, nor in the basis of the strong ones. In the eigenstate basis of a given interaction for the case of three generations, the Yukawa coupling constants are 3×3 matrixes:

$$
\begin{aligned}
\mathcal{L}_Y \;=\; & -\sum_{j,k=1}^{3} \left\{ (\bar{u}_{jL}\bar{d}_{jL}) \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} y_{jk}^{(d)} d_{kR} + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} y_{jk}^{(u)} u_{kR} \right] \right. \\
& + \left. (\bar{\nu}_{jL}\bar{l}_{jL}) \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} y_{jk}^{(l)} l_{kR} + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} y_{jk}^{(\nu)} \nu_{kR} \right] \right\} + h.c.
\end{aligned}
$$

where indexes j and k mark the generation number.

Charged lepton mixing is formally allowed in the SM, but not (yet) observed experimentally. Searches for lepton flavour violating processes, like the $\mu \to e\gamma$ decay, are being performed. The Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix for (Dirac) neutrinos can be embedded in the SM.

3.8   Short form of the SM Lagrangian

At CERN one can buy souvenirs with the Standard Model Lagrangian represented in a very short compressed form:

$$
\begin{aligned}
\mathcal{L}_{\text{SM}} \;=\; & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + \bar{\Psi}\, \slashed{D}\, \Psi + h.c. \\
& + \Psi_i y_{ij} \Psi_j \Phi + h.c. \\
& + |D_\mu \Phi|^2 - V(\Phi).
\end{aligned}
\tag{63}
$$

We can understand now the meaning of each term. First of all, we see that the Lagrangian is given in the initial form before the spontaneous symmetry breaking. Summation over $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge groups is implicitly assumed in the first term. The second line represents the gauge interactions of fermions provided by the covariant derivative(s). The third line is the Yukawa interaction of fermions with the scalar field. And the fourth line represent the kinetic and potential term of the primary scalar doublet field.

EXERCISE: Find two 'misprints' in the Lagrangian (63) which break the commonly accepted QFT notation discussed in Sect. 2.

3.9   Axial anomaly

There are *axial-vector* currents in the SM:

$$
J_\mu^A = \overline{\Psi}\gamma_\mu \gamma_5 \Psi.
\tag{64}
$$

In the case of massless fermions, the unbroken global symmetry (via the Noether theorem) leads to conservation of these currents: $\partial_\mu J_\mu = 0$. For massive fermions $\partial_\mu J_\mu^A = 2im\overline{\Psi}\gamma_5 \Psi$. But *one-loop corrections*, see Fig. 6, give

$$
\partial_\mu J_\mu^A = 2im\overline{\Psi}\gamma_5 \Psi + \frac{\alpha}{2\pi} F_{\mu\nu}\tilde{F}_{\mu\nu}, \qquad \tilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}.
\tag{65}
$$

That fact is known as *axial* or *chiral* or *triangular* Adler–Bell–Jackiw anomaly. So at the quantum level the classical symmetry is lost. That is a problem for the theory. In simple words, such a symmetry breaking makes the classical and quantum levels of the theory being inconsistent to each other.
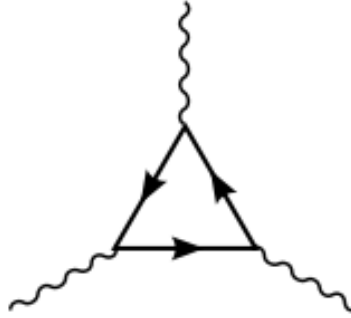
Fig. 6: Triangular anomaly diagram.

But in the SM the axial anomalies apparently cancel out. This can be seen for all possible combination of external gauge bosons:

1) $(W\,W\,W)$ and $(W\,W\,B)$ — automatically since left leptons and quarks are doublets;

2) $(B\,W\,W)$ — since $Q_e + Q_u + Q_d = 0$;

3) $(B\,B\,B)$ — since $Q_e = -1$, $Q_\nu = 0$, $Q_u = \frac{2}{3}$, $Q_d = -\frac{1}{3}$;

4) $(B\,g\,g)$ — automatically (g = gluon);

5) $(B\,gr\,gr)$ — the same as '3)' (gr = graviton).
Here B and W are the primary $U(1)$ and $SU(2)_L$ gauge bosons. Note that anomalies cancel out in each generation separately. It is interesting to note that condition '2)' means that the hydrogen atom is neutral.

It is very important that the axial anomalies cancel out in the *complete SM*: with the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetries. So there is a nontrivial connection between the QCD and EW sectors of the model.

QUESTION: Where is γ_5 in the $(B\,B\,B)$ case?

3.10 Parameters and interactions in the SM

The SM has quite a lot of parameters. We do not know (yet) where do they come from and have to define their values from observations. Let us fist count the number of independent free parameters in the SM. It is convenient to perform this exercise by looking at the initial form of the SM Lagrangian before the change of variables invoked by the spontaneous symmetry breaking. So, we have:

 – 3 gauge charges (g_1, g_2, g_s);
 – 2 parameters in the Higgs potential;
 – 9 Yukawa couplings for charged fermions;
 – 4 parameters in the CKM matrix.

It makes in total 18 free parameters for the *canonical* Standard Model. Usually, we add also as a free parameter the Λ_{QCD} scale, which is required for calculations in perturbative QCD (see lectures by A. Mitov). Moreover, we can include neutrino masses and mixing, as described above. That would give in addition 4 (or 6 for the Majorana case) parameters in the PMNS matrix and 3 more Yukawa couplings.

QUESTION: How many independent dimensionful parameters is there in the SM?

Most likely that many of the listed parameters are not true independent ones. There should be some hidden symmetries and relations. Those certainly go beyond the SM. In spite of a large number of parameters the SM is distinguished between many other models by its minimality and predictive power. For example, the supersymmetric extension of the SM formally has more than one hundred free parameters, and for this reason it is not able to provide unambiguous predictions for concrete observables.

Let us now count the interactions in the SM. Obviously, we should do that in accord with the QFT rules. The key point is to exploit symmetries, first of all the gauge ones. But looking at the Lagrangian it might be not clear what actually should be counted:
— number of *different vertexes* in Feynman rules?
— number of particle which *mediate* interactions?
— number of *coupling constants*?

Our choice here is to count coupling constants. In fact that will automatically help us to avoid double coupling of the same interactions. So we have:

- 3 gauge charges (g_1, g_2, g_s);
- 1 self-coupling λ in the Higgs potential;
- 9 Yukawa couplings for charged fermions.

If required we can add 3 Yukawa couplings for neutrinos. We see that the SM contains 5 types of interactions: 3 gauge one, the self-interaction of scalar bosons, and the Yukawa interactions of the scalar bosons with fermions. Note also that even we like some interactions e.g., the gauge ones, in the SM more than others, we can not say that any of them is more fundamental than others just since they all are in the same Lagrangian.

3.11 The naturalness problem in the SM

The most serious, actually the only one real theoretical problem of the SM is the *naturalness* known also as *fine-tuning* or *hierarchy problem*. Note that all but one masses in the SM are generated due to the spontaneous symmetry breaking in the Higgs sector. While the scalar boson mass itself has been introduced *by hands* (of Peter Higgs et al.) from the beginning. The tachyon mass term breaks the scale invariance (the conformal symmetry) *explicitly*.

So the running of all but one masses is suppressed by the classical symmetries. As the result, the masses run with energy only logarithmically, but the Higgs mass runs quadratically. In the one-loop approximation we get

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 \, - \, 4m_t^2\right].$$

It is unnatural to have $\Lambda \gg M_H$. The most natural option would be $\Lambda \sim M_H$ e.g., everything is defined by the EW scale. But that is not the case of the SM... There are two general ways to solve the problem:
— either to exploit some (super)symmetry to cancel out the huge terms;
— or to introduce some new physics at a scale not very far from the electroweak one, i.e., making Λ being not large. One can find in the literature quite a lot of models for both options. But the experimental data coming from modern accelerators and rare decay studies disfavors most of scenarios of new physics with scales up to about 1 TeV and even higher. Moreover, it was shown that the measured value of the Higgs boson mass makes the SM being self-consistent up to very high energies up to the Planck mass scale [13]. Direct and indirect experimental searches push up and up possible energy scale of new physical phenomena. So the naturalness problem becomes nowadays more and more prominent. And the question, why the top quark mass, the Higgs boson mass and and vacuum expectation value v are of the same order becomes more and more intriguing.

After the discovery of the Higgs boson and measurement of its mass, we found some remarkable empirical relation between parameters of the SM. In particular the equality

$$v = \sqrt{M_H^2 + M_W^2 + M_Z^2 + m_t^2} \tag{66}$$

holds within the experimental errors: $246.22 = 246\pm 1$ GeV. Obviously, there should be some tight clear relation between the top quark mass and the Higgs boson one (or the EW scale in general). The present version of the SM does not explain this puzzle.

EXERCISE: Divide both sides of Eq. (66) by v and find a relation between coupling constants.

Another interesting relation also involves the Higgs boson and the top quark:

$$2\frac{m_h^2}{m_t^2} = 1.05 \approx 1 \approx 2\frac{m_t^2}{v^2} \equiv y_t^2 = 0.99. \tag{67}$$

It might be that these relations are of a pure numerological nature, but they certainly indicate some hidden properties of the SM.

4 Phenomenology of the Standard Model

Let us discuss input parameters of the SM. It was convenient to count their number in the *primary* form of the Lagrangian. But for *practical* applications we use different sets, see e.g., Table 1. Different *EW schemes* with different sets of practical input parameters are possible, since there are relations between them. One should keep in mind that the result of calculations does depend on the choice because we usually work in a limited order of the perturbation theory, while the true relations between the parameters (and between observed quantities) involve the complete series. So simple relations appear at the lowest order, quantum effects (radiative corrections) make them complicated.

Table 1: Input parameters of the SM.

18(19)=	1	1	1	1	1	9	4	(1)
primary:	g'	g	g_s	m_Φ	λ	y_f	y_{jk}	none
practical:	α	M_W	α_s	G_{Fermi}	M_H	m_f	V_{CKM}	Λ_{QCD}

A comprehensive up-to-date set of the SM parameters can be found in the Review of Particle Physics published by the Particle Data Group Collaboration [14]. Let us look at some values of input parameters extracted from experiments:

  – The *fine structure constant*: $\alpha^{-1}(0) = 137.035999074(44)$ from $(g-2)_e$;
  – The SM predicts $M_W = M_Z \cos\theta_w \;\Rightarrow\; M_W < M_Z$, we have now
    $M_Z = 91.1876(21)$ GeV from LEP1/SLC, $M_W = 80.385(15)$ GeV from LEP2/Tevatron/LHC;
  – The Fermi coupling constant: $G_{\text{Fermi}} = 1.1663787(6) \cdot 10^{-5}$ GeV^{-2} from muon decay,
  – The top quark mass: $m_t = 173.21(51)(71)$ GeV from Tevatron/LHC;
  – The Higgs boson mass: $M_H = 125.09(21)(11)$ GeV from ATLAS & CMS (March 2015).

One can see that the precision in definition of the parameters varies by several orders of magnitude. That is related to experimental uncertainties and to the limited accuracy of theoretical calculations which are required to extract the parameter values from the data. *QUESTION: What parameter of the (canonical) SM is known now with the least precision?*

4.1 The muon decay

Let us consider a few examples of particle interaction processes and start with the muon decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, see Fig. 7. It is the most clean weak interaction process. One can say that this process is one of keystones of particle physics. The muon decay width reads

$$\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_{\text{Fermi}}^2 m_\mu^5}{192\pi^3}\left[f(m_e^2/m_\mu^2) + \mathcal{O}(m_\mu^2/M_W^2) + \mathcal{O}(\alpha)\right],$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x,$$

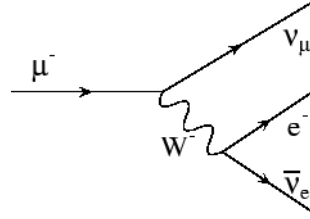$$\mathcal{O}(m_\mu^2/M_W^2) \sim 10^{-6}, \qquad \mathcal{O}(\alpha) \sim 10^{-3},$$

Fig. 7: The Feynman diagram for muon decay in the SM.

where $\mathcal{O}(\alpha)$ includes effects of radiative corrections due to loop (virtual)effects and real photon and/or e^+e^- pair emission.

As mentioned above, the value of the Fermi coupling constant is extracted from the data on muon lifetime, $G_{\mathrm{Fermi}} = 1.1663787(6) \cdot 10^{-5}\,\mathrm{GeV}^{-2}$. The high precision is provided by a large experimental statistics, low systematical errors of the final state electron observation, and by accurate theoretical calculations of radiative corrections. But impressive precision ($\sim 1 \cdot 10^{-6}$) in the measurement of the muon life time doesn't give by itself any valuable test of the SM. *QUESTION: Why is that so?* On the other hand, studies of differential distributions in electron energy and angle do allow to test the $V - A$ structure of weak interactions and look for other possible types of interactions which can be parameterized in a model-independent way by the so-called *Michel parameters*.

4.2 Electron and muon anomalous magnetic moments

The Dirac equations predict gyromagnetic ratio $g_f = 2$ in the fermion magnetic moment $\vec{M} = g_f \frac{e}{2m_f} \vec{s}$. Julian Schwinger in 1948 found that one-loop QED correction the vertex function gives the so called *anomalous magnetic moment*:

$$a_f \equiv \frac{g_f - 2}{2} \approx \frac{\alpha}{2\pi} = 0.001\,161\,\ldots \tag{68}$$

For the electron case, the Harvard experiment [15] obtained

$$a_e^{\mathrm{exp}} = 1\,159\,652\,18\mathbf{0.73}\,(28) \cdot 10^{-12}\ [0.24\mathrm{ppb}].$$

The SM predicts [16]

$$a_e^{\mathrm{SM}} = 1\,159\,652\,18\mathbf{1.643}\,(25)_{8th}(23)_{10th}(16)_{EW+had.}(763)_{\delta\alpha} \cdot 10^{-12}.$$

The perfect agreement between the measurement and the theoretical prediction is a triumph of Quantum electrodynamics. In particular, we note that $a_f \neq 0$ is a pure quantum loop effect which is absent as in classical physics as well as in Quantum mechanics.

It is worth to note that the extremely high precision in the experimental measurement of the electron anomalous magnetic moment allows to use it as a reference point for definition of the fine structure constant: $a_e^{\mathrm{exp}} \Rightarrow \alpha^{-1}(0) = 137.035999074(44)$.

For the anomalous magnetic moment of muon, the E821 experiment at BNL in 2006 published the following result of data analysis:

$$a_\mu^{\mathrm{exp}} = 116\,59\mathbf{2\,089}\,(54)(33) \cdot 10^{-11}\ [0.5\mathrm{ppm}].$$

The corresponding theoretical value and the difference are

$$a_\mu^{\mathrm{SM}} = 116\,59\mathbf{1\,840}\,(59) \cdot 10^{-11}\ [0.5\mathrm{ppm}] \tag{69}$$

$$\Delta a_\mu \equiv a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = \mathbf{249}\,(87) \cdot 10^{-11}\ [\sim 3\sigma].$$
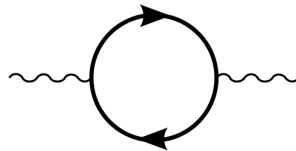
Fig. 8: The one-loop Feynman diagram for QED vacuum polarization.

First, one can see that both experimental and theoretical values are very accurate. Second, there is a discrepancy of the order of three standard deviations. That is a rather rare case for the SM tests. Moreover, this discrepancy remains for a long period of time in spite of intensive efforts of instrumentalists and theoreticians.

The SM prediction consists of the QED, hadronic and weak contributions:

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{hadronic}) + a_\mu(\text{weak}), \tag{70}$$
$$a_\mu(\text{QED}) = 116\,584\,718\,845\,(9)(19)(7)(30) \cdot 10^{-14} \qquad [\text{5 loops}],$$
$$a_\mu(\text{hadronic}) = a_\mu(\text{had. vac.pol.}) + a_\mu(\text{had. l.b.l}), = 6949\,(37)(21) \cdot 10^{-11} + 116\,(40) \cdot 10^{-11},$$
$$a_\mu(\text{weak}) = 154\,(2) \cdot 10^{-11} \qquad [\text{2 loops}].$$

Note that the QED contribution to the muon anomalous magnetic moment is essentially the same as the one to the electron magnetic moment. The only difference is coming from the dependence on electron and muon masses. As concerning the hadronic and weak interaction contributions, they are enhanced by the factor m_μ^2/m_e^2 with respect to the electron case. The same factor typically appears for hypothetical contributions of new interactions beyond the SM. For this reason anomalous magnetic moments of muon and tau lepton are potentially more sensitive to new physics contributions.

One can see that the difference between the theoretical prediction and the experimental data is almost twice the contribution of weak interactions: $\Delta a_\mu \sim 2 \times a_\mu(\text{weak})$. Here by 'weak' we mean the complete electroweak calculation minus the pure QED contribution. The weak interactions have been directly tested with high precision experimentally. So it is not so simple to attribute the difference to an effect of new physics. Nevertheless, there is a bunch of theoretical models that try to resolve the problem by introduction of new interactions and/or new particles.

4.3 Vacuum polarization

By direct calculation in QED, one can see that virtual charged fermion anti-fermion pairs provide a screening effect for the electric force between probe charges. Resummation of bubbles, see Fig. 8, gives

$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Pi(q^2)}, \qquad \text{e.g. } \alpha^{-1}(M_Z^2) \approx 128.944(19),$$
$$\Pi(q^2) = \frac{\alpha(0)}{\pi}\left(\frac{1}{3}\ln\left(\frac{-q^2}{m_e^2}\right) - \frac{5}{9} + \delta(q^2)\right) + \mathcal{O}(\alpha^2),$$
$$\delta(q^2) = \delta_\mu(q^2) + \delta_\tau(q^2) + \delta_W(q^2) + \delta_{\text{hadr.}}(q^2). \tag{71}$$

The hadronic contribution to vacuum polarization $\delta_{\text{hadr.}}(q^2)$ for $|q^2|$ below a few GeV^2 is not calculable within the perturbation theory. Now we get it from experimental data on $e^+e^- \to$ hadrons and $\tau \to \nu_\tau +$ hadrons with the help of dispersion relations, see e.g., review [17]. Lattice results for this quantity are approaching.

Note that screening, i.e., an effective reduction of the observed charge with increasing of distance, is related to the minus sign attributed to a fermion loop in the Feynman rules.

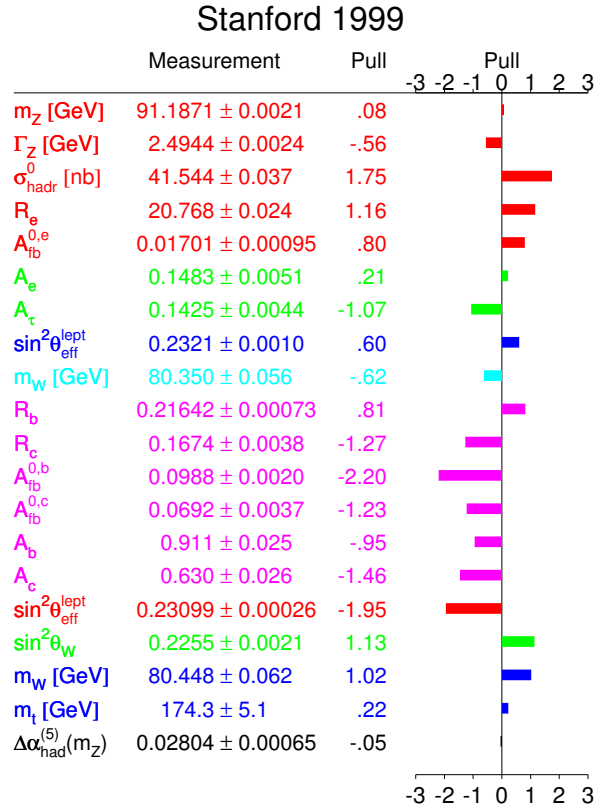*QUESTION: Estimate the value of q_0^2 at which $\alpha(q_0^2) = \infty$.*

Fig. 9: Pulls of pseudo-observables at LEP [18].

This singularity is known as the *Landau pole*. Formally, such a behaviour of QED brakes unitarity at large energies. But that happens at energies much higher than any practical energy scale including the Planck mass and the mass of the visible part of the Universe. So we keep this problem in mind as a theoretical issue which stimulates our searches for a more fundamental description of Nature.

4.4 Experimental tests of the SM at LEP

After the analysis of LEP1 and LEP2 experimental data, the LEP Electroweak Working Group (LEP-EWWG) [18] illustrated the overall status of the Standard Model by the so-called *pulls*, see Fig. 9. The pulls are defined as differences between the measurement and the SM prediction calculated for the central values of the fitted SM input parameters $[\alpha(M_Z^2) = 1/128.878, \alpha_s(M_Z^2) = 0.1194, M_Z = 91.1865\,\text{GeV}, m_t = 171.1\,\text{GeV}]$ divided by the experimental error. Although there are several points where deviations between the theory and experiment approach two standard deviations, the average situation should be ranked as extremely good. We note that the level of precision reached is of the order of $\sim 10^{-3}$, and that it is extremely non-trivial to control all experimental systematics at this level.

Through quantum effects the observed cross sections of electron-positron annihilation at LEP depend on all parameters of the Standard Model including the Higgs boson mass. The so called yellow band plot Fig. 10 shows the fit of M_H performed by LEPEWWG [18] with the LEP data in March 2012. The left yellow area has been excluded by direct searches at LEP, and the right one was also excluded by LHC. The plot is derived from a combined fit of all the world experimental data to the SM exploiting the best knowledge of precision theoretical calculations which is realized in computer codes ZFITTER [19] and TOPAZ0 [20]. One can see that the data was not very sensitive to M_H, but the fit unambiguously
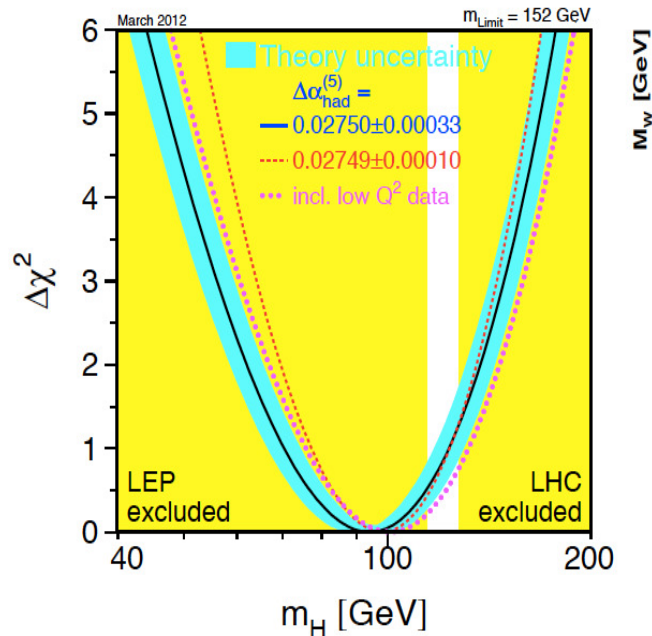
Fig. 10: The curve shows $\Delta\chi^2_{\min}(M_H^2) = \chi^2_{\min}(M_H^2) - \chi^2_{\min}$ as a function of M_H. The width of the shaded band around the curve shows the theoretical uncertainty. The vertical bands show the 95% CL exclusion limit on M_H from the direct searches at LEP (left) and at LHC (right). The dashed curve is the result obtained using the evaluation of $\Delta\alpha^{(5)}(M_Z^2)$. The dotted curve corresponds to a fit including also the low-Q^2 data.

prefers a relatively light Higgs boson. Now we can say that the measured value of this parameter agrees very well with the LEP fit. That indirectly confirms again the consistency and the power of the Standard Model.

It is interesting also to look at the behavior of the cross sections of electron-positron annihilation into hadrons as a function of energy Fig. 11. A clear peak at the Z boson mass is seen. The excellent agreement of the experimental data with the SM predictions is achieved only after inclusion of QCD and electroweak radiative corrections which reach dozens of percent in the vicinity of the peak.

A peculiar result was obtained at LEP for number of (light) neutrinos, see Fig. 12. Even so that the final state neutrinos in the process $e^+ + e^- \to Z \to \nu + \bar\nu$ was not observed, the corresponding cross section was restored with the help of the separately measured total, hadronic, and leptonic cross sections [18].

It appears that to dependence of LEP observables on quantum loop effects involving top quark is rather strong. So even without approaching the direct production of top quark, LEP experiments were able to extract information about its mass. The top quark mass 'history' (till 2006) is shown by Fig. 13.

In general, all LEP measurements of various cross-sections of electroweak SM processes were found in a very good agreement with theoretical predictions obtained within the SM, see plot Fig. 14 from the LEPEWWG [18] 2013 report. The dots show the measurements and curves are the SM predictions with radiative corrections taken into account.

4.5 Measurements of SM processes at LHC

The Large Hadron Collider is not only a discovery machine. In fact the large luminosity and advanced detectors allow to perform there high-precision tests of the Standard Model. High statistics on many SM
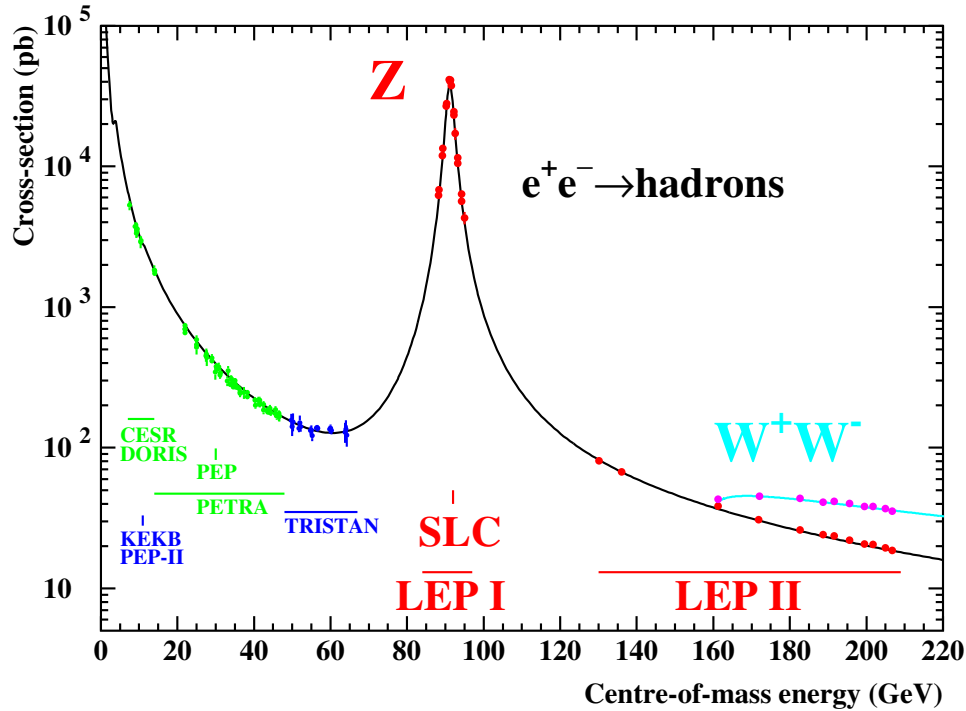
Fig. 11: Measurements of the $e^+e^- \to$ hadrons cross section.

processes is collected. Plots Fig. 15 and Fig. 16 show the public preliminary results of the ATLAS and CMS collaborations. One can see that we again have a good agreement for all channels. Certainly, the tests of the SM will be continued at LHC at higher energies and luminosity. That is one of the main tasks the LHC physical programme. The proton-antiproton collider Tevatron has proven that hadronic colliders can do high-precision studies of the SM. In particular, CDF and D0 experiments at Tevatron managed to exceed LEP in the precision of the W boson mass measurement.

At LHC the best precision in SM processes measurement is reached for the Drell–Yan-like processes. A schematic diagram for such a process is shown on Fig. 17. These processes are distinguished by production of final state leptons which can be accurately detected. We distinguish the neutral current (NC) Drell–Yan-like processes which involve intermediate Z bosons and photons, and the charged current (CC) ones which go through W bosons. The main contribution to the (observed) total cross section of these processes comes from the domain where the invariant mass of the final state lepton pair is close to the masses of Z and W bosons. So these processes are also known as single Z and W production reactions. The CC and NC Drell–Yan-like processes at LHC are used for:

– luminosity monitoring;
– W mass and width measurements;
– extraction of parton density functions;
– detector calibration;
– background to many other processes;
– and new physics searches.

In particular, a new peak in the observed invariant-mass distribution of final leptons can indicate the presence of a new intermediate particle.
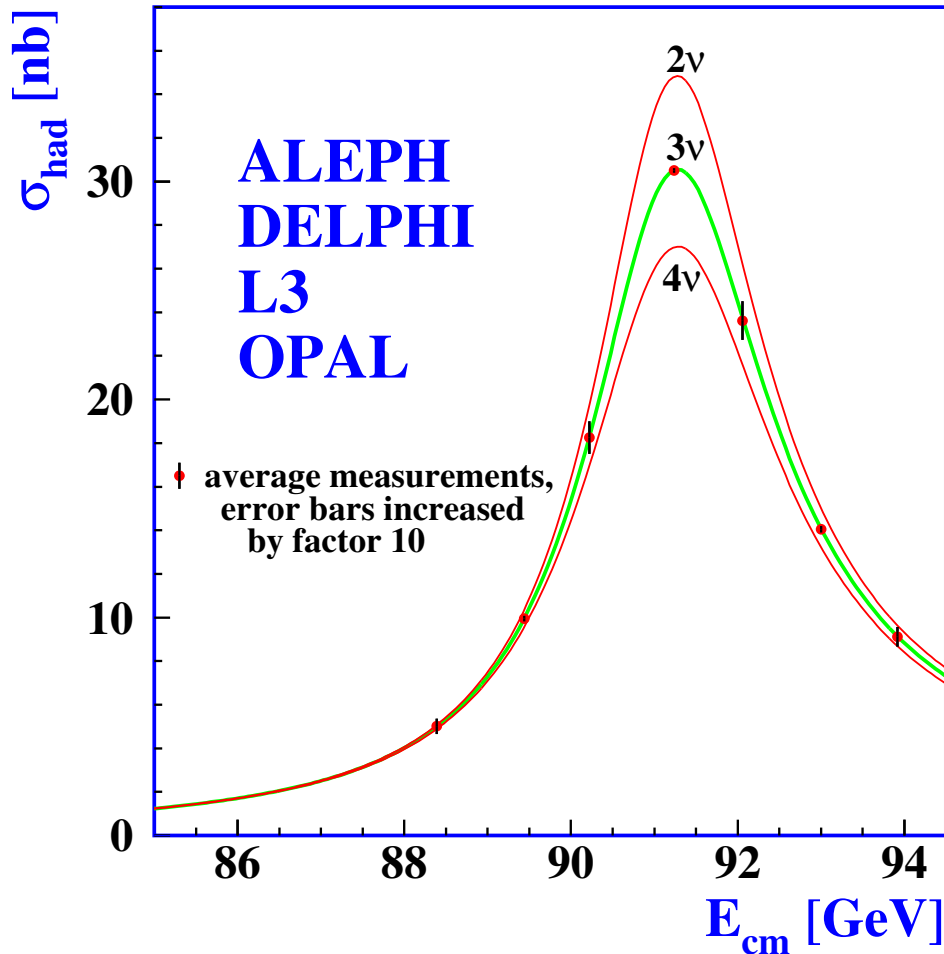
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Fig. 12: Annihilation cross section $e^+ e^- \to$ hadrons for different numbers of light neutrinos: measured distribution vs. the SM prediction.

5 Conclusions

Let us summarize the status of the SM. We see that it is a rather elegant construction which allows making systematic predictions for an extremely wide range of observables in particle physics. The energy range of its applicability covers the whole domain which is explored experimentally while the limits remain unknown. We do not understand all features of the model, the origin of its symmetries, and parameter values. But we see that the SM has the highest predictive power among all models in particle physics and it successfully passed verification at thousands of experiments.

There are several particularly nice features of the SM:

– it is renormalizable and unitary $\Rightarrow$ it gives finite predictions;
– its predictions do agree with experimental data;
– symmetry principles are extensively exploited;
– it is minimal;
– all its particles are discovered;
– the structure of interactions is fixed (but not yet tested everywhere);
– not so many free parameters, all are fixed;
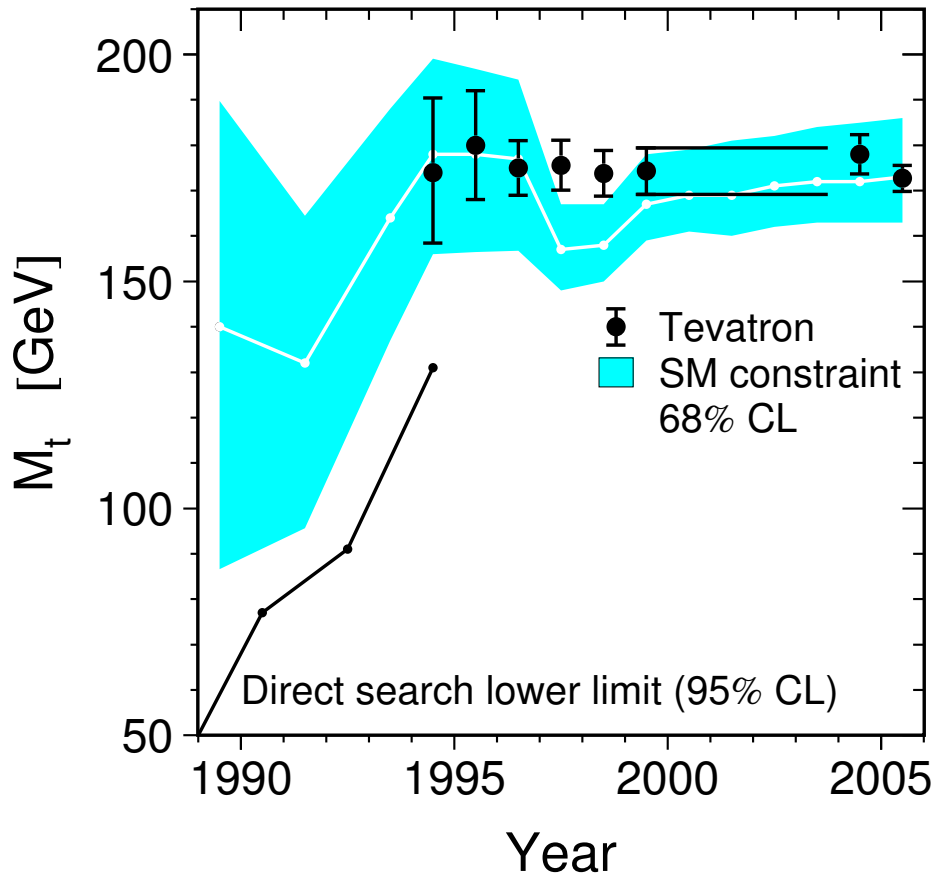– CP violation is allowed;

Fig. 13: Indirect (LEP) and direct (Tevatron) measurements of the top quark mass.

- tree-level flavor-changing neutral currents are not present;
- there is a room to incorporate neutrino masses and mixing.

In principle in the future, the SM can be embedded into a more general theory as an effective low-energy approximation.

For many reasons we do not believe that the SM is the final 'theory of everything'. Of course first of all, we have to mention that the SM is not joined with General Relativity. But frankly speaking, that is mostly the problem of GR, the SM itself is ready to be incorporated into a generalized joint QFT construction. The naturalness problem discussed above in Sect. 3.11 indicates that either some new physics should be very close to the EW energy scale, or we do not understand features of the renormalization procedure in the SM. In general, we have a lot of open questions within the SM:

- the origin of symmetries;
- the origin of energy scales;
- the origin of 3 fermion generations;
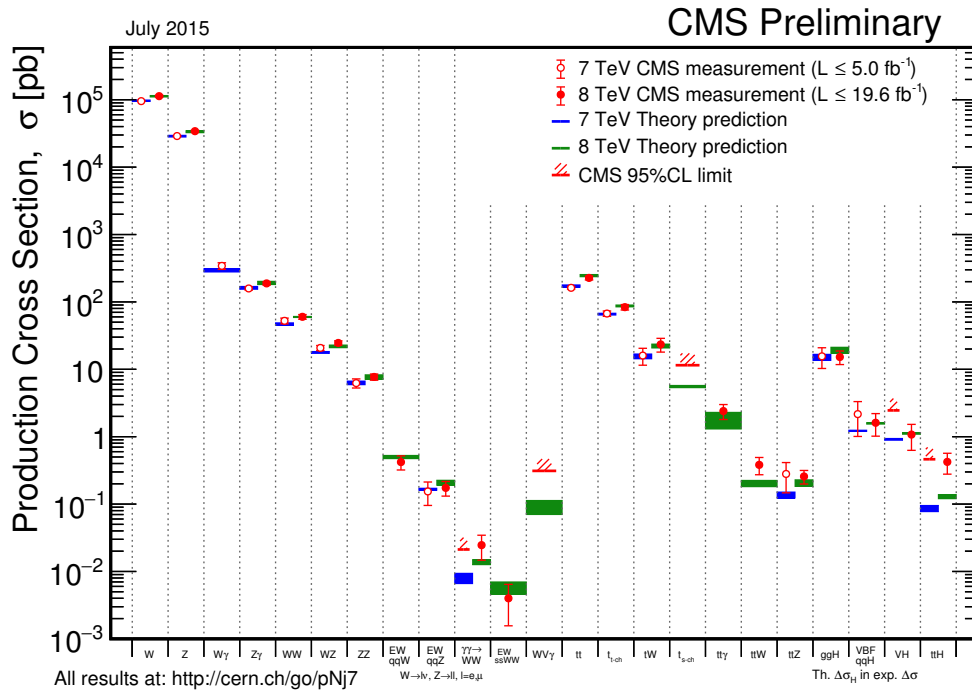- the origin of neutrino masses;
- the hierarchy of lepton masses;

Fig. 14: Cross-sections of electroweak SM processes at LEP2.

Fig. 15: SM cross sections measured by ATLAS (public results).

Fig. 16: SM cross sections measured by CMS (public results).
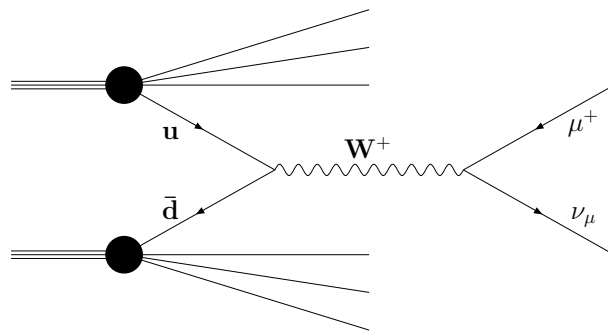

Fig. 17: Schematic Feynman diagram for the charged current Drell–Yan-like process.

– the absence of strong CP violation in the QCD sector;
– confinement in QCD, and so on...

There are also some phenomenological issues:

– the baryon asymmetry in the Universe;
– the dark matter;
– the dark energy;
– the proton charge radius, $(g-2)_\mu$, and not much else...

The first three items above are related to Cosmology, see lectures by A. De Simone. We should note also that most of observations in Cosmology and Astrophysics are well described within the Standard Model (and General Relativity). But for the listed cases we need most likely something beyond the SM. The last item in the list claims that there are some tensions in the predictions of the SM and measurements at experiments in particle physics.

So we see that the SM is build using nice fundamental principles but with a substantial phenomenological input. The most *valuable task* for high-energy physicists now is to find the energy domain limit(s)

of the SM applicability. We hope to discover soon new physical phenomena. But any kind of new physics has to preserve the correspondence to the SM. The SM contains good mechanisms to generate masses of vector bosons and fermions, but it doesn't show the *origin(s)* of the energy scales.

So, the SM can not be the full story in particle physics, we still have a lot to explore.

References

[1] S. L. Glashow, Nucl. Phys. **22** (1961) 579.

[2] S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264.

[3] A. Salam, Conf. Proc. C **680519** (1968) 367.

[4] N. N. Bogoliubov, D. V. Shirkov, *Introduction to the theory of quantized fields,* Interscience Publishers, 1959.

[5] L. H. Ryder, *Quantum Field Theory,* 2nd Edition, Cambridge University Press, 1996.

[6] M. E. Peskin, D. V. Schroeder, *An Introduction to Quantum Field Theory,* Westview Press, 1995.

[7] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716** (2012) 1.

[8] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716** (2012) 30.

[9] D. Y. Bardin and G. Passarino, *The standard model in the making: Precision study of the electroweak interactions,* Oxford University Press, International series of monographs on physics, 104 Oxford, 1999.

[10] F. Englert and R. Brout, Phys. Rev. Lett. **13** (1964) 321.

[11] P. W. Higgs, Phys. Rev. Lett. **13** (1964) 508.

[12] Scientific Background on the Nobel Prize in Physics 2013: The BEH-Mechanism, Interactions with Short Range Forces and Scalar Particles, `http://www.nobelprize.org/nobel_prizes/physics/laureates/2013/advanced-physicsprize2013.pdf`.

[13] A. V. Bednyakov, B. A. Kniehl, A. F. Pikelner and O. L. Veretin, Phys. Rev. Lett. **115** (2015) 20, 201802.

[14] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38** (2014) 090001.

[15] D. Hanneke, S. F. Hoogerheide and G. Gabrielse, Phys. Rev. A **83** (2011) 052122.

[16] T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. **109** (2012) 111807.

[17] S. Actis *et al.* [Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies Collaboration], Eur. Phys. J. C **66** (2010) 585.

[18] The LEP Electroweak Working Group, `http://lepewwg.web.cern.ch/LEPEWWG/`.

[19] A. B. Arbuzov, M. Awramik, M. Czakon, A. Freitas, M. W. Grunewald, K. Monig, S. Riemann and T. Riemann, Comput. Phys. Commun. **174** (2006) 728.

[20] G. Montagna, O. Nicrosini, F. Piccinini and G. Passarino, Comput. Phys. Commun. **117** (1999) 278.