A ν hope

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Abstract
The Standard Model has been successful beyond expectations in predicting the results of almost all the experimental tests done so far. In it, neutrinos are massless. Nonetheless, in recent years we have collected solid proofs indicating little but non zero masses for the neutrinos (when contrasted with those of the charged leptons). These masses permit neutrinos to change their flavour and oscillate, indeed a unique treat. In these lectures, I discuss the properties and the amazing potential of neutrinos in and beyond the Standard Model. I review also the pieces of evidence that do not fit with the vanilla three neutrino picture and the prospects to discover the physics that hides beyond the Standard Model (if any) using neutrinos.

Keywords
Lectures; neutrinos; neutrino oscillations; mass; flavor; Majorana particles.

1 Introduction

Last decade witnessed a brutal transformation in neutrino physics. It has been proven beyond any doubt that neutrinos have non-zero masses, implying that leptons mix. This fact was demonstrated by the experimental evidence that neutrinos can change from one state, or "flavour", to another. All the information we have gathered about neutrinos is relatively new. Less than thirty years old. Neutrino physics as a solid science is in its teenage years and therefore as any adolescent, in a wild and very exciting (and excited) state. However, before jumping into the latest "news" about neutrinos, let's understand how and why neutrinos were conceived.

The '20s saw the death of numerous sacred cows, and physics was no exception. One of physics most holly principles, energy conservation, apparently showed up not to hold inside the subatomic world. For some radioactive nuclei, it appeared that a non-negligible fraction of its energy simply vanished, leaving no trace of its presence.

In 1920, in a (by now famous) letter to a meeting [1], Pauli quasi apologetically wrote,"Dear radioactive Ladies and Gentlemen, . . . as a desperate remedy to save the principle of energy conservation in beta decay, . . . I propose the idea of a neutral particle of spin half". Pauli hypothesised that the missing energy was taken off by another particle, whose properties were such that made it invisible and impossible to detect: it had no electric charge, no mass and only very rarely interacted with matter. With these properties, the neutrino was introduced as one of the few inhabitants of the particle zoo. Before long, Fermi postulated the four-Fermi Hamiltonian in order to describe beta decay utilising the neutrino, electron, neutron and proton. Another field was born: weak interactions took the stage to never leave it. Closing the loop, twenty something years after Pauli's letter, Cowan and Reines got the experimental signature of anti-neutrinos emitted by a nuclear power plant.

As more particles who participated in weak interactions were found in the years following neutrino discovery, weak interactions got credibility as an authentic new force of nature and the neutrino got to be a key element of it.

Further experimental tests spanning the following years demonstrated that there was not one but three sort, or "flavours" of neutrinos (tagged according to the charged lepton they were produces in association with: electron neutrinos (ν_e), muon neutrinos (ν_μ) and tau neutrinos (ν_τ)) and that, to the extent we could test, had no mass (and no charge) whatsoever.

The neutrino adventure could have easily finished there, but new tests using neutrinos coming from the sun have shown that the neutrino saga was just beginning . . .

In the canonical Standard Model, neutrinos are completely massless and as a consequence are flavour eigenstates:

$$\begin{array}{ccc} W^+ \longrightarrow e^+ \, + \, \nu_e & ; & Z \longrightarrow \nu_e \, + \, \bar{\nu}_e \quad , \\ W^+ \longrightarrow \mu^+ \, + \, \nu_\mu & ; & Z \longrightarrow \nu_\mu \, + \, \bar{\nu}_\mu \quad , \\ W^+ \longrightarrow \tau^+ \, + \, \nu_\tau & ; & Z \longrightarrow \nu_\tau \, + \, \bar{\nu}_\tau \quad . \end{array} \tag{1}$$

Precisely because they are massless, they travel at the speed of light. But the masslessness not only defines the speed at which they propagate, it fixes its flavour as they travel as well. It is evident then, that as flavour is concerned, zero mass neutrinos are not an attractive object to study, specially when contrasted with quarks.

However, if neutrinos were massive, and these masses were not degenerate (degenerate masses flavour-wise is identical to the zero mass case) would mean that neutrino mass eigenstates exist $\nu_i, i = 1, 2, \ldots$, each with a mass m_i. The impact of leptonic mixing becomes apparent by looking at the leptonic decays, $W^+ \longrightarrow \nu_i + \overline{\ell_\alpha}$ of the charged vector boson W, where $\alpha = e, \mu$, or τ, and ℓ_e refers to the electron, ℓ_μ the muon, or ℓ_τ the tau.

We call particle ℓ_α as the charged lepton of flavour α. Mixing basically implies that when the charged boson W^+ decays to a given kind of charged lepton $\overline{\ell_\alpha}$, the neutrino that goes along is not generally the same mass eigenstate ν_i. *Any* of the different ν_i can appear.

The amplitude for the decay of a vector boson W^+ to a particular mix $\overline{\ell_\alpha} + \nu_i$ is given by $U^*_{\alpha i}$. The neutrino that is emitted in this decay alongside the given charged lepton $\overline{\ell_\alpha}$ is then

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} \, |\nu_i\rangle \quad . \tag{2}$$

This specific mixture of mass eigenstates yields the neutrino of flavour α.

The different $U_{\alpha i}$ can be gathered in a unitary matrix (in the same way they were collected in the CKM matrix in the quark sector) that receives the name of the leptonic mixing matrix or U_{PNMS} [2]. The unitarity of U ensures that each time a neutrino of flavour α through its interaction produces a charged lepton, the produced charged lepton will always be ℓ_α, the charged lepton of flavour α. That is, a ν_e produces exclusively an e, a ν_μ exclusively a μ, and in a similar way ν_τ a τ.

The expression (2), portraying each neutrino of a given flavour as a linear combination of the three mass eigenstates, can be easily inverted to depict every mass eigenstate ν_i as an analogous linear combination of the three flavours:

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} \, |\nu_\alpha\rangle \quad . \tag{3}$$

The amount of α-flavour (or the α-fraction) of ν_i is obviously $|U_{\alpha i}|^2$. When a ν_i interacts and creates a charged lepton, this α-content (or fraction) expresses the probability that the created charged lepton be of flavour α.

2 Neutrino oscillations basics

The phenomenon of neutrino flavour transitions or in short oscillation, can be understood in the following form. A neutrino is created or emitted by a source along with a charged lepton $\overline{\ell_\alpha}$ of flavour α. In this way, at the emission point, the neutrino does have a definite flavour. It is a ν_α. After that point, the neutrino propagates, i.e., covers some distance L until it is absorbed.

At this point, when it has already reached the detector, the neutrino (sometimes) interacts and these interactions create another charged lepton ℓ_β of flavour β, which we can detect. In this way, at the target, we can know that the neutrino is again a neutrino of definite flavour, a ν_β. Of course not always both flavours are identical, sometimes $\beta \neq \alpha$ (for instance, if ℓ_α is a μ however ℓ_β is a τ), then, all along his journey from the source to the identification point, the neutrino has changed or mutated from a ν_α into a ν_β.

This transition from one flavour to the other, $\nu_\alpha \longrightarrow \nu_\beta$, is only one more realization of the widely known quantum-mechanical effect present in a variety of two state systems and not a particular property of neutrinos.

Since, as shown clearly by Eq. (2), a ν_α is truly a coherent superposition of the three mass eigenstates ν_i, the neutrino that travels since it is produced until it is detected, can be any of the three ν_i's. Because of that, we should include the contributions of each of the ν_i in a coherent way. As a consequence, the transition amplitude, Amp($\nu_\alpha \longrightarrow \nu_\beta$) receives a contribution of each ν_i and turns out to be the product of three pieces. The first factor is the amplitude for the neutrino created at the generation point along with a charged lepton $\overline{\ell_\alpha}$ to be, particularly, a ν_i and is given by $U^*_{\alpha i}$.

The second component of our product is the amplitude for the ν_i made by the source to cover the distance up to the detector. We will name this element Prop(ν_i) for the time being and will postpone the calculation of its value until later. The last (third) piece is the amplitude for the charged lepton born out of the interaction of the neutrino ν_i with the target to be, particularly, a ℓ_β.

Being the Hamiltonian that describes the interactions between neutrinos, charged leptons and charged bosons W hermitian (otherwise probabilities wouldn't be conserved), it follows that if Amp($W \longrightarrow \overline{\ell_\alpha}\nu_i$) $= U^*_{\alpha i}$, then Amp ($\nu_i \longrightarrow \ell_\beta W$) $= U_{\beta i}$. In this way, the third and last component of the product the ν_i contribution is given by $U_{\beta i}$, and

$$\mathrm{Amp}(\nu_\alpha \longrightarrow \nu_\beta) = \sum_i U^*_{\alpha i} \ \ \mathrm{Prop}(\nu_i) \ \ U_{\beta i} \quad . \tag{4}$$

It still remains to be established the value of Prop(ν_i). To determine it, we'd better study the ν_i in its rest frame. We will label the time in that system τ_i. If ν_i does have a rest mass m_i, then in this frame of reference its state vector satisfies the Schrödinger equation

$$i\frac{\partial}{\partial \tau_i}|\nu_i(\tau_i)\rangle \ = m_i|\nu_i(\tau_i)\rangle \ \ , \tag{5}$$

whose solution is given clearly by

$$|\nu_i(\tau_i)\rangle \ = e^{-im_i\tau_i}|\nu_i(0)\rangle \ \ . \tag{6}$$

Then, the amplitude for a given mass eigenstate ν_i to travel freely during a time τ_i, is simply the amplitude $\langle\nu_i(0)|\nu_i(\tau_i)\rangle$ for observing the initial state ν_i, $|\nu_i(0)\rangle$ after some time as the evolved state $|\nu_i(\tau_i)\rangle$, *i.e.* $\exp[-im_i\tau_i]$. Thus Prop(ν_i) is only this amplitude where we have used that the time taken by ν_i to cover the distance from the source to the detector is just τ_i, the proper time.

Nevertheless, if we want Prop(ν_i) to be of any use to us, we must write it first in terms of variables we can measure, this means to express it, in variables in the lab frame. The natural choice is obviously the distance, L, that the neutrino covers between the source and the detector as seen in the lab frame, and the time, t, that slips away during the journey, again in the lab frame. The distance L is set by the experimentalists through the selection of the place of settlement of the source and that of the detector and is unique to each experimental setting. Likewise, the value of t is selected by the experimentalists through their election for the time at which the neutrino is made and that when it dies (or gets detected). Therefore, L and t are determined by the experimental set up, and are the same for all the ν_i in the beam. The different ν_i do travel through an identical distance L, in an identical time t.

We still have two additional lab frame variables to determine, the energy E_i and three momentum p_i of the neutrino mass eigenstate ν_i. By using the Lorentz invariance of the four component internal product (scalar product), we can obtain the expression for the $m_i\tau_i$ appearing in the ν_i propagator Prop(ν_i) in terms of the (easy to measure) lab frame variables we have been looking for, which is given by

$$m_i\tau_i = E_i t - p_i L \quad . \tag{7}$$

At this point however one may argue that, in real life, neutrino sources are basically constant in time, and that the time t that slips away since the neutrino is produced till it dies in the detector is actually not measured. This argument is absolutely right. In reality, an experiment smears over the time t used by the neutrino to complete its route. However, let's consider that two constituents of the neutrino beam, the first one with energy E_1 and the second one with energy E_2 (both measured in the lab frame), add up coherently to the neutrino signal produced in the detector. Now, let us call t the time used by the neutrino to cover the distance separating the production and detection points. Then by the time the constituent whose energy is E_j $(j = 1, 2)$ arrives to the detector, it has raised a phase factor $\exp[-iE_j t]$. Therefore, we will have an interference between the E_1 and E_2 beam participants that will include a phase factor $\exp[-i(E_1 - E_2)t]$. When smeared over the non-observed travel time t, this factor goes away, *except when* $E_2 = E_1$. Therefore, only those constituents of the neutrino beam that share the same energy contribute coherently to the neutrino oscillation signal [3,4]. Specifically, only the different mass eigenstates constituents of the beams that have the same energy weight in. The rest gets averaged out.

Courtesy to its dispersion relation, a mass eigenstate ν_i, with mass m_i, and energy E, has a three momentum p_i whose absolute value is given by

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \quad . \tag{8}$$

Where, we have utilised that as the masses of the neutrinos are miserably small, $m_i^2 \ll E^2$ for a typical energy E attainable at any experiment (the lowest energy neutrinos have are MeV energies while masses are sub-eV). From Eqs. (7) and (8), it is easy to see that at a given energy E the phase $m_i\tau_i$ appearing in Prop(ν_i) takes the value

$$m_i\tau_i \cong E(t - L) + \frac{m_i^2}{2E} L \quad . \tag{9}$$

As the phase $E(t - L)$ appears in all the interfering terms it will eventually disappear when calculating the transition amplitude. After all it is a common phase factor (its absolute value is one). Thus, we can get rid of it already now and use

$$\mathrm{Prop}(\nu_i) = \exp[-im_i^2 \frac{L}{2E}] \quad . \tag{10}$$

Plugging this into Eq. (4), we can obtain that the amplitude for a neutrino born as a ν_α to be detected as a ν_β after covering a distance L with energy E yields

$$\mathrm{Amp}(\nu_\alpha \longrightarrow \nu_\beta) = \sum_i U_{\alpha i}^* \, e^{-im_i^2 \frac{L}{2E}} U_{\beta i} \quad . \tag{11}$$

The expression above is valid for an arbitrary number of neutrino flavours and an identical number of mass eigenstates, as far as they travel through vacuum. The probability P($\nu_\alpha \longrightarrow \nu_\beta$) for $\nu_\alpha \longrightarrow \nu_\beta$ can be found by squaring it, giving

$$\begin{aligned} &\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \longrightarrow \nu_\beta)|^2 = \delta_{\alpha\beta} - \\ &4\sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) + 2\sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right) \end{aligned} \quad ,(12)$$

with $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$. In order to get Eq. (12) we have used that the mixing matrix U is unitary.

The oscillation probability $\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$ we have just obtained corresponds to that of a *neutrino*, and not to an *antineutrino*, as we have used that the oscillating neutrino was produced along with a charged *antilepton* $\bar{\ell}$, and gives birth to a charged *lepton* ℓ once it reaches the detector. The corresponding probability $\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta})$ for an antineutrino oscillation can be obtained from $\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$ taking advantage of the fact that the two transitions $\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}$ and $\nu_\beta \longrightarrow \nu_\alpha$ are CPT conjugated processes. Thus, assuming that neutrino interactions respect CPT [5],

$$\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}) = \mathrm{P}(\nu_\beta \longrightarrow \nu_\alpha) \ . \tag{13}$$

Then it is clear that if the mixing matrix U is complex, $\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta})$ and $\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$ will not be identical, in general. As $\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}$ and $\nu_\alpha \longrightarrow \nu_\beta$ are CP conjugated processes, $\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}) \neq \mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$ would provide evidence of CP violation in neutrino oscillations (if Nature has chosen its mixing parameters so that the mixing matrix is indeed complex). Until now, CP violation has been observed only in the quark sector, so its measurement in neutrino physics would be quite exciting.

So far, we have been working in natural units. A fact that becomes transparent by looking at the dispersion relation Eq. (9). If we restore now the $\hbar$'s and c factors (we have happily set to one) into the oscillation probability we find that

$$\sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \quad \longrightarrow \quad \sin^2\left(\Delta m_{ij}^2 c^4 \frac{L}{4\hbar c E}\right) \ . \tag{14}$$

Having done that, it is easy and instructive to explore the semi-classical limit, $\hbar \longrightarrow 0$. In this limit the oscillation length goes to zero (the oscillation phase goes to infinity) and the oscillations are averaged to 1/2. The interference pattern is lost. A similar situation appears if we let the mass difference Δm^2 become large. This is exactly what happens in the quark sector (and the reason why we never study quark oscillations despite knowing that mass eigenstates do not coincide with flavour eigenstates).

In terms of real life units (which are not "natural" units), the oscillation phase is given by

$$\Delta m_{ij}^2 \frac{L}{4E} = 1.27\, \Delta m_{ij}^2 (\mathrm{eV}^2) \frac{L\,(\mathrm{km})}{E\,(\mathrm{GeV})} \ . \tag{15}$$

then, since $\sin^2[1.27\, \Delta m_{ij}^2 (\mathrm{eV}^2) L\,(\mathrm{km})/E\,(\mathrm{GeV})]$ can be experimentally observed (*ie.* not smeared out) only if its argument is in a ballpark around one, an experimental set-up with a baseline L (km) and an energy E (GeV) is sensitive to neutrino mass squared differences $\Delta m_{ij}^2(\mathrm{eV}^2)$ of order $\sim [L\,(\mathrm{km})/E\,(\mathrm{GeV}]^{-1}$. For example, an experiment with a baseline of $L \sim 10^4$ km, roughly the size of Earth's diameter, and $E \sim 1$ GeV would explore mass differences Δm_{ij}^2 down to $\sim 10^{-4}$ eV^2. This fact makes it clear that neutrino long-baseline experiments can test even miserably small neutrino mass differences. It does so by exploiting the quantum mechanical interference between amplitudes whose relative phases are given precisely by these super tiny neutrino mass differences, which can be transformed into sizeable effects by choosing L/E appropriately.

But let's keep analysing the oscillation probability and see whether we can learn more about neutrino oscillations by studying its expression.

It is clear from $\mathrm{P}(\overset{(-)}{\nu}_\alpha \longrightarrow \overset{(-)}{\nu}_\beta)$ that if neutrinos have zero mass, in such a way that all $\Delta m_{ij}^2 = 0$, then, $\mathrm{P}(\overset{(-)}{\nu}_\alpha \longrightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$. Therefore, the experimental observation that neutrinos can morph from one flavour to a different one indicates that neutrinos are not only massive but also that their masses are not degenerate. Actually, it was precisely this evidence the one that proved beyond any reasonable doubt that neutrinos are massive.

However, every neutrino oscillation seen so far has involved at some point neutrinos that travel through matter. But the expression we derived is valid only for flavour change in vacuum, and does not take into account any interaction between the neutrinos and the matter traversed between their source and their detector. Thus, the question remains whether it may be that some unknown flavour changing interactions between neutrinos and matter are indeed responsible of the observed flavour transitions, and not neutrino masses. Regarding this question, a couple of things should be said. First, although it is true that the Standard Model of elementary particle physics contains only massless neutrinos, it provides an amazingly well corroborated description of weak interactions, and therefore of all the ways a neutrino interacts. Such a description does not include flavour change. Second, for some of the processes experimentally observed where neutrinos do change flavour, matter effects are expected to be miserably small, and on those cases the evidence points towards a dependence on L and E in the flavour transition probability through the combination L/E, as anticipated by the oscillation hypothesis. Modulo a constant, L/E is precisely the proper time that goes by in the rest frame of the neutrino as it covers a distance L possessing an energy E. Therefore, these flavour transitions behave as if they were a true progression of the neutrino itself over time, and not a result of an interaction with matter.

Now, lets explore the case where the leptonic mixing were trivial. This would imply that in the charged boson decay $W^+ \longrightarrow \overline{\ell_\alpha} + \nu_i$, which as we established has an amplitude $U^*_{\alpha i}$, the emerging charged antilepton $\overline{\ell_\alpha}$ of flavour α comes along always with the *same* neutrino mass eigenstate ν_i. That is, if $U^*_{\alpha i} \neq 0$, then due to unitarity, $U_{\alpha j}$ becomes zero for all $j \neq i$. Therefore, from Eq. (12) it is clear that, $\mathrm{P}(\overset{(-)}{\nu}_\alpha \longrightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$. Thus, the observation that neutrinos morph indicates non trivial a mixing matrix.

Then, we are left with basically two ways to detect neutrino flavour change. The first one is to observe, in a beam of neutrinos which are all created with the same flavour, say α, some amount of neutrinos of a new flavour β that is different from the flavour α we started with. This goes under the name of appearance experiments. The second way is to start with a beam of identical ν_αs, whose flux is either measured or known, and observe that after travelling some distance this flux is depleted. Such experiments are called disappearance experiments.

As Eq. (12) shows, the transition probability in vacuum does not only depend on L/E but also oscillates with it. It is because of this fact that neutrino flavour transitions are named "neutrino oscillations". Now notice also that neutrino transition probabilities do not depend on the individual neutrino masses (or masses squared) but on the squared-mass *differences*. Thus, oscillation experiments can only measure the neutrino mass squared spectrum. Not its absolute scale. Experiments can test the pattern but cannot determine the distance above zero the whole spectra lies.

It is clear that neutrino transitions cannot modify the total flux in a neutrino beam, but simply alter its distribution between the different flavours. Actually, from Eq. (12) and the unitarity of the U matrix, it is obvious that

$$\sum_\beta \mathrm{P}(\overset{(-)}{\nu}_\alpha \longrightarrow \overset{(-)}{\nu}_\beta) = 1 \ , \tag{16}$$

where the sum runs over all flavours β, including the original one α. Eq. (16) makes it transparent that the probability that a neutrino morphs its flavour, added to the probability that it keeps the flavour it had at birth, is one. Ergo, flavour transitions do not modify the total flux. Nevertheless, some of the flavours $\beta \neq \alpha$ into which a neutrino can oscillate into may be *sterile* flavours; that is, flavours that do not take part in weak interactions and therefore escape detection. If any of the original (active) neutrino flux turns into sterile, then an experiment able to measure the total *active* neutrino flux—that is, the flux associated to those neutrinos that couple to the weak gauge bosons: ν_e, ν_μ, and ν_τ— will observe it to be not exactly the original one, but smaller than it. In the experiments performed up today, no clear evidence of missing fluxes was found (although there are some hints in this direction).

In the literature, description of neutrino oscillations normally assume that the different mass eigenstates ν_i that contribute coherently to a beam share the same *momentum*, rather than the same *energy* as we have argued they must have. While the supposition of equal momentum is technically wrong, it is an inoffensive mistake, since, as can easily be shown, it conveys to the same oscillation probabilities as the ones we have obtained.

A relevant and interesting case of the (not that simple) formula for P($\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}$) is the case where only two flavours participate in the oscillation. The only-two-neutrino scenario is a rather rigorous description of a vast number of experiments. In fact only recently (and in few experiments) a more sophisticated (three neutrino description) was needed to fit observations. Lets assume then, that only two mass eigenstates, which we will name ν_1 and ν_2, and two reciprocal flavour states, which we will name ν_μ and ν_τ, are relevant, in such a way that only one squared-mass difference, $m_2^2 - m_1^2 \equiv \Delta m^2$ arises. Even more, neglecting phase factors that can be proven to have no impact on oscillation probabilities, the mixing matrix U can be written as

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad . \tag{17}$$

The unitary mixing matrix U of Eq. (17) is just a 2×2 rotation matrix, and as such, parameterized by a single rotation angle θ which is named (in neutrino physics) as the mixing angle. Plugging the U of Eq. (17) and the unique Δm^2 into the general formula of the transition probability P($\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}$), we can readily see that, for $\beta \neq \alpha$, when only two neutrinos are relevant,

$$\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 \, L}{4E} \right) \quad . \tag{18}$$

Moreover, the survival probability, *ie.* the probability that the neutrino remains with the same flavour its was created with is, as expected, one minus the probability that it changes flavour.

3 Neutrino oscillations in a medium

When we create a beam of neutrinos on earth through an accelerator and send it up to thousand kilometres away to meet a detector, the beam does not move through vacuum, but through matter, earth matter. The beam of neutrinos then scatters from the particles it meets along the way. Such a coherent forward scattering can have a large effect on the transition probabilities. We will assume for the time being that neutrino interactions with matter are flavour conserving, as described by the Standard Model, and comment on the possibility of flavour changing interactions later. Then as there are only two types of weak interactions (mediated by charged and neutral currents) there would be accordingly only two possibilities for this coherent forward scattering from matter particles to take place. Charged current mediated weak interactions will occur only if and only if the incoming neutrino is an electron neutrino. As only the ν_e can exchange charged boson W with an Earth electron. Thus neutrino-electron coherent forward scattering via W exchange opens up an extra source of interaction energy V_W suffered exclusively by electron neutrinos. Obviously, this additional energy being from weak interactions origin has to be proportional to G_F, the Fermi coupling constant. In addition, the interaction energy coming from $\nu_e - e$ scattering grows with the number of targets, N_e, the number of electrons per unit volume (given by the density of the Earth). Putting everything together it is not difficult to see that

$$V_W = +\sqrt{2}\, G_F \, N_e \quad , \tag{19}$$

clearly, this interaction energy affects also antineutrinos (in a opposite way though). It changes sign if we switch the ν_e by $\overline{\nu_e}$.

The interactions mediated by neutral currents correspond to the case where a neutrino in matter interacts with a matter electron, proton, or neutron by exchanging a neutral Z boson. According to the Standard Model weak interactions are flavour blind. Every flavour of neutrino enjoys them, and the amplitude for this Z exchange is always the same. It also teaches us that, at zero momentum transfer, electrons and protons couple to the Z boson with equal strength. The interaction has though, opposite sign. Therefore, counting on the fact that the matter through which our neutrino moves is electrically neutral (it contains equal number of electrons and protons), the contribution of both, electrons and protons to coherent forward neutrino scattering through Z exchange will add up to zero. Consequently only interactions with neutrons will survive so that, the effect of the Z exchange contribution to the interaction potential energy V_Z reduces exclusively to that with neutrons and will be proportional to N_n, the number density of neutrons. It goes without saying that it will be equal to all flavours. This time, we find that

$$V_Z = -\frac{\sqrt{2}}{2} G_F N_n \quad , \tag{20}$$

as was the case before, for V_W, this contribution will flip sign if we replace the neutrinos by antineutrinos.

But if, as we said, the Standard Model interactions do not change neutrino flavour, neutrino flavour transitions or neutrino oscillations point undoubtedly to neutrino mass and mixing even when neutrinos are propagating through matter. Unless non-Standard-Model flavour changing interactions play a role.

Neutrino propagation in matter is easy to understand when analysed through the time dependent Schrödinger equation in the lab frame

$$i\frac{\partial}{\partial t}|\nu(t)\rangle \ = \mathcal{H}|\nu(t)\rangle \quad . \tag{21}$$

where, $|\nu(t)\rangle\rangle$ is a (three component) neutrino vector state, in which each neutrino flavour corresponds to one component. In the same way, the Hamiltonian $\mathcal{H}$ is a (three $\times$ three) matrix in flavour space. To make our lives easy, lets analyse the case where only two neutrino flavours are relevant, say ν_e and ν_μ. Then

$$|\nu(t)\rangle \ = \begin{pmatrix} f_e(t) \\ f_\mu(t) \end{pmatrix} \quad , \tag{22}$$

with $f_i(t)^2$ the amplitude of the neutrino to be a ν_i at time t. This time the Hamiltonian, $\mathcal{H}$, is a $2{\times}2$ matrix in neutrino flavour space, i.e., $\nu_e - \nu_\mu$ space.

It will prove to be clarifying to work out the two flavour case in vacuum first, and add matter effects afterwards. Using Eq. (2) to express $|\nu_\alpha\rangle$ as a linear combination of mass eigenstates, we can see that the $\nu_\alpha - \nu_\beta$ matrix element of the Hamiltonian in vacuum, $\mathcal{H}_{\rm Vac}$, can be written as

$$\langle\nu_\alpha|\mathcal{H}_{\rm Vac}|\nu_\beta\rangle = \langle\sum_i U^*_{\alpha i}\nu_i|\mathcal{H}_{\rm Vac}|\sum_j U^*_{\beta j}\nu_j\rangle = \sum_j U_{\alpha j}U^*_{\beta j}\sqrt{p^2+m_j^2} \quad . \tag{23}$$

where we are supposing that the neutrinos belong to a beam where all its mass components (the mass eigenstates) share the same definite momentum p. As we have already mentioned, despite this supposition being technically wrong, it leads anyway to the right transition amplitude. In the second line of Eq. (23), we have used that the neutrinos ν_j with momentum p, the mass eigenstates, are the asymptotic states of the hamiltonian, $\mathcal{H}_{\rm Vac}$ for which constitute an orthonormal basis, satisfy

$$\mathcal{H}_{\rm Vac}|\nu_j\rangle = E_j|\nu_j\rangle \tag{24}$$

and have the standard dispersion relation, $E_j = \sqrt{p^2+m_j^2}$.

As we have already mentioned, neutrino oscillations are the archetype quantum interference phenomenon, where only the *relative* phases of the interfering states play a role. Therefore, only the *relative* energies of these states, which set their relative phases, are relevant. As a consequence, if it proves to be convenient (and it will), we can feel free to happily remove from the Hamiltonian $\mathcal{H}$ any contribution proportional to the identity matrix I. As we have said, this subtraction will leave unaffected the differences between the eigenvalues of $\mathcal{H}$, and therefore will leave unaffected the prediction of $\mathcal{H}$ for flavour transitions.

It goes without saying that as in this case only two neutrinos are relevant, there are only two mass eigenstates, ν_1 and ν_2, and only one mass splitting $\Delta m^2 \equiv m_2^2 - m_1^2$, and therefore there should be, as before a unitary U matrix given by Eq. (17) which rotates from one basis to the other. Inserting it into Eq. (23), and assuming that our neutrinos have low masses as compared to their momenta, i.e., $(p^2 + m_j^2)^{1/2} \cong p + m_j^2/2p$, and removing from $\mathcal{H}_{\text{Vac}}$ a term proportional to the the identity matrix (a removal we know is going to be harmless), we get

$$\mathcal{H}_{\text{Vac}} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \quad . \tag{25}$$

To write this expression, the highly relativistic approximation, which says that $p \cong E$ is used. Where E is the average energy of the neutrino mass eigenstates in our neutrino beam of ultra high momentum p.

It is not difficult to corroborate that the Hamiltonian $\mathcal{H}_{\text{Vac}}$ of Eq. (25) for the two neutrino scenario would give an identical oscillation probability, Eq. (18), as the one we have already obtained in a different way. An easy way to do it is to analyse the transition probability for the process $\nu_e \longrightarrow \nu_\mu$. From Eq. (17) it is clear that in terms of the mixing angle, the electron and muon neutrino states composition is

$$|\nu_e\rangle \;=\; |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta \;\; , \quad |\nu_\mu\rangle \;= -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta \;\; . \tag{26}$$

In the same way, we can also write the eigenvalues of the vacuum hamiltonian $\mathcal{H}_{\text{Vac}}$, Eq.25, in terms of the mass squared differences as

$$\lambda_1 = -\frac{\Delta m^2}{4E} \;\; , \; \lambda_2 = +\frac{\Delta m^2}{4E} \;\; . \tag{27}$$

The mass eigenbasis of this Hamiltonian, $|\nu_1\rangle$ and $|\nu_2\rangle$, can also be written in terms of flavour eigenbasis $|\nu_e\rangle$ and $|\nu_\mu\rangle$ by means of Eqs. (26). Therefore, the Schrödinger equation of Eq. (21), with the identification of $\mathcal{H}$ in this case with $\mathcal{H}_{\text{Vac}}$ tells us that if at time $t = 0$ we begin from a $|\nu_e\rangle$, then once some time t elapses this $|\nu_e\rangle$ will progress into the state given by

$$|\nu(t)\rangle \;= |\nu_1\rangle e^{+i\frac{\Delta m^2}{4E}t} \cos\theta + |\nu_2\rangle e^{-i\frac{\Delta m^2}{4E}t} \sin\theta \;\; . \tag{28}$$

Thus, the probability $\mathrm{P}(\nu_e \longrightarrow \nu_\mu)$ that this evoluted neutrino be detected as a different flavour ν_μ, from Eqs. (26) and (28), is given by,

$$\begin{aligned} \mathrm{P}(\nu_e \longrightarrow \nu_\mu) &= |\langle \nu_\mu | \nu(t)\rangle|^2 = |\sin\theta\cos\theta(-e^{i\frac{\Delta m^2}{4E}t} + e^{-i\frac{\Delta m^2}{4E}t})|^2 \\ &= \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4E}\right) \quad . \end{aligned} \tag{29}$$

Where we have substituted the time t travelled by our highly relativistic state by the distance L it has covered. The flavour transition or oscillation probability of Eq. (29), as expected, is exactly the same we have found before, Eq. (18).

We can now move on to analyse neutrino propagation in matter. In this case, the 2×2 Hamiltonian representing the propagation in vacuum $\mathcal{H}_{\text{Vac}}$ receives the two additional contributions we have discussed before, and becomes $\mathcal{H}_M$, which is given by

$$\mathcal{H}_M = \mathcal{H}_{\text{Vac}} + V_W \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + V_Z \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad . \tag{30}$$

In the new Hamiltonian, the first additional contribution corresponds to the interaction potential due to the charged bosons exchange, Eq. (19). As this interaction is suffered only by ν_e, this contribution is different from zero only in the $\mathcal{H}_M$(1,1) element or the $\nu_e - \nu_e$ element. The second additional contribution, the last term of Eq. (30) comes from the Z boson exchange, Eq. (20). Since this interaction is flavour blind, it affects every neutrino flavour in the same way, its contribution to $\mathcal{H}_M$ is proportional to the identity matrix, and can be safely neglected. Thus

$$\mathcal{H}_M = \mathcal{H}_{\rm Vac} + \frac{V_W}{2} + \frac{V_W}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \tag{31}$$

where (for reasons that are going to become clear later) we have divided the W-exchange contribution into two pieces, one proportional to the identity (that we will disregarded in the next step) and, a piece that it is not proportional to the identity, that we will keep. Disregarding the first piece as promised, we have from Eqs. (25) and (31)

$$\mathcal{H}_M = \frac{\Delta m^2}{4E}\begin{pmatrix} -(\cos 2\theta - A) & \sin 2\theta \\ \sin 2\theta & (\cos 2\theta - A) \end{pmatrix} \quad , \tag{32}$$

where we have defined

$$A \equiv \frac{V_W/2}{\Delta m^2/4E} = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2} \quad . \tag{33}$$

Clearly, A parameterizes the relative size of the matter effects as compared to the vacuum contribution given by the neutrino squared-mass splitting and signals the situations when they become important.

Now, if we introduce (a physically meaningful) short-hand notation

$$\Delta m_M^2 \equiv \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2} \tag{34}$$

and

$$\sin^2 2\theta^M \equiv \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \quad , \tag{35}$$

then the Hamiltonian in a medium $\mathcal{H}_M$ turns out to be

$$\mathcal{H}_M = \frac{\Delta m_M^2}{4E}\begin{pmatrix} -\cos 2\theta^M & \sin 2\theta^M \\ \sin 2\theta^M & \cos 2\theta^M \end{pmatrix} \quad . \tag{36}$$

and can be diagonalised by inspection, i.e., as a result of our choice, the Hamiltonian in a medium, $\mathcal{H}_M$, becomes formally indistinguishable to the vacuum one, $\mathcal{H}_{\rm Vac}$, Eq. (25). The difference being that in this case what used to be the vacuum parameters Δm^2 and θ are presently given by the matter ones, Δm_M^2 and θ^M, respectively.

Obviously, the mass eigenstates and eigenvalues (which determine the mass differences and mixing angle) of $\mathcal{H}_M$ are not identical to the ones in vacuum. The eigenstates in matter, *ie.* the files of the unitary matrix that rotates from the flavour basis to the mass basis, are different from the vacuum eigenvalues that form the vacuum mixing matrix, and therefore θ_M is not θ. But, the matter Hamiltonian $\mathcal{H}_M$ does indeed contain all about the propagation of neutrinos in matter, in the same way $\mathcal{H}_{\rm Vac}$ contains all about the propagation in vacuum.

According to Eq. (36), $\mathcal{H}_M$ has the same functional dependence on the matter parameters Δm_M^2 and θ^M as the vacuum Hamiltonian $\mathcal{H}_{\rm Vac}$, Eq. (25), on the vacuum ones, Δm^2 and θ. Therefore, Δm_M^2 can be identified with an effective mass squared difference in matter, and accordingly θ^M can be unidentified with an effective mixing angle in matter.

In a typical experimental set-up where the neutrino beam is generated by an accelerator and sent away to a detector that is several hundred, or even thousand kilometres away, it traverses through earth matter, but only superficially, it does not get deep into the earth. Then, during this voyage the matter density encountered by such a beam can be taken to be approximately constant [1]. But if the density of

[1]This approximation is clearly not valid for neutrinos that cross the Earth

the earth's matter is constant, the same happens with the electron density N_e, and the A parameter in which it is incorporated. And it is also true about the Hamiltonian $\mathcal{H}_M$. They all become approximately constant, and therefore quite identical to the vacuum Hamiltonian $\mathcal{H}_{\text{Vac}}$, except for the particular values of their parameters. By comparing Eqs. (36) and (25), we can immediately conclude that exactly in the same way $\mathcal{H}_{\text{Vac}}$ gives rise to vacuum oscillations with probability $\text{P}(\nu_e \longrightarrow \nu_\mu)$ of Eq. (29), $\mathcal{H}_M$ must give rise to matter oscillations with probability

$$\text{P}_M(\nu_e \longrightarrow \nu_\mu) = \sin^2 2\theta^M \sin^2\left(\Delta m_M^2 \frac{L}{4E}\right) \ . \tag{37}$$

Namely, the transition and survival probabilities in matter are the same as those in vacuum, except that the vacuum parameters Δm^2 and θ are now replaced by their matter counterparts, Δm_M^2 and θ^M.

In theory, judging simply by its potential, matter effects can have very drastic repercussions in the oscillation probabilities. The exact impact (if any) can be estimated only after the details of the experimental set-up of the experiment in question are given. As a rule of thumb, to guess the importance of matter effects, we should keep in mind that for neutrinos propagating through the earth's mantle (not deeper than 200 km below the surface) and if the kinematic phase associated to the solar mass difference is still negligible,

$$A \cong \frac{E}{13 \ \text{GeV}} \tag{38}$$

so that only for beam energies of several GeV matter effects do matter.

And how much do they matter? They matter a lot! From Eq. (35) for the matter mixing angle, θ^M, we can appreciate that even when the vacuum mixing angle θ is incredible small, say, $\sin^2 2\theta = 10^{-4}$, if we get to have $A \cong \cos 2\theta$, i.e., for energies of a few tens of GeV, then $\sin^2 2\theta^M$ can be brutally enhanced as compared to its vacuum value and can even reach maximal mixing, *ie.* $\sin^2 2\theta^M = 1$. This wild enhancement of a small mixing angle in vacuum up to a sizeable (even maximal) one in matter is the "resonant" enhancement, the largest possible version of the Mikheyev-Smirnov-Wolfenstein effect [6–9]. In the beginning of solar neutrino experiments, people entertained the idea that this brutal enhancement was actually taking place while neutrinos crossed the sun. Nonetheless, as we will see soon the mixing angle associated with solar neutrinos is quite sizeable ($\sim 34°$) already in vacuum [10]. Then, although matter effects on the sun are important and they do enhance the solar mixing angle, unfortunately they are not as drastic as we once dreamt. Nevertheless, for long-baselines they will play (they are already playing!) a key role in the determination of the ordering of the neutrino spectrum.

4 Evidence for neutrino oscillations

4.1 Atmospheric and accelerator neutrinos

Almost twenty years have elapsed since we were presented solid and convincing evidence of neutrino masses and mixings, and since then, the evidence has only grown. SuperKamiokande (SK) was the first experiment to present compelling evidence of ν_μ disappearance in their atmospheric neutrino fluxes, see Ref. [11]. In Fig. 1 the zenith angle (the angle subtended with the horizontal) dependence of the multi-GeV ν_μ sample is shown together with the disappearance as a function of L/E. These data fit amazingly well the naive two component neutrino hypothesis with

$$\Delta m^2_{\text{atm}} = 2 - 3 \times 10^{-3} \text{eV}^2 \quad \text{and} \quad \sin^2\theta_{\text{atm}} = 0.50 \pm 0.13 \ . \tag{39}$$

Roughly speaking SK corresponds to an L/E for oscillations of 500 km/GeV and almost maximal mixing (the mass eigenstates are nearly even admixtures of muon and tau neutrinos). No signal of an involvement of the third flavour, ν_e is found so the assumption is that atmospheric neutrino disappearance is basically $\nu_\mu \longrightarrow \nu_\tau$. Notice however, that the first NOvA results seem to point toward a mixing angle which is not maxima.

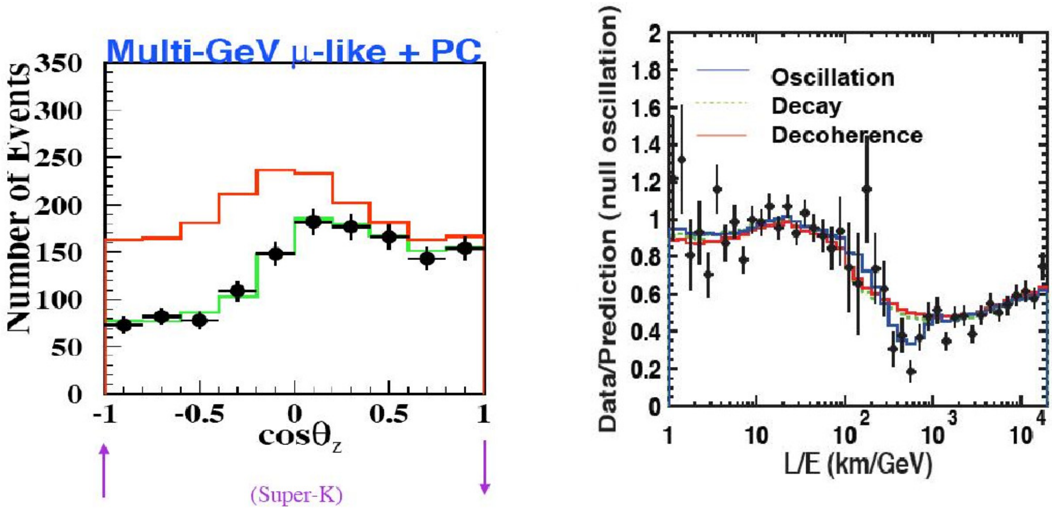

Fig. 1: Superkamiokande's evidence for neutrino oscillations both in the zenith angle and L/E plots

After atmospheric neutrino oscillations were established, a new series of neutrino experiments were built, sending (man-made) beams of ν_μ neutrinos to detectors located at large distances: the K2K (T2K) experiment [12, 13], sends neutrinos from the KEK accelerator complex to the old SK mine, with a baseline of 120 (235) km while the MINOS (NOvA) experiment [14, 15], sends its beam from Fermilab, near Chicago, to the Soudan mine (Ash river) in Minnesota, a baseline of 735 (810) km. All these experiments have seen evidence for ν_μ disappearance consistent with the one found by SK. Their results are summarised in Fig. 2.

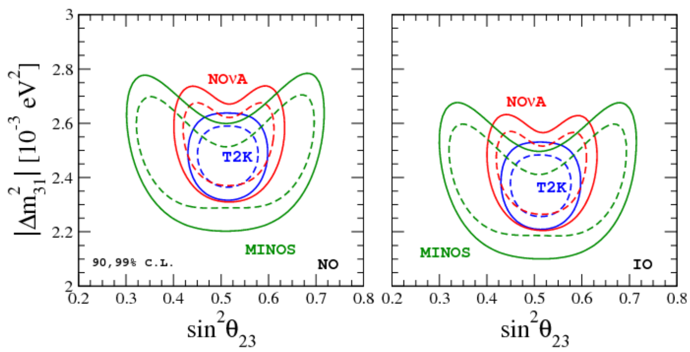

Fig. 2: Allowed regions in the Δm^2_{atm} vs $\sin^2\theta_{\text{atm}}$ plane for MINOS and NOVA data as well as for T2K data and two of the SK analyses. Results from https://globalfit.astroparticles.es/.

4.2 Reactor and solar neutrinos

The KamLAND reactor experiment, an antineutrino disappearance experiment, receiving neutrinos from sixteen different reactors, at distances ranging from hundred to thousand kilometres, with an average baseline of 180 km and neutrinos of a few eV [16, 17], has seen evidence of neutrino oscillations. Such evidence was collected not only at a different L/E than the atmospheric and accelerator experiments but also consists on oscillations involving electron neutrinos, ν_e, the ones which were not involved before. These oscillations have also been seen for neutrinos coming from the sun (the sun produces only electron neutrinos). However,in order to compare the two experiments we should assume that neutrinos (solar) and antineutrinos (reactor) behave in the same way, *ie.* assume CPT conservation. The best fit values in the two neutrino scenario for the KamLAND experiment are

$$\Delta m^2_{\odot} = 7.55 \pm 0.2 \times 10^{-5}\text{eV}^2 \quad \text{and} \quad \sin^2\theta_{\odot} = 0.32 \pm 0.03 \ . \tag{40}$$

In this case, the L/E involved is 15 km/MeV which is more than an order of magnitude larger than the atmospheric scale and the mixing angle, although large, is clearly not maximal.

Figure 3 shows the disappearance probability for the $\bar{\nu}_e$ for KamLAND as well as several older reactor experiments with shorter baselines [2].The second panel depicts the flavour content of the ^{8}Boron solar neutrino flux (with GeV energies) measured by SNO [18] and SK [19]. The reactor outcome can be explained in terms of two flavour oscillations in vacuum, given that the fit to the disappearance probability, is appropriately averaged over E and L.

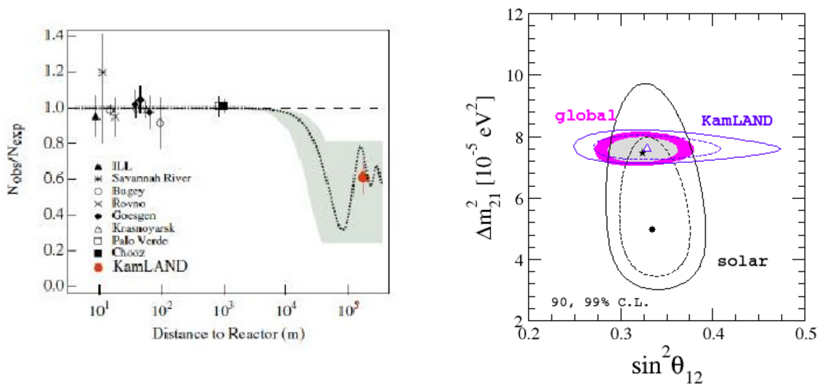

Fig. 3: Disappearance of $\bar{\nu}_e$ observed in reactor experiments as a function of distance from the reactor (favoured region for all solar and reactor experiments). Results from https://globalfit.astroparticles.es/.

The analysis of neutrinos originating from the sun is marginally more complex than the one we did before because it should incorporate the matter effects that the neutrinos endure since they are born (at the centre of the sun) until they abandon it, which are imperative at least for the ^{8}Boron neutrinos. The pp and ^{7}Be neutrinos are less energetic and therefore are not significantly altered by the presence of matter and leave the sun as though it were ethereal. ^{8}Boron neutrinos on the other hand, leave the sun unequivocally influenced by the presence of matter and this is evidenced by the fact that they leave the sun as ν_2, the second mass eigenstate and therefore do not experience oscillations. This distinction among neutrinos coming from different reaction chains is, as mentioned, due mainly to their disparities at birth. While pp (^{7}Be) neutrinos are created with an average energy of 0.2 MeV (0.9 MeV), ^{8}B are born with 10 MeV and as we have seen the impact of matter effects grows with the energy of the neutrino.

However, we ought to emphasise that we do not really see solar neutrino oscillations. To trace the oscillation pattern, to be able to test is distinctive shape, we need a kinematic phase of order one otherwise the oscillations either do not develop or get averaged to 1/2. In the case of neutrinos coming from the sun the kinematic phase is

$$\Delta_{\odot} = \frac{\Delta m^2_{\odot} L}{4E} = 10^{7\pm 1} \ . \tag{41}$$

Consequently, solar neutrinos behave as "effectively incoherent" mass eigenstates once they leave the sun, and remain so once they reach the earth. Consequently the ν_e disappearance or survival probability

[2] Shorter baseline reactor neutrino experiments, which has seen no evidence of flux depletion suffer the so-called reactor neutrino anomaly, which may point toward the existence of light sterile states

is given by

$$\langle P_{ee}\rangle = f_1\cos^2\theta_\odot + f_2\sin^2\theta_\odot \tag{42}$$

where f_1 is the ν_1 content or fraction of ν_μ and f_2 is the ν_2 content of ν_μ and therefore both fractions satisfy $f_1 + f_2 = 1$.

Nevertheless, as we have already mentioned, solar neutrinos originating from the pp and ^{7}Be chains are not affected by the solar matter and oscillate as in vacuum and thus, in their case $f_1 \approx \cos^2\theta_\odot = 0.69$ and $f_2 \approx \sin^2\theta_\odot = 0.31$. In the ^{8}B a neutrino case, however, the impact of solar matter is sizeable and the corresponding fractions are substantially altered. In a two neutrino scenario, the day-time CC/NC measured by SNO, which is roughly identical to the day-time average ν_e survival probability, $\langle P_{ee}\rangle$, reads

$$\left.\frac{CC}{NC}\right|_{\text{day}} = \langle P_{ee}\rangle = f_1\cos^2\theta_\odot + f_2\sin^2\theta_\odot \quad , \tag{43}$$

where f_1 and $f_2 = 1 - f_1$ are the ν_1 and ν_2 contents of the muon neutrino, respectively, averaged over the ^{8}B neutrino energy spectrum appropriately weighted with the charged current current cross section. Therefore, the ν_1 fraction (or how much f_2 differs from 100%) is given by

$$f_1 = \frac{\left(\frac{CC}{NC}\big|_{\text{day}} - \sin^2\theta_\odot\right)}{\cos 2\theta_\odot} = \frac{(0.347 - 0.311)}{0.378} \approx 10\% \tag{44}$$

where the central values of the last SNO analysis [18] were used. As there are strong correlations between the uncertainties of the CC/NC ratio and $\sin^2\theta_\odot$ it is not obvious how to estimate the uncertainty on f_1 from their analysis. Note, that if the fraction of ν_2 were 100%, then $\frac{CC}{NC}\big|_{\text{day}} = \sin^2\theta_\odot$.

Utilising the analytic analysis of the Mikheyev-Smirnov-Wolfenstein (MSW) effect, gave in Ref. [20], one can obtain the mass eigenstate fractions in a medium, which are given by

$$f_2 = 1 - f_1 = \langle \sin^2\theta_\odot^M + P_x\cos 2\theta_\odot^M\rangle_{^8\text{B}} \quad , \tag{45}$$

with $\theta_\odot^M$ being the mixing angle as given at the ν_e production point and P_x is the probability of the neutrino to hop from one mass eigenstate to the second one during the Mikheyev-Smirnov resonance crossing. The average $\langle\ldots\rangle_{^8\text{B}}$ is over the electron density of the ^{8}B ν_e production region in the centre of the Sun as given by the Solar Standard Model and the energy spectrum of ^{8}B neutrinos has been appropriately weighted with SNO's charged current cross section. All in all, the ^{8}B energy weighted average content of ν_2's measured by SNO is

$$f_2 = 91 \pm 2\% \text{ at the 95 \% C.L.} \quad . \tag{46}$$

Therefore, it is obvious that the ^{8}B solar neutrinos are the purest mass eigenstate neutrino beam known so far and SK super famous picture of the sun taken (from underground) with neutrinos is made with approximately 90% of ν_2, *ie.* almost a pure beam of mass eigenstates.

Last but not least, six years ago, a newly built reactor neutrino experiment, the Daya Bay experiment, located in China, announced the measurement of the third mixing angle [22], the only one which was still missing and found it to be

$$\sin^2(2\theta_{12}) = 0.092 \pm 0.017 \quad . \tag{47}$$

Following this announcement, several experiments confirmed the finding and during the last years the last mixing angle to be measured became the best (most precisely) measured one. The fact that this angle, although smaller that the other two, is still sizeable opens the door to a new generation of neutrino experiments aiming to answer the open questions in the field.

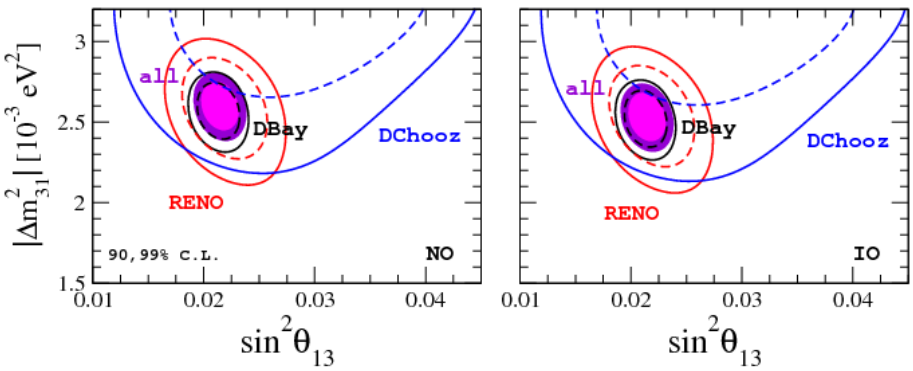

Fig. 4: Reactor mixing angle. Results from https://globalfit.astroparticles.es/

5 ν Standard Model

Now that we have comprehended the physics behind neutrinos oscillations and have leaned the experimental evidence about the parameters driving these oscillations, we can move ahead and construct the Neutrino Standard Model: it comprises three light ($m_i < 1$ eV) neutrinos, *ie.* it involves just two mass differences.

So far we have not seen any solid experimental indication (or need) for additional neutrinos [3]. As we have measured long time ago the invisible width of the Z boson and found it to be 3, within errors, if additional neutrinos are going to be incorporated into the model, they cannot couple to the Z boson, *ie.* they cannot enjoy weak interactions, so we call them sterile. However, as sterile neutrinos have not been seen (although they may have been hinted), and are not needed to explain any solid experimental evidence, our Neutrino Standard Model will contain just the three active flavours: e, μ and τ.

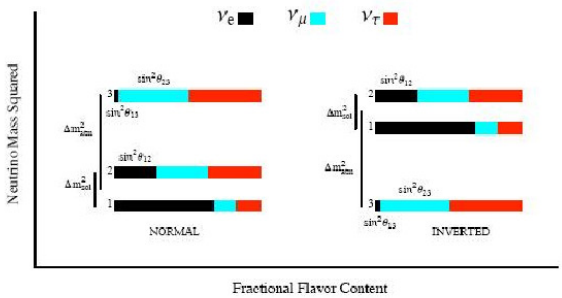

Fig. 5: Flavour content of the three neutrino mass eigenstates (not including the dependence on the cosine of the CP violating phase δ). If CPT is conserved, the flavour content must be the same for neutrinos and anti-neutrinos. Notice that oscillation experiments cannot tell us how far above zero the entire spectrum lies.

The unitary mixing matrix which rotates from the flavour to the mass basis, called the PMNS matrix, comprises three mixing angles (the so called solar mixing angle:θ_{12}, the atmospheric mixing angle θ_{23}, and the last to be measured, the reactor mixing angleθ_{13}), one Dirac phase (δ) and potentially two Majorana phases (α, β) and is given by

$$| \nu_\alpha \rangle = U_{\alpha i} \mid \nu_i \rangle$$

[3] Although it must be noted that there are several not significant hint pointing in this direction

$$U_{\alpha i} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. Courtesy of the hierarchy in mass differences (and to a less extent to the smallness of the reactor mixing angle) we are permitted to recognise the (23) label in the three neutrino scenario as the atmospheric Δm^2_{atm} we obtained in the two neutrino scenario, in a similar fashion the (12) label can be assimilated to the solar $\Delta m^2_{\odot}$. The (13) sector drives the ν_e flavour oscillations at the atmospheric scale, and the depletion in reactor neutrino fluxes, see Ref. [22]. According to the experiments done so far, the three sigma ranges for the neutrino mixing angles are

$$0.273 < \sin^2\theta_{12} < 0.379 \quad ; \quad 0.445 < \sin^2\theta_{23} < 0.599 \quad ; \quad 0.0196 < \sin^2\theta_{13} < 0.0241$$

while the corresponding ones for the mass splittings are

$2.41 \times 10^{-3}\text{eV}^2 < |\ \Delta m^2_{32}\ | < 2.60 \times 10^{-3}\text{eV}^2$ and $7.05 \times 10^{-5}\text{eV}^2 < \Delta m^2_{21} < 8.14 \times 10^{-5}\text{eV}^2$. These mixing angles and mass splittings are summarised in Fig. 5.

As oscillation experiments only explore the two mass differences, two orderings are possible, as shown in Fig. 5. They are called normal and inverted hierarchy and roughly identify whether the mass eigenstate with the smaller electron neutrino content is the lightest or the heaviest.

The absolute mass scale of the neutrinos, or the mass of the lightest neutrino is not known yet, but cosmological bounds already say that the heaviest one must be lighter than about 0.3 eV.

As transition or survival probabilities depend on the combination $U^*_{\alpha i}U_{\beta i}$ no trace of the Majorana phases could appear on oscillation phenomena, however they will have observable effects in those processes where the Majorana character of the neutrino is essential for the process to happen, like neutrinoless double beta decay.

6 Neutrino mass and character

6.1 Absolute neutrino mass

The absolute mass scale of the neutrino, *ie.* the mass of the lightest/heaviest neutrino, cannot be obtained from oscillation experiments, however this does not mean we have no access to it. Direct experiments like tritium beta decay, or neutrinoless double beta decay and indirect ones, like cosmological observations, have the potential to give us the information on the absolute scale of neutrino mass, we so desperately need. The KATRIN tritium beta decay experiment [23] has sensitivity down to 200 meV for the "mass" of ν_e defined as

$$m_{\nu_e} = |\ U_{e1}\ |^2\ m_1 + |\ U_{e2}\ |^2\ m_2 + |\ U_{e3}\ |^2\ m_3 \quad . \tag{48}$$

Neutrino-less double beta decay experiments, see Ref. [24] for a review, do not measure the absolute mass of the neutrino directly but a particular combination of neutrino masses and mixings,

$$m_{\beta\beta} = |\sum m_i U^2_{ei}| = |\ m_a c^2_{13} c^2_{12} + m_2 c^2_{13} s^2_{12} e^{2i\alpha} + m_3 s^2_{13} e^{2i\beta}\ | \quad , \tag{49}$$

where it is understood that neutrinos are taken to be Majorana particles, *ie.* truly neutral particles (having all their quantum numbers to be zero). The new generation of experiments seeks to reach below 10 meV for $m_{\beta\beta}$ in double beta decay.

Cosmological probes (CMB and Large Scale Structure experiments) measure the sum of the neutrino masses

$$m_{\text{cosmo}} = \sum_i m_i \quad . \tag{50}$$

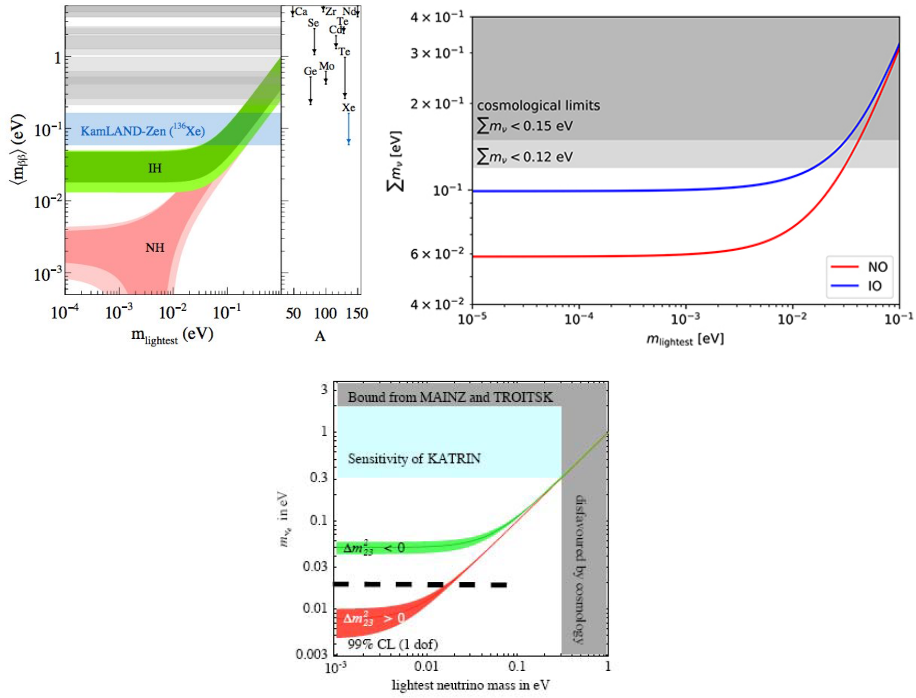

Fig. 6: The effective mass measured in double β decay, in cosmology and in Tritium β decay versus the mass of the lightest neutrino. Below the dashed lines, only the normal hierarchy is allowed. Notice that while double β decay experiments bound the neutrino mass only in the Majorana case, Planck bounds apply for either case.

and may have a say on the mass ordering (direct or inverted spectrum) as well as test other neutrino properties like neutrino asymmetries [25]. If $\sum m_i \approx 10$ eV, the energy balance of the universe saturates the bound coming from its critical density. The current limit [26], is a few % of this number, ~ 0.3 eV. These bounds are model dependent but they do all give numbers of the same order of magnitude. However, given the systematic uncertainties characteristic of cosmology, a solid limit of less that 200 meV seems way too aggressive. Figure 6 shows the allowed parameter space for the neutrino masses (as a function of the absolute scale) for both the normal and inverted hierarchy.

6.2 Majorana vs Dirac

A fermion mass is nothing but a coupling between a left handed state and a right handed one. Thus, if we examine a massive fermion at rest, then one can regard this state as a linear combination of two massless particles, one right handed and one left handed. If the particle we are examining is electrically charged, like an electron or a muon, both particles, the left handed as well as the right handed must have the same charge (we want the mass term to be electrically neutral). This is a Dirac mass term. However, for a neutral particle, like the neutrino, a new possibility opens up, the left handed particle can be coupled to the right handed anti-particle, (a term which would have a net charge, if the fields are not absolutely and totally neutral) this is a Majorana mass term.

Thus a truly and absolutely neutral particle (who will inevitably be its own antiparticle) does have two ways of getting a mass term, a la Dirac or a la Majorana, and if there are no reasons to forbid one of them, will have them both.

In the case of a neutrino, the left chiral field couples to $SU(2) \times U(1)$ implying that a Majorana mass term is forbidden by gauge symmetry. However, the right chiral field carries no quantum numbers, is totally and absolutely neutral. Then, the Majorana mass term is unprotected by any symmetry and it is expected to be very large, of the order of the largest scale in the theory. On the other hand, Dirac mass terms are expected to be of the order of the electroweak scale times a Yukawa coupling, giving a mass of the order of magnitude of the charged lepton or quark masses. Putting all the pieces together, the mass matrix for the neutrinos results as in Fig. 7.

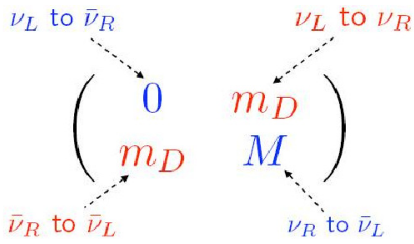

Fig. 7: The neutrino mass matrix with the various right to left couplings, M_D is the Dirac mass terms while 0 and M are Majorana masses for the charged and uncharged (under $SU(2) \times U(1)$) chiral components.

To get the mass eigenstates we need to diagonalise the neutrino mass matrix. By doing so, one is left with two Majorana neutrinos, one super-heavy Majorana neutrino with mass $\simeq M$ and one super-light Majorana neutrino with mass m_D^2/M, *ie.* one mass goes up while the other sinks, this is what we call the seesaw mechanism [27–29][4]. The light neutrino(s) is(are) the one(s) observed in current experiments (its mass differences) while the heavy neutrino(s) are not accessible to current experiments and could be responsible for explaining the baryon asymmetry of the universe through the generation of a lepton asymmetry at very high energy scales since its decays can in principle be CP violating (they depend on the two Majorana phases on the PNMS matrix which are invisible for oscillations). The super heavy Majorana neutrinos being their masses so large can play a role at very high energies and can be related to inflation [30].

If neutrinos are Majorana particles lepton number is no longer a good quantum number and a plethora of new processes forbidden by lepton number conservation can take place, it is not only neutrino-less double beta decay. For example, a muon neutrino can produce a positively charged muon. However, this process and any processes of this kind, would be suppressed by $(m_\nu/E)^2$ which is tiny, 10^{-20}, and therefore, although they are technically allowed, are experimentally unobservable. To most stringent limit nowadays comes from KamLAND-zen [31], and constraints the half-life of neutrino-less double beta decay to be $T^{0\nu}_{1/2} > 1.07 \times 10^{26}$ years at 90% C.L. Forthcoming experiments such as GERDA-PhaseII, Majorana, SuperNEMO, CUORE, and nEXO will improve this sensitivity by one order of magnitude.

Recently low energy sewsaw models [32] have experienced a revival and are actively being explored [33]. In such models the heavy states, of only few tens of TeV can be searched for at the LHC. The heavy right handed states in these models will be produced at LHC either through Yukawa couplings of through gauge coupling to right handed gauge bosons. Some models contain also additional scalar that can be looked for.

7 Conclusions

The experimental observations of neutrino oscillations, meaning that neutrinos have mass and mix, have answered questions that have been present since the establishment of the Standard Model. As those veils have disappeared, new questions open up and challenge our understanding:

[4]Depending on the envisioned high energy theory, the simplest see saw mechanism can be categorised into three different classes or types (as they are called) depending on their scalar content.

– What is the true nature of the neutrinos? Are they Majorana or Dirac particles?
– Is there any new scale associated to neutrinos masses? Is this new scale accessible using particle colliders?
– Is the spectrum normal or inverted? Is the lightest neutrino the one with the least electron content on it, or is it the heaviest one?
– Is CP violated (is $\sin\delta \neq 0$)? If so, is this phase related at any rate with the baryon asymmetry of the Universe? What about the other two phases? Which is the absolute mass scale of the neutrinos?
– Are there new interactions? Are neutrinos related to the open questions in cosmology like dark matter and/or dark energy? Do (presumably heavy) neutrinos play a role in inflation?
– Can neutrinos violate CPT [34]? What about Lorentz invariance? If we ever measure a different spectrum for neutrinos and antineutrinos (after matter effects are properly taken into account), how can we distinguish whether it is due to a true (genuine) CTP violation or to a non-standard neutrino interaction?
– Are these intriguing signals in short baseline reactor neutrino experiments (the missing fluxes) a real effect? Do they imply the existence of sterile neutrinos?

We would like to answer these questions, so we plan to do new experiments. These experiments will for sure bring some answers and clearly open new and pressing questions. Only one thing is clear—our journey into the neutrino world has just begun.

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