

Hadron spectroscopy

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If you query the arXiv electronic database of preprints and look at the topics of published articles, you can see that roughly one in five deals with hadron spectroscopy. During its development, this area of particle physics has experienced several rises and falls. Today, it is a rapidly developing branch of science, which comprises a significant part of the research program of almost every accelerator experiment. It is especially valid for B -, $c\tau$ -factories and, certainly, the LHC, where exciting results have been obtained. The purpose of this lecture is to try giving an overview of the current state of hadron spectroscopy through the eyes of an experimentalist.

1 Introduction: why do we call it spectroscopy?

The term ‘spectroscopy’ was coined by Robert Bunsen and Gustav Kirchhoff in the 1860s, when they built the first *spectroscope*. The idea of the device was to use a prism to split the light into a spectrum and study the resulting picture at the screen. Pretty soon it became clear that the atom of a chemical element is capable of emitting and absorbing light of only a definite frequency (or photon energy, as we say today). In the experiment, this was expressed as the presence in the spectrum of clear emission lines (for heated substances) or absorption lines (for white light going through a cold gas), with a unique line pattern for each element. As a result, the victorious march of atomic spectroscopy began with the discovery of several new chemical elements and the study of the chemical composition of stars. Further, it led to the emergence of the Bohr model of the atom, which made it possible to describe in detail the spectrum of the hydrogen atom and became a keystone of quantum mechanics. At present, atomic spectroscopy remains a powerful and sensitive method of chemical analysis used in almost every laboratory.

After the discovery of radioactivity, the method of spectroscopy was applied to the investigation of nuclear radiation. Naturally, the technique of the experiment has changed a lot. The energy of the radioactive decay made it possible to abandon the use of a gas burner, and more complex instruments began to be used to detect radiation instead of a prism and a screen. The radiation energy was measured either by deflecting charged rays by a magnetic field (so-called *mass spectrometry*) or by measuring the energy released during the absorption of radiation in a scintillator. It is curious that both methods, although with much more sophisticated equipment, are used in the experiment even today. Nuclear spectroscopy made it possible to ‘weigh’ various nuclei and study their energy levels. The problem turned out to be difficult, and the study of the spectra of nuclei and their theoretical description remain on the agenda of nuclear physics till now.

Advances in the study of atomic nuclei led, by the end of the Second World War, to a clear and consistent picture of the structure of matter. The nucleus consists of nucleons (protons and neutrons) bound by the strong nuclear force, carried by the exchange of pions. The weak force is responsible for

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the beta decay. Indeed, the remaining problems were the lack of a detailed theoretical description of the nuclear force and the problem of the existence of the muon. The role of the muon in the structure of matter was unclear. The development of technology during the war opened up opportunities for the construction of new, much more powerful accelerators and the start of a series of experiments on them. However, contrary to expectations, instead of solving existing problems, new experiments have brought new mysteries. A large number of new elementary particles was discovered, including a great many short-lived resonances. Among the new particles, analogs of nucleons (antiprotons, antineutrons, hyperons) and interaction carrier particles (kaons, ρ , η and ω mesons, etc.) were found. It became evident that nuclear matter has a much more complicated structure. Note that the Particle Data Group counts over 250 established particles now, of which more than 50 have been detected in the past two decades.

It is this reason that led to the appearance of hadron spectroscopy. By analogy with atomic and nuclear spectroscopy, hadron spectroscopy aims to study the possible quantum states of strongly interacting particles – hadrons. The purpose of hadron spectroscopy is essentially to find answers to the questions: which hadrons exist? What properties do they possess? What reactions can they undergo? Just like atomic spectroscopy for atoms or nuclear spectroscopy for nuclei, hadron spectroscopy is a key source of experimental data for understanding the structure of hadrons and, ultimately, the strong interaction.

2 What are we really measuring?

Since we are talking about an experiment, we should start by discussing what exactly we want and can measure. First of all, we can experimentally establish the very existence of the particle. Then, we can determine its electric charge, mass and lifetime (or particle width). Finally, we can study its possible decays, the relative probabilities of different decay channels, and the angular and energy distributions of the decay products.

From the decay information, one can extract the P-symmetry and C-symmetry of the particle, its total angular momentum J , as well as the quantum numbers associated with its internal composition: the flavor numbers (strangeness, charm, beauty, isospin), as well as the baryon number (that is, the difference between the number of quarks and antiquarks). Already difficult, these measurements are further complicated by the quantum nature of the experiment: we are measuring a state that can be a superposition of several or many quantum states, and we usually have no way to distinguish them.

Now let us consider the most important experimental techniques.

The invariant mass spectrum of a system of particles is perhaps the simplest and most powerful tool for hadron spectroscopy. Indeed, by registering particles, measuring their energies or momenta, and identifying (or assuming) their types, we can easily calculate the mass of a hypothetical parent particle. Having plotted the distribution of this mass, it is easy to detect peaks in the resulting spectrum if the parent particle really exists. At the same time, the spectrum of the invariant mass gives the mass of the particle and, if the resolution of the detector allows it, its width.

The Dalitz plot, invented by R. H. Dalitz [1], is a more sophisticated technique used in the analysis of multiparticle decays. The Dalitz plot is constructed as a correlation of the momenta and energies of product particles in the decay. Thus, in the case of a decay into three particles, each decay event is

represented in the Dalitz plot by a point, the position of which is determined by the invariant mass of two pairs of particles in this event. Obviously, the area populated by decay events is confined according to the energy and momentum conservation laws.

Analyzing the population of the Dalitz plot by events, one can establish the presence of intermediate states during the decay of the parent particle to the final state under study, derive their quantum numbers, and quantify the probabilities of individual decay channels (that is, measure the decay matrix elements). It is important to emphasize that all variations in the event density distribution are due to the dynamics, which defines the dependence of the decay matrix elements on the momenta (or energies) of the decay particles, and not by their particular kinematics. This is one of the most important properties of the Dalitz plot.

However, the most complex technique is **the partial-wave analysis, or PWA**. This technique is based on the idea that the interaction of particles is modeled by a coherent sum of resonances. The resulting set of partial waves is fitted to the experimental data, and the energy and angular information are used simultaneously in the fit. As a result of the fit, the individual contributions of partial waves to the final state are determined. This method allows one to deal with broad and overlapping resonances that cannot be separated, for example, in the invariant mass spectrum. An important advantage of partial wave analysis is the proper accounting for the interference between different states with the same quantum numbers. Nevertheless, the PWA is not at all a simple and easy method. Its great technical complexity, heavy computation, and the need for strong theoretical support are the main obstacles to starting to use it. In many cases, you have to work with sets of dozens of waves, which leads to the need to adjust up to hundreds of free parameters in the fit. Therefore, as in any multi-parameter fit, in PWA one has to overcome the instability of the fit, to solve the problem of finding a global minimum, and also deal with the ambiguity due to the inevitable presence of multiple solutions. Interpretation of PWA results is often hampered by rescattering effects, although some methods like using the K-matrix formalism and simultaneous analysis of several complementary reactions may, of course, help. Lastly, experimental conditions, such as background processes or detector resolution, often present a serious problem.

Finally, note that even simple methods should be used with care. As an illustration, we can consider the observation of the $\tilde{X}(3872)$ state in the COMPASS experiment [2]. The experiment studied the hadron production by a virtual photon in the reaction $\mu^+ N \rightarrow \mu^+ (J/\psi \pi^+ \pi^-) \pi^\pm N'$, using COMPASS data collected with incoming muons of 160 GeV/c and 200 GeV/c momentum. In the invariant mass spectrum of the system $(J/\psi \pi^+ \pi^-)$, a peak with a mass of 3860.0 ± 10.4 MeV/c² and width of less than 51 MeV/c² was found near the $\psi(3686)$ resonance with a statistical significance of 4.1 σ . While the mass and width were compatible with the $X(3872)$ state observed in other experiments before, this turned out to be not enough to draw a conclusion that it was the same particle. The reason was the fact that the shape of the $\pi^+ \pi^-$ mass distribution from the observed decay into $J/\psi \pi^+ \pi^-$ had shown disagreement with previous observations for $X(3872)$. The observed state was designated as $\tilde{X}(3872)$ and interpreted as a possible evidence of a new charmonium state. Its nature is still not clear.

3 Conventional hadron spectra

The first successful attempt to describe the spectrum of hadrons using a simple model was made by M. Gell-Mann [3] and independently by G. Zweig [4] in 1964. The basis of the model is the assumption about the constituent parts of hadrons, which Gell-Mann called *quarks*. It turned out that three varieties (flavors) of quarks – u (up), d (down) and s (strange) – were enough to describe all hadrons known at that time as their various combinations and even to predict a few yet unknown. Quarks have a fractional electric charge. In the quark model, all baryons are combinations of three quarks, and all mesons are combinations of a quark and an antiquark. The quark model explained the whole variety of particles. From the three constituents you can add up not so many combinations. However, the same combination of quarks can form particles of different masses. These are ‘excited states’, where quarks move around each other in a higher orbit. Over time, the number of quarks that make up hadrons increased to five: c (charm) and b (beauty, or bottom) quarks were discovered, along with entire families of charmed and beautiful particles. In the beginning, the quark model was just a convenient way to classify particles. However, after the experimental observation of partons inside the nucleon, and especially after the successful construction of the fundamental theory of the strong interaction – quantum chromodynamics, it is difficult to doubt the reality of quarks, despite their astonishing properties, such as confinement and fractional electric charge.

The mesons are classified in J^{PC} multiplets. The states with orbital angular momentum equal to zero are the pseudoscalars (0^{-+}) and the vectors (1^{--}), depending on the quark spins. The orbital excitations with orbital angular momentum equal to 1 are the scalars (0^{++}), the axial vectors (1^{++} and 1^{+-}), and the tensors (2^{++}). The mesons in the multiplet have different quark compositions and are close in mass and in their properties. Thus, light mesons form multiplets of nine states each (nonets):

- $J^{PC} = 0^{-+} : (\pi, K, \eta, \eta')$
- $J^{PC} = 1^{--} : (\rho, K^*, \omega, \phi)$
- $J^{PC} = 1^{+-} : (b_1, K_1, h_1, h'_1)$

and so on.

Introducing the heavier charm and beauty quarks makes this scheme more involved. For instance, including the charm quark transforms the meson nonet into a 16-plet, which includes both mesons with open charm (D, D_s) and charmonium states ($c\bar{c}$), where the charm quantum number is hidden. Taking into account the b quark makes this picture even more complicated, though keeping the general principle untouched.

Mesons consisting of two heavy quarks – charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) – deserve special attention. The large mass of quarks makes it possible to use various potential models or effective theories, like HQET or NRQCD, to calculate the spectrum of such a system. It turns out that these calculations are in excellent agreement with experiment, at least up to the production threshold of mesons with an open charm or beauty. This makes it possible to rely on a good understanding of the excited states of quarkonium in order to use them, for example, to verify calculations using lattice QCD, or to search for unconventional hadrons.

The mechanism for baryons is the same as for the mesons. Of course, for baryons, a much larger

number of combinations is obtained. In the ground state, only for light quarks, multiplets of 8 baryons (octet) with $J^P = 1/2^+$ and 10 baryons (decuplet) with $J^P = 3/2^+$ are formed. With the addition of heavy b or c quarks, the number of combinations increases even more. All baryons of the ground state multiplets are known. Many of their properties, in particular their masses, are in good agreement even with the most basic versions of the quark model. The picture for excited states is much less clear.

The conventional hadron spectra, derived from the quark model, make it possible to describe the properties of hundreds of hadrons and do not contradict the available experimental facts. However, today hadronic spectroscopy is as far as possible from drawing a conclusion about a complete understanding of the structure of hadrons and from limiting itself to a compilation of reference books with higher and higher precision. Fortunately, the existing mysteries in the spectrum of long-known particles and the discoveries made in recent decades do not give grounds for such conclusions and turn hadron spectroscopy into a rapidly developing and fascinating field of research.

4 Exotic hadrons

The assumption that hadrons can be formed not only by combinations of two or three quarks but also by a larger number of them, was made by Gell-Mann in the very first work devoted to the quark model. Subsequently, in the framework of QCD, it was shown that the existence of hadrons including one or more valence gluons is possible. Such particles are called *hybrids*, and the ones consisting only of gluons are called *glueballs*. Naturally, such particles, called exotic ones, have been sought and tried to be discovered experimentally for more than 30 years. The main problem faced in this search by experimentalists concerns not so much the discovery itself of new particles, as the proof of their ‘exotic’ nature.

Indeed, how can we identify exotic hadrons? Unfortunately, there are not many distinct signatures. Essentially, there are only three main ways to go. First, we can search for particles with quantum numbers that are not compatible with the assumption of two- or three-quark composition. For instance, for mesons it could be $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}$, etc. which cannot be constructed having only a quark and an antiquark.

Secondly, we can look for superfluous states. That is, those particles whose existence does not match theoretical predictions, or several particles found instead of one predicted. In the latter case, it will be necessary to find out which of the extra states is a conventional hadron and which is an exotic one. Unfortunately, we are on shaky ground here, as theoretical predictions are still not accurate enough, especially for excited states.

Finally, one can look for unexpected decay patterns. For example, in a decay $X^+ \rightarrow J/\psi\pi^+$, we need charm quark and antiquark to form charmonium, as well as at least two more light quarks to get the unit electric charge of a pion. That is, such decay can be interpreted as the decay of a multiquark state of four or more quarks.

The lack of experimental findings over the decades has been exacerbated several times by reports of false observations. The most striking case was the report of the observation of a narrow pentaquark state $\Theta^+(1530)$ in 2002. The first indication of a pentaquark in the nK^+ system in the photoproduction on nuclei received more than ten independent confirmations within two years before it was proved that

this was an erroneous result [6]. All this, together with the undoubted successes of QCD in explaining the spectrum of conventional hadrons, gave every reason to doubt the existence of anything other than generally accepted mesons and baryons.

4.1 Light hadrons: hunt for glueballs, search for diquarks

The relatively poor theoretical understanding of hadrons at low energies is not particularly surprising, since it is known that QCD becomes nonperturbative at low energies and does not allow one to calculate the spectrum of light hadrons. The progress of lattice QCD calculations is impressive and encouraging, but the results have not yet reached sufficient precision and reliability either. Combined with the experimental difficulties of studying broad mixing resonances, this explains why there are still quite a few unsolved problems in the spectrum of light hadrons. Let's consider some of them.

Among the light hadrons, the lightest $f_0(500)$ meson, also known as the σ meson, attracts special attention. The existence and properties of this meson have been controversial for almost six decades. The existence of this meson was suggested in 1955, even before the quark model was formulated, to explain short-range nucleon-nucleon interactions by two-pion exchange. Experimental difficulties of studying the light and broad state in a $\pi\pi$ system, together with the model-dependent interpretation, resulted in a highly uncertain mass range, from 400 to 1200 MeV, and a similarly large range, from 500 to 1000 MeV, for the width, for a long time. General agreement on the σ meson properties was reached only in the mid-2000s after extensive theoretical and experimental efforts. However, the composition of this resonance in terms of quarks and gluons is still a puzzle. It is well established that it cannot be interpreted as predominantly made of a quark and an antiquark. Whether it is a glueball or a tetraquark, there is a lot of controversies, but no firm conclusion is made yet. More details can be found in an excellent review paper, see Ref. [7]. Similar difficulties, theoretical and experimental, exist when considering the light strange meson, $K^*(700)$ or κ , showing up as an S-wave in πK scattering. However, unlike the $f_0(500)$, they have not yet been finally resolved [8].

Among the light mesons, the search for possible glueballs and other exotic hadrons has been going on for a long time. In particular, among scalar mesons with masses less than 1 GeV, the mesons $a_0(980)$ and $f_0(980)$ are known, which have almost identical masses, just below the $K\bar{K}$ production threshold, but different isospin. These two particles have long been considered as candidates for exotic hadrons – tetraquarks, meson molecules, etc. or a mixture of exotic and ordinary mesons. In the spectrum of light hadrons above 1 GeV, the same can be said about the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ mesons, at least one of which looks as a surplus state and possibly a glueball. This corresponds, among other things, to predictions by lattice QCD calculations that the mass of the lightest scalar glueball lays the range 1500 to 1800 MeV. Just below the $p\bar{p}$ production threshold, the BES experiment observed the $X(1835)$ state [9], later confirmed by BESIII in several different reactions, which is also rather difficult to interpret within the framework of the classical quark model. These are just some examples of the existing difficulties in interpreting the spectrum of light mesons.

As regards the light baryons, there is an opposite problem. There is a great inconsistency between theoretical predictions and the experimentally observed spectrum of baryons [10–12]. The number of experimentally discovered baryons is much smaller than those predicted. Of course, this can be ex-

plained by the lack of experimental data and the difficulty of their interpretation. Indeed, the number of established excited states of light baryons increases with time. An alternative explanation for the problem of missing baryons is that our ideas about the structure of baryons need to be refined. There is an assumption that two valence quarks can combine into a colored correlation, or diquark, which leads to a decrease in possible combinations in the construction of excited baryons. The experimental verification of this assumption is complicated, since diquarks are not a color singlet and, like quarks, cannot exist as free particles.

4.2 Heavy hadrons: multiquarks and other exotics

The revolution in the study of exotic hadrons began rather quietly, with the discovery of the $X(3872)$ state. This state was found in the BELLE experiment and appeared as a clear peak in the spectrum of invariant masses of the system $(\pi^+\pi^-J/\psi)$ during the study of the decay $B^+ \rightarrow K^+\pi^+\pi^-J/\psi$. The mass of the new state was $M = 3872.0 \pm 0.6(\text{stat}) \pm 0.5(\text{syst})$ MeV and the width was narrow, less than 2.3 MeV at 90% C.L. This state did not fit into the conventional charmonium spectrum and became a vivid example of an extra, unexpected particle. Of course, a great many assumptions have been made about the nature of the $X(3872)$ since then, but none of them was confirmed or rejected. After 20 years, we know more. The quantum numbers of the $X(3872)$ have been measured: $J^{PC} = 1^{++}$ [14]. The mass is suspiciously (within 1 MeV) close to the mass of a D^*D pair, but the comparable decay rate to D^*D and $\gamma\psi(3686)$ tells us that the $X(3872)$ is unlikely to be a D^*D bound state. The decay rates of $X(3872)$ to $\omega J/\psi$ and $\rho J/\psi$ are approximately the same, which suggests a large violation of isospin. A charged partner of the $X(3872)$ is not found. So, despite all these findings, we still do not know the nature of this mysterious state.

After the $X(3872)$, the epoch of charged charmonium-like and bottomonium-like states began. These are resonances decaying into charmonium or bottomonium and a charged meson, which hints at their multiquark nature. It was the BELLE experiment at the B-factory that made the first observations of states like these. Charged charmonium-like states, $Z_c^\pm(4430)$, $Z_c^\pm(4050)$, $Z_c^\pm(4250)$, and bottomonium-like states, $Z_b^\pm(10610)$ and $Z_b^\pm(10650)$, have been reported [15–17]. However, this evidence was not confirmed by the BaBar experiment in similar conditions and remained contradictory. The first charged charmonium-like state beyond doubt was the $Z_c^\pm(3900)$ in the BESIII experiment, discovered in 2013 and quickly confirmed by the BELLE and CLEO-c experiments [18]. Since then, the existence of a large number of exotic charmonium-like and bottomonium-like states has been firmly established, mainly in the BELLE, BESIII, and LHCb experiments [19]. Since the new states did not fit into the traditional charmonium and bottomonium spectra, they were named X , Y , or Z , depending on their electrical charges and quantum numbers, and they were collectively referred to as the XYZ states.

Pentaquarks have been found in a similar way, in decays to charmonium and baryons. In 2015, at the LHCb experiment, the $P_c^+(4450)$ and $P_c^+(4380)$ pentaquarks were discovered in $\Lambda_b^0 \rightarrow J/\psi p K^-$ decays in the invariant mass spectrum of the $(J/\psi p)$ system [20]. Later, also at the LHCb experiment, when analyzing samples of increased statistics, another pentaquark, $P_c^+(4312)$, was found, and the pentaquark $P_c^+(4450)$ was reinterpreted as two separate states, $P_c^+(4440)$ and $P_c^+(4457)$ [21]. In addition to pentaquarks, more than 10 candidates for other exotic states, including two possible tetraquarks, and about 50 excited states of ordinary mesons and baryons were discovered at the LHC [22].

Thus, the existence of a large number of hadrons that are not mesons nor baryons is now firmly established. They are not ‘exotic’ any more. No doubt that the number of such hadrons will increase in the coming years, both among heavy and light hadrons. Discussions whether these particles are multiquarks, diquark molecules, hybrids or some more complicated structures are still ongoing and it will probably take a long time before the nature of the new states becomes clear. However, we can say right now that these discoveries once again confirmed the validity of the quark model and provided rich material for refining our understanding of hadronic spectra. Ultimately, the discovery of new hadronic states does not simply aim at adding the properties of more particle to the set of particles known to us. The main result is the verification and refinement of the theory that describes the properties of these particles, based on fundamental concepts of nuclear matter, and a deeper understanding of the laws of strong interaction.

5 Where do we stand?

Currently, the data accumulated in the B factory experiments (Belle, BaBar), the τ -charm experiment (BESIII), and the LHC experiments (LHCb, ATLAS, CMS) are the main source of knowledge about new hadrons. In 2017, the GlueX experiment began, aimed at high-sensitivity searches for pentaquarks and other exotic mesons in photoproduction processes. The Belle II experiment started data taking in 2019 after a major upgrade. There is no doubt that it is the Belle II, BESIII, GlueX, LHCb and other experiments at the LHC that will determine the future of hadron spectroscopy in the next 10–15 years. In addition, some new experiments are expected to join exotic hadron research efforts in the near future: the PANDA experiment in Darmstadt, Germany, the AMBER experiment at CERN, the τ -charm Super Factories STCF in Hefei, China, and the SCTF in Sarov, Russia. Later, experiments at the planned electron-ion collider EIC might also contribute a lot to our understanding of baryon structure and hadron spectra.

As a final note, we can add that, throughout the history of particle physics, hadron spectroscopy has repeatedly led to significant changes in our understanding of the structure of matter. It remains a unique tool for gaining knowledge about the intrinsic properties and composition of hadrons today. There is no doubt that the discovery of a large number of exotic, supposedly multiquark states is one of the most exciting events in hadron physics recently, perhaps comparable to the discovery of the J/ψ meson. To understand the nature of exotic states, and to solve the problem of extra mesons and missing baryons, both more accurate and high-quality experimental data and new theoretical insights are needed. Special hopes are placed here on lattice QCD, which is making impressive progress; but there is still a long way to go before constructing a theory that fully describes hadronic spectra.

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