Flavour physics

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We explain the reasons for the interest in flavor physics. We describe flavor physics and the related CP violation within the Standard Model, with emphasis on the predictions of the model related to features such as flavor universality and flavor diagonality. We describe the flavor structure of flavor changing charged current interactions, and how they are used to extract the CKM parameters. We describe the structure of flavor changing neutral current interactions, and explain why they are highly suppressed in the Standard Model. We explain how the B-factories proved that the CKM (KM) mechanism dominates the flavor changing (CP violating) processes that have been observed in meson decays. We explain the implications of flavor physics for new physics, with emphasis on the "new physics flavor puzzle", and present the idea of minimal flavor violation as a possible solution. We explain the "Standard Model flavor puzzle", and present the Froggatt–Nielsen mechanism as a possible solution. We show that measurements of the Higgs boson decays may provide new opportunities for making progress on the various flavor puzzles. We briefly discuss two sets of measurements and some of their possible theoretical implications: $R(K^{(*)})$ and $R(D^{(*)})$.

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1 Introduction

1.1 What is flavor?

The term "**flavors**" is used, in the jargon of particle physics, to describe several mass eigenstates of the same gauge representation, namely several fields that are assigned the same quantum charges under the unbroken symmetries. Within the Standard Model (SM), when thinking of its unbroken $SU(3)_{\rm C} \times U(1)_{\rm EM}$ gauge group, there are four different types of fermions, each coming in three flavors:

– Up-type quarks in the $(3)_{+2/3}$ representation: u, c, t;
– Down-type quarks in the $(3)_{-1/3}$ representation: d, s, b;
– Charged leptons in the $(1)_{-1}$ representation: e, μ, τ;
– Neutrinos in the $(1)_0$ representation: ν_1, ν_2, ν_3.

The term "**flavor physics**" refers to interactions that distinguish between flavors. By definition, gauge interactions, namely interactions that are related to unbroken symmetries and mediated therefore by massless gauge bosons, do not distinguish among the flavors and do not constitute part of flavor physics. Within the Standard Model, flavor-physics refers to the weak and Yukawa interactions.

The term "**flavor parameters**" refers to parameters that carry flavor indices. Within the Standard Model, these are the nine masses of the charged fermions and the four "mixing parameters" (three angles and one phase) that describe the interactions of the charged weak-force carriers ($W^\pm$) with quark–anti-quark pairs. If one augments the Standard Model with Majorana mass terms for the neutrinos, one should add to the list three neutrino masses and six mixing parameters (three angles and three phases) for the $W^\pm$ interactions with lepton–anti-lepton pairs.

The term "**flavor universal**" refers to interactions with couplings (or to parameters) that are proportional to the unit matrix in flavor space. Thus, the strong and electromagnetic interactions are flavor-universal. An alternative term for "flavor-universal" is "**flavor-blind**".

The term "**flavor diagonal**" refers to interactions with couplings (or to parameters) that are diagonal, but not necessarily universal, in the flavor space. Within the Standard Model, the Yukawa interactions of the Higgs boson are flavor diagonal.

The term "**flavor changing**" refers to processes where the initial and final flavor-numbers (that is, the number of particles of a certain flavor minus the number of anti-particles of the same flavor) are different. In "**flavor changing charged current**" (FCCC) processes, both up-type and down-type flavors, and/or both charged lepton and neutrino flavors are involved. Examples are (i) $\mu \to e\bar{\nu}_e\nu_\mu$, (ii) $K^- \to \mu^-\bar{\nu}_\mu$ (which corresponds, at the quark level, to $s\bar{u} \to \mu^-\bar{\nu}_\mu$), and (iii) $B \to \psi K$ ($b \to c\bar{c}s$). Within the Standard Model, these processes are mediated by the W-bosons and occur at tree level. In "**flavor changing neutral current**" (FCNC) processes, either up-type or down-type flavors but not both, and/or either charged lepton or neutrino flavors but not both, are involved. Example are (i) $\mu \to e\gamma$, (ii) $K_L \to \mu^+\mu^-$ (which corresponds, at the quark level, to $s\bar{d} \to \mu^+\mu^-$), and (iii) $B \to \phi K$ ($b \to s\bar{s}s$). Within the Standard Model, these processes do not occur at tree level, and are often highly suppressed.

Another useful term is "**flavor violation**". We will explain it later in these lectures.

1.2 Why is flavor physics interesting?

Flavor physics is interesting, on one hand, as a tool for discovery and, on the other hand, because of intrinsic puzzling features:

- Flavor physics can discover new physics or probe it before it is directly observed in experiments. Here are some examples from the past:
  - The smallness of $\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)}$ led to predicting a fourth (the charm) quark;
  - The size of Δm_K led to a successful prediction of the charm mass;
  - The size of Δm_B led to a successful prediction of the top mass;
  - The measurement of ε_K led to predicting the third generation;
  - The measurement of neutrino flavor transitions led to the discovery of neutrino masses.
- CP violation is closely related to flavor physics. Within the Standard Model, there is a single CP violating parameter, the Kobayashi–Maskawa phase $\delta_{\rm KM}$. Baryogenesis tells us, however, that there must exist new sources of CP violation. Measurements of CP violation in flavor changing processes might provide evidence for such sources.
- The fine-tuning problem of the Higgs mass, and the puzzle of the dark matter suggest that there may exist new physics at, or below, the TeV scale. If such new physics had a generic flavor structure, it would contribute to flavor changing neutral current (FCNC) processes orders of magnitude above the observed rates. The question of why this does not happen constitutes the *new physics flavor puzzle*.
- Most of the charged fermion flavor parameters are small and hierarchical. The Standard Model does not provide any explanation of these features. This is the *Standard Model flavor puzzle*. The puzzle became even deeper after neutrino masses and lepton mixing were measured because, so far, neither smallness nor hierarchy in these parameters have been established.

2 The Standard Model

A model of elementary particles and their interactions is defined by the following ingredients: (i) The symmetries of the Lagrangian and the pattern of spontaneous symmetry breaking (SSB); (ii) The representations of fermions and scalars. The Standard Model (SM) is defined as follows:

- The symmetry is a local
$$G_{\rm SM}=SU(3)_C\times SU(2)_L\times U(1)_Y. \tag{2.1}$$
- It is spontaneously broken,
$$G_{\rm SM}\to SU(3)_C\times U(1)_{\rm EM}\quad (Q_{\rm EM}=T_3+Y), \tag{2.2}$$
  by the VEV of a single scalar field,
$$\phi(1,2)_{+1/2},\quad (\langle\phi^0\rangle=v/\sqrt{2}). \tag{2.3}$$

– There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q_{Li}(3,2)_{+1/6},\;\; U_{Ri}(3,1)_{+2/3},\;\; D_{Ri}(3,1)_{-1/3},\;\; L_{Li}(1,2)_{-1/2},\;\; E_{Ri}(1,1)_{-1}. \tag{2.4}$$

The SM scalar field is called the Higgs field. The $SU(3)_C$-triplet fermions fields are called quark fields, and the $SU(3)_C$-singlet fermions fields are called lepton fields.

2.1 The Lagrangian

The most general renormalizable Lagrangian with scalar and fermion fields can be decomposed into

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_\psi + \mathcal{L}_{\rm Yuk} + \mathcal{L}_\phi. \tag{2.5}$$

Here $\mathcal{L}_{\rm kin}$ describes free propagation in spacetime, as well as gauge interactions, $\mathcal{L}_\psi$ gives fermion mass terms, $\mathcal{L}_{\rm Yuk}$ describes the Yukawa interactions, and $\mathcal{L}_\phi$ gives the scalar potential. We now find the specific form of the Lagrangian made of the fermion fields Q_{Li}, U_{Ri}, D_{Ri}, L_{Li} and E_{Ri} (2.4), and the scalar field (2.3), subject to the gauge symmetry (2.1) and leading to the SSB of Eq. (2.2).

2.1.1 $\mathcal{L}_{\rm kin}$

The local symmetry requires the following gauge boson degrees of freedom:

$$G_a^\mu(8,1)_0, \quad W_a^\mu(1,3)_0, \quad B^\mu(1,1)_0. \tag{2.6}$$

The corresponding field strengths are given by

$$\begin{aligned}
G_a^{\mu\nu} &= \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu, \\
W_a^{\mu\nu} &= \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu, \\
B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu.
\end{aligned} \tag{2.7}$$

The covariant derivative is

$$D^\mu = \partial^\mu + i g_s G_a^\mu L_a + i g W_b^\mu T_b + i g' B^\mu Y, \tag{2.8}$$

where the L_a's are $SU(3)_C$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_L$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the Y's are the $U(1)_Y$ charges. Explicitly, the covariant derivatives acting on the various scalar and fermion fields are given by

$$\begin{aligned}
D^\mu \phi &= \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{2} g' B^\mu\right)\phi, \\
D^\mu Q_{Li} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{6} g' B^\mu\right) Q_{Li}, \\
D^\mu U_{Ri} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{2i}{3} g' B^\mu\right) U_{Ri},
\end{aligned}$$

$$\begin{aligned} D^\mu D_{Ri} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a - \frac{i}{3} g' B^\mu\right) D_{Ri}, \\ D^\mu L_{Li} &= \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b - \frac{i}{2} g' B^\mu\right) L_{Li}, \\ D^\mu E_{Ri} &= \left(\partial^\mu - i g' B^\mu\right) E_{Ri}. \end{aligned} \tag{2.9}$$

$\mathcal{L}_{\rm kin}$ is given by

$$\begin{aligned} \mathcal{L}_{\rm kin}^{\rm SM} = & -\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} - \frac{1}{4} W_b^{\mu\nu} W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \\ & + i\overline{Q_{Li}} \not{D} Q_{Li} + i\overline{U_{Ri}} \not{D} U_{Ri} + i\overline{D_{Ri}} \not{D} D_{Ri} + i\overline{L_{Li}} \not{D} L_{Li} + i\overline{E_{Ri}} \not{D} E_{Ri} \\ & + (D^\mu \phi)^\dagger (D_\mu \phi). \end{aligned} \tag{2.10}$$

This part of the interaction Lagrangian is generation-universal. In addition, it conserves CP.

2.1.2 $\mathcal{L}_\psi$

There are no mass terms for the fermions in the SM. We cannot write Dirac mass terms for the fermions because they are assigned to chiral representations of the gauge symmetry. We cannot write Majorana mass terms for the fermions because they all have $Y \neq 0$. Thus,

$$\mathcal{L}_\psi^{\rm SM} = 0. \tag{2.11}$$

2.1.3 $\mathcal{L}_{\rm Yuk}$

The Yukawa part of the Lagrangian is given by

$$-\mathcal{L}_{\rm Y}^{\rm SM} = Y_{ij}^d \overline{Q_{Li}} \phi D_{Rj} + Y_{ij}^u \overline{Q_{Li}} \tilde{\phi} U_{Rj} + Y_{ij}^e \overline{L_{Li}} \phi E_{Rj} + {\rm h.c.}, \tag{2.12}$$

where $\tilde{\phi}_a = \epsilon_{ab} \phi_b^*$ (a, b are the $SU(2)$-indices). The Y^f's are general complex 3×3 matrices of dimensionless couplings. This part of the Lagrangian is, in general, generation-dependent (that is, $Y^f \not\propto \mathbf{1}$) and CP violating.

We now present three special interaction bases. Without loss of generality, we can use a bi-unitary transformation,

$$Y^e \to \hat{Y}_e = U_{eL} Y^e U_{eR}^\dagger, \tag{2.13}$$

to change the basis to one where Y^e is diagonal and real:

$$\hat{Y}^e = {\rm diag}(y_e, y_\mu, y_\tau). \tag{2.14}$$

In the basis defined in Eq. (2.14), we denote the components of the lepton $SU(2)$-doublets, and the three lepton $SU(2)$-singlets, as follows:

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}; \quad e_R, \quad \mu_R, \quad \tau_R, \tag{2.15}$$

where e, μ, τ are ordered by the size of $y_{e,\mu,\tau}$ (from smallest to largest).

Similarly, without loss of generality, we can use a bi-unitary transformation,

$$Y^u \to \hat{Y}^u = V_{uL} Y^u V_{uR}^\dagger, \tag{2.16}$$

to change the basis to one where $\hat{Y}^u$ is diagonal and real:

$$\hat{Y}^u = \mathrm{diag}(y_u, y_c, y_t). \tag{2.17}$$

In the basis defined in Eq. (2.17), we denote the components of the quark $SU(2)$-doublets, and the quark up $SU(2)$-singlets, as follows:

$$\begin{pmatrix} u_L \\ d_{uL} \end{pmatrix}, \quad \begin{pmatrix} c_L \\ d_{cL} \end{pmatrix}, \quad \begin{pmatrix} t_L \\ d_{tL} \end{pmatrix}; \quad u_R, \quad c_R, \quad t_R, \tag{2.18}$$

where u, c, t are ordered by the size of $y_{u,c,t}$ (from smallest to largest).

We can use yet another bi-unitary transformation,

$$Y^d \to \hat{Y}^d = V_{dL} Y^d V_{dR}^\dagger, \tag{2.19}$$

to change the basis to one where $\hat{Y}^d$ is diagonal and real:

$$\hat{Y}^d = \mathrm{diag}(y_d, y_s, y_b). \tag{2.20}$$

In the basis defined in Eq. (2.20), we denote the components of the quark $SU(2)$-doublets, and the quark down $SU(2)$-singlets, as follows:

$$\begin{pmatrix} u_{dL} \\ d_L \end{pmatrix}, \quad \begin{pmatrix} u_{sL} \\ s_L \end{pmatrix}, \quad \begin{pmatrix} u_{bL} \\ b_L \end{pmatrix}; \quad d_R, \quad s_R, \quad b_R, \tag{2.21}$$

where d, s, b are ordered by the size of $y_{d,s,b}$ (from smallest to largest).

2.1.4 $\mathcal{L}_\phi$

The scalar potential is given by

$$\mathcal{L}_\phi^{\mathrm{SM}} = -\mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2. \tag{2.22}$$

Choosing $\mu^2 < 0$ and $\lambda > 0$ leads to the required spontaneous symmetry breaking. This part of the Lagrangian is also CP conserving.

2.2 The spectrum

The fermion masses arise from the Yukawa couplings as a result of the spontaneous symmetry breaking. The mass matrices are given by

$$M_f = (v/\sqrt{2}) Y^f \quad (f = e, u, d). \tag{2.23}$$

Table 1: The SM particles.

particle	spin	color	$Q_{\rm EM}$	mass $[v]$
$W^\pm$	1	(1)	±1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
A^0	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$
e,μ,τ	1/2	(1)	-1	$y_{e,\mu,\tau}/\sqrt{2}$
ν_e,ν_μ,ν_τ	1/2	(1)	0	0
u,c,t	1/2	(3)	$+2/3$	$y_{u,c,t}/\sqrt{2}$
d,s,b	1/2	(3)	$-1/3$	$y_{d,s,b}/\sqrt{2}$

It is clear then that the bases of diagonal Yukawa matrices—the $\hat{Y}^e$ basis of Eq. (2.14), the $\hat{Y}^u$ basis of Eq. (2.17), and the $\hat{Y}^d$ basis of Eq. (2.20)—are mass bases for, respectively, the charged leptons, the up quarks and the down quarks, with $m_f = (v/\sqrt{2})y_f$. The spectrum of the Standard Model is presented in Table 1.

All masses are proportional to the VEV of the scalar field, v. For the three massive gauge bosons, and for the fermions, this is expected: In the absence of spontaneous symmetry breaking, the former would be protected by the gauge symmetry and the latter by their chiral nature. For the Higgs boson, the situation is different, as a mass-squared term does not violate any symmetry.

For the charged fermions, the spontaneous symmetry breaking allows their masses because they are in vector-like representations of the $SU(3)_C \times U(1)_{\rm EM}$ group: The LH and RH charged lepton fields, e, μ and τ, are in the $(1)_{-1}$ representation; The LH and RH up-type quark fields, u, c and t, are in the $(3)_{+2/3}$ representation; The LH and RH down-type quark fields, d, s and b, are in the $(3)_{-1/3}$ representation. On the other hand, the neutrinos remain massless in spite of the fact that they are in the $(1)_0$ representation of $SU(3)_C \times U(1)_{\rm EM}$, which allows for Majorana masses. Such masses require a VEV carried by a scalar field in the $(1,3)_{+1}$ representation of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry, but there is no such field in the SM.

The experimental values of the charged fermion masses are [1][1]

$$\begin{aligned}
&m_e = 0.510998946(3)\ \text{MeV},\ \ m_\mu = 105.6583745(24)\ \text{MeV},\ \ m_\tau = 1776.86(12)\ \text{MeV},\\
&m_u = 2.2^{+0.5}_{-0.3}\ \text{MeV},\ \ m_c = 1.27\pm0.02\ \text{GeV},\ \ m_t = 172.9\pm0.4\ \text{GeV},\\
&m_d = 4.7^{+0.5}_{-0.2}\ \text{MeV},\ \ m_s = 93^{+11}_{-5}\ \text{MeV},\ \ m_b = 4.18^{+0.03}_{-0.02}\ \text{GeV}.
\end{aligned} \tag{2.24}$$

2.2.1 The CKM matrix

In the derivation above, there is an important difference between the analysis of the quark spectrum and the analysis of the lepton spectrum. For the leptons, there exists a basis that is simultaneously an

[1]See Ref. [1] for detailed explanations of the quoted quark masses. For $q = u,d,s,c,b$, m_q are the running quark masses in the $\overline{\rm MS}$ scheme, with $m_{u,d,s} = m_{u,d,s}(\mu = 2\ \text{GeV})$ and $m_{c,b} = m_{c,b}(\mu = m_{c,b})$.

Table 2: The SM fermion interactions.

interaction	fermions	force carrier	coupling	flavor
Electromagnetic	u, d, ℓ	A^0	eQ	universal
Strong	u, d	g	g_s	universal
NC weak	u, d, e, ν	Z^0	$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$	universal
CC weak (q)	$\bar{u}d$	$W^\pm$	gV	non-universal
CC weak (l)	$\bar{\ell}\nu$	$W^\pm$	g	universal
Yukawa	u, d, ℓ	h	y_q	diagonal

interaction basis and a mass basis for both the charged leptons and the neutrinos, that is the $\hat{Y}_e$ basis. In contrast, for the quarks, in general there is no interaction basis that is also a mass basis for both up-type and down-type quarks. To see that, we denote $u^i = (u, c, t)$ and $d^i = (d, s, b)$, and write the relation of these mass eigenstates to the interaction eigenstates:

$$u_L^i = (V_{uL})_{ij} U_L^j, \qquad u_R^i = (V_{uR})_{ij} U_R^j, \qquad d_L^i = (V_{dL})_{ij} D_L^j, \qquad d_R^i = (V_{dR})_{ij} D_R^j. \tag{2.25}$$

If $V_{uL} \neq V_{dL}$, as is the general case, then the interaction basis defined by Eq. (2.17) is different from the interaction basis defined by Eq. (2.20). In the former, Y^d can be written as a unitary matrix times a diagonal one,

$$Y^u = \hat{Y}^u, \qquad Y^d = V\hat{Y}^d. \tag{2.26}$$

In the latter, Y^u can be written as a unitary matrix times a diagonal one,

$$Y^d = \hat{Y}^d, \qquad Y^u = V^\dagger \hat{Y}^u. \tag{2.27}$$

In either case, the unitary matrix V is given by

$$V = V_{uL} V_{dL}^\dagger, \tag{2.28}$$

where V_{uL} and V_{dL} are defined in Eqs. (2.16) and (2.19), respectively. Note that each of V_{uL}, V_{uR}, V_{dL} and V_{dR} depends on the basis from which we start the diagonalization. The combination $V = V_{uL} V_{dL}^\dagger$, however, does not. This is a hint that V is physical. The matrix V is called the Cabibbo–Kobayashi–Maskawa (CKM) matrix [2, 3]. Its physical significance becomes clear in Section 2.3.3.

2.3 The interactions

Within the SM, the fermions have five types of interactions. These interactions are summarized in Table 2. In the next few subsections, we explain the entries of this table.

2.3.1 EM and strong interactions

By construction, a local $SU(3)_C \times U(1)_{\rm EM}$ symmetry survives the SSB. The SM has thus the photon and gluon massless gauge fields. All charged fermions interact with the photon:

$$\mathcal{L}_{{\rm QED},\psi} = -\frac{2e}{3}\overline{u_i}\not{A}u_i + \frac{e}{3}\overline{d_i}\not{A}d_i + e\overline{\ell_i}\not{A}\ell_i, \tag{2.29}$$

where $u_{1,2,3} = u, c, t$, $d_{1,2,3} = d, s, b$ and $\ell_{1,2,3} = e, \mu, \tau$. We emphasize the following points:

1. The photon couplings are *vector-like*.
2. The EM interactions are *P, C and T conserving*.
3. *Diagonality:* The photon couples to e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$, but not to $e^\pm\mu^\mp$, $e^\pm\tau^\mp$ or $\mu^\pm\tau^\mp$ pairs, and similarly in the up and down sectors.
4. *Universality:* The couplings of the photon to different generations are universal.

All colored fermions (namely, quarks) interact with the gluon:

$$\mathcal{L}_{{\rm QCD},\psi} = -\frac{g_s}{2}\overline{q}\lambda_a \not{G}_a q, \tag{2.30}$$

where $q = u, c, t, d, s, b$. We emphasize the following points:

1. The gluon couplings are *vector-like*.
2. The strong interactions are *P, C and T conserving*.
3. *Diagonality:* The gluon couples to $\bar{t}t$, $\bar{c}c$, *etc.*, but not to $\bar{t}c$ or any other flavor changing pair.
4. *Universality:* The couplings of the gluon to different quark generations are universal.

The universality of the photon and gluon couplings is a result of the $SU(3)_C \times U(1)_{\rm EM}$ gauge invariance, and thus holds in any model, and not just within the SM.

2.3.2 Neutral current weak interactions

All SM fermions couple to the Z-boson:

$$\begin{aligned}\mathcal{L}_{Z,\psi} \;=\; & \frac{e}{s_W c_W}\left[-\left(\frac{1}{2} - s_W^2\right)\overline{e_{Li}}\not{Z}e_{Li} + s_W^2\,\overline{e_{Ri}}\not{Z}e_{Ri} + \frac{1}{2}\,\overline{\nu_{L\alpha}}\not{Z}\nu_{L\alpha}\right. \\ & \left. + \left(\frac{1}{2} - \frac{2}{3}s_W^2\right)\overline{u_{Li}}\not{Z}u_{Li} - \frac{2}{3}s_W^2\,\overline{u_{Ri}}\not{Z}u_{Ri} - \left(\frac{1}{2} - \frac{1}{3}s_W^2\right)\overline{d_{Li}}\not{Z}d_{Li} + \frac{1}{3}s_W^2\,\overline{d_{Ri}}\not{Z}d_{Ri}\right].\end{aligned} \tag{2.31}$$

where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$. We emphasize the following points:

1. The Z-boson couplings are *chiral* and *parity violating*.
2. *Diagonality:* The Z-boson couples diagonally. For example, in the lepton sector, the Z-boson couples to e^+e^- and to $\mu^+\mu^-$ but not to $e^\pm\mu^\mp$ pairs. The diagonality in the lepton sector holds to all orders in perturbation theory, due to an accidental $[U(1)]^3$ symmetry of the SM (see below).
3. *Universality:* The couplings of the Z-boson in each of the seven sectors $(\nu_L, \ell_L, \ell_R, d_L, d_R, u_L, u_R)$ are universal. This is a result of a special feature of the SM:

all fermions of given chirality, EM charge and $SU(3)_C$ representation come from the same $SU(2)_L \times U(1)_Y$ representation (see below).

As an example to experimental tests of diagonality and universality, we can take the leptonic sector. The branching ratios of the Z-boson into charged lepton pairs [1],

$$\begin{aligned} \mathrm{BR}(Z \to e^+e^-) &= (3.363 \pm 0.004)\%\,, \\ \mathrm{BR}(Z \to \mu^+\mu^-) &= (3.366 \pm 0.007)\%\,, \\ \mathrm{BR}(Z \to \tau^+\tau^-) &= (3.370 \pm 0.008)\%\,, \end{aligned} \tag{2.32}$$

beautifully confirms universality:

$$\begin{aligned} \Gamma(\mu^+\mu^-)/\Gamma(e^+e^-) &= 1.0001 \pm 0.0024, \\ \Gamma(\tau^+\tau^-)/\Gamma(e^+e^-) &= 1.002 \pm 0.003. \end{aligned}$$

Diagonality is also tested by the following experimental searches:

$$\begin{aligned} \mathrm{BR}(Z \to e^+\mu^-) &< 7.5 \times 10^{-7}, \\ \mathrm{BR}(Z \to e^+\tau^-) &< 5.0 \times 10^{-6}, \\ \mathrm{BR}(Z \to \mu^+\tau^-) &< 6.5 \times 10^{-6}. \end{aligned} \tag{2.33}$$

Thus, for example,

$$\begin{aligned} \Gamma(e^+\mu^-)/\Gamma(\ell^+\ell^-) &< 2.2 \times 10^{-5}, \\ \Gamma(\mu^+\tau^-)/\Gamma(\tau^+\tau^-) &< 1.9 \times 10^{-4}. \end{aligned} \tag{2.34}$$

2.3.3 Charged current weak interactions

We now study the couplings of the charged vector bosons, $W^\pm$, to fermion pairs. For the lepton mass eigenstates, things are simple, because there exists an interaction basis that is also a mass basis. Thus,

$$\mathcal{L}_{W,\ell} = -\frac{g}{\sqrt{2}} \left(\overline{\nu_{eL}}\, \not{W}^+ e_L^- + \overline{\nu_{\mu L}}\, \not{W}^+ \mu_L^- + \overline{\nu_{\tau L}}\, \not{W}^+ \tau_L^- + \text{h.c.} \right). \tag{2.35}$$

Eq. (2.35) reveals some important features of the model:

1. *Parity violation*: The W-boson couplings are chiral. More specifically, only left-handed particles take part in charged-current interactions. Consequently, parity is violated.
2. *Universality*: the couplings of the W-boson to $\tau\bar{\nu}_\tau$, to $\mu\bar{\nu}_\mu$ and to $e\bar{\nu}_e$ are equal. This is a result of the local nature of the imposed $SU(2)$: a global symmetry would have allowed an independent coupling to each lepton pair.

All of these predictions have been experimentally tested. As an example of how well universality works, consider the decay rates of the W-bosons to the three lepton pairs [1]:

$$\mathrm{BR}(W^+ \to e^+\nu_e) = (10.71 \pm 0.16) \times 10^{-2},$$

$$\begin{aligned}
\mathrm{BR}(W^+ \to \mu^+ \nu_\mu) &= (10.63 \pm 0.15) \times 10^{-2}, \\
\mathrm{BR}(W^+ \to \tau^+ \nu_\tau) &= (11.38 \pm 0.21) \times 10^{-2}.
\end{aligned} \tag{2.36}$$

These results confirm universality:

$$\begin{aligned}
\Gamma(\mu^+\nu)/\Gamma(e^+\nu) &= 0.996 \pm 0.008, \\
\Gamma(\tau^+\nu)/\Gamma(\mu^+\nu) &= 1.043 \pm 0.024.
\end{aligned} \tag{2.37}$$

As concerns quarks, things are more complicated, since there is no interaction basis that is also a mass basis. In the interaction basis, the W interactions have the following form:

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}} \overline{U^i_L}\, \not{W}^+ D^i_L + \mathrm{h.c.}. \tag{2.38}$$

Using Eq. (2.25) to write $U^i_L = (V^\dagger_{uL})_{ij} u^j_L$ and $D^i_L = (V^\dagger_{dL})_{ij} d^j_L$, we can rewrite $\mathcal{L}_{W,q}$ in terms of the mass eigenstates:

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}\, \overline{u^k_L} (V_{uL})_{ki}\, \not{W}^+ (V^\dagger_{dL})_{il} d^l_L + \mathrm{h.c.} = -\frac{g}{\sqrt{2}}\, \overline{u^k_L} V_{kl}\, \not{W}^+ d^l_L + \mathrm{h.c.}, \tag{2.39}$$

where V is the CKM matrix defined in Eq. (2.28).

Eq. (2.39) reveals some important features of the model:

1. Only left-handed particles take part in charged-current interactions. Consequently, parity is violated by these interactions.
2. The W couplings to the quark mass eigenstates are not universal. The universality of gauge interactions is hidden in the unitarity of the CKM matrix, V.
3. The W couplings are not diagonal. This is a manifestation of the fact that no pair of an up-type and a down-type mass eigenstates fits into an $SU(2)_L$ doublet. For example, the d and u mass eigenstates are not members of a single $SU(2)_L$ doublet.

The matrix V is called the CKM matrix [2,3]. The (hidden) universality within the quark sector is tested by the prediction

$$\Gamma(W \to uX) = \Gamma(W \to cX) = \frac{1}{2}\Gamma(W \to \mathrm{hadrons}). \tag{2.40}$$

Experimentally,

$$\Gamma(W \to cX)/\Gamma(W \to \mathrm{hadrons}) = 0.49 \pm 0.04. \tag{2.41}$$

2.3.4 Yukawa interactions

The Yukawa interactions are given by

$$\begin{aligned}
\mathcal{L}_{\mathrm{Yuk}} = &- \frac{h}{v} \big(m_e\, \overline{e_L}\, e_R + m_\mu\, \overline{\mu_L}\, \mu_R + m_\tau\, \overline{\tau_L}\, \tau_R \\
&+ m_u\, \overline{u_L}\, u_R + m_c\, \overline{c_L}\, c_R + m_t\, \overline{t_L}\, t_R + m_d\, \overline{d_L}\, d_R + m_s\, \overline{s_L}\, s_R + m_b\, \overline{b_L}\, b_R + \mathrm{h.c.} \big).
\end{aligned}$$

To see that the Higgs boson couples diagonally to the fermion mass eigenstates, let us take the example of the down quarks, and start from an arbitrary interaction basis:

$$\begin{aligned} h\overline{D_L}Y^d D_R &= h\overline{D_L}(V_{dL}^\dagger V_{dL})Y^d(V_{dR}^\dagger V_{dR})D_R \\ &= h(\overline{D_L}V_{dL}^\dagger)(V_{dL}Y^d V_{dR}^\dagger)(V_{dR}D_R) \\ &= h(\overline{d_L}\ \overline{s_L}\ \overline{b_L})\hat{Y}^d(d_R\ s_R\ b_R)^T. \end{aligned} \tag{2.42}$$

We conclude that the Higgs couplings to the fermion mass eigenstates have the following features:

1. *Diagonality.*
2. *Non-universality.*
3. *Proportionality* to the fermion masses: the heavier the fermion, the stronger the coupling. The factor of proportionality is m_f/v.
4. *CP conservation.*

Thus, the Higgs boson decay is dominated by the heaviest particle which can be pair-produced in the decay. For $m_h \sim 125$ GeV, this is the bottom quark. Indeed, the SM predicts the following branching ratios for the leading decay modes:

$$\text{BR}_{\bar{b}b} : \text{BR}_{WW^*} : \text{BR}_{gg} : \text{BR}_{\tau^+\tau^-} : \text{BR}_{ZZ^*} : \text{BR}_{c\bar{c}} = 0.58 : 0.21 : 0.09 : 0.06 : 0.03 : 0.03. \tag{2.43}$$

The following comments are in order with regard to Eq. (2.43):

1. From the six branching ratios, three (b, τ, c) stand for two-body tree-level decays. Thus, at tree level, the respective branching ratios obey $\text{BR}_{\bar{b}b} : \text{BR}_{\tau^+\tau^-} : \text{BR}_{c\bar{c}} = 3m_b^2 : m_\tau^2 : 3m_c^2$. QCD radiative corrections somewhat suppress the two modes with the quark final states (b, c) compared to one with the lepton final state (τ).
2. The WW^* and ZZ^* modes stand for the three-body tree-level decays ($W\overline{f}f'$ and $Z\overline{f}f$, respectively), where one of the vector bosons is on-shell and the other off-shell.
3. The Higgs boson does not have a tree-level coupling to gluons since it carries no color (and the gluons have no mass). The decay into final gluons proceeds via loop diagrams. The dominant contribution comes from the top-quark loop.
4. Similarly, the Higgs decays into final two photons via loop diagrams with small ($\text{BR}_{\gamma\gamma} \sim 0.002$), but observable, rate. The dominant contributions come from the W and the top-quark loops which interfere destructively.

Experimentally, the decays into final ZZ^*, WW^*, $\gamma\gamma$, $\bar{b}b$ and $\tau^+\tau^-$ have been established with rates that are consistent with the SM predictions.

2.4 Global symmetries

In the absence of the Yukawa matrices, $\mathcal{L}_{\text{Yuk}} = 0$, the SM has a $[U(3)]^5$ global symmetry:

$$G_{\text{global}}^{\text{SM}}(Y^{u,d,e} = 0) = SU(3)_q^3 \times SU(3)_\ell^2 \times U(1)^5, \tag{2.44}$$

where

$$
\begin{aligned}
SU(3)^3_q &= SU(3)_Q \times SU(3)_U \times SU(3)_D, \\
SU(3)^2_\ell &= SU(3)_L \times SU(3)_E, \\
U(1)^5 &= U(1)_Q \times U(1)_U \times U(1)_D \times U(1)_L \times U(1)_E. \qquad (2.45)
\end{aligned}
$$

The point that is important for our purposes is that $\mathcal{L}_{\rm kin}$ respects the non-Abelian flavor symmetry $SU(3)^3_q \times SU(3)^2_\ell$, under which

$$
Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R, \quad L_L \to V_L L_L, \quad E_R \to V_E E_R, \qquad (2.46)
$$

where the V_i are unitary matrices. The Yukawa interactions (2.12) break the global symmetry,

$$
G^{\rm SM}_{\rm global}(Y^{u,d,e} \neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau. \qquad (2.47)
$$

Thus, the transformations of Eq. (2.46) are not a symmetry of $\mathcal{L}_{\rm SM}$. Instead, they correspond to a change of the interaction basis. These observations also offer an alternative way of defining flavor physics: it refers to interactions that break the $[SU(3)]^5$ symmetry (2.46). Thus, the term "**flavor violation**" is often used to describe processes or parameters that break the symmetry.

One can think of the quark Yukawa couplings as spurions that break the global $SU(3)^3_q$ symmetry (but are neutral under $U(1)_B$),

$$
Y^u \sim (3, \bar{3}, 1)_{SU(3)^3_q}, \quad Y^d \sim (3, 1, \bar{3})_{SU(3)^3_q}, \qquad (2.48)
$$

and of the lepton Yukawa couplings as spurions that break the global $SU(3)^2_\ell$ symmetry (but are neutral under $U(1)_e \times U(1)_\mu \times U(1)_\tau$),

$$
Y^e \sim (3, \bar{3})_{SU(3)^2_\ell}. \qquad (2.49)
$$

The spurion formalism is convenient for several purposes: parameter counting (see below), identification of flavor suppression factors (see Section 7), and the idea of minimal flavor violation (see Section 7.3).

2.5 Counting parameters

How many independent parameters are there in $\mathcal{L}^q_{\rm Yuk}$? The two Yukawa matrices, Y^u and Y^d, are 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. The pattern of $G_{\rm global}$ breaking means that there is freedom to remove 9 real and 17 imaginary parameters (the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$). For example, we can use the unitary transformations $Q_L \to V_Q Q_L$, $U_R \to V_U U_R$ and $D_R \to V_D D_R$, to lead to the following interaction basis:

$$
Y^d = \hat{Y}^d, \quad Y^u = V^\dagger \hat{Y}^u, \qquad (2.50)
$$

where $\hat{Y}^{d,u}$ are diagonal,

$$\hat{Y}^d = \mathrm{diag}(y_d, y_s, y_b), \quad \hat{Y}^u = \mathrm{diag}(y_u, y_c, y_t), \tag{2.51}$$

while V is a unitary matrix that depends on three real angles and one complex phase. We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. In the mass basis, we identify the nine real parameters as six quark masses and three mixing angles, while the single phase is $\delta_{\rm KM}$.

How many independent parameters are there in $\mathcal{L}^{\ell}_{\rm Yuk}$? The Yukawa matrix Y^e is 3×3 and complex. Consequently, there are 9 real and 9 imaginary parameters in this matrix. There is, however, freedom to remove 6 real and 9 imaginary parameters (the number of parameters in two 3×3 unitary matrices minus the phases related to $U(1)^3$). For example, we can use the unitary transformations $L_L \to V_L L_L$ and $E_R \to V_E E_R$, to lead to the following interaction basis:

$$Y^e = \hat{Y}^e = \mathrm{diag}(y_e, y_\mu, y_\tau). \tag{2.52}$$

We conclude that there are 3 real lepton flavor parameters. In the mass basis, we identify these parameters as the three charged lepton masses. We must, however, modify the model when we take into account the evidence for neutrino masses.

3 Flavor changing charged current (FCCC) processes

3.1 The CKM matrix

Among the SM interactions, the W-mediated interactions are the only ones that are not diagonal in the mass basis. Consequently, all flavor changing processes depend on the CKM parameters. The fact that there are only four independent CKM parameters, while the number of measured flavor changing processes is much larger, allows for stringent tests of the CKM mechanism for flavor changing processes.

3.1.1 The standard parametrization

The CKM matrix is defined in Eq. (2.28). Its explicit form is not unique. First, there is freedom in defining V in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, i.e. $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. We then write the W interaction of Eq. (2.39) as

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} \end{pmatrix} V \not{W}^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \mathrm{h.c.}. \tag{3.1}$$

The elements of V are therefore written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{3.2}$$

Second, we can redefine the phases of the quark fields in such a way that the masses remain real but the phase structure of the CKM matrix changes. This freedom can be used to choose an explicit parametrization that depends on three real and one imaginary parameters. For example, the standard parametrization [4, 5], used by the PDG, is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{3.3}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The three θ_{ij} are the three mixing angles while δ is the Kobayashi–Maskawa phase. With the fixed mass ordering explained above, we have $\theta_{ij} \in \{0, \pi/2\}$ and $\delta \in \{0, 2\pi\}$. The mixing angles θ_{ij} are often referred to as the real parameters, and δ as the imaginary one, or the CP violating one.

The fitted values of the four parameters are given by

$$\begin{aligned} \sin\theta_{12} &= 0.2250 \pm 0.0007, \\ \sin\theta_{23} &= 0.0418 \pm 0.0008, \\ \sin\theta_{13} &= 0.0037 \pm 0.0001, \\ \delta &= 1.20 \pm 0.04. \end{aligned} \tag{3.4}$$

This translates into the following ranges for the magnitude of the CKM elements:

$$|V| = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361 \pm 0.00010 \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}. \tag{3.5}$$

We discuss some of the ways in which these entries are determined below.

3.1.2 The Wolfenstein parametrization

Equation (3.5) implies that the CKM matrix is numerically close to a unit matrix, with small off-diagonal terms that obey the following hierarchy:

$$|V_{ub}|, |V_{td}| \ll |V_{cb}|, |V_{ts}| \ll |V_{us}|, |V_{cd}|. \tag{3.6}$$

This situation inspires an approximate parametrization, known as the Wolfenstein parametrization. The Wolfenstein parameters consist of the three real parameters λ, A and ρ, and the imaginary (CP violating) parameter $i\eta$. The expansion is in the small parameter,

$$\lambda = |V_{us}| \approx 0.23. \tag{3.7}$$

The order of magnitude of each element can be read from the power of λ. To $\mathcal{O}(\lambda^3)$, the CKM matrix is written in terms of the Wolfenstein parameters as follows [6,7]:

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{3.8}$$

The relations between the standard parameters and the Wolfenstein parameters are given by

$$\lambda = s_{12}, \qquad A\lambda^2 = s_{23}, \qquad A\lambda^3(\rho - i\eta) = s_{13}e^{-i\delta}. \tag{3.9}$$

The fitted values of the four parameters can be read from Eq. (3.4)

$$\begin{aligned} \rho &= 0.16 \pm 0.01, \\ \eta &= 0.35 \pm 0.01, \\ A &= 0.83 \pm 0.02, \\ \lambda &= 0.2250 \pm 0.0007. \end{aligned} \tag{3.10}$$

The experimental fact that the CKM matrix is close to a unit matrix is one of the ingredients of *the SM* that are far from a generic SM. The hierarchy in the quark masses constitutes another such ingredient.

3.1.3 *CP violation*

Various parameterizations differ in the way that the freedom of phase rotation is used to leave a single phase in V. One can define, however, a CP violating quantity in V that is independent of the parametrization. This quantity, the Jarlskog invariant [8,9], $J_{\rm CKM}$, is defined through

$$\mathcal{I}m(V_{ij}V_{kl}V_{il}^*V_{kj}^*) = J_{\rm CKM}\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln}, \qquad (i,j,k,l=1,2,3). \tag{3.11}$$

(There is no sum over the i,j,k,l indices.) In terms of the explicit parameterizations given in Eqs. (3.3) and (3.8), the Jarlskog invariant is given by

$$J_{\rm CKM} = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta \approx \lambda^6 A^2 \eta. \tag{3.12}$$

Note that $|J_{\rm CKM}|$ is bounded from above,

$$|J_{\rm CKM}| \leq \frac{1}{6\sqrt{3}} \sim 0.1. \tag{3.13}$$

The current best fit for $J_{\rm CKM}$ is given by

$$J_{\rm CKM} = (3.00^{+0.15}_{-0.09}) \times 10^{-5}, \tag{3.14}$$

which is much smaller than the upper bound of Eq. (3.13). More significantly, the experimental value is much smaller than the value it would have if all relevant parameters were $O(1)$. This is one more demonstration that, within the flavor sector, the SM has non-generic features.

While a generic SM violates CP, specific realizations of it could still conserve CP. In order that *the SM* violates CP, the following necessary and sufficient condition must be fulfilled:

$$X_{CP} \equiv \Delta m^2_{tc}\Delta m^2_{tu}\Delta m^2_{cu}\Delta m^2_{bs}\Delta m^2_{bd}\Delta m^2_{sd}J_{\rm CKM} \neq 0, \tag{3.15}$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$. Equation (3.15) puts the following requirements on the SM in order that CP is violated:

1. Within each quark sector, there should be no mass degeneracy;
2. The Jarlskog invariant does not vanish.

These conditions can also be written as a single requirement on the quark mass matrices in any interaction basis [8, 9]:

$$X_{CP} = \mathcal{I}m\left\{\det\left[M_d M_d^\dagger, M_u M_u^\dagger\right]\right\} \neq 0 \;\Leftrightarrow\; CP \text{ violation}. \tag{3.16}$$

This is a convention independent condition.

3.1.4 SM2: CP conserving

Consider a two generation Standard Model, SM2. This model is similar to the one defined in Section 2, which in this section will be referred to as SM3, except that there are two, rather than three fermion generations. Many features of SM2 are similar to SM3, but there is one important difference: CP is a good symmetry of SM2, but not of SM3. To see how this difference comes about, let us examine the accidental symmetries of SM2. We follow here the line of analysis of SM3 in Section 2.5.

If we set the Yukawa couplings to zero, $\mathcal{L}^{\rm SM2}_{\rm Yuk} = 0$, SM2 gains an accidental global symmetry:

$$G^{\rm global}_{\rm SM2}(Y^{u,d,e} = 0) = U(2)_Q \times U(2)_U \times U(2)_D \times U(2)_L \times U(2)_E, \tag{3.17}$$

where the two generations of each gauge representation are a doublet of the corresponding $U(2)$. The Yukawa couplings break this symmetry into the subgroup

$$G^{\rm global}_{\rm SM2} = U(1)_B \times U(1)_e \times U(1)_\mu. \tag{3.18}$$

A-priori, the Yukawa terms depend on three 2×2 complex matrices, namely $12_R + 12_I$ parameters. The global symmetry breaking, $[U(2)]^5 \to [U(1)]^3$, implies that we can remove $5 \times (1_R + 3_I) - 3_I = 5_R + 12_I$ parameters. Thus the number of physical flavor parameters is 7 real parameters and no imaginary parameter. The real parameters can be identified as two charged lepton masses, four quark masses, and the single real mixing angle, $\sin\theta_c = |V_{us}|$.

The important conclusion for our purposes is that all imaginary couplings can be removed from SM2, and CP is an accidental symmetry of the model.

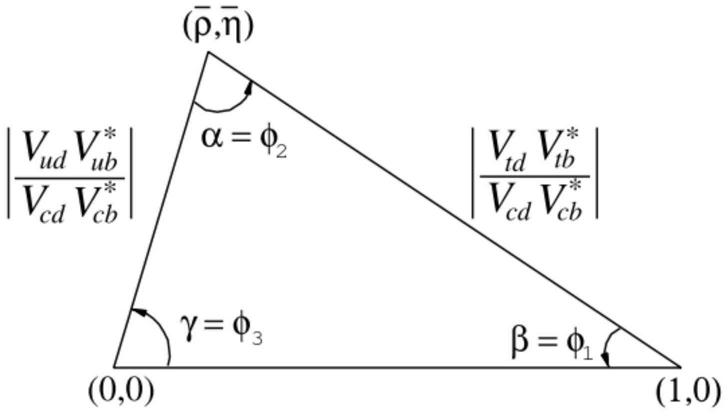

Fig. 1: The rescaled unitarity triangle.

3.1.5 Unitarity triangles

A very useful concept with regard to CP violation is that of the unitarity triangles. The unitarity of the CKM matrix leads to various relations among its elements. Of particular interest are the six relations:

$$\begin{aligned} \sum_{i=u,c,t} V_{iq}V_{iq'}^* &= 0 \qquad (qq' = ds, db, sb), \\ \sum_{i=d,s,b} V_{qi}V_{q'i}^* &= 0 \qquad (qq' = uc, ut, ct). \end{aligned} \tag{3.19}$$

Each of these relations requires the sum of three complex quantities to vanish. Therefore, they can be geometrically represented in the complex plane as triangles and are called "unitarity triangles". It is a feature of the CKM matrix that all six unitarity triangles have equal areas. Moreover, the area of each unitarity triangle equals $|J_{\rm CKM}|/2$ while the sign of $J_{\rm CKM}$ gives the direction of the complex vectors around the triangles.

The triangle which corresponds to the relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \tag{3.20}$$

has its three sides of roughly the same length, of $\mathcal{O}(\lambda^3)$—see Eq. (3.8). Furthermore, both the lengths of its sides and its angles are experimentally accessible. For these reasons, the term "the unitarity triangle" is reserved for Eq. (3.20).

We further define the rescaled unitarity triangle. It is derived from Eq. (3.20) by choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real and dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. The rescaled unitarity triangle is similar to the unitarity triangle. Two vertices of the rescaled unitarity triangle are fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ, η). The rescaled unitarity triangle is shown in Fig. 1. The lengths of the two complex sides are

$$R_u \equiv \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2+\eta^2}, \qquad R_t \equiv \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right| = \sqrt{(1-\rho)^2+\eta^2}. \tag{3.21}$$

The three angles of the unitarity triangle are defined as follows:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \qquad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \qquad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{3.22}$$

They are physical quantities and can be independently measured, as we discuss below. Another commonly used notation is $\phi_1 = \beta$, $\phi_2 = \alpha$, and $\phi_3 = \gamma$. Note that in the standard parametrization $\gamma = \delta$.

3.2 Tree level determination of the CKM parameters

The charged current weak interactions allow the determination of CKM parameters from tree level processes. There is an inherent difficulty in determining the CKM parameters: While the SM Lagrangian has the quarks as its degrees of freedom, in Nature they appear only within hadrons. There are various tools to overcome this difficulty, particularly for semileptonic decays, such as isospin symmetry and heavy quark symmetry.

At tree level, the W-mediated interactions lead to only FCCC processes. These suffice, however, to over-constrain the CKM parameters. The most useful processes are semileptonic ones. Here we give a short summary of the results:

– Processes related to $d \to u\ell^-\bar\nu$ transitions give $|V_{ud}| = 0.97370 \pm 0.00014$.
– Processes related to $s \to u\ell^-\bar\nu$ transitions give $|V_{us}| = 0.2245 \pm 0.0008$.
– Processes related to $c \to d\ell^+\nu$ or to $\nu_\mu + d \to c + \mu^-$ transitions give $|V_{cd}| = 0.221 \pm 0.004$.
– Processes related to $c \to s\ell^+\nu$ or to $c\bar s \to \ell^+\nu$ transitions give $|V_{cs}| = 0.987 \pm 0.011$.
– Processes related to $b \to c\ell^-\bar\nu$ transitions give $|V_{cb}| = 0.0410 \pm 0.0014$.
– Processes related to $b \to u\ell^-\bar\nu$ transitions give $|V_{ub}| = 0.00382 \pm 0.00024$.

There are two additional classes of tree level processes that depend on the CKM parameters:

– Processes related to single top production in hadron colliders give $|V_{tb}| = 1.013 \pm 0.030$.
– Processes related to $b \to sc\bar u$ and $b \to su\bar c$ transitions give $\gamma = (72 \pm 5)^o$.

These eight distinct classes of processes depend on only four CKM parameters. The system is thus over-constrained and tests the SM.

The values of λ and A can be straightforwardly extracted from the measurements of $|V_{us}|$ and $|V_{cb}|$, respectively:

$$\lambda = 0.2250 \pm 0.0007, \qquad A = 0.83 \pm 0.02. \tag{3.23}$$

The values of ρ and η are extracted mainly from combining the measurements of $|V_{ub}|$ and γ, as shown in Fig. 2:

$$\rho = 0.13 \pm 0.03, \qquad \eta = 0.38 \pm 0.02. \tag{3.24}$$

The fact that the ranges of the four parameters in Eqs. (3.23) and (3.24) are consistent with all the measurements means that the SM passes the test successfully.

Note that the error bars on the determination here, Eqs. (3.23) and (3.24), is larger than the one in Eq. (3.10). The reason is that here we only consider tree level processes.

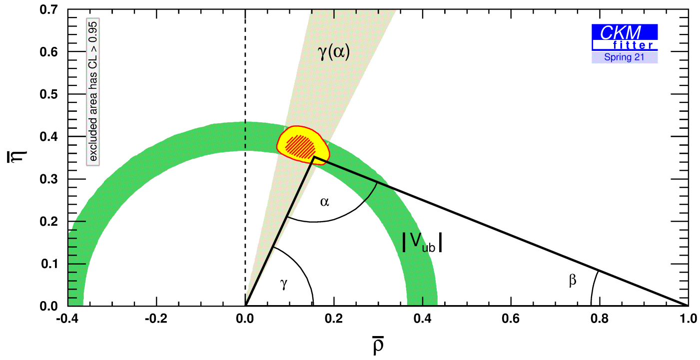

Fig. 2: Allowed region in the (ρ, η) plane from SM tree level processes (taken from Ref. [10]).

4 Flavor changing neutral current (FCNC) processes

4.1 No FCNC at tree level

Historically, the strong suppression of FCNC played a very important role in constructing the SM. At present it continues to play a significant role in testing the SM and in searching for new physics. In this subsection we explain why, within the SM, there are no tree level contributions to FCNC processes. Since there is no symmetry that forbids FCNC in the quark sector, there are loop contributions to these processes. These are discussed in the following subsections.

The W-boson cannot mediate FCNC processes at tree level, since it couples to up–down pairs, or to neutrino–charged-lepton pairs. Only neutral bosons could mediate FCNC at tree level. The SM has four neutral bosons: the gluon, the photon, the Z-boson and the Higgs-boson. As derived explicitly in Section 2, within the SM all of them couple diagonally in the mass basis, and therefore cannot mediate FCNC at tree level. Here we explain the qualitative features of the SM that lead to this situation.

4.1.1 Photon- and gluon-mediated FCNC

As concerns the massless gauge bosons, the gluon and the photon, their couplings are flavor-universal and, in particular, flavor-diagonal. This is guaranteed by gauge invariance. The universality of the kinetic terms in the canonical basis requires universality of the gauge couplings related to the unbroken symmetries. Hence neither the gluon nor the photon can mediate flavor changing processes at tree level. Since we require that extensions of the SM respect the local $SU(3)_C \times U(1)_{\rm EM}$ symmetry, this result holds in all such extensions.

4.1.2 Z-mediated FCNC

The Z-boson, similarly to the W-boson, corresponds to a broken gauge symmetry (as manifest in the fact that it is massive). Hence, there is no fundamental symmetry principle that forbids flavor changing Z couplings. Yet, as we explicitly find in Section 2.3.2, in the SM the Z couplings are universal and diagonal.

The key point is the following. The Z couplings are proportional to $T_3 - Q\sin^2\theta_W$. A sector of mass eigenstates is characterized by spin, $SU(3)_C$ representation and $U(1)_{\rm EM}$ charge. While Q must be the same for all the flavors in a given sector, there are two possibilities regarding T_3:

1. All mass eigenstates in this sector originate from interaction eigenstates in the same $SU(2)_L \times U(1)_Y$ representation, and thus have the same T_3 and Y.
2. The mass eigenstates in this sector mix interaction eigenstates with the same $Q = T_3 + Y$ but different $SU(2)_L \times U(1)_Y$ representations and, more specifically, different T_3 and Y.

Let us examine the Z couplings in the interaction and mass bases for several flavors of (hypothetical) fermions in the same $SU(3)_C \times U(1)_{\rm EM}$ representation:

1. In the first class, the Z couplings in the fermion interaction basis are universal, namely they are proportional to the unit matrix (times $T_3 - Q\sin^2\theta_W$ of the relevant interaction eigenstates). The rotation to the mass basis maintains the universality:

$$V_{fM} \times \mathbf{1} \times V_{fM}^\dagger = \mathbf{1}, \qquad (f = u, d, e; M = L, R). \tag{4.1}$$

2. In the second class, the Z couplings in the fermion interaction basis are diagonal but not universal. Each diagonal entry is proportional to the relevant $T_3 - Q\sin^2\theta_W$. Generally in this case, the rotation to the mass basis does not maintain the diagonality:

$$V_{fM} \times \hat{G}_{\rm diagonal} \times V_{fM}^\dagger = G_{\rm non-diagonal}, \qquad (f = u, d, e; M = L, R). \tag{4.2}$$

The SM fermions belong to the first class: All fermion mass eigenstates with a given chirality and in a given $SU(3)_C \times U(1)_{\rm EM}$ representation come from the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ representation. For example, all the left-handed up quark mass eigenstates, which are in the $(3)_{+2/3}$ representation, come from interaction eigenstates in the $(3,2)_{+1/6}$ representation. This is the reason that the SM predicts universal Z couplings to fermions. If, for example, Nature had also left-handed quarks in the $(3,1)_{+2/3}$ representation, then the Z couplings in the left-handed up sector would be non-universal and the Z-boson could mediate FCNC, such as $t \to cZ$ decay, at tree level.

4.1.3 Higgs-mediated FCNC

The Yukawa couplings of the Higgs boson are not universal. In fact, in the interaction basis, they are given by completely general 3×3 matrices. Yet, as explained in Section 2.3.4, in the fermion mass basis they are diagonal. The reason is that the fermion mass matrix is proportional to the corresponding Yukawa matrix and, consequently, the mass matrix and the Yukawa matrix are simultaneously diagonalized. The general condition for the absence of Higgs-mediated FCNC at tree level is that the only source of masses for any fermion type is a single Higgs field.

The relevant features of the SM are the following:

1. All the SM fermions are chiral and charged (under $SU(2)_L \times U(1)_Y$), and therefore there are no bare mass terms.
2. The scalar sector has a single Higgs doublet.

In contrast, either of the following possible extensions would lead to flavor changing Higgs couplings:

1. There are quarks and/or leptons in vector-like representations, and thus there are bare mass terms.
2. There is more than one $SU(2)_L$-doublet scalar that couples to a specific type of fermions.

Subsection 4.1.4 provides an example of the first case. Subsection 4.1.5 provides an example of the second case.

We conclude that, within the SM, all FCNC processes in the quark sector are loop suppressed (while in the lepton sector they are forbidden). However, in extensions of the SM, FCNC can appear at the tree level, mediated by the Z-boson, the Higgs boson, or by new massive bosons.

To summarize, FCNC processes cannot be mediated at tree level in the SM. Yet, since there is no symmetry that forbids them in the quark sector, they are mediated at the loop level. Concretely, the W-mediated interactions lead to FCNC at the one-loop level. Since the W-boson couplings are charged current flavor changing, an even number of insertions of W-boson couplings are needed to generate an FCNC process. We consider two classes of FCNC based on the change in F (the charge under the global $[U(1)]^6$ flavor symmetry of the QCD Lagrangian):

– FCNC decays ($\Delta F = 1$ processes) have two insertions of W-couplings.
– Neutral meson mixings ($\Delta F = 2$ processes) have four insertions of W-couplings.

4.1.4 SM1.5: FCNC at tree level

Consider a model with the SM gauge group and pattern of SSB, but with only three quark flavors: u, d, s. Such a situation cannot fit into a model with all left-handed quarks in doublets of $SU(2)_L$. How can we incorporate the interactions of the strange quark in this picture? The solution that we now describe is wrong. Yet, it is of historical significance and, moreover, helps us to understand some of the unique properties of the SM described above. In particular, it leads to FCNC at tree level. We define the three flavor Standard Model (SM1.5) as follows (we ignore the lepton sector):

– The symmetry is a local

$$G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{4.3}$$

– It is spontaneously broken by the VEV of a single Higgs scalar,

$$\phi(1,2)_{+1/2}, \quad (\langle \phi^0 \rangle = v/\sqrt{2}), \tag{4.4}$$

$$G_{\rm SM} \to SU(3)_C \times U(1)_{\rm EM} \quad (Q_{\rm EM} = T_3 + Y). \tag{4.5}$$

– The colored fermion representations are the following:

$$Q_L(3,2)_{+1/6}, \;\; D_L(3,1)_{-1/3}, \;\; U_R(3,1)_{+2/3}, \;\; D_{Ri}(3,1)_{-1/3} \;\; (i=1,2). \tag{4.6}$$

We point out two important ingredients that are different from the SM:

1. There are quarks in a vector-like representation ($D_L + D_R$);
2. Not all $(3)_{-1/3}$ quarks come from the same type of $SU(2)_L \times U(1)_Y$ representations.

We first note that D_L does not couple to the W-bosons:

$$\mathcal{L}_W = \frac{g}{2}\overline{Q_L}\not{W}_b\tau_b Q_L. \tag{4.7}$$

The Yukawa interactions are given by

$$\mathcal{L}_{\rm Yuk} = -y_u\overline{Q_L}\tilde{\phi}U_R - Y_i^d\overline{Q_L}\phi D_{Ri} + {\rm h.c.}. \tag{4.8}$$

Unlike the SM, we now have bare mass terms for fermions:

$$\mathcal{L}_q = -m_{di}\overline{D_L}D_{Ri} + {\rm h.c.}. \tag{4.9}$$

Given that there is a single up generation, the interaction basis is also the up mass basis. Explicitly, we identify the up-component of Q_L with u_L (and denote the down component of the doublet as d_{uL}), and U_R with u_R. With the SSB, we have the following mass terms:

$$-\mathcal{L}_{\rm mass} = (\overline{d_{uL}}\ \overline{D_L})\begin{pmatrix} Y_{d1}\frac{v}{\sqrt{2}} & Y_{d2}\frac{v}{\sqrt{2}} \\ m_{d1} & m_{d2}\end{pmatrix}\begin{pmatrix} D_{R1} \\ D_{R2}\end{pmatrix} + y_u\frac{v}{\sqrt{2}}\overline{u_L}u_R + {\rm h.c.}. \tag{4.10}$$

We now rotate to the down mass basis:

$$V_{dL}\begin{pmatrix} Y_{d1}\frac{v}{\sqrt{2}} & Y_{d2}\frac{v}{\sqrt{2}} \\ m_{d1} & m_{d2}\end{pmatrix}V_{dR}^\dagger = \begin{pmatrix} m_d & \\ & m_s\end{pmatrix}. \tag{4.11}$$

The resulting mixing matrix for the charged current interactions is a 1×2 matrix:

$$-\mathcal{L}_{W,q} = \frac{g}{\sqrt{2}}\overline{u_L}\not{W}^+(\cos\theta_C\ \ \sin\theta_C)\begin{pmatrix} d_L \\ s_L\end{pmatrix} + {\rm h.c.}, \tag{4.12}$$

where θ_C is the rotation angle of V_{dL}. The neutral current interactions in the left-handed down sector are neither universal nor diagonal:

$$\begin{aligned}\mathcal{L}_{Z,q} \ &= \ \frac{g}{c_W}\left[\left(\frac{1}{2}-\frac{2}{3}s_W^2\right)\ \overline{u_L}\not{Z}u_L - \frac{2}{3}s_W^2\ \overline{u_R}\not{Z}u_R + \frac{1}{3}s_W^2(\overline{d_L}\not{Z}d_L + \overline{s_L}\not{Z}s_L + \overline{d_R}\not{Z}d_R + \overline{s_R}\not{Z}s_R)\right] \\ &\quad -\frac{g}{2c_W}(\overline{d_L}\ \overline{s_L})\not{Z}\begin{pmatrix} \cos^2\theta_C & \cos\theta_C\sin\theta_C \\ \cos\theta_C\sin\theta_C & \sin^2\theta_C\end{pmatrix}\begin{pmatrix} d_L \\ s_L\end{pmatrix}.\end{aligned} \tag{4.13}$$

The Higgs interactions in the down sector are neither proportional to the mass matrix nor diagonal:

$$\mathcal{L}_{\rm Yuk}^q = y_u h\overline{u_L}u_R + h(\overline{d_L}\ \overline{s_L})\left[V_{dL}\begin{pmatrix} Y_{d1} & Y_{d2} \\ 0 & 0\end{pmatrix}V_{dR}^\dagger\right]\begin{pmatrix} d_R \\ s_R\end{pmatrix} + {\rm h.c.}. \tag{4.14}$$

Thus, in this model, both the Z-boson and the h-boson mediate FCNC at tree level. For example, $K_L\to\mu^+\mu^-$ and $K^0-\overline{K}^0$ mixing get Z- and h-mediated tree-level contributions.

4.1.5 2HDM: FCNC at tree level

Consider a model with two Higgs doublets. The symmetry structure, the pattern of spontaneous symmetry breaking, and the fermion content are the same as in the SM. However, the scalar content is extended:

– The scalar representations are

$$\phi_i(1,2)_{+1/2}, \quad i=1,2. \tag{4.15}$$

We are particularly interested in the modification of the Yukawa terms:

$$\mathcal{L}_{\rm Yuk} = (Y_k^u)_{ij}\overline{Q_{Li}}U_{Rj}\,\tilde{\phi}_k + (Y_k^d)_{ij}\overline{Q_{Li}}D_{Rj}\,\phi_k + (Y_k^e)_{ij}\overline{L_{Li}}E_{Rj}\,\phi_k + {\rm h.c.}. \tag{4.16}$$

Without loss of generality, we can work in a basis (commonly called "the Higgs basis") (ϕ_A, ϕ_M), where one the Higgs doublets carries the VEV, $\langle\phi_M\rangle = v$, while the other has zero VEV, $\langle\phi_A\rangle = 0$. In this basis, Y_M^f is known and related to the fermions masses in the same way as the Yukawa matrices of the SM:

$$Y_M^f = \sqrt{2}M_f/v. \tag{4.17}$$

The entries in the Yukawa matrices Y_A^f are, however, free parameters and, in general, unrelated to the fermion masses. The rotation angle from the Higgs basis to the basis of neutral CP-even Higgs states, (ϕ_h, ϕ_H), is denoted by $(\alpha-\beta)$. The Yukawa matrix of the light Higgs field h is given by

$$Y_h^f = c_{\alpha-\beta}Y_A^f - s_{\alpha-\beta}Y_M^f. \tag{4.18}$$

Given the arbitrary structure of Y_A^f, the Higgs boson can have couplings that are neither proportional to the mass matrix nor diagonal.

It is interesting to note, however, that not all multi Higgs doublet models lead to flavor changing Higgs couplings. If all the fermions of a given sector couple to one and the same doublet, then the Higgs couplings in that sector would still be diagonal. For example, in a model with two Higgs doublets, ϕ_1 and ϕ_2, and Yukawa terms of the form

$$\mathcal{L}_{\rm Yuk} = Y_{ij}^u\overline{Q_{Li}}U_{Rj}\,\tilde{\phi}_2 + Y_{ij}^d\overline{Q_{Li}}D_{Rj}\,\phi_1 + Y_{ij}^e\overline{L_{Li}}E_{Rj}\,\phi_1 + {\rm h.c.}, \tag{4.19}$$

the Higgs couplings are flavor diagonal:

$$Y_h^u = (c_\alpha/s_\beta)Y_M^u, \quad Y_h^d = -(s_\alpha/c_\beta)Y_M^d, \quad Y_h^e = -(s_\alpha/c_\beta)Y_M^e, \tag{4.20}$$

where β [α] is the rotation angle from the (ϕ_1, ϕ_2) basis to the (ϕ_A, ϕ_M) [(ϕ_h, ϕ_H)] basis. In the physics jargon, we say that such models have *natural flavor conservation* (NFC) [11–13].

4.2 CKM and GIM suppressions in FCNC decays

In this section, we discuss FCNC meson decays, which are $\Delta F = 1$ processes. To demonstrate the generic features of one-loop FCNC, we consider the example of $s \to d$ transitions. Since the change of flavor QNs is $\Delta s = -\Delta d = 1$, this transition belongs to the class of $\Delta F = 1$ processes. The FCNC part

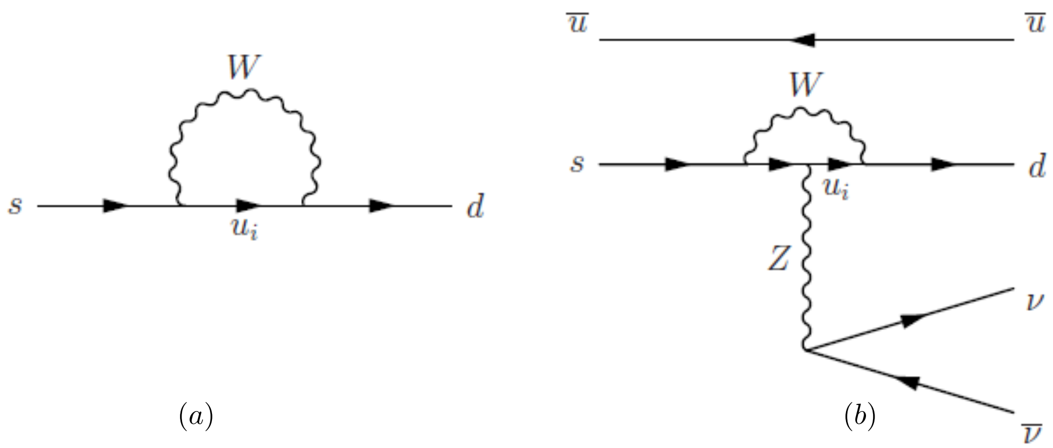

Fig. 3: (a) One loop diagrams for $\Delta F = 1$ $s \to d$ FCNC. (b) A one loop diagram that contributes to $K^- \to \pi^- \nu\bar{\nu}$.

of any process that involves $s \to d$ transition is plotted in Fig. 3(a). For example, in Fig. 3(b) we show a full diagram that contributes to the decay $K \to \pi\nu\bar{\nu}$, and which includes Fig. 3(a) as a sub-diagram (diagrams with such a topology are usually called penguin diagrams).

By inspecting the diagram in Fig. 3(a), we learn that its flavor structure is given by

$$\mathcal{A}_{s\to d} \sim \sum_{i=u,c,t} (V_{is}V_{id}^*)\ f(x_i), \qquad x_i = \frac{m_i^2}{m_W^2}, \tag{4.21}$$

where $f(x_i)$ depends on the specific decay. CKM unitarity implies

$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0. \tag{4.22}$$

We can then use this unitarity condition to eliminate one of the three CKM terms in the sum in Eq. (4.21). We choose to eliminate the u-term and write

$$\mathcal{A}_{s\to d} \sim \sum_{i=c,t} [f(x_i) - f(x_u)]\, V_{is} V_{id}^*. \tag{4.23}$$

We draw the following lessons:

– The contribution of the m_i-independent terms in $f(x_i)$ to $\mathcal{A}_{s\to d}$ vanishes when summed over all internal quarks.
– $\mathcal{A}_{s\to d}$ would vanish if the up-type quarks were all degenerate and, therefore, it must depend on the mass-splittings among the up-type quarks.

The explicit dependence on the mass-splittings among the quarks depends on the process. In many cases, for small x_i we have

$$f(x_i) \sim x_i. \tag{4.24}$$

Using this crude approximation we can write

$$\mathcal{A}_{s\to d} \sim [(x_t - x_u)V_{ts}V_{td}^* + (x_c - x_u)V_{cs}V_{cd}^*] \,. \tag{4.25}$$

Inspecting Eq. (4.25), we identify two suppression factors:

(i) *CKM suppression:* The amplitude is proportional to at least one off-diagonal CKM matrix element. Given the specific structure of the CKM matrix, off-diagonal elements are small. Specifically, $|V_{ts}V_{td}^*| \sim \lambda^5$ and $|V_{cs}V_{cd}^*| \sim \lambda$ where λ is the Wolfenstein parameter defined in Eq. (3.7).

(ii) *GIM suppression:* The amplitude is proportional to mass-squared differences between the up-type quarks. In particular, $(x_c - x_u) \sim (m_c/m_W)^2$. The suppression by factors of small quark masses is called the Glashow–Iliopoulos–Maiani (GIM) mechanism [14].

While we derive the results based on one specific example, the CKM and GIM suppressions play a role in all FCNC processes. For the other FCNC in the down sector, $b \to q$ with $q = d, s$, we have

$$\mathcal{A}_{b\to q} \sim (x_t - x_u)V_{tb}V_{tq}^* + (x_c - x_u)V_{cb}V_{cq}^*. \tag{4.26}$$

With regard to FCNC in the up sector, for $c \to u$, we have

$$\mathcal{A}_{c\to u} \sim (x_b - x_d)V_{cb}^*V_{ub} + (x_s - x_d)V_{cs}^*V_{us}, \tag{4.27}$$

and, for $t \to q$ with $q = u, c$, we have

$$\mathcal{A}_{t\to q} \sim (x_b - x_d)V_{tb}^*V_{qb} + (x_s - x_d)V_{ts}^*V_{qs}. \tag{4.28}$$

The CKM suppression applies to FCNC decay rates. It does not necessarily apply, however, to the corresponding branching ratios. The reason is that branching ratios depend on the ratio between the FCNC decay rate and the full decay width which, in the down sector, is CKM-suppressed, and thus the ratio of CKM factors is not necessarily small. In particular, the leading (FCCC) K decay rate is suppressed by $|V_{us}| \sim |V_{cs}V_{cd}|$ and the leading (FCCC) B decay rate is suppressed by $|V_{cb}| \sim |V_{tb}V_{ts}|$.

Several remarks are in order:

1. While the $f(x_i) \sim x_i$ approximation is not valid for the top quark, it gives a reasonable order of magnitude estimate, and we use it for the purpose of demonstration. For example, while $x_t/x_c \sim 10^4$, we have for $f(x)$ defined in Eq. (4.31), $f(x_t)/f(x_c) \approx 10^3$.
2. The exact form of the dependence on the mass splitting is process dependent, but in all cases the amplitude vanishes when the internal quarks are degenerate. We refer to quadratic dependence $[x_i - x_j]$ as hard GIM, and to logarithmic dependence $[\log(x_i/x_j)]$ as soft GIM.
3. The size of FCNC amplitudes increases with the mass of the internal quark. The reason that this does not violate the decoupling theorem is that the mass comes from SSB, so larger masses correspond to larger Yukawa couplings.

4.2.1 Examples: $K \to \pi\nu\bar{\nu}$ and $B \to \pi\nu\bar{\nu}$

As examples of $\Delta F = 1$ processes, we consider the semileptonic decays

$$K^+ \to \pi^+\nu\bar{\nu}, \qquad B^+ \to \pi^+\nu\bar{\nu}, \tag{4.29}$$

which proceed via the $\bar{s} \to \bar{d}\nu\bar{\nu}$ and $\bar{b} \to \bar{d}\nu\bar{\nu}$ transitions.

Consider the following ratios of FCNC-to-FCCC semileptonic decay rates:

$$\begin{aligned} R_{K\pi} &= \frac{\Gamma(K^+ \to \pi^+\nu\bar{\nu})}{\Gamma(K^+ \to \pi^0 e^+\nu)} = \frac{3}{2}\frac{g^4}{16\pi^2}\left|\frac{V_{ts}^* V_{td}[f(x_t) - f(x_u)] + V_{cs}^* V_{cd}[f(x_c) - f(x_u)]}{V_{us}}\right|^2, \\ R_{B\pi} &= \frac{\Gamma(B^+ \to \pi^+\nu\bar{\nu})}{\Gamma(B^+ \to \pi^0 e^+\nu)} = \frac{3}{2}\frac{g^4}{16\pi^2}\left|\frac{V_{tb}^* V_{td}[f(x_t) - f(x_u)] + V_{cb}^* V_{cd}[f(x_c) - f(x_u)]}{V_{ub}}\right|^2, \end{aligned} \tag{4.30}$$

where

$$f(x) = \frac{x}{8}\left[\frac{2+x}{1-x} - \frac{3x-6}{(1-x)^2}\log x\right]. \tag{4.31}$$

For small x, we have

$$f(x \ll 1) \approx \frac{x(3\log x + 1)}{4}. \tag{4.32}$$

Since $f(x_u) \ll 1$, we have to a very good approximation $f(x_i) - f(x_u) \approx f(x_i)$ for $i = c, t$.

A few comments are in order with regard to Eq. (4.30):

1. The factor of 3 comes from summing over the neutrino flavors, while the factor of $1/2$ is an isospin factor between the $P^+ \to \pi^+$ and $P^+ \to \pi^0$ ($P = K, B$) transitions.
2. The $g^4/16\pi^2$ is the loop suppression factor.
3. For $R_{K\pi}$, the t-term is CKM-suppressed, $|V_{ts}V_{td}/V_{us}| \sim \lambda^4$, but not GIM-suppressed, $f(x_t) = \mathcal{O}(1)$. The c-term is GIM-suppressed, $f(x_c) = \mathcal{O}(m_c^2/m_W^2)$, but not CKM-suppressed, $|V_{cs}V_{cd}/V_{us}| \simeq 1$. The two terms contribute comparably.
4. For $R_{B\pi}$, the t-term is neither CKM-suppressed, $|V_{tb}V_{td}/V_{ub}| = \mathcal{O}(1)$, nor GIM-suppressed, $f(x_t) = \mathcal{O}(1)$. The c-term is not CKM-suppressed, $|V_{cb}V_{cd}/V_{ub}| = \mathcal{O}(1)$, but it is GIM-suppressed, $f(x_c) = \mathcal{O}(m_c^2/m_W^2)$. Thus, the contribution of the c-term is negligible.
5. As a result of the different CKM and GIM suppression factors, we obtain numerically very different predictions:

   $$R_{K\pi} \sim 10^{-9}, \qquad R_{B\pi} \sim 10^{-4}. \tag{4.33}$$

   These predictions have not been fully tested yet, as we have only experimental upper bounds, $R_{K\pi} \lesssim 10^{-8}$ and $R_{B\pi} \lesssim 0.18$.

The comparison of $R_{K\pi}$ and $R_{B\pi}$ demonstrates how the CKM and GIM suppression factors depend crucially on the specific quarks involved, and how they come into play in determining the various FCNC rates. While for the two specific examples presented here there exist only experimental upper bounds, many FCNC decays have been observed and their rates measured. To date, all measured FCNC decay rates in the quark sector agree with the SM predictions.

Table 3: Measurements related to neutral meson mixing

Sector	CP-conserving	CP-violating
sd	$\Delta m_K/m_K = 7.0 \times 10^{-15}$	$\epsilon_K = 2.3 \times 10^{-3}$
cu	$\Delta m_D/m_D = 8.7 \times 10^{-15}$	$A_\Gamma/y_{\rm CP} \lesssim 0.05$
bd	$\Delta m_B/m_B = 6.3 \times 10^{-14}$	$S_{\psi K} = +0.699 \pm 0.017$
bs	$\Delta m_{B_s}/m_{B_s} = 2.1 \times 10^{-12}$	$S_{\psi\phi} = +0.046 \pm 0.020$

4.3 CKM and GIM suppressions in neutral meson mixing

A very useful class of FCNC is that of neutral meson mixing. Nature provides us with four pairs of neutral mesons: $K^0 - \overline{K}^0$, $B^0 - \overline{B}^0$, $B_s^0 - \overline{B}_s^0$, and $D^0 - \overline{D}^0$. Mixing in this context refers to a transition such as $K^0 \to \overline{K}^0$ ($\bar{s}d \to \bar{d}s$).[2] The experimental results for CP conserving and CP violating observables related to neutral meson mixing (mass splittings and CP asymmetries in tree level decays, respectively) are given in Table 3.

Neutral meson mixing is a $\Delta F = 2$ process. This phenomenon is observed and measured via meson oscillations, as discussed in Appendix B. In Appendix A we present the explicit SM calculation. In this section we show that the general lessons learned from $\Delta F = 1$ processes about the loop, CKM, and GIM suppression factors of FCNC, mostly carry over to $\Delta F = 2$ processes.

To demonstrate the features of $\Delta F = 2$ FCNC processes, we consider the example of $K^0 - \overline{K}^0$ mixing. It is generated by the $s\bar{d} \to d\bar{s}$ transition which is a $\Delta s = -\Delta d = 2$ process. The leading diagram for this transition is plotted in Fig. 4. By inspecting this diagram, we learn that its flavor structure is given by

$$\mathcal{A}_{s\bar{d}\to d\bar{s}} \sim \sum_{i,j=u,c,t} (V_{is}V_{id}^* V_{js}V_{jd}^*)\, S(x_i, x_j), \tag{4.34}$$

where $S(x_j, x_i)$ [$x_i \equiv m_i^2/m_W^2$] is given explicitly in Eq. (A.6). We draw the following lessons:

- The contribution of the m_i-independent terms in $S(x_i, x_j)$ to $\mathcal{A}_{s\bar{d}\to d\bar{s}}$ vanishes when summed over all internal quarks.
- $\mathcal{A}_{s\bar{d}\to d\bar{s}}$ would vanish if the up-type quarks were all degenerate and, therefore, it must depend on the mass-splittings among the up-type quarks.

To proceed, we use the unitarity condition of Eq. (4.22) and approximate $x_u = 0$ to eliminate the u-terms in the sum. We obtain:

$$\mathcal{A}_{s\bar{d}\to d\bar{s}} \sim (V_{cs}V_{cd}^*)^2 S(x_c, x_c) + 2V_{cs}V_{cd}^* V_{ts}V_{td}^* S(x_c, x_t) + (V_{ts}V_{td}^*)^2 S(x_t, x_t). \tag{4.35}$$

We conclude that $\Delta F = 2$ amplitudes have, in addition to the loop suppression factor, also the following suppression factors:

[2]These transitions involve four-quark operators. When calculating the matrix elements of these operators between meson-antimeson states, approximate symmetries of QCD are of no help. Instead, one uses lattice calculations to relate, for example, the $B^0 \to \overline{B}^0$ transition to the corresponding quark process, $\bar{b}d \to \bar{d}b$.

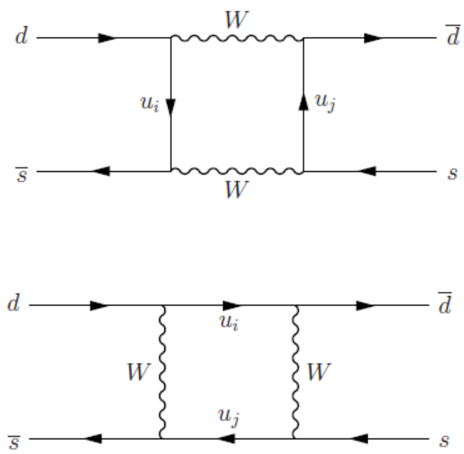

Fig. 4: The one loop diagrams for $\Delta F = 2$ FCNC.

(i) *CKM suppression:* The amplitude is proportional to a least two off-diagonal CKM matrix elements.

(ii) *GIM suppression:* The amplitude depends on the mass-squared differences between the up-type quarks.

While we derive the results based on one example, the CKM and GIM suppressions play a role in the mixing of all neutral mesons. In fact, there are three more $\Delta F = 2$ amplitudes that we should consider:

$$\begin{aligned}
\mathcal{A}_{b\bar{d}\to d\bar{b}} &\sim (V_{cb}V_{cd}^*)^2 S(x_c, x_c) + 2V_{cb}V_{cd}^*V_{tb}V_{td}^* S(x_c, x_t) + (V_{tb}V_{td}^*)^2 S(x_t, x_t), \\
\mathcal{A}_{b\bar{s}\to s\bar{b}} &\sim (V_{cb}V_{cs}^*)^2 S(x_c, x_c) + 2V_{cb}V_{cs}^*V_{tb}V_{ts}^* S(x_c, x_t) + (V_{tb}V_{ts}^*)^2 S(x_t, x_t), \\
\mathcal{A}_{c\bar{u}\to u\bar{c}} &\sim (V_{cs}^*V_{us})^2 S(x_s, x_s) + 2V_{cs}^*V_{us}V_{cb}^*V_{ub} S(x_s, x_b) + (V_{cb}^*V_{ub})^2 S(x_b, x_b),
\end{aligned} \tag{4.36}$$

which correspond to $B^0 - \overline{B}^0$, $B_s^0 - \overline{B_s}^0$, and $D^0 - \overline{D}^0$ mixing, respectively.

4.3.1 *Examples: Δm_K, Δm_B and Δm_{B_s}*

The hadronic process of $K^0 - \overline{K}^0$ mixing proceeds via the $s\bar{d} \to d\bar{s}$ quark transition, and leads to the mass splitting Δm_K between the two neutral kaon mass eigenstates. The SM calculation gives [see Eq. (A.10)]

$$\frac{|\Delta m_K|}{m_K} = \frac{g^4}{96\pi^2}\frac{m_K^2}{m_W^2}\frac{B_K f_K^2}{m_K^2}\left|(V_{cs}^*V_{cd})^2 S(x_c, x_c) + 2V_{cs}^*V_{cd}V_{ts}^*V_{td} S(x_c, x_t) + (V_{ts}^*V_{td})^2 S(x_t, x_t)\right|. \tag{4.37}$$

To estimate the relative size of the three terms, we note that

$$\frac{|V_{ts}^*V_{td}|}{|V_{cs}^*V_{cd}|} \sim 10^{-3}, \qquad \frac{S(x_c, x_t)}{S(x_c, x_c)} \sim 10, \qquad \frac{S(x_t, x_t)}{S(x_c, x_c)} \sim 10^4. \tag{4.38}$$

We conclude that the contributions of the terms proportional to $S(x_t, x_t)$ are smaller by a factor of $\mathcal{O}(100)$ than the contribution of the $S(x_c, x_c)$ term and can thus be neglected:

$$\frac{\Delta m_K}{m_K} \approx \frac{B_K f_K^2}{m_K^2} \times \frac{g^4}{96\pi^2} \times \frac{m_K^2}{m_W^2} \times |V_{cs}V_{cd}|^2 \times \frac{m_c^2}{m_W^2}. \tag{4.39}$$

The $B_K f_K^2/m_K^2 \sim O(1)$ factor encodes the QCD hadronic matrix element. The m_K^2/m_W^2 factor is related to the fact that the flavor changing processes are W-mediated, so we get the scale suppression. This factor is also present in FCCC tree level processes. The other three factors are the following:

- The $g^4/(96\pi^2)$ factor represents the one-loop suppression.
- The $|V_{cs}V_{cd}|^2$ factor represents the CKM suppression.
- The m_c^2/m_W^2 factor represents the GIM suppression.

The $B^0 - \overline{B}^0$ and $B_s - \overline{B}_s$ mixing amplitudes are given in Eqs. (A.7) and (A.8), respectively. In both cases the $S(x_t, x_t)$ is the largest of the S-functions while the CKM factors are of the same order in all three terms. We thus have

$$\frac{\Delta m_B}{m_B} \propto \frac{g^4}{96\pi^2}\left(\frac{m_t^2}{m_W^2}\right)|V_{tb}V_{td}|^2, \qquad \frac{\Delta m_{B_s}}{m_{B_s}} \propto \frac{g^4}{96\pi^2}\left(\frac{m_t^2}{m_W^2}\right)|V_{tb}V_{ts}|^2. \tag{4.40}$$

The GIM- and CKM-suppression factors are thus different among the various neutral meson systems of the down sector:

- $B_s^0 - \overline{B}_s^0$ mixing: CKM suppression by $|V_{tb}V_{ts}|^2 \sim 2 \times 10^{-3}$, and no GIM suppression;
- $B^0 - \overline{B}^0$ mixing: CKM suppression by $|V_{tb}V_{td}|^2 \sim 10^{-4}$, and no GIM suppression;
- $K^0 - \overline{K}^0$ mixing: CKM and GIM suppression by $|V_{cs}V_{cd}|^2(m_c^2/m_W^2) \sim 10^{-5}$.

We learn that the SM predicts hierarchy among the $\Delta F = 2$ processes:

$$\frac{\Delta m_K}{m_K} \ll \frac{\Delta m_B}{m_B} \ll \frac{\Delta m_{B_s}}{m_{B_s}}. \tag{4.41}$$

The experimental results,

$$\frac{\Delta m_K}{m_K} = 7.0 \times 10^{-15}, \qquad \frac{\Delta m_B}{m_B} = 6.3 \times 10^{-14}, \qquad \frac{\Delta m_{B_s}}{m_{B_s}} = 2.1 \times 10^{-12}, \tag{4.42}$$

show that this pattern is indeed realized in Nature.

4.3.2 CPV suppression

In some cases, CP violating (CPV) observables are CKM suppressed beyond their CP conserving (CPC) counterparts. Whether this is the case can be understood by examining the relevant unitarity triangle: The CPV observables depend on the area of it, while CPC observables depend on the length-squared of one side. Thus, in cases where the unitarity triangle is squashed (such as the sd and bs triangles), we can have a situation where the area of the triangle, $|J_{\rm CKM}|/2 \sim \lambda^6$, is much smaller than the length-squared of one of its sides, resulting in an extra suppression for CPV observables. Explicitly, for FCNC in the down sector, we have

$$\begin{aligned} sd: \; J_{\rm CKM}/|V_{us}V_{ud}|^2 &= \mathcal{O}(\lambda^4), \\ bs: \; J_{\rm CKM}/|V_{tb}V_{ts}|^2 &= \mathcal{O}(\lambda^2), \\ bd: \; J_{\rm CKM}/|V_{tb}V_{td}|^2 &= \mathcal{O}(1). \end{aligned} \tag{4.43}$$

CP asymmetries measure the ratios between the CPV difference between two CP-conjugate rates and the CPC sum of these rates:

- CPV in $K^0-\overline{K}^0$ mixing is the source of δ_L, the CP asymmetry in $K_L \to \pi\ell\nu$ defined in Eq. (C.13);
- CPV in the interference of $B_s - \overline{B_s}$ mixing with $b \to c\bar{c}s$ decay is the source of $\mathcal{I}m(\lambda_{\psi\phi})$, the CP asymmetry in $B_s \to \psi\phi$ defined similarly to Eq. (C.15);
- CPV in the interference of $B^0 - \overline{B}^0$ mixing with $b \to c\bar{c}d$ decay is the source of $\mathcal{I}m(\lambda_{D^+D^-})$, the CP asymmetry in $B \to D^+D^-$ defined in Eq. (C.15).

The pattern of a possible significant CP suppression in the sd sector, possible intermediate CP suppression in the bs sector, and no CP suppression in the bd sector, is manifest in the SM predictions:

$$\begin{aligned} \delta_L &\propto J_{\rm CKM}/|V_{us}V_{ud}|^2 \sim 10^{-3}, \\ \mathcal{I}m(\lambda_{\psi\phi}) &\propto J_{\rm CKM}/|V_{tb}V_{ts}|^2 \sim 10^{-2}, \\ \mathcal{I}m(\lambda_{D^+D^-}) &\propto J_{\rm CKM}/|V_{tb}V_{td}|^2 \sim 1. \end{aligned} \tag{4.44}$$

Experiments confirm this pattern:

$$\begin{aligned} \delta_L &= (3.34 \pm 0.07) \times 10^{-3}, \\ \mathcal{I}m(\lambda_{\psi\phi}) &= (5.0 \pm 2.0) \times 10^{-2}, \\ \mathcal{I}m(\lambda_{D^+D^-}) &= -0.76^{+0.15}_{-0.13}. \end{aligned} \tag{4.45}$$

4.4 Summary

Within the SM, we identify four possible suppression factors of FCNC processes relative to FCCC ones:

1. Loop suppression.
2. CKM suppression.
3. GIM suppression in processes that are not dominated by the top quark contribution.
4. CPV suppression in some of the processes related to squashed unitarity triangles.

5 Testing the CKM sector

Within the SM, the CKM matrix is the only source of flavor changing processes and of CP violation. In Section 3.2 we use only tree level processes to extract the values of CKM parameters. Here we add

FCNC to the set of CKM measurements to form a global test of the SM. The primary question is whether the long list of measurements can be fitted by the four CKM parameters.

5.1 $S_{\psi K_S}$

The CP asymmetry in $B \to \psi K_S$ decays plays a major role in testing the KM mechanism. Before we explain the test itself, we should understand why the theoretical interpretation of the asymmetry is exceptionally clean, and what are the theoretical parameters on which it depends, within and beyond the Standard Model.

The CP asymmetry in neutral B meson decays into final CP eigenstates f_{CP} is defined as follows:

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}] - d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}]}{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}] + d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}]} . \tag{5.1}$$

A detailed evaluation of this asymmetry is given in Appendix B. It leads to the following form:

$$\begin{aligned} \mathcal{A}_{f_{CP}}(t) &= S_{f_{CP}} \sin(\Delta m_B t) - C_{f_{CP}} \cos(\Delta m_B t), \\ S_{f_{CP}} &\equiv \frac{2\,\mathcal{I}m(\lambda_{f_{CP}})}{1+|\lambda_{f_{CP}}|^2}, \quad C_{f_{CP}} \equiv \frac{1-|\lambda_{f_{CP}}|^2}{1+|\lambda_{f_{CP}}|^2}, \end{aligned} \tag{5.2}$$

where

$$\lambda_{f_{CP}} = e^{-i\phi_B}(\overline{A}_{f_{CP}}/A_{f_{CP}}) . \tag{5.3}$$

Here ϕ_B refers to the phase of $M_{B\bar{B}}$ [see Eq. (C.3)]. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_B} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*) . \tag{5.4}$$

The decay amplitudes A_f and $\overline{A}_f$ are defined in Eq. (B.1).

The $B^0 \to J/\psi K^0$ decay [15, 16] proceeds via the quark transition $\bar{b} \to \bar{c}c\bar{s}$. There are contributions from both tree (t) and penguin (p^{q_u}, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 5) which carry different weak phases:

$$A_f = (V_{cb}^* V_{cs})\, t_f + \sum_{q_u=u,c,t} (V_{q_u b}^* V_{q_u s})\, p_f^{q_u} \tag{5.5}$$

(the distinction between tree and penguin contributions is a heuristic one, the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, Ref. [17]). Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations:

$$A_{\psi K} = (V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u, \tag{5.6}$$

where $T_{\psi K} = t_{\psi K} + p_{\psi K}^c - p_{\psi K}^t$ and $P_{\psi K}^u = p_{\psi K}^u - p_{\psi K}^t$. A subtlety arises in this decay that is related to the fact that $B^0 \to J/\psi K^0$ and $\overline{B}^0 \to J/\psi \overline{K}^0$. A common final state, e.g. $J/\psi K_S$, can be reached via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor corresponding to neutral K mixing,

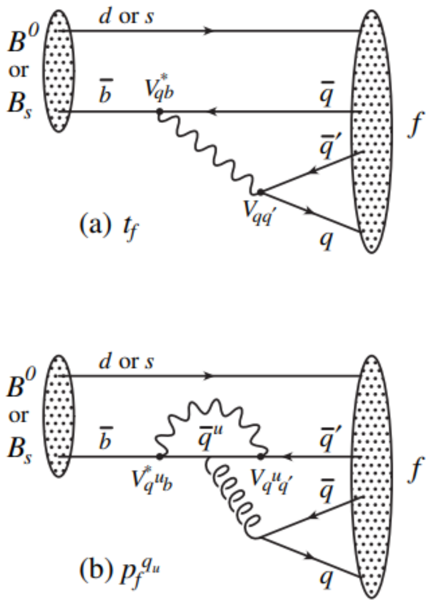

Fig. 5: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s \to f$ via a $\bar{b} \to \bar{q}q\bar{q}'$ quark-level process.

$e^{-i\phi_K} = (V_{cd}^* V_{cs})/(V_{cd} V_{cs}^*)$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb} V_{cs}^*)\, T_{\psi K} + (V_{ub} V_{us}^*)\, P_{\psi K}^u}{(V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}. \tag{5.7}$$

The crucial point is that, for $B \to J/\psi K_S$ and other $\bar{b} \to \bar{c}c\bar{s}$ processes, we can neglect the P^u contribution to $A_{\psi K}$, in the SM, to an approximation that is better than one percent:

$$|P_{\psi K}^u / T_{\psi K}| \times |V_{ub}/V_{cb}| \times |V_{us}/V_{cs}| \sim (\text{loop factor}) \times 0.1 \times 0.23 \lesssim 0.005. \tag{5.8}$$

Thus, to an accuracy better than one percent,

$$\lambda_{\psi K_S} = \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) \left(\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}\right) = -e^{-2i\beta}, \tag{5.9}$$

where β is defined in Eq. (3.22), and consequently

$$S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0 \tag{5.10}$$

(below the percent level, several effects modify this equation [18–21]).

Exercise 1: *Show that, if the $B \to \pi\pi$ decays were dominated by tree diagrams, then $S_{\pi\pi} = \sin 2\alpha$.*

Exercise 2: *Estimate the accuracy of the predictions $S_{\phi K_S} = \sin 2\beta$ and $C_{\phi K_S} = 0$.*

The experimental measurements give the following ranges [22]:

$$S_{\psi K_S} = +0.70 \pm 0.02, \quad C_{\psi K_S} = -0.005 \pm 0.015\ . \tag{5.11}$$

5.2 Is the CKM picture self-consistent?

The present status of our knowledge of the absolute values of the various entries in the CKM matrix is given in Eq. (3.5). The values there take into account all the relevant tree-level and loop processes. Yet, as explained above, the test of the SM is stronger when we reduce the above to the four CKM parameters. Indeed, the following ranges for the four Wolfenstein parameters are consistent with all measurements:

$$\lambda = 0.2265 \pm 0.0005, \qquad A = 0.790 \pm 0.015, \qquad \rho = 0.14 \pm 0.02, \qquad \eta = 0.36 \pm 0.01. \tag{5.12}$$

For the purpose of demonstration, it is useful to project the individual constraints onto the (ρ, η) plane:

– Charmless semileptonic B decays can be used to extract

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 = \lambda^2(\rho^2 + \eta^2). \tag{5.13}$$

– $B \to DK$ decays can be used to extract

$$\tan\gamma = \left(\frac{\eta}{\rho}\right). \tag{5.14}$$

– $S_{\psi K_S}$, the CP asymmetry in $B \to \psi K_S$, is used to extract

$$\sin 2\beta = \frac{2\eta(1-\rho)}{(1-\rho)^2 + \eta^2}. \tag{5.15}$$

– The CP asymmetries of various $B \to \pi\pi$, $B \to \rho\pi$, and $B \to \rho\rho$ decays depend on the phase

$$\alpha = \pi - \beta - \gamma. \tag{5.16}$$

– The ratio between the mass splittings in the B and B_s systems depends on

$$\left|\frac{V_{td}}{V_{ts}}\right|^2 = \lambda^2[(1-\rho)^2 + \eta^2] \tag{5.17}$$

– The CP violation in $K \to \pi\pi$ decays, ϵ_K, depends in a complicated way on ρ and η.

The resulting constraints are shown in Fig. 6. The consistency of the various constraints is impressive. This is a triumph of the SM in that such a variety of measurements, with different sources of uncertainties, all agree to a high precision. We conclude that the flavor structure of the SM passes a highly non-trivial test.

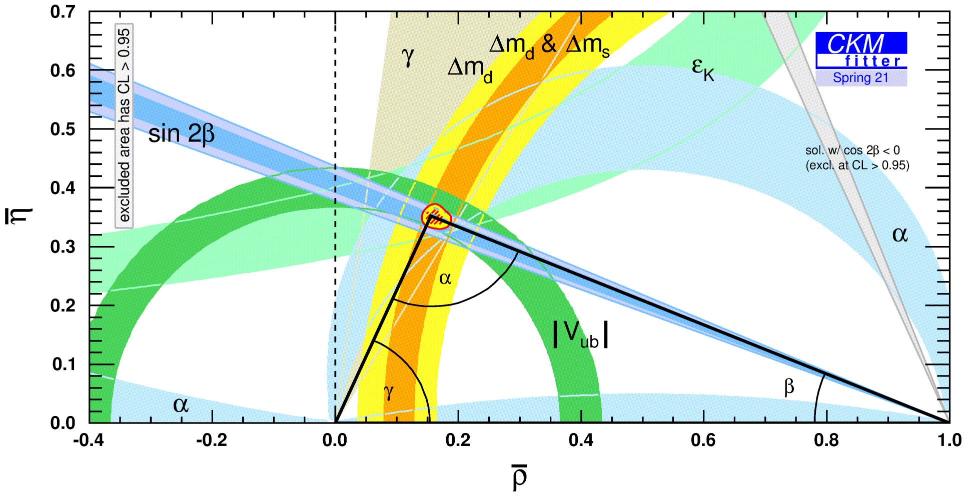

Fig. 6: Allowed region in the ρ, η plane. Superimposed are the individual constraints from charmless semileptonic B decays ($|V_{ub}|$), mass differences in the B^0 (Δm_d) and B_s (Δm_s) neutral meson systems, and CP violation in $K \to \pi\pi$ (ε_K), $B \to \psi K$ ($\sin 2\beta$), $B \to \pi\pi, \rho\pi, \rho\rho$ (α), and $B \to DK$ (γ). Taken from Ref. [10].

6 Probing BSM

In spite of the enormous experimental success of the SM, we know that the SM is not a complete theory of Nature. In this section, we explain this statement and discuss the formalism and the experimental probes to be used in case that the physics that extends the SM takes place at a high energy scale.

One obvious reason that we know that the SM is not the full theory of Nature is that it does not include gravity. There are, however, reasons to think that, beyond gravity and the SM list of elementary particles and fundamental interactions, there must exist degrees of freedom that are yet unknown to us. These reasons can be roughly divided into four classes:

1. Experiments: measurements that are inconsistent with the SM predictions.
2. Cosmology and astrophysics: observations that cannot be explained by the SM.
3. Fine-tuning: parameters whose values can be explained in the SM only with accidental fine-tuned cancellations among several contributions.
4. Clues: various non-generic features that are just parameterized in the SM, but not explained.

We elaborate on this list with specific examples in Sections 6.1 and 6.2.

Models that extend the SM by adding degrees of freedom (DoF), and often by imposing larger symmetries, come under the general name of "Beyond the SM," or BSM for short. The fact that experiments have not observed any particles related to such hypothetical new fields tells us that either these new particles are very heavy, or that their couplings to the SM particles are very weak. In light BSM scenarios, where the new DoF are at or below the weak scale, the SM is not a good low energy effective theory. Each such feebly coupled BSM scenario requires a specific discussion of how to probe it. We do not discuss such theories any further.

The situation is different for heavy BSM scenarios, where the new DoF are much above the weak scale. There is a unified framework that allows one to understand the possible probes of heavy BSM scenarios, while remaining agnostic about the details of the new degrees of freedom. We present this framework in Section 6.4 and employ it in our discussion of BSM flavor physics.

Direct searches for BSM physics aim to produce the new particles on shell and study their properties. Numerous such searches have been conducted but, as of now, no BSM particle has been discovered. Instead, these searches have set combination of lower bounds on the masses and upper bounds on the couplings of such states to SM states. Roughly speaking, the lower bounds on the masses of particles with order one couplings to the SM particles are of order 1 TeV. What sets this scale is the center of mass energy of the most powerful accelerator in action (the LHC). Indirect searches for BSM physics aim to observe virtual effects of the new states at low energies. We discuss this method below.

6.1 Experimental and observational problems

There are several experimental results and observational data that cannot be explained within the SM. They provide the most direct evidence that we need to extend the SM. Here we present the three that are the most robust.

Neutrino masses. There are several, related, pieces of experimental evidence for BSM physics from the neutrino sector. All of these measurements prove that the neutrinos are massive, in contrast to the SM prediction that they are massless. First, measurements of the ratio of ν_μ to ν_e fluxes of atmospheric neutrinos and the directional dependence of the ν_μ flux are different from the SM predictions. Both facts are beautifully explained by neutrino masses and mixing which lead to $\nu_\mu - \nu_\tau$ oscillations. Second, measurements of the solar neutrino flux find that, while the Sun produces only electron-neutrinos, their flux on Earth is significantly smaller than the total flux of neutrinos. This puzzle is beautifully explained by $\nu_e - \nu_{\mu,\tau}$ mixing. Both the atmospheric neutrino result and the solar neutrino result are now confirmed by terrestrial accelerator and reactor neutrino experiments.

The baryon asymmetry of the universe (BAU). There exists observational evidence for BSM physics from cosmology. The features of the Cosmic Microwave Background (CMB) radiation imply a certain baryon asymmetry of the Universe. Similarly, the standard Big Bang Nucleosynthesis (BBN) scenario is consistent with the observed abundances of light elements only for a certain range of the baryon asymmetry, consistent with the CMB constraint. Baryogenesis, the dynamical generation of a baryon asymmetry, requires CP violation. The CP violation in the SM generates baryon asymmetry that is smaller by at least ten orders of magnitude than the observed asymmetry. This implies that there must exist new sources of CP violation, beyond the SM. Furthermore, baryogenesis requires a departure from thermal equilibrium at a very early time after the Big Bang, and the one provided by the SM is not of the right kind.

Dark matter (DM). The evidence for dark matter—particles that are EM neutral and do not carry the color charge of the strong interactions—comes from several observations: Rotation curves in galaxies, gravitational lensing, the CMB, and the large scale structure of the Universe. The neutrinos of the SM do constitute dark matter, but their abundance is too small to be all the dark matter abundance. Thus, there must exist DoF beyond those of the SM.

6.2 Theoretical considerations

Some of the SM parameters are small. We distinguish between two classes of small parameters. "Technically natural" small parameters are those that, if set to zero, the theory gains an extra symmetry. The small parameters that are not technically natural are those where the symmetry of the theory is not enlarged when setting them to zero. An equivalent way to distinguish the two classes is based on their renormalization properties: For technically natural parameters the renormalization is multiplicative, while for non-technically natural parameters it is additive. For a technically natural parameter, loop corrections are proportional to the parameter itself, and if the parameter is set to be small at tree level, it remains small to all orders in perturbation theory. For a non-technically natural parameter, the radiative corrections are not proportional to the tree level parameter, and in cases where the radiative corrections are much larger than the measured value of the parameter, the smallness of the parameter can only be maintained by fine-tuned cancellation between the tree level and loop level contributions.

The existence in the SM of small parameters that are not technically natural is suggestive of BSM frameworks, where the smallness of the parameters is protected against large radiative corrections by some symmetry. There are two parameters of this kind in the SM: m_h^2 and $\theta_{\rm QCD}$.

The Higgs fine-tuning problem. Within the SM, the mass-squared of the Higgs μ^2, gets additive, quadratically divergent, radiative corrections. Given that the SM is an effective theory, the radiative corrections must be finite and proportional to the scale above which the SM is no longer valid. The higher the cutoff scale above the weak scale, the stronger the fine-tuned cancellation between the tree-level mass-squared term and the radiative corrections must be. In particular, if there is no BSM physics below $m_{\rm Pl}$, the bare mass-squared term and the loop contributions have to cancel each other to an accuracy of about thirty four orders of magnitude. Among the theories that aim to solve the Higgs fine-tuning problem, supersymmetry and composite Higgs have been intensively studied and searched for.

The strong CP problem. The CP violating $\theta_{\rm QCD}$ parameter contributes to the electric dipole moment of the neutron d_N. The experimental upper bound on d_N puts an upper bound on $\theta_{\rm QCD}$ of $\mathcal{O}(10^{-9})$. The smallness of $\theta_{\rm QCD}$ is not technically natural. Among the theories that aim to solve the strong CP problem, the Peccei–Quinn mechanism, and its prediction of an ultra-light pseudoscalar, the axion, have been intensively studied and searched for.

Other features of the SM parameters provide hints for the existence of BSM physics because they are non-generic, but they are not related to non-technically natural small parameters. We mention two of them below.

The flavor parameters. The Yukawa couplings are small (except for y_t) and hierarchical. For example, the electron Yukawa is of $\mathcal{O}(10^{-5})$. These are technically natural small numbers, but their non-generic structure—smallness and hierarchy—is suggestive of BSM physics. Among the theories that aim to solve this puzzle, the Froggatt–Nielsen mechanism, $U(2)$ flavor models, and models of extra dimensions, have been intensively studied.

Grand unification. The three gauge couplings of the strong, weak, and electromagnetic interactions seem to converge to a unified value at a high energy scale. The SM cannot explain this fact, which is just accidental within this model. Yet, it can be explained if the gauge group of the SM is part of a larger simple group. This idea is called Grand Unified Theory, or GUT, and among the relevant unifying groups, $SU(5)$ and $SO(10)$ have been intensively studied.

6.3 The BSM scale

The SM has a single mass scale that we call the weak scale and denote by $\Lambda_{\rm EW}$. It can be represented by the masses of the weak force carriers, m_W or m_Z, or by the VEV of the Higgs field, v. As an order of magnitude estimate, we take $\Lambda_{\rm EW} \sim 10^2$ GeV.

Some of the problems of the SM presented above are suggestive of where the BSM scale lies. We present these well-motivated scales in decreasing order. Of course, there could be more than one scale for the BSM physics.

The Planck scale, $m_{\rm Pl} \sim 10^{19}$ **GeV**. The Planck scale constitutes a cut-off scale of all QFTs. At this scale, gravitational effects become as important as the known gauge interactions and cannot be neglected.

The GUT scale, $\Lambda_{\rm GUT} \sim 10^{16}$ **GeV**. The GUT scale is the one where the three gauge couplings of the SM roughly unify. It is an indication that at that scale, the GUT symmetry group is broken into the SM symmetry group. For example, in $SU(5)$ GUT models, $\Lambda_{\rm GUT}$ can be represented by the VEV of the scalar field that breaks $SU(5) \to SU(3) \times SU(2) \times U(1)$, or by the masses of the gauge bosons that correspond to the broken $SU(5)$ generators.

The seesaw scale, $\Lambda_\nu \sim 10^{15}$ **GeV**. The value of the neutrino masses m_ν hints that new degrees of freedom appear at or below the so-called seesaw scale, $\Lambda_\nu \sim v^2/m_\nu$. This scale is intriguingly close to the GUT scale, and thus the two might be in fact related to the same BSM physics.

The Higgs fine-tuning scale $\Lambda_{\rm FT} \sim 1$ **TeV**. No fine tuning is necessary to explain the smallness of m_h^2 if radiative corrections are cut-off at a scale $\Lambda_{\rm FT}$ of order $4\pi m_h/y_t \sim$ TeV.

The WIMP scale $\Lambda_{\rm DM} \sim 1$ **TeV**. If the DM particles are weakly interacting massive particles (WIMPs), the cross section of their annihilation that is required to explain the DM abundance is of order $1/(20~{\rm TeV})^2$. If the relevant coupling is of order α_W, the relevant scale is of order 1 TeV.

6.4 The SMEFT

As argued above, the SM is not a full theory of Nature. If the BSM degrees of freedom are at a scale $\Lambda \gg \Lambda_{\rm EW}$, then the SM is a good low energy effective theory which is valid below Λ. In such a case, the SM Lagrangian should be extended to include all nonrenormalizable terms, suppressed by powers of Λ:

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda} O_{d=5} + \frac{1}{\Lambda^2} O_{d=6} + \cdots . \tag{6.1}$$

Here $\mathcal{L}_{\rm SM}$ is the renormalizable SM Lagrangian and $O_{d=n}$ represents operators that are products of SM fields, of overall dimension n in the fields, and transforming as singlets under the SM gauge group. The SM extended to include such non-renormalizable term is called the SM effective field theory, or SMEFT for short. For physics at an energy scale E well below Λ, the effects of operators of dimension $n > 4$ are suppressed by $(E/\Lambda)^{n-4}$. Thus, in general, the higher the dimension of an operator, the smaller its effect at low energies.

Nonrenormalizable operators are generated by extensions of the SM, which introduce new degrees of freedom that are much heavier than the electroweak scale. By studying nonrenormalizable operators, we allow the most general extension of the SM and remain agnostic about its specific structure. At the same time, constraints on nonrenormalizable terms can be translated into constraints on specific BSM models.

The low energy effects of nonrenormalizable operators are small. Thus, when we study them, we have to consider also loop effects in the SM. We can classify the effects of including loop corrections and nonrenormalizable terms into three broad categories:

1. *Forbidden processes:* Various processes are forbidden by the accidental symmetries of the SM. Nonrenormalizable terms, but not loop corrections, can break these accidental symmetries and allow the forbidden processes to occur. Examples include neutrino masses, and proton decay. In particular, neutrino masses and mixing violate the lepton flavor symmetries of the SM.
2. *Rare processes:* Within the SM, various processes that are not forbidden do not occur at tree level. Here both loop corrections and nonrenormalizable terms can contribute. Examples include FCNC processes.
3. *Tree level processes:* Often tree level processes in a particular sector depend on a small subset of the SM parameters. This situation leads to relations among different processes within this sector. These relations are violated by both loop effects and nonrenormalizable terms. Here, precision measurements and precision theory calculations are needed to observe these small effects. Examples include electroweak precision measurements.

As concerns the last two types of effects, where loop corrections and nonrenormalizable terms may both contribute, their use in phenomenology can be divided into two eras. Before all the SM particles have been directly discovered and all the SM parameters measured, one could assume the validity of the renormalizable SM and indirectly measure the properties of the yet unobserved SM particles. Indeed, the masses of charm quark, the top quark, and the Higgs boson were first indirectly measured in this way. Once all the SM particles have been observed and the parameters measured directly, the loop corrections can be quantitatively determined, and the effects of nonrenormalizable terms in the SMEFT can be unambiguously probed. Thus, at present, all three classes of processes serve to search for BSM physics.

In this section, we go beyond testing the self-consistency of the CKM picture of flavor physics and CP violation. The aim is to quantify how much room is left for BSM physics in the flavor sector and to translate these constraints into lower bounds on the scale of higher-dimension flavor-violating operators in the SMEFT. We make the following working assumption:

– The contribution of new physics to FCCC processes, where the SM contributions are tree-level, can be neglected.

On the other hand, we allow BSM physics of arbitrary size and phase to contribute to FCNC processes.

6.5 New physics contributions to $B^0 - \overline{B^0}$ mixing

We consider BSM effects in the FCNC process of $B^0 - \overline{B^0}$ mixing, which plays a role in the mass splitting Δm_B and in the CP asymmetry $S_{\psi K_S}$. The SM amplitude is given by Eq. (A.7). The modification of the mixing amplitude by general BSM physics can be parameterized as follows:

$$M_{B\overline{B}} = M^{\rm SM}_{B\overline{B}}\Delta_d, \tag{6.2}$$

where Δ_d is a dimensionless complex parameter. BSM physics will be signalled by $\Delta_d \neq 1$. Our aim is to find the phenomenological constraints on Δ_d.

Our first step is to use all relevant tree level processes which, under our assumption, can be used to determine the CKM parameters. This was done in Section 3.2 and the results of this fit were shown in Fig. 2. Our second step is to use $\Delta B = 2$ processes to determine Δ_d:

– The mass splitting between the two neutral B-mesons is given by

$$\Delta m_B = 2|M^{\rm SM}_{B\overline{B}}(\rho,\eta)| \times |\Delta_d|. \tag{6.3}$$

– The CP asymmetry in $B \to \psi K_S$ is given by

$$S_{\psi K_S} = \sin\left[2\mathrm{arctan}\left(\frac{\eta}{1-\rho}\right) + \arg(\Delta_d)\right]. \tag{6.4}$$

The results of the fit are (see Fig. 7)

$$\mathcal{R}e(\Delta_d) = +0.94^{+0.18}_{-0.15}, \qquad \mathcal{I}m(\Delta_d) = -0.11^{+0.11}_{-0.05}. \tag{6.5}$$

We learn that BSM physics can contribute to the $B^0 - \overline{B}^0$ mixing amplitude up to about 20% of the SM contribution.

Analogous upper bounds can be obtained for BSM contributions to the $K^0 - \overline{K}^0$ and $B^0_s - \overline{B}^0_s$ mixing amplitudes.

6.6 Probing the SMEFT

Assuming that new degrees of freedom that have flavor changing couplings to quarks are much heavier than the electroweak breaking scale, their effects on low energy processes, such as neutral meson mixing, can be presented as higher dimension operators. Then, bounds such as Eq. (6.5), constrain the coefficients of such operators.

Consider a simple example, where we have a single dimension-six operator that contributes to Δ_d:

$$\frac{z_{bd}}{\Lambda^2}(\overline{Q_{Ld}}\gamma_\mu Q_{Lb})(\overline{Q_{Ld}}\gamma^\mu Q_{Lb}). \tag{6.6}$$

where Q_{Ld} and Q_{Lb} are the $SU(2)$-doublet quark fields whose $T_3 = -1/2$ members are the d_L and b_L fields [see Eq. (2.21)], and where we separated the coefficient into a dimensionless complex coupling, z_{bd}, and a high energy scale, Λ. We further define $\tilde\Lambda = \Lambda/\sqrt{z_{bd}}$. We consider the bound that can be obtained from Δm_B. Comparing Eqs. (A.1) and (6.6), we obtain

$$|\Delta_d| - 1 = \left|\frac{1}{\tilde\Lambda^2 C_{\rm SM}}\right| \approx \frac{1}{|\tilde\Lambda^2|} \times \frac{2\pi^2}{|V^*_{td}V_{tb}|^2 S(x_t,x_t)G_F^2 m_W^2}. \tag{6.7}$$

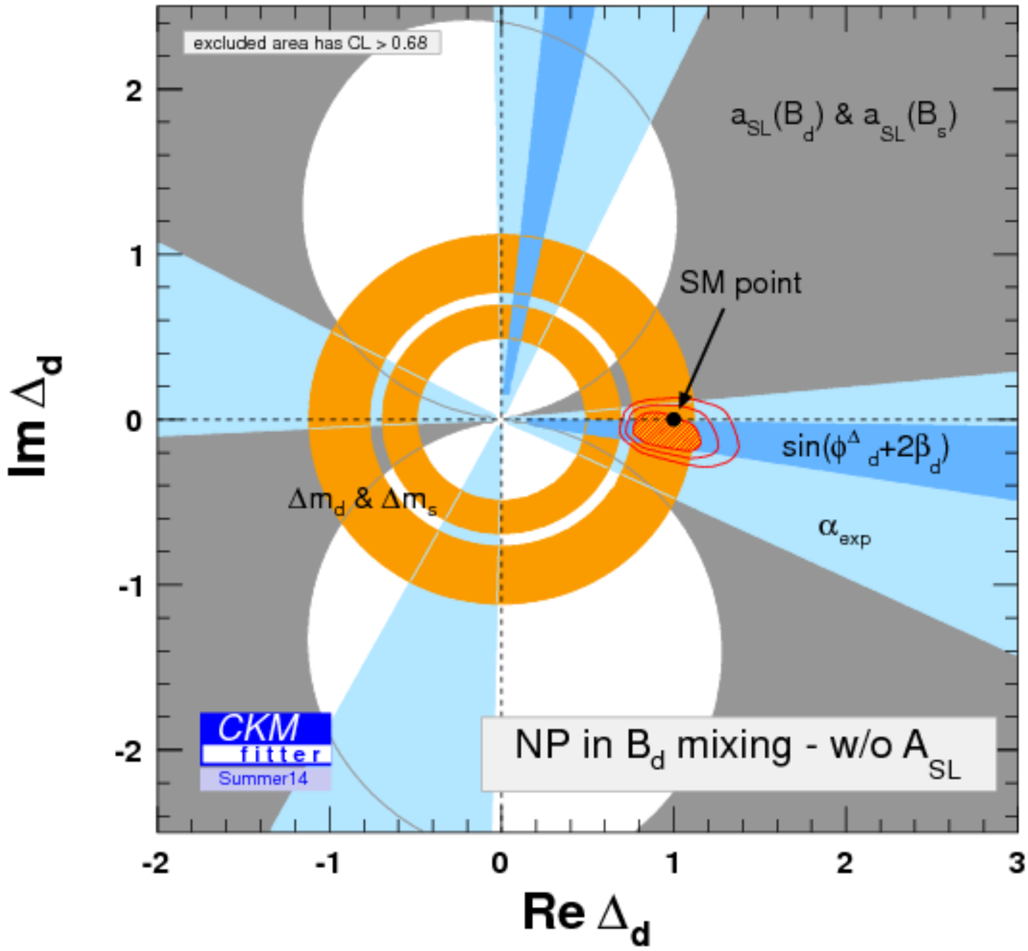

Fig. 7: Allowed region in the $(\mathcal{R}e\Delta_d, \mathcal{I}m\Delta_d)$ plane. Superimposed are the individual constraints from the mass differences in the B^0 (Δm_d), CP violation in $B \to \psi K$ ($S_{\psi K}$), and CP violation in semileptonic B^0 decay ($a_{\rm SL}$). Taken from Ref. [10].

The bound of Eq. (6.5) translates into a lower bound on $\tilde\Lambda$:

$$\tilde\Lambda \gtrsim 10^3 \text{ TeV}. \tag{6.8}$$

Using $S_{\psi K_S}$, one can obtain analogous bounds for various operators that contribute to CP violation in $B^0 - \overline{B}^0$ mixing. We can also obtain bounds for operators that affect other $\Delta F = 2$ processes. Some of these bounds are given in Table 4.

The following points are worth emphasizing:

1. BSM physics can contribute to FCNC at a level comparable to the SM contributions even if it takes place at a scale that is five orders of magnitude above the electroweak scale.
2. If the BSM physics has a generic flavor structure, that is $z_{ij} = \mathcal{O}(1)$, then its scale must be above 10^4 TeV.

Table 4: Lower bounds from CPC and CPV $\Delta F = 2$ processes on the scale of new physics $\tilde{\Lambda}$, for $(\overline{Q_{Li}}\gamma_\mu Q_{Lj})(\overline{Q_{Li}}\gamma^\mu Q_{Lj})$ operators.

i,j	$\tilde{\Lambda}$ [TeV] CPC	$\tilde{\Lambda}$ [TeV] CPV	Observables
s,d	9.8×10^2	1.6×10^4	Δm_K; ϵ_K
b,d	6.6×10^2	9.3×10^2	Δm_B; $S_{\psi K}$
b,s	1.4×10^2	2.5×10^2	Δm_{B_s}; $S_{\psi\phi}$

3. It could be that there are new particles with mass of order a TeV, but then their flavor structure must be far from generic, $|z_{ij}| \ll 1$.
4. The pattern of the bounds—those from K^0 are stronger than those from B^0 which are stronger than those from B_s—is directly related to the strength of the flavor (CKM and GIM) suppression we have in the SM, as discussed in Section 4.3.1. The reason for that is that the experimental accuracy and the QCD uncertainties are similar for the three cases.

7 The new physics flavor puzzle

7.1 A model independent discussion

Given that the SM is only an effective low energy theory, non-renormalizable terms must be added to $\mathcal{L}_{\rm SM}$. These are terms of dimension higher than four in the fields which, therefore, have couplings that are inversely proportional to the scale of new physics $\Lambda_{\rm NP}$.

The lowest dimension non-renormalizable terms are dimension-five:

$$-\mathcal{L}^{\rm dim-5}_{\rm Seesaw} = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} L_{Li} L_{Lj} \phi\phi + {\rm h.c.}. \tag{7.1}$$

These are the seesaw terms, leading to neutrino masses.

Exercise 3: *How does the global symmetry breaking pattern in Eq. (2.47) change when Eq. (7.1) is taken into account?*

Exercise 4: *What is the number of physical lepton flavor parameters in this case? Identify these parameters in the mass basis.*

As concerns quark flavor physics, consider, for example, the following dimension-six set of operators:

$$\mathcal{L}^{\rm dim-6}_{\Delta F=2} = \sum_{i\neq j} \frac{z_{ij}}{\Lambda^2} (\overline{Q_{Li}}\gamma_\mu Q_{Lj})^2, \tag{7.2}$$

where the z_{ij} are dimensionless couplings. These terms contribute to the mass splittings between the corresponding two neutral mesons. As discussed in the previous section, the consistency of the experimental results with the SM predictions for neutral meson mixing allows us to impose the condition

$|M_{P\bar{P}}^{\rm NP}| < |M_{P\bar{P}}^{\rm SM}|$ for $P = K, B, B_s$, which implies that

$$\Lambda > \frac{3.4 \text{ TeV}}{|V_{ti}^* V_{tj}|/|z_{ij}|^{1/2}} \sim \begin{cases} 9 \times 10^3 \text{ TeV} \times |z_{sd}|^{1/2} \\ 4 \times 10^2 \text{ TeV} \times |z_{bd}|^{1/2} \\ 7 \times 10^1 \text{ TeV} \times |z_{bs}|^{1/2} \end{cases} \tag{7.3}$$

The first lesson that we draw from these bounds on Λ is that new physics can contribute to FCNC at a level comparable to the SM contributions even if it takes place at a scale that is six orders of magnitude above the electroweak scale. A second lesson is that if the new physics has a generic flavor structure, that is $z_{ij} = \mathcal{O}(1)$, then its scale must be above $10^4 - 10^5$ TeV (or, if the leading contributions involve electroweak loops, above $10^3 - 10^4$ TeV). *If indeed $\Lambda \gg$ TeV, it means that we have misinterpreted the hints from the fine-tuning problem and the dark matter puzzle.*

A different lesson can be drawn from the bounds on z_{ij}. *It could be that the scale of new physics is of order TeV, but its flavor structure is far from generic.* Specifically, if new particles at the TeV scale couple to the SM fermions, then there are two ways in which their contributions to FCNC processes, such as neutral meson mixing, can be suppressed: degeneracy and alignment. Either of these principles, or a combination of both, signifies non-generic structure.

One can use the language of effective operators also for the SM, integrating out all particles significantly heavier than the neutral mesons (that is, the top, the Higgs and the weak gauge bosons). Thus, the scale is $\Lambda_{\rm SM} \sim m_W$. Since the leading contributions to neutral meson mixings come from box diagrams, the z_{ij} coefficients are suppressed by α_2^2. To identify the relevant flavor suppression factor, one can employ the spurion formalism. For example, the flavor transition that is relevant to $B^0 - \overline{B}^0$ mixing involves $\overline{d_L} b_L$ which transforms as $(8,1,1)_{SU(3)_q^3}$. The leading contribution must then be proportional to $(Y^u Y^{u\dagger})_{13} \propto y_t^2 V_{tb} V_{td}^*$. Indeed, an explicit calculation (using VIA for the matrix element and neglecting QCD corrections) gives[3]

$$\frac{2M_{B\bar{B}}}{m_B} \approx -\frac{\alpha_2^2}{12} \frac{f_B^2}{m_W^2} S_0(x_t) (V_{tb} V_{td}^*)^2, \tag{7.4}$$

where $x_i = m_i^2/m_W^2$ and

$$S_0(x) = \frac{x}{(1-x)^2} \left[1 - \frac{11x}{4} + \frac{x^2}{4} - \frac{3x^2 \ln x}{2(1-x)} \right]. \tag{7.5}$$

Similar spurion analyses, or explicit calculations, allow us to extract the weak and flavor suppression factors that apply in the SM:

$$\begin{aligned} \mathcal{I}m(z_{sd}^{\rm SM}) &\sim \alpha_2^2 y_t^2 |V_{td} V_{ts}|^2 \sim 1 \times 10^{-10}, \\ z_{sd}^{\rm SM} &\sim \alpha_2^2 y_c^2 |V_{cd} V_{cs}|^2 \sim 5 \times 10^{-9}, \\ \mathcal{I}m(z_{cu}^{\rm SM}) &\sim \alpha_2^2 y_b^2 |V_{ub} V_{cb}|^2 \sim 2 \times 10^{-14}, \\ z_{bd}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{td} V_{tb}|^2 \sim 7 \times 10^{-8}, \\ z_{bs}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{ts} V_{tb}|^2 \sim 2 \times 10^{-6}. \end{aligned} \tag{7.6}$$

[3] A detailed derivation can be found in Appendix B of Ref. [23].

(We did not include $z_{cu}^{\rm SM}$ in the list because it requires a more detailed consideration. The naively leading short distance contribution is $\propto \alpha_2^2(y_s^4/y_c^2)|V_{cs}V_{us}|^2 \sim 5\times10^{-13}$. However, higher dimension terms can replace a y_s^2 factor with $(\Lambda/m_D)^2$ [24]. Moreover, long distance contributions are expected to dominate. In particular, peculiar phase space effects [25, 26] have been identified which are expected to enhance Δm_D to within an order of magnitude of the its measured value. The CP violating part, on the other hand, is dominated by short distance physics.)

It is clear then that contributions from new physics at $\Lambda_{\rm NP} \sim 1$ TeV should be suppressed by factors that are comparable or smaller than the SM ones. Why does that happen? This is the new physics flavor puzzle.

The fact that the flavor structure of new physics at the TeV scale must be non-generic means that flavor measurements are a good probe of the new physics. Perhaps the best-studied example is that of supersymmetry. Here, the spectrum of the superpartners and the structure of their couplings to the SM fermions will allow us to probe the mechanism of dynamical supersymmetry breaking.

7.2 Lessons for supersymmetry from neutral meson mixing

We consider, as an example, the contributions from the box diagrams involving the squark doublets of the second and third generations, $\tilde{Q}_{L2,3}$, to the $B_s-\overline{B_s}$ mixing amplitude. The contributions are proportional to $K_{3i}^{d*}K_{2i}^dK_{3j}^{d*}K_{2j}^d$, where K^d is the mixing matrix of the gluino couplings to a left-handed down quark and their supersymmetric squark partners ($\propto [(\delta_{LL}^d)_{23}]^2$ in the mass insertion approximation). We work in the mass basis for both quarks and squarks. A detailed derivation can be found in Ref. [27]. It gives:

$$M_{B_s\overline{B_s}} = \frac{\alpha_s^2 m_{B_s} f_{B_s}^2 B_{B_s}\eta_{\rm QCD}}{108 m_{\tilde{d}}^2}[11\tilde{f}_6(x)+4xf_6(x)]\frac{(\Delta\tilde{m}_{\tilde{d}}^2)^2}{\tilde{m}_{\tilde{d}}^4}(K_{32}^{d*}K_{22}^d)^2. \tag{7.7}$$

Here $m_{\tilde{d}}$ is the average mass of the two squark generations, $\Delta m_{\tilde{d}}^2$ is the mass-squared difference, and $x = m_{\tilde{g}}^2/m_{\tilde{d}}^2$.

Equation (7.7) can be translated into our generic language:

$$\Lambda_{\rm NP} = m_{\tilde{q}}, \tag{7.8}$$

$$z_1^{bs} = \frac{11\tilde{f}_6(x)+4xf_6(x)}{18}\alpha_s^2\left(\frac{\Delta\tilde{m}_{\tilde{d}}^2}{m_{\tilde{d}}^2}\right)^2(K_{32}^{d*}K_{22}^d)^2 \approx 10^{-4}(\delta_{23}^{LL})^2,$$

where, for the last approximation, we took the example of $x = 1$ [and used, correspondingly, $11\tilde{f}_6(1) + 4f_6(1) = 1/6$], and defined

$$\delta_{23}^{LL} = \left(\frac{\Delta\tilde{m}_{\tilde{d}}^2}{m_{\tilde{d}}^2}\right)(K_{32}^{d*}K_{22}^d). \tag{7.9}$$

Similar expressions can be derived for the dependence of $K^0-\overline{K^0}$ on $(\delta_{MN}^d)_{12}$, $B^0-\overline{B^0}$ on $(\delta_{MN}^d)_{13}$, and $D^0-\overline{D^0}$ on $(\delta_{MN}^u)_{12}$. Then we can use the constraints of Table 4 to put upper bounds on $(\delta_{MN}^q)_{ij}$. Some examples are given in Table 5 (see Ref. [28] for details and list of references).

We learn that, in most cases, we need $\delta_{ij}^q/m_{\tilde{q}} \ll 1/\text{TeV}$. One can immediately identify three generic ways in which supersymmetric contributions to neutral meson mixing can be suppressed:

Table 5: The phenomenological upper bounds on $(\delta^q_{LL})_{ij}$ and $\langle\delta^q_{ij}\rangle = \sqrt{(\delta^q_{LL})_{ij}(\delta^q_{RR})_{ij}}$. Here $q = u, d$ and $M = L, R$. The constraints are given for $m_{\tilde{q}} = 1$ TeV and $x = m^2_{\tilde{g}}/m^2_{\tilde{q}} = 1$. We assume that the phases could suppress the imaginary part by a factor of ~ 0.3. Taken from Ref. [28].

q	ij	$(\delta^q_{LL})_{ij}$	$\langle\delta^q_{ij}\rangle$
d	12	0.03	0.002
d	13	0.2	0.07
d	23	0.2	0.07
u	12	0.1	0.008

1. Heaviness: $m_{\tilde{q}} \gg 1$ TeV;
2. Degeneracy: $\Delta m^2_{\tilde{q}} \ll m^2_{\tilde{q}}$;
3. Alignment: $K^q_{ij} \ll 1$.

When heaviness is the only suppression mechanism, as in split supersymmetry [29], the squarks are very heavy and supersymmetry no longer solves the fine tuning problem. If we want to maintain supersymmetry as a solution to the fine tuning problem, either degeneracy or alignment or a combination of both is needed. This means that the flavor structure of supersymmetry is not generic, as argued in the previous section.

Take, for example, $(\delta^d_{LL})_{12} \leq 0.03$. Naively, one might expect the alignment to be of order $(V_{cd}V^*_{cs}) \sim 0.2$, which is far from sufficient by itself. Barring a very precise alignment ($|K^d_{12}| \ll |V_{us}|$) and accidental cancellations, we are led to conclude that the first two squark generations must be quasi-degenerate. Actually, by combining the constraints from $K^0 - \overline{K^0}$ mixing and $D^0 - \overline{D^0}$ mixing, one can show that this is the case independently of assumptions about the alignment [30–32]. Analogous conclusions can be drawn for many TeV-scale new physics scenarios: a strong level of degeneracy is required (for definitions and detailed analysis, see Ref. [33]).

Exercise 5: *Does $K^d_{31} \sim |V_{ub}|$ suffice to satisfy the Δm_B constraint with neither degeneracy nor heaviness? (Use the two generation approximation and ignore the second generation.)*

Is there a natural way to make the squarks degenerate? Degeneracy requires that the 3×3 matrix of soft supersymmetry breaking mass-squared terms $\tilde{m}^2_{Q_L} \simeq \tilde{m}^2_{\tilde{q}}\mathbf{1}$. We have mentioned already that flavor universality is a generic feature of gauge interactions. Thus, the requirement of degeneracy is perhaps a hint that supersymmetry breaking is *gauge mediated* to the MSSM fields.

7.3 Minimal flavor violation (MFV)

If supersymmetry breaking is gauge mediated, the squark mass matrices for $SU(2)_L$-doublet and $SU(2)_L$-singlet squarks have the following form at the scale of mediation m_M:

$$\begin{aligned}\tilde{M}^2_{U_L}(m_M) &= \left(m^2_{\tilde{Q}_L} + D_{U_L}\right)\mathbf{1} + M_u M_u^\dagger,\\ \tilde{M}^2_{D_L}(m_M) &= \left(m^2_{\tilde{Q}_L} + D_{D_L}\right)\mathbf{1} + M_d M_d^\dagger,\end{aligned}$$

$$\begin{aligned} \tilde{M}^2_{U_R}(m_M) &= \left(m^2_{\tilde{U}_R} + D_{U_R}\right)\mathbf{1} + M_u^\dagger M_u, \\ \tilde{M}^2_{D_R}(m_M) &= \left(m^2_{\tilde{D}_R} + D_{D_R}\right)\mathbf{1} + M_d^\dagger M_d, \end{aligned} \tag{7.10}$$

where $D_{q_A} = [(T_3)_{q_A} - (Q_{\rm EM})_{q_A} s_W^2] m_Z^2 \cos 2\beta$ are the D-term contributions. Here, the only source of the $SU(3)_q^3$ breaking are the SM Yukawa matrices.

This statement holds also when the renormalization group evolution is applied to find the form of these matrices at the weak scale. Taking the scale of the soft breaking terms $m_{\tilde{q}_A}$ to be somewhat higher than the electroweak breaking scale m_Z allows us to neglect the D_{q_A} and M_q terms in Eq. (7.10). Then we obtain

$$\begin{aligned} \tilde{M}^2_{Q_L}(m_Z) &\sim m^2_{\tilde{Q}_L}\left(r_3\mathbf{1} + c_u Y^u Y^{u\dagger} + c_d Y^d Y^{d\dagger}\right), \\ \tilde{M}^2_{U_R}(m_Z) &\sim m^2_{\tilde{U}_R}\left(r_3\mathbf{1} + c_{uR} Y^{u\dagger} Y^u\right), \\ \tilde{M}^2_{D_R}(m_Z) &\sim m^2_{\tilde{D}_R}\left(r_3\mathbf{1} + c_{dR} Y^{d\dagger} Y^d\right). \end{aligned} \tag{7.11}$$

Here r_3 represents the universal RGE contribution that is proportional to the gluino mass ($r_3 = \mathcal{O}(6) \times (M_3(m_M)/m_{\tilde{q}}(m_M))$) and the c-coefficients depend logarithmically on m_M/m_Z and can be of $\mathcal{O}(1)$ when m_M is not far below the GUT scale.

Models of gauge mediated supersymmetry breaking (GMSB) provide a concrete example of a large class of models that obey a simple principle called *minimal flavor violation* (MFV) [34]. This principle guarantees that low energy flavor changing processes deviate only very little from the SM predictions. The basic idea can be described as follows. The gauge interactions of the SM are universal in flavor space. The only breaking of this flavor universality comes from the three Yukawa matrices, Y^u, Y^d and Y^e. If this remains true in the presence of the new physics, namely Y^u, Y^d and Y^e are the only flavor non-universal parameters, then the model belongs to the MFV class.

Let us now formulate this principle in a more formal way, using the language of spurions that we presented in section 2.4. The Standard Model with vanishing Yukawa couplings has a large global symmetry, see Eqs. (2.44, 2.45). In this section we concentrate only on the quarks. The non-Abelian part of the flavor symmetry for the quarks is $SU(3)_q^3$ of Eq. (2.45) with the three generations of quark fields transforming as follows:

$$Q_L(3,1,1), \quad U_R(1,3,1), \quad D_R(1,1,3). \tag{7.12}$$

The Yukawa interactions,

$$\mathcal{L}^q_{\rm Yuk} = \overline{Q_L} Y^d D_R H + \overline{Q_L} Y^u U_R H_c, \tag{7.13}$$

($H_c = i\tau_2 H^*$) break this symmetry. The Yukawa couplings can thus be thought of as spurions with the following transformation properties under $SU(3)_q^3$ [see Eq. (2.48)]:

$$Y^u \sim (3, \bar{3}, 1), \qquad Y^d \sim (3, 1, \bar{3}). \tag{7.14}$$

When we say "spurions", we mean that we pretend that the Yukawa matrices are fields which transform under the flavor symmetry, and then require that all the Lagrangian terms, constructed from the SM fields, Y^d and Y^u, must be (formally) invariant under the flavor group $SU(3)^3_q$. Of course, in reality, $\mathcal{L}^q_{\rm Yuk}$ breaks $SU(3)^3_q$ precisely because $Y^{d,u}$ are *not* fields and do not transform under the symmetry.

The idea of minimal flavor violation is relevant to extensions of the SM, and can be applied in two ways:

1. If we consider the SM as a low energy effective theory, then all higher-dimension operators, constructed from SM-fields and Y-spurions, are formally invariant under $G_{\rm global}$.
2. If we consider a full high-energy theory that extends the SM, then all operators, constructed from SM and the new fields, and from Y-spurions, are formally invariant under $G_{\rm global}$.

Exercise 6: *Use the spurion formalism to argue that, in MFV models, the $K_L \to \pi^0\nu\bar\nu$ decay amplitude is proportional to $y_t^2 V_{td}V_{ts}^*$.*

Exercise 7: *Find the flavor suppression factors in the z_i^{bs} coefficients, if MFV is imposed, and compare to the bounds in Table 4.*

Examples of MFV models include models of supersymmetry with gauge-mediation or with anomaly-mediation of its breaking.

8 The Standard Model flavor puzzle

The SM has thirteen flavor parameters: six quark Yukawa couplings, four CKM parameters (three angles and a phase), and three charged lepton Yukawa couplings (one can use fermions masses instead of the fermion Yukawa couplings, $y_f = \sqrt{2}m_f/v$). The orders of magnitudes of these thirteen dimensionless parameters are as follows:

$$\begin{aligned} y_t &\sim 1, \quad y_c \sim 10^{-2}, \quad y_u \sim 10^{-5}, \\ y_b &\sim 10^{-2}, \quad y_s \sim 10^{-3}, \quad y_d \sim 10^{-4}, \\ y_\tau &\sim 10^{-2}, \quad y_\mu \sim 10^{-3}, \quad y_e \sim 10^{-6}, \\ |V_{us}| &\sim 0.2, \quad |V_{cb}| \sim 0.04, \quad |V_{ub}| \sim 0.004, \quad \delta_{\rm KM} \sim 1. \end{aligned} \tag{8.1}$$

Only two of these parameters are clearly of $\mathcal{O}(1)$, the top-Yukawa and the KM phase. The other flavor parameters exhibit smallness and hierarchy. Their values span six orders of magnitude. It may be that this set of numerical values are just accidental. More likely, the smallness and the hierarchy have a reason. The question of why there is smallness and hierarchy in the SM flavor parameters constitutes "The Standard Model flavor puzzle."

The motivation to think that there is indeed a structure in the flavor parameters is strengthened by considering the values of the four SM parameters that are not flavor parameters, namely the three gauge couplings and the Higgs self-coupling:

$$g_s \sim 1, \quad g \sim 0.6, \quad e \sim 0.3, \quad \lambda \sim 0.12. \tag{8.2}$$

This set of values does seem to be a random distribution of order-one numbers, as one would naively expect.

A few examples of mechanisms that were proposed to explain the observed structure of the flavor parameters are the following:

- An approximate Abelian symmetry (“The Froggatt–Nielsen mechanism" [35]);
- An approximate non-Abelian symmetry (see e.g. Ref. [36]);
- Conformal dynamics (“The Nelson–Strassler mechanism" [37]);
- Location in an extra dimension [38];
- Loop corrections (see e.g. Ref. [39]).

We take as an example the Froggatt–Nielsen mechanism.

8.1 The Froggatt–Nielsen (FN) mechanism

Small numbers and hierarchies are often explained by approximate symmetries. For example, the small mass splitting between the charged and neutral pions finds an explanation in the approximate isospin (global $SU(2)$) symmetry of the strong interactions.

Approximate symmetries lead to selection rules which account for the size of deviations from the symmetry limit. Spurion analysis is particularly convenient to derive such selection rules. The Froggatt–Nielsen mechanism postulates a $U(1)_H$ symmetry, that is broken by a small spurion ϵ_H. Without loss of generality, we assign ϵ_H a $U(1)_H$ charge of $H(\epsilon_H) = -1$. Each SM field is assigned a $U(1)_H$ charge. In general, different fermion generations are assigned different charges, hence the term ‘horizontal symmetry’. The rule is that each term in the Lagrangian, made of SM fields and the spurion, should be formally invariant under $U(1)_H$.

The approximate $U(1)_H$ symmetry thus leads to the following selection rules:

$$\begin{aligned} Y^u_{ij} &= \epsilon_H^{|H(\bar{Q}_i)+H(U_j)+H(\phi_u)|}, \\ Y^d_{ij} &= \epsilon_H^{|H(\bar{Q}_i)+H(D_j)+H(\phi_d)|}, \\ Y^e_{ij} &= \epsilon_H^{|H(\bar{L}_i)+H(E_j)-H(\phi_d)|}. \end{aligned} \tag{8.3}$$

As a concrete example, we take the following set of charges:

$$\begin{aligned} H(\bar{Q}_i) &= H(U_i) = H(E_i) = (2,1,0), \\ H(\bar{L}_i) &= H(D_i) = (0,0,0), \\ H(\phi_u) &= H(\phi_d) = 0. \end{aligned} \tag{8.4}$$

It leads to the following parametric suppressions of the Yukawa couplings:

$$Y^u \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \quad Y^d \sim (Y^e)^T \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix}. \tag{8.5}$$

We emphasize that for each entry we give the parametric suppression (that is the power of ϵ), but each entry has an unknown (complex) coefficient of order one, and there are no relations between the order one coefficients of different entries.

The structure of the Yukawa matrices dictates the parametric suppression of the physical observables:

$$\begin{aligned} y_t &\sim 1, \quad y_c \sim \epsilon^2, \quad y_u \sim \epsilon^4, \\ y_b &\sim 1, \quad y_s \sim \epsilon, \quad y_d \sim \epsilon^2, \\ y_\tau &\sim 1, \quad y_\mu \sim \epsilon, \quad y_e \sim \epsilon^2, \\ |V_{us}| &\sim \epsilon, \quad |V_{cb}| \sim \epsilon, \quad |V_{ub}| \sim \epsilon^2, \quad \delta_{\rm KM} \sim 1. \end{aligned} \tag{8.6}$$

For $\epsilon \sim 0.05$, the parametric suppressions are roughly consistent with the observed hierarchy. In particular, this set of charges predicts that the down and charged lepton mass hierarchies are similar, while the up hierarchy is the square of the down hierarchy. These features are roughly realized in Nature.

Exercise 8: *Derive the parametric suppression and approximate numerical values of Y^u, its eigenvalues, and the three angles of V_L^u, for $H(Q_i) = 4, 2, 0$, $H(U_i) = 3, 2, 0$ and $\epsilon_H = 0.2$.*

Could we explain any set of observed values with such an approximate symmetry? If we could, then the FN mechanism cannot be really tested. The answer however is negative. Consider, for example, the quark sector. Naively, we have 11 $U(1)_H$ charges that we are free to choose. However, the $U(1)_Y \times U(1)_B \times U(1)_{\rm PQ}$ symmetry implies that there are only 8 independent choices that affect the structure of the Yukawa couplings. On the other hand, there are 9 physical parameters. Thus, there should be a single relation between the physical parameters that is independent of the choice of charges. Assuming that the sum of charges in the exponents of Eq. (8.3) is of the same sign for all 18 combinations, the relation is

$$|V_{ub}| \sim |V_{us} V_{cb}|, \tag{8.7}$$

which is fulfilled to within a factor of 2. There are also interesting inequalities (here $i < j$):

$$|V_{ij}| \gtrsim m(U_i)/m(U_j), \; m(D_i)/m(D_j). \tag{8.8}$$

All six inequalities are fulfilled. Finally, if we order the up and the down masses from light to heavy, then the CKM matrix is predicted to be $\sim \mathbf{1}$, namely the diagonal entries are not parametrically suppressed. This structure is also consistent with the observed CKM structure.

8.2 The flavor of neutrinos

Five neutrino flavor parameters have been measured in recent years (see e.g. Ref. [40]): two mass-squared differences,

$$\Delta m_{21}^2 = (7.4 \pm 0.2) \times 10^{-5}\ {\rm eV}^2, \quad |\Delta m_{32}^2| = (2.51 \pm 0.03) \times 10^{-3}\ {\rm eV}^2, \tag{8.9}$$

and the three mixing angles,

$$\sin^2\theta_{12} = 0.310 \pm 0.013, \quad \sin^2\theta_{23} = 0.56 \pm 0.03, \quad \sin^2\theta_{13} = 0.0224 \pm 0.0007. \tag{8.10}$$

These parameters constitute a significant addition to the thirteen SM flavor parameters and provide, in principle, tests of various ideas to explain the SM flavor puzzle.

The numerical values of the parameters show various surprising features:

- $|U_{\mu 3}| >$ any $|V_{ij}|$;
- $|U_{e2}| >$ any $|V_{ij}|$;
- $|U_{e3}|$ is not particularly small ($|U_{e3}| \not\ll |U_{e2}U_{\mu 3}|$);
- $m_2/m_3 \gtrsim 1/6 >$ any m_i/m_j for charged fermions.

These features can be summarized by the statement that, in contrast to the charged fermions, neither smallness nor hierarchy have been observed so far in the neutrino related parameters.

One way of interpretation of the neutrino data comes under the name of neutrino mass anarchy [41–43]. It postulates that the neutrino mass matrix has no special structure, namely all entries are of the same order of magnitude. Normalized to an effective neutrino mass scale, $v^2/\Lambda_{\rm seesaw}$, the various entries are random numbers of order one. Note that anarchy means neither hierarchy nor degeneracy.

If true, the contrast between neutrino mass anarchy and quark and charged lepton mass hierarchy may be a deep hint for a difference between the flavor physics of Majorana and Dirac fermions. The source of both anarchy and hierarchy might, however, be explained by a much more mundane mechanism. In particular, neutrino mass anarchy could be a result of a FN mechanism, where the three left-handed lepton doublets carry the same FN charge. In that case, the FN mechanism predicts parametric suppression of neither neutrino mass ratios nor leptonic mixing angles, which is quite consistent with (8.9) and (8.10). Indeed, the viable FN model presented in Section 8.1 belongs to this class.

Another possible interpretation of the neutrino data is to take $m_2/m_3 \sim |U_{e3}| \sim 0.15$ to be small, and require that they are parametrically suppressed (while the other two mixing angles are order one). Such a situation is impossible to accommodate in a large class of FN models [44].

The same data, and in particular the proximity of $(|U_{\mu 3}|, |U_{\tau 3}|)$ to $(1/\sqrt{2}, 1/\sqrt{2})$, and the proximity of $|U_{e2}|$ to $1/\sqrt{3} \simeq 0.58$, led to a very different interpretation. This interpretation, termed 'tribimaximal mixing' (TBM), postulates that the leptonic mixing matrix is parametrically close to the following special form [45]:

$$|U|_{\rm TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} . \tag{8.11}$$

Such a form is suggestive of discrete non-Abelian symmetries, and indeed numerous models based on an A_4 symmetry have been proposed [46, 47]. A significant feature of of TBM is that the third mixing angle should be close to $|U_{e3}| = 0$. Until 2012, there have been only upper bounds on $|U_{e3}|$, consistent with the models in the literature. In recent years, however, a value of $|U_{e3}|$ close to the previous upper bound has been established [48], see Eq. (8.10). Such a large value (and the consequent significant deviation of $|U_{\mu 3}|$ from maximal bimixing) puts in serious doubt the TBM idea. Indeed, it is difficult in this framework, if not impossible, to account for $\Delta m^2_{12}/\Delta m^2_{23} \sim |U_{e3}|^2$ without fine-tuning [49].

9 Higgs physics: the new flavor arena

The SM relates the Yukawa couplings to the corresponding mass matrices:

$$Y^f = \sqrt{2}M_f/v. \tag{9.1}$$

Examining the Yukawa couplings in the mass basis, this simple equation implies four features:

1. *Proportionality:* $y_i \equiv Y^f_{ii} \propto m_i$;
2. *Factor* of proportionality: $y_i/m_i = \sqrt{2}/v$;
3. *Diagonality:* $Y^f_{ij} = 0$ for $i \neq j$.
4. *CP*: $\mathcal{I}m(y_i/m_i) = 0$.

In extensions of the SM, each of these four features might be violated. Thus, testing these features might provide a window to new physics and to allow progress in understanding the flavor puzzles.

The Higgs boson h was discovered by the ATLAS and CMS experiments at the LHC [50,51]. The experiments normalize their results for Higgs production and decays to the SM rates:

$$\mu_f \equiv \frac{\sigma(pp \to h)\text{BR}(h \to f)}{[\sigma(pp \to h)\text{BR}(h \to f)]^{\text{SM}}}. \tag{9.2}$$

The measurements give:

$$\begin{aligned}
\mu_{\gamma\gamma} &= 1.10 \pm 0.07, \\
\mu_{ZZ^*} &= 1.02 \pm 0.08, \\
\mu_{WW^*} &= 1.00 \pm 0.08, \\
\mu_{b\bar{b}} &= 0.99 \pm 0.12, \\
\mu_{\tau\tau} &= 0.91 \pm 0.09, \\
\mu_{\mu\mu} &= 1.21 \pm 0.33,
\end{aligned} \tag{9.3}$$

and the bounds [52–54]

$$\begin{aligned}
\mu_{c\bar{c}} &\in [1.2, 26], \\
\mu_{c\bar{c}} &\leq 20\mu_{b\bar{b}}, \\
\text{BR}_{ee} &< 3.6 \times 10^{-4}.
\end{aligned} \tag{9.4}$$

Given that $m_b/m_c \simeq 4.58$ [55], and that the upper bound on $\mu_{c\bar{c}}/\mu_{b\bar{b}}$ implies that $\kappa_c/\kappa_b < 4.5$, it is now experimentally established that $y_c < y_b$. Given $\text{BR}^{\text{SM}}_{ee} = 5 \times 10^{-9}$, the latter translates into $\mu_{ee} < 7.2 \times 10^4$.

As concerns quark flavor changing Higgs couplings, these have been searched for in $t \to qh$ decays ($q = c, u$) [56,57]:

$$\text{BR}(t \to ch) < 7.3 \times 10^{-4},$$

$$\text{BR}(t \to uh) \quad < \quad 1.9 \times 10^{-4}. \tag{9.5}$$

As concerns lepton flavor violating Higgs decays, the current bounds are

$$\begin{aligned} \text{BR}(h \to \tau\mu) &< 1.5 \times 10^{-3}, \\ \text{BR}(h \to \tau e) &< 2.2 \times 10^{-3}, \\ \text{BR}(h \to \mu e) &< 6.1 \times 10^{-5}. \end{aligned} \tag{9.6}$$

CPV has been searched for in the Higgs couplings to $t\bar{t}$ and to $\tau^+\tau^-$, yielding uppr bounds on the relative CP-odd fraction [58–60]:

$$\begin{aligned} \sin\theta_{htt} &= 0.00 \pm 0.33, \\ \sin\theta_{h\tau\tau} &= -0.02 \pm 0.32. \end{aligned} \tag{9.7}$$

The measurements quoted in Eq. (9.3) can be presented in the $y_i - m_i$ plane. We do so in Fig. 8. The first two features quoted above are already being tested. The upper bounds on flavor violating decays quoted in Eqs. (9.5) and (9.6) test the third feature. The allowed ranges in Eq. (9.7) test the fourth feature. We can make the following statements:

- $y_e \lesssim y_\mu < y_\tau$. This goes in the direction of proportionality.
- The third generation Yukawa couplings, y_t, y_b, y_τ, as well as the second generation y_μ, obey $y_i/m_i \approx \sqrt{2}/v$. This is in agreement with the predicted factor of proportionality.
- There are strong upper bounds on violation of diagonality: $Y_{tc} \lesssim 0.02$ and $Y_{\tau\mu} \lesssim 0.002$.
- There are upper bounds on CPV in y_t/m_t and y_τ/m_τ.

Beyond the search for new physics via Higgs decays, it is interesting to ask whether the measurements of the Higgs couplings to quarks and leptons can shed light on the Standard Model and/or new physics flavor puzzles. If eventually the values of y_b and/or y_τ deviate from their SM values, the most likely explanation of such deviations will be that there are more than one Higgs doublets, and that the doublet(s) that couple to the down and charged lepton sectors are not the same as the one that couples to the up sector. A more significant test of our understanding of flavor physics comes from the double ratio

$$X_{\mu^+\mu^-} \equiv \frac{\text{BR}(h \to \mu^+\mu^-)/\text{BR}(h \to \tau^+\tau^-)}{m_\mu^2/m_\tau^2}, \tag{9.8}$$

which is predicted within the SM with impressive theoretical cleanliness. To leading order, it is given by 1, and the corrections of order α_W and of order m_μ^2/m_τ^2 to this leading result are known, and reduce the value to 0.98. The current experimental value is given by

$$X_{\mu^+\mu^-} = 1.03 \pm 0.31, \tag{9.9}$$

consistent with the SM prediction (as well as with the predictions of 2HDMs with NFC, the MSSM and MFV models), and excluding the possibility that Y_μ and Y_τ arise from terms of different dimensions in the SMEFT [61]. It is also interesting to test diagonality via the search for the SM-forbidden decay

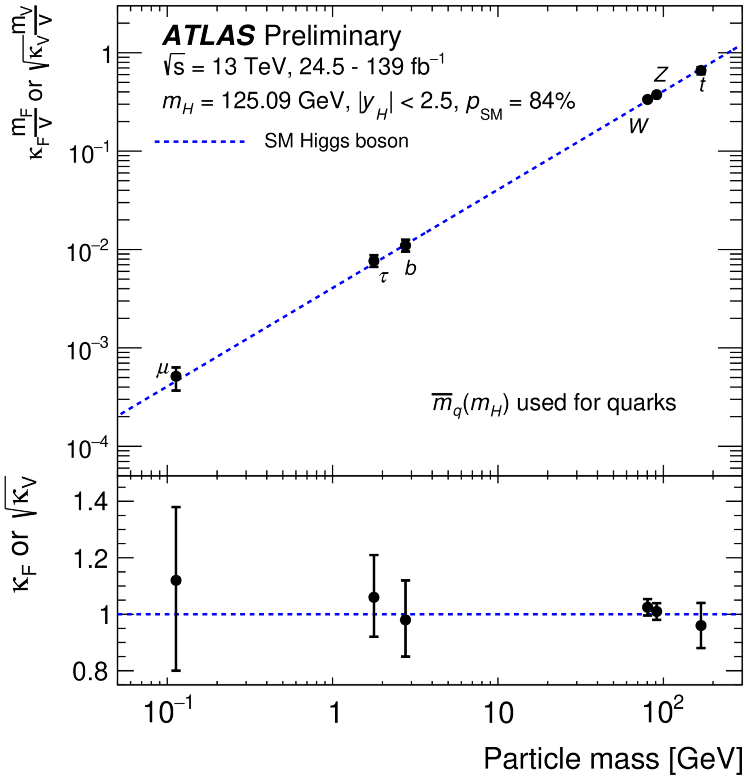

Fig. 8: The allowed ranges for the Higgs couplings. The SM prediction is presented by the dashed line.

modes, $h \to \mu^\pm\tau^\mp$. A measurement of, or an upper bound on

$$X_{\mu\tau} \equiv \frac{\text{BR}(h \to \mu^+\tau^-) + \text{BR}(h \to \mu^-\tau^+)}{\text{BR}(h \to \tau^+\tau^-)}, \tag{9.10}$$

would provide additional information relevant to flavor physics. The current experimental value is given by

$$X_{\mu\tau} < 0.04. \tag{9.11}$$

We demonstrate below the potential power of Higgs flavor physics to lead to progress in our understanding of the flavor puzzles by focusing on the measurements of $\mu_{\tau^+\tau^-}$, $X_{\mu^+\mu^-}$ and $X_{\mu\tau}$ [61].

Let us take as an example how we can use the set of these three measurements if there is a single light Higgs boson. A violation of the SM relation $Y_{ij}^{\rm SM} = \frac{\sqrt{2}m_i}{v}\delta_{ij}$, is a consequence of non renormalizable terms. The leading ones are the $d = 6$ terms. In the interaction basis, we have

$$\begin{aligned} \mathcal{L}_Y^{d=4} &= -\lambda_{ij}\bar{f}_L^i f_R^j \phi + \text{h.c.}, \\ \mathcal{L}_Y^{d=6} &= -\frac{\lambda'_{ij}}{\Lambda^2}\bar{f}_L^i f_R^j \phi(\phi^\dagger\phi) + \text{h.c.}\,, \end{aligned} \tag{9.12}$$

where expanding around the vacuum we have $\phi = (v+h)/\sqrt{2}$. Defining $V_{L,R}$ via

$$\sqrt{2}m = V_L \left(\lambda + \frac{v^2}{2\Lambda^2}\lambda'\right) V_R^\dagger v, \tag{9.13}$$

where $m = \text{diag}(m_e, m_\mu, m_\tau)$, and defining $\hat{\lambda}$ via

$$\hat{\lambda} = V_L \lambda' V_R^\dagger, \tag{9.14}$$

we obtain

$$Y_{ij} = \frac{\sqrt{2}m_i}{v}\delta_{ij} + \frac{v^2}{\Lambda^2}\hat{\lambda}_{ij}. \tag{9.15}$$

To proceed, one has to make assumptions about the structure of $\hat{\lambda}$. In what follows, we consider first the assumption of minimal flavor violation (MFV) and then a Froggatt–Nielsen (FN) symmetry.

Exercise 9: *Find the predictions of models with Natural Flavor Conservation (NFC) for $\mu_{\tau^+\tau^-}$, $X_{\mu^+\mu^-}$ and $X_{\tau\mu}$.*

9.1 MFV

MFV requires that the leptonic part of the Lagrangian is invariant under an $SU(3)_L \times SU(3)_E$ global symmetry, with the left-handed lepton doublets transforming as $(3,1)$, the right-handed charged lepton singlets transforming as $(1,3)$ and the charged lepton Yukawa matrix Y is a spurion transforming as $(3,\bar{3})$.

Specifically, MFV means that, in Eq. (9.12),

$$\lambda' = a\lambda + b\lambda\lambda^\dagger\lambda + \mathcal{O}(\lambda^5), \tag{9.16}$$

where a and b are numbers. Note that, if V_L and V_R are the diagonalizing matrices for λ, $V_L\lambda V_R^\dagger = \lambda^{\text{diag}}$, then they are also the diagonalizing matrices for $\lambda\lambda^\dagger\lambda$: $V_L\lambda\lambda^\dagger\lambda V_R^\dagger = (\lambda^{\text{diag}})^3$. Then, Eqs. (9.13), (9.14), and (9.15) become

$$\begin{aligned}
\frac{\sqrt{2}m}{v} &= \left(1+\frac{av^2}{2\Lambda^2}\right)\lambda^{\text{diag}} + \frac{bv^2}{2\Lambda^2}(\lambda^{\text{diag}})^3, \\
\hat{\lambda} &= a\lambda^{\text{diag}} + b(\lambda^{\text{diag}})^3 = a\frac{\sqrt{2}m}{v} + \frac{2\sqrt{2}bm^3}{v^3}, \\
Y_{ij} &= \frac{\sqrt{2}m_i}{v}\delta_{ij}\left[1+\frac{av^2}{\Lambda^2}+\frac{2bm_i^2}{\Lambda^2}\right],
\end{aligned} \tag{9.17}$$

where, in the expressions for $\hat{\lambda}$ and Y, we included only the leading universal and leading non-universal corrections to the SM relations.

We learn the following points about the Higgs-related lepton flavor parameters in this class of models:

1. h has no flavor off-diagonal couplings:

$$Y_{\mu\tau}, Y_{\tau\mu} = 0. \tag{9.18}$$

2. The values of the diagonal couplings deviate from their SM values. The deviation is small, of order v^2/Λ^2:

$$y_\tau \approx \left(1 + \frac{av^2}{\Lambda^2}\right) \frac{\sqrt{2}m_\tau}{v}. \tag{9.19}$$

3. The ratio between the Yukawa couplings to different charged lepton flavors deviates from its SM value. The deviation is, however, very small, of order m_ℓ^2/Λ^2:

$$\frac{y_\mu}{y_\tau} = \frac{m_\mu}{m_\tau}\left(1 - \frac{2b(m_\tau^2 - m_\mu^2)}{\Lambda^2}\right). \tag{9.20}$$

The predictions of the SM with MFV non-renormalizable terms are then the following:

$$\begin{aligned} \mu_{\tau^+\tau^-} &= 1 + 2av^2/\Lambda^2, \\ X_{\mu^+\mu^-} &= 1 - 4bm_\tau^2/\Lambda^2, \\ X_{\tau\mu} &= 0. \end{aligned} \tag{9.21}$$

Thus, MFV will be excluded if experiments observe the $h \to \mu\tau$ decay. On the other hand, MFV allows for a universal deviation of $\mathcal{O}(v^2/\Lambda^2)$ of the flavor-diagonal dilepton rates, and a smaller non-universal deviation of $\mathcal{O}(m_\tau^2/\Lambda^2)$.

9.2 FN

An attractive explanation of the smallness and hierarchy in the Yukawa couplings is provided by the Froggatt–Nielsen (FN) mechanism [35]. In this framework, a $U(1)_H$ symmetry, under which different generations carry different charges, is broken by a small parameter ϵ_H. Without loss of generality, ϵ_H is taken to be a spurion of charge -1. Then, various entries in the Yukawa mass matrices are suppressed by different powers of ϵ_H, leading to smallness and hierarchy.

Specifically for the leptonic Yukawa matrix, taking the Higgs field to be neutral under $U(1)_H$, $H(\phi) = 0$, we have

$$\lambda_{ij} \propto \epsilon_H^{H(E_j)-H(L_i)}. \tag{9.22}$$

We emphasize that the FN mechanism dictates only the parametric suppression. Each entry has an arbitrary order-one coefficient. The resulting parametric suppression of the masses and leptonic mixing angles is given by [62]

$$m_{\ell_i}/v \sim \epsilon_H^{H(E_i)-H(L_i)}, \quad |U_{ij}| \sim \epsilon_H^{H(L_j)-H(L_i)}. \tag{9.23}$$

Since $H(\phi^\dagger\phi) = 0$, the entries of the matrix λ' have the same parametric suppression as the corresponding entries in λ [63], though the order-one coefficients are different:

$$\lambda'_{ij} = \mathcal{O}(1) \times \lambda_{ij}. \tag{9.24}$$

This structure allows us to estimate the entries of $\hat{\lambda}_{ij}$ in terms of physical observables:

$$\begin{aligned} \hat{\lambda}_{33} &\sim m_\tau/v, \\ \hat{\lambda}_{22} &\sim m_\mu/v, \\ \hat{\lambda}_{23} &\sim |U_{23}|(m_\tau/v), \\ \hat{\lambda}_{32} &\sim (m_\mu/v)/|U_{23}|. \end{aligned} \tag{9.25}$$

We learn the following points about the Higgs-related lepton flavor parameters in this class of models:

1. h has flavor off-diagonal couplings:

$$\begin{aligned} Y_{\mu\tau} &= \mathcal{O}\left(\frac{|U_{23}|vm_\tau}{\Lambda^2}\right), \\ Y_{\tau\mu} &= \mathcal{O}\left(\frac{vm_\mu}{|U_{23}|\Lambda^2}\right). \end{aligned} \tag{9.26}$$

2. The values of the diagonal couplings deviate from their SM values:

$$y_\tau \approx \frac{\sqrt{2}m_\tau}{v} \left[1 + \mathcal{O}\left(\frac{v^2}{\Lambda^2}\right)\right]. \tag{9.27}$$

3. The ratio between the Yukawa couplings to different charged lepton flavors deviates from its SM value:

$$\frac{y_\mu}{y_\tau} = \frac{m_\mu}{m_\tau}\left[1 + \mathcal{O}\left(\frac{v^2}{\Lambda^2}\right)\right]. \tag{9.28}$$

The predictions of the SM with FN-suppressed non-renormalizable terms are then the following:

$$\begin{aligned} \mu_{\tau^+\tau^-} &= 1 + \mathcal{O}(v^2/\Lambda^2), \\ X_{\mu^+\mu^-} &= 1 + \mathcal{O}(v^2/\Lambda^2), \\ X_{\tau\mu} &= \mathcal{O}(v^4/\Lambda^4). \end{aligned} \tag{9.29}$$

Thus, FN will be excluded if experiments observe deviations from the SM of the same size in both flavor-diagonal and flavor-changing h decays. On the other hand, FN allows non-universal deviations of $\mathcal{O}(v^2/\Lambda^2)$ in the flavor-diagonal dilepton rates, and a smaller deviation of $\mathcal{O}(v^4/\Lambda^4)$ in the off-diagonal rate.

10 New physics?

In this section we discuss two sets of recent measurements of flavor changing processes that arouse much interest: lepton flavor universality in semileptonic B decays, $R(D^{(*)})$, and direct CP violation in

D decays, ΔA_{CP}.

10.1 $B \to D^{(*)}\tau\nu$

Within the Standard Model (SM), the electroweak interactions of the leptons are flavor universal. Violation of lepton flavor universality arises from Yukawa interactions, that are negligible in this context, and from phase space effects, which are calculable. A test of the SM prediction of lepton flavor universality between the τ-lepton and the light ℓ-leptons ($\ell = e, \mu$) is provided by the ratios

$$R(D^{(*)}) \equiv \frac{\Gamma(B \to D^{(*)}\tau\bar{\nu})}{\Gamma(B \to D^{(*)}\ell\bar{\nu})}, \quad (\ell = e \text{ or } \mu). \tag{10.1}$$

The SM predictions, derived by naive averaging [22] over the results reported in Refs. [64–67], are

$$\begin{aligned} R(D) &= 0.299 \pm 0.003, \\ R(D^*) &= 0.258 \pm 0.005. \end{aligned} \tag{10.2}$$

The current world averages for $R(D)$ and $R(D^*)$, combining the results reported in Refs. [68–76] are as follows [22]:

$$\begin{aligned} R(D) &= 0.340 \pm 0.027 \pm 0.013, \\ R(D^*) &= 0.295 \pm 0.011 \pm 0.008. \end{aligned} \tag{10.3}$$

The difference of the experimental measurements from the SM predictions corresponds to about 3.1σ (p-value of 2.7×10^{-3}). We thus aim to explain

$$R(D^{(*)})/R(D^{(*)})^{\rm SM} \approx 1.14 \pm 0.05. \tag{10.4}$$

In this section we entertain the idea that a deviation from the SM will indeed be established. We describe the analysis of Ref. [77]. The quark transition via which the $B \to D^{(*)}\tau\nu$ proceeds is $b \to c\tau\nu$. Note, however, that the flavor of the neutrino is, of course, unobservable. It could be ν_τ, in which case the process respects the accidental lepton flavor symmetry of the SM. There is no reason, however, that the symmetry is respected by new physics, particularly when the new physics violates lepton flavor universality, so that the neutrino could also be ν_μ or ν_e or some combination of the three flavors [77, 78]. Let us ask two questions:

- Could the $R(D^{(*)})$ puzzle be solved via new physics contributions to $b \to c\tau\nu_{e,\mu}$?
- If not, how precise should the alignment of ν with ν_τ be?

We assume that the new physics contributions originate at a scale $\Lambda \gg v$, and consider the following two terms in the SMEFT Lagrangian [78]:

$$\mathcal{L}_{\rm NP} = \frac{C_1^{ilkm}}{\Lambda^2}(\overline{L_i}\gamma_\sigma L_l)(\overline{Q_k}\gamma^\sigma Q_m) + \frac{C_3^{ilkm}}{\Lambda^2}(\overline{L_i}\gamma_\sigma\tau^a L_l)(\overline{Q_k}\gamma^\sigma\tau^a Q_m), \tag{10.5}$$

where L is the $SU(2)$-doublet lepton field, Q is the $SU(2)$-doublet quark field, and i, l, k, m are flavor indices. For the sake of definiteness, and to avoid the strongest constraints from flavor changing neutral current (FCNC) processes, we take $i = \tau$, $k = s$, and $m = b$, while l runs over e, μ, τ. We denote $C_{1,3}^{\tau lsb}$ by $C_{1,3}^{l}$. The $C_{1,3}^{l}$-dependent terms can be rewritten as follows:

$$\begin{aligned}\Lambda^2\mathcal{L}_{\rm NP} &= (C_1^l + C_3^l)V_{is}V_{jb}^*(\overline{u_{Li}}\gamma^\mu u_{Lj})(\overline{\nu_\tau}\gamma_\mu\nu_l) + (C_1^l + C_3^l)(\overline{s_L}\gamma^\mu b_L)(\overline{\tau_L}\gamma_\mu l_L)\\ &+ (C_1^l - C_3^l)V_{is}V_{jb}^*(\overline{u_{Li}}\gamma^\mu u_{Lj})(\overline{\tau_L}\gamma_\mu l_L) + (C_1^l - C_3^l)(\overline{s_L}\gamma^\mu b_L)(\overline{\nu_\tau}\gamma_\mu\nu_l)\\ &+ 2C_3^l V_{is}(\overline{u_{Li}}\gamma^\mu b_L)(\overline{\tau_L}\gamma_\mu\nu_l) + 2C_3^l V_{jb}(\overline{u_{Lj}}\gamma^\mu s_L)(\overline{\tau_L}\gamma_\mu\nu_l) + {\rm h.c.}.\end{aligned} \tag{10.6}$$

Thus, the SMEFT Lagrangian terms that contribute to $b \to c\tau\nu$ are

$$\mathcal{L} = \left(\frac{4G_F V_{cb}\delta_{l\tau}}{\sqrt{2}} + \frac{2C_3^l V_{cs}}{\Lambda^2}\right)(\overline{c_L}\gamma^\mu b_L)(\overline{\tau_L}\gamma_\mu\nu_l). \tag{10.7}$$

We obtain:

$$\frac{R(D^{(*)})}{R(D^{(*)})^{\rm SM}} = 1 + \frac{\sqrt{2}}{G_F}\mathcal{R}e\left(\frac{V_{cs}}{V_{cb}}\frac{C_3^\tau}{\Lambda^2}\right) + \frac{\sum_{\ell=e,\mu}|C_3^\ell|^2}{2G_F^2\Lambda^4}\left|\frac{V_{cs}}{V_{cb}}\right|^2, \tag{10.8}$$

where we assume that the contribution of the term quadratic in C_3^τ is negligible compared to the term linear in C_3^τ.

Thus, to account for the $R(D^{(*)})$ puzzle by purely $b \to c\tau\nu_\ell$, $\ell = e, \mu$, we need

$$\left(\frac{\sum_{\ell=e,\mu}|C_3^\ell|^2}{\Lambda^4}\right)^{1/2} = (0.24 \pm 0.04)\ {\rm TeV}^{-2} = \frac{1}{[(2.0 \pm 0.2)\ {\rm TeV}]^2}. \tag{10.9}$$

On the other hand, to account for the $R(D^{(*)})$ puzzle by purely $b \to c\tau\nu_\tau$, we need

$$\frac{C_3^\tau}{\Lambda^2} = (0.046 \pm 0.016)\ {\rm TeV}^{-2} \approx \frac{1}{[(4.7 \pm 0.8)\ {\rm TeV}]^2}. \tag{10.10}$$

If the $R(D^{(*)})$ puzzle is accounted for by purely $b \to c\tau\bar{\nu}_\ell$, Eq. (10.9) implies that we need $|C_3^\ell|/\Lambda^2 \sim 1/(2\ {\rm TeV})^2$. Eq. (10.6) implies that the C_3^ℓ term contributes, via four Fermi operators with the flavor structures $\bar{s}b\bar{\tau}\ell$ and $b\bar{s}\bar{\nu}_\tau\nu_\ell$, to various flavor changing neutral current and lepton flavor violating processes which are forbidden in the SM. The strongest constraints are the following:

- The experimental upper bound on ${\rm BR}(B^+ \to K^+\tau^+\mu^-)$ [79] implies

$$\frac{|C_1^\mu + C_3^\mu|}{\Lambda^2} < 0.058\ {\rm TeV}^{-2}. \tag{10.11}$$

- The experimental upper bound on ${\rm BR}(B^+ \to K^+e^-\tau^+)$ [79] implies

$$\frac{|C_1^e + C_3^e|}{\Lambda^2} < 0.044\ {\rm TeV}^{-2}. \tag{10.12}$$

- The experimental upper bound on ${\rm BR}(B^+ \to K^+\nu\bar{\nu})$ [80, 81] implies

$$\frac{|C_1^\ell - C_3^\ell|}{\Lambda^2} < 0.031\ {\rm TeV}^{-2}. \tag{10.13}$$

The effective operators of Eq. (10.5) will contribute to the scattering process $pp \to \tau^\pm \mu^\mp X_h$, where X_h stands for final hadrons. At present, however, these bounds are not competitive with the ones extracted from the LFV B decays.

From the upper bounds on $|C_1^\ell \pm C_3^\ell|$ we can obtain upper bounds on C_3^ℓ alone:

$$\frac{|C_3^\mu|}{\Lambda^2} < 0.044 \text{ TeV}^{-2}, \quad \frac{|C_3^e|}{\Lambda^2} < 0.037 \text{ TeV}^{-2}. \tag{10.14}$$

We reach the following conclusions:

- Given that, to account for the central value of $R(D^{(*)})$, it is required that $|C_3^\ell|/\Lambda^2 \simeq 0.24 \text{ TeV}^{-2}$, but other constraints require that $|C_3^\mu|/\Lambda^2 < 0.044 \text{ TeV}^{-2}$, the contribution of $b \to c\tau\nu_\ell$, with $\ell = e, \mu$, to $R(D^{(*)})/R(D^{(*)})^{\rm SM} - 1$ cannot exceed about 4% of the required shift.
- Given that, to account for the central value of $R(D^{(*)})$, it is required that $|C_3^\tau|/\Lambda^2 \simeq 0.046 \text{ TeV}^{-2}$, but phenomenological constraints require that $|C_3^\mu|/\Lambda^2 < 0.044 \text{ TeV}^{-2}$, and $|C_3^e|/\Lambda^2 < 0.037 \text{ TeV}^{-2}$, we learn that no special alignment with the τ-direction is needed to explain the $R(D^{(*)})$ puzzle.
- Conversely, if operators of the form

$$\frac{C_3^l}{\Lambda^2}(\overline{L_\tau}\gamma_\sigma\tau^a L_l)(\overline{Q_s}\gamma^\sigma\tau^a Q_b) \tag{10.15}$$

  have C_3^τ, C_3^μ and C_3^e all of the same order of magnitude, $C_3^l/\Lambda^2 \sim 0.04 \text{ TeV}^{-2}$, then the shift in $R(D^{(*)})$ will be dominated by a factor of order 30 by C_3^τ, and all phenomenological constraints satisfied.

10.2 Direct CP violation in charm decays

Direct CP violation can be measured in charm decays to final CP eigenstates [82] via

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-), \tag{10.16}$$

where

$$A_{CP}(f) = \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D}^0 \to f)}{\Gamma(D^0 \to f) + \Gamma(\overline{D}^0 \to f)}. \tag{10.17}$$

The LHCb collaboraion measured [83, 84]

$$\begin{aligned} \Delta A_{CP} &= (-1.54 \pm 0.29) \times 10^{-3}, \\ A_{CP}(K^+K^-) &= (+0.77 \pm 0.57) \times 10^{-3}, \\ A_{CP}(\pi^+\pi^-) &= (+2.32 \pm 0.61) \times 10^{-3}. \end{aligned} \tag{10.18}$$

The CP asymmetry arises from interference between a strong penguin and tree diagrams. It is thus loop suppressed, and CKM suppressed by a factor of

$$2\mathcal{I}m\left(\frac{V_{ub}V_{cb}^*}{V_{us}V_{cs}^*}\right)\approx 1.4\times 10^{-3}. \tag{10.19}$$

Within the SM, and assuming the U-spin symmetry of QCD, one can then estimate the size of the asymmetry:

$$\begin{aligned} \Delta A_{CP}^{\rm SM} &\approx -2.8\times 10^{-3}\times(\alpha_s/\pi)r_{QCD}, \\ A_{CP}(K^+K^-) &= -A_{CP}(\pi^+\pi^-). \end{aligned} \tag{10.20}$$

Thus, to accommodate the experimental results within the SM, two surprising features should arise in the relevant strong interactions:

– The ratio of penguin to tree should be enhanced by a surprisingly large factor, $r_{QCD}\sim 10$ [85].
– U-spin should be strongly broken. with $A_{USV}/A_{USC}\sim 1.7$ [86].

If these features are not realized in nature, the measured values call for new physics (see e.g. Ref. [87, 88]). Relevant models include the 2HDM, the MSSM, vector-like up quarks and Z' models. Within the SMEFT, the scale of the CP violating new physics is bounded:

$$\Lambda_{\rm NP}\lesssim 40\ {\rm TeV}. \tag{10.21}$$

11 Conclusions

(i) The symmetry principles that define the Standard Model have a very strong predictive power concerning flavor physics. They predict that the photon-, gluon- and Z-mediated interactions are flavor universal, that the W-mediated interactions in the lepton sector are flavor universal, and in the quark sector depend on a unitary matrix, and that the Higgs mediated interactions are flavor diagonal.

(ii) Experimental results are consistent with all of these predictions, except for the lepton flavor universality of the leptonic W interactions. The observed lepton flavor transitions established that the neutrinos are massive.

(iii) Measurements of CP violating B-meson decays have established that the Kobayashi–Maskawa mechanism is the dominant source of the observed CP violation.

(iv) Measurements of flavor changing B-meson decays have established the the Cabibbo–Kobayashi–Maskawa mechanism is the dominant source of the observed quark flavor violation.

(v) The consistency of all these measurements with the CKM predictions sharpens the new physics flavor puzzle: If there is new physics at, or below, the TeV scale, then its flavor structure must be highly non-generic.

(vi) Measurements of neutrino flavor parameters have not only not clarified the Standard Model flavor puzzle, but actually deepened it. Whether they imply an anarchical structure, or a tribimaximal mixing, it seems that the neutrino flavor structure is very different from that of quarks.

(vii) If the LHC experiments, ATLAS and CMS, discover new particles that couple to the Standard

Model fermions, then, in principle, they will be able to measure new flavor parameters. Consequently, the new physics flavor puzzle is likely to be understood.

(viii) If the flavor structure of such new particles is affected by the same physics that sets the flavor structure of the Yukawa couplings, then the LHC experiments (and future flavor factories) may be able to shed light also on the Standard Model flavor puzzle.

(ix) The Higgs program provides an opportunity to make progress in our understanding of the flavor puzzle(s).

(x) Extensions of the SM where new particles couple to quark- and/or lepton-pairs are constrained by flavor.

(xi) There are experimental hints that lepton flavor universality is violated in B decays. These hints will be further tested in the coming years.

The huge progress in flavor physics in recent years has provided answers to many questions. At the same time, new questions arise. The LHC experiments, Belle-II and neutrino experiments are likely to provide more answers and more questions.

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Appendices

A SM calculations of the mixing amplitude

We present the SM calculation of the mixing amplitude $M_{B\overline{B}}$ and its generalization to the other meson systems. The leading diagrams that contribute to $M_{B\overline{B}}$ are one loop diagrams that are called "box diagrams" and are displayed in Fig. 4. We can write the transition amplitude as

$$\mathcal{A}_{B^0\to\overline{B}^0} = C_{\rm SM}(\bar{d}_L\gamma_\mu b_L)(\bar{d}_L\gamma^\mu b_L). \tag{A.1}$$

The normalized matrix element is related to the amplitude via

$$M_{B\overline{B}} = \frac{1}{2m_B}\langle B^0|\mathcal{A}_{B^0\to\overline{B}^0}|\overline{B}^0\rangle, \tag{A.2}$$

and thus

$$M_{B\overline{B}} = \frac{C_{\rm SM}}{2m_B}\langle B^0|(\bar{d}_L\gamma_\mu b_L)(\bar{d}_L\gamma^\mu b_L)|\overline{B}^0\rangle. \tag{A.3}$$

The non-perturbative QCD effects are encoded in the hadronic matrix element, which we parameterize as follows:

$$\langle B^0|(\bar{d}_L\gamma_\mu b_L)(\bar{d}_L\gamma^\mu b_L)|\overline{B}^0\rangle = -\frac{1}{3}m_B^2 B_B f_B^2, \tag{A.4}$$

where B_B is a number and f_B is the B-meson decay constant. Lattice calculations give $\sqrt{B_B}f_B \approx 0.22$ GeV. This is where the hadronic uncertainties lie.

The weak interactions effects are encoded in $C_{\rm SM}$, which is calculated from the box diagrams:

$$C_{\rm SM} = \frac{G_F^2 m_W^2}{2\pi^2}\times\left[(V_{cb}V_{cd}^*)^2 S(x_c,x_c) + (V_{tb}V_{td}^*)^2 S(x_t,x_t) + (V_{cb}V_{cd}^*)(V_{tb}V_{td}^*)S(x_t,x_c)\right], \tag{A.5}$$

where $x_i = m_i^2/m_W^2$, we approximate $x_u = 0$, and S is the loop function:

$$\begin{aligned} S(x_i,x_j) \quad = \quad & x_i x_j\left[-\frac{3}{4(1-x_i)(1-x_j)} + \frac{\log x_i}{(x_i-x_j)(1-x_i)^2}\left(1-2x_i+\frac{x_i^2}{4}\right) + \right. \\ & \left. \frac{\log x_j}{(x_j-x_i)(1-x_j)^2}\left(1-2x_j+\frac{x_j^2}{4}\right)\right]. \end{aligned} \tag{A.6}$$

Note that $S(0,x) = 0$. Taking into account the values of the quark masses and CKM elements, we conclude that the term proportional to $S(x_t,x_t)$ dominates over those proportional to $S(x_t,x_c)$ and $S(x_c,x_c)$, and thus

$$M_{B\overline{B}} \approx \frac{G_F^2}{12\pi^2}m_B m_W^2(B_B f_B^2)(V_{tb}V_{td}^*)^2 S(x_t,x_t). \tag{A.7}$$

This result is subject to known radiative correction that are of $O(1)$.

Eq. (A.7) can be straightforwardly generalized to other systems. For $M_{B_s\overline{B_s}}$, we replace $d\to s$:

$$M_{B_s\overline{B_s}} \approx \frac{G_F^2}{12\pi^2}m_{B_s}m_W^2(B_{B_s}f_{B_s}^2)(V_{tb}V_{ts}^*)^2 S(x_t,x_t)\,. \tag{A.8}$$

Table A.1: The experimental values of the neutral meson mixing parameters. In all cases (including the K meson system) we define x and y as in Eqs. (B.12). For the K^0 system, the error on y is well below a permill and thus we do not include an error. For the B^0 system, there is only an upper bound on $|y|$.

P	m [GeV]	Γ [GeV]	x	y
K^0	0.498	3.68×10^{-15}	0.945 ± 0.001	-0.997
D^0	1.86	1.60×10^{-13}	0.0039 ± 0.0018	$+0.0065 \pm 0.0009$
B^0	5.28	4.33×10^{-13}	0.775 ± 0.006	-0.007 ± 0.009
B_s	5.37	4.34×10^{-13}	26.82 ± 0.23	-0.061 ± 0.008

The ratio $\Delta m_B/\Delta m_{B_s}$ is particularly interesting:

$$\frac{\Delta m_B}{\Delta m_{B_s}} = \frac{m_B B_B f_B^2}{m_{B_s} B_{B_s} f_{B_s}^2} \left| \frac{V_{td}}{V_{ts}} \right|^2 . \tag{A.9}$$

In the $SU(3)_F$ limit, the hadronic matrix elements of B and B_s are the same. Consequently, in the ratio of Eq. (A.9), the hadronic uncertainty is only in the correction to the $SU(3)_F$ limit, and is therefore small. Thus, the ratio $\Delta m_B/\Delta m_{B_s}$ provides an excellent measurement of $|V_{td}/V_{ts}|$.

For $M_{K\overline{K}}$, we replace $b \to s$:

$$M_{K\overline{K}} = \frac{G_F^2}{12\pi^2} m_K m_W^2 (B_K f_K^2) \left[(V_{cs}V_{cd}^*)^2 S(x_c, x_c) + (V_{ts}V_{td}^*)^2 S(x_t, x_t) + (V_{cs}V_{cd}^*)(V_{ts}V_{td}^*) S(x_t, x_c) \right] . \tag{A.10}$$

Lattice results gives $B_K = 0.86 \pm 0.24$.

For the four systems, $P = B, B_s, D, K$, the calculation of $M_{P\overline{P}}$ translates into the calculation of the mass splitting $\Delta M_P = 2|M_{P\overline{P}}|$ (in the D system, however, the calculation of $M_{D\overline{D}}$ is complicated, and we do not discuss it here).

The numerical values of the mixing parameters are presented in Table A.1. The SM calculations outlined above agree well with the data.

B Neutral meson oscillations

We define decay amplitudes of B (which could be charged or neutral) and its CP conjugate $\overline{B}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$A_f = \langle f|\mathcal{H}|B\rangle \quad , \quad \overline{A}_f = \langle f|\mathcal{H}|\overline{B}\rangle \quad , \quad A_{\overline{f}} = \langle \overline{f}|\mathcal{H}|B\rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{B}\rangle \; , \tag{B.1}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases ξ_B and ξ_f according to

$$\begin{aligned} CP\,|B\rangle &= e^{+i\xi_B}\,|\overline{B}\rangle \quad , \quad CP\,|f\rangle = e^{+i\xi_f}\,|\overline{f}\rangle \; , \\ CP\,|\overline{B}\rangle &= e^{-i\xi_B}\,|B\rangle \quad , \quad CP\,|\overline{f}\rangle = e^{-i\xi_f}\,|f\rangle \; , \end{aligned} \tag{B.2}$$

so that $(CP)^2 = 1$. The phases ξ_B and ξ_f are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP, \mathcal{H}] = 0$, then A_f and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_B)} A_f \ . \tag{B.3}$$

A state that is initially a superposition of B^0 and $\overline{B}^0$, say

$$|\psi(0)\rangle = a(0)|B^0\rangle + b(0)|\overline{B}^0\rangle \ , \tag{B.4}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\overline{B}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots \ . \tag{B.5}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times t in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [89]. The simplified time evolution is determined by a 2×2 effective Hamiltonian $\mathcal{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathcal{H}$, can be written in terms of Hermitian matrices M and Γ as

$$\mathcal{H} = M - \frac{i}{2}\Gamma \ . \tag{B.6}$$

M and Γ are associated with $(B^0, \overline{B}^0) \leftrightarrow (B^0, \overline{B}^0)$ transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. Diagonal elements of M and Γ are associated with the flavor-conserving transitions $B^0 \to B^0$ and $\overline{B}^0 \to \overline{B}^0$ while off-diagonal elements are associated with flavor-changing transitions $B^0 \leftrightarrow \overline{B}^0$.

The eigenvectors of $\mathcal{H}$ have well defined masses and decay widths. We introduce complex parameters p and q to specify the components of the strong interaction eigenstates, B^0 and $\overline{B}^0$, in the light (B_L) and heavy (B_H) mass eigenstates:

$$|B_{L,H}\rangle = p|B^0\rangle \pm q|\overline{B}^0\rangle \tag{B.7}$$

with the normalization $|p|^2 + |q|^2 = 1$. The special form of Eq. (B.7) is related to the fact that CPT imposes $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. Solving the eigenvalue problem gives

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}} \ . \tag{B.8}$$

If either CP or T is a symmetry of $\mathcal{H}$, then M_{12} and Γ_{12} are relatively real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_B} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1 \ , \tag{B.9}$$

where ξ_B is the arbitrary unphysical phase introduced in Eq. (B.2).

The real and imaginary parts of the eigenvalues of $\mathcal{H}$ corresponding to $|B_{L,H}\rangle$ represent their masses and decay-widths, respectively. The mass difference Δm_B and the width difference $\Delta\Gamma_B$ are defined as follows:

$$\Delta m_B \equiv M_H - M_L, \quad \Delta\Gamma_B \equiv \Gamma_H - \Gamma_L. \tag{B.10}$$

Note that here Δm_B is positive by definition, while the sign of $\Delta\Gamma_B$ is to be experimentally determined. The average mass and width are given by

$$m_B \equiv \frac{M_H + M_L}{2}, \quad \Gamma_B \equiv \frac{\Gamma_H + \Gamma_L}{2}. \tag{B.11}$$

It is useful to define dimensionless ratios x and y:

$$x \equiv \frac{\Delta m_B}{\Gamma_B}, \quad y \equiv \frac{\Delta\Gamma_B}{2\Gamma_B}. \tag{B.12}$$

Solving the eigenvalue equation gives

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \quad \Delta m_B \Delta\Gamma_B = 4\mathcal{R}e(M_{12}\Gamma_{12}^*). \tag{B.13}$$

All CP-violating observables in B and $\overline{B}$ decays to final states f and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of A_f, $\overline{A}_f$, $A_{\overline{f}}$ and $\overline{A}_{\overline{f}}$, together with, for neutral-meson decays only, q/p. CP violation in charged-meson decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in neutral-meson decays is complicated by $B^0 \leftrightarrow \overline{B}^0$ oscillations and depends, additionally, on $|q/p|$ and on

$$\lambda_f \equiv (q/p)(\overline{A}_f/A_f). \tag{B.14}$$

For neutral D, B, and B_s mesons, $\Delta\Gamma/\Gamma \ll 1$ and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|B^0\rangle$ or $|\overline{B}^0\rangle$ after an elapsed proper time t as $|B^0{}_{\rm phys}(t)\rangle$ or $|\overline{B}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, we obtain

$$\begin{aligned} |B^0{}_{\rm phys}(t)\rangle &= g_+(t)\,|B^0\rangle - \frac{q}{p}\,g_-(t)|\overline{B}^0\rangle, \\ |\overline{B}^0_{\rm phys}(t)\rangle &= g_+(t)\,|\overline{B}^0\rangle - \frac{p}{q}\,g_-(t)|B^0\rangle\,, \end{aligned} \tag{B.15}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \tag{B.16}$$

One obtains the following time-dependent decay rates:

$$\begin{aligned} \frac{d\Gamma[B^0{}_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \left(|A_f|^2 + |(q/p)\overline{A}_f|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\overline{A}_f|^2\right)\cos(x\Gamma t) \\ &+ 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t)\,, \end{aligned} \tag{B.17}$$

$$\begin{aligned} \frac{d\Gamma[\overline{B}^0_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \left(|(p/q)A_f|^2 + |\overline{A}_f|^2\right)\cosh(y\Gamma t) - \left(|(p/q)A_f|^2 - |\overline{A}_f|^2\right)\cos(x\Gamma t) \\ &+ 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t)\,, \end{aligned} \tag{B.18}$$

where $\mathcal{N}_f$ is a common normalization factor. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (B.17,B.18). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $B \leftrightarrow \overline{B}$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$ are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (B.17,B.18) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

C CP violation in neutral meson decays

CP asymmetries arise when two processes related by CP conjugation differ in their rates. Given the fact that CP violation is related to a phase in the Lagrangian, all CP asymmetries must arise from interference effects.

To date, CP violation has been observed (at a level higher than 5σ) in about thirty different hadron decay modes, involving b or c or s decays. It has not been established in other quark decays, nor in the leptonic sector, nor in flavor diagonal processes. Here we present the formalism relevant to measuring CP asymmetries in meson decays.

C.1 Notations and formalism

We discuss here the specific case of B-meson decays, but our discussion applies to all meson decays. Our starting points are Eqs. (B.17,B.18), which give the time-dependent decay rates of B^0 and $\overline{B}^0$. We also use the parameter λ_f, defined in Eq. (B.14).

Consider A_f, the $B \to f$ decay amplitude, and $\overline{A}_{\overline{f}}$, the amplitude of the CP conjugate process, $\overline{B} \to \overline{f}$. There are two types of phases that may appear in these decay amplitudes:

- CP-odd phases, also known as weak phases. They are complex parameters in any Lagrangian term that contributes to A_f, and appear in a complex conjugate form in $\overline{A}_{\overline{f}}$. In other words, CP violating phases change sign between A_f and $\overline{A}_{\overline{f}}$. In the SM, these phases appear only in the couplings of the $W^{\pm}$-bosons, hence the CP violating phases are called "weak phases".
- CP-even phases, also known as strong phases. Phases can appear in decay amplitudes even when the Lagrangian parameters are all real. They arise from contributions of intermediate on-shell states, and can be identified with the e^{-iHt} term in the time evolution Schrödinger equation. These CP conserving phases appear with the same sign in A_f and $\overline{A}_{\overline{f}}$. In meson decays, the intermediate states are typically hadronic state with the same flavor QN as the final state, and their dynamics is driven by strong interactions, hence the CP conserving phases are called "strong phases".

It is useful to factorize an amplitude into three parts: the magnitude a_i, the weak phase ϕ_i, and the strong phase δ_i. If there are two such contributions we write

$$A_f = a_1 e^{i(\delta_1+\phi_1)} + a_2 e^{i(\delta_2+\phi_2)}, \qquad \overline{A}_{\overline{f}} = a_1 e^{i(\delta_1-\phi_1)} + a_2 e^{i(\delta_2-\phi_2)}. \tag{C.1}$$

where we always can choose $a_1 \geq a_2$. It is further useful to define

$$\phi_f \equiv \phi_2 - \phi_1, \qquad \delta_f \equiv \delta_2 - \delta_1, \qquad r_f \equiv \frac{a_2}{a_1}. \tag{C.2}$$

For neutral meson mixing, it is useful to write

$$M_{B\overline{B}} = |M_{B\overline{B}}|e^{i\phi_M}, \qquad \Gamma_{B\overline{B}} = |\Gamma_{B\overline{B}}|e^{i\phi_\Gamma}\ , \tag{C.3}$$

and define

$$\theta_B = \phi_M - \phi_\Gamma. \tag{C.4}$$

Note that each of the phases appearing in Eqs. (C.1) and (C.3) is convention dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$, are physical.

In neutral meson decays, the phenomenology of CP violation is particularly rich thanks to the fact that meson mixing can contribute to the CP violating interference effects. One distinguishes three types of CP violation in meson decays, depending on which amplitudes interfere:

1. In decay: The interference is between two decay amplitudes. It corresponds to interference between a_1 and a_2.
2. In mixing: The interference is between the absorptive and dispersive mixing amplitudes. It corresponds to interference between $M_{B\overline{B}}$ and $\Gamma_{B\overline{B}}$.
3. In interference of decays with and without mixing: The interference is between the direct decay amplitude and a first-mix-then-decay amplitude. It corresponds to interference between $\overline{A}_f$ and $M_{B\overline{B}}A_f$.

We discuss these three types below.

For the discussion of CP violation in the $K^0 - \overline{K}^0$ system, we use a somewhat different notation. The reason is that, since the lifetimes of K_S and K_L are so different, experiments often identify these mass eigenstates, rather than the flavor-tagged decays, as done in most measurements of CP violation in the $B^0 - \overline{B}^0$ system. Thus, for K-mesons, we define

$$\epsilon_f \equiv \frac{1-\lambda_f}{1+\lambda_f}. \tag{C.5}$$

The converse relation reads

$$\lambda_f \equiv \frac{1-\epsilon_f}{1+\epsilon_f}. \tag{C.6}$$

Historically, CP violation was first observed in the $K_L \to \pi^+\pi^-$ decay and thus we denote $\epsilon_{\pi^+\pi^-} = \epsilon_K$. For modes with $|\bar{A}_f/A_f| - 1 \ll |q/p| - 1$, as is the case for $f = \pi^+\pi^-$, we can set $|\bar{A}_f/A_f| = 1$ and then we have $|q/p| = |\lambda_f|$.

C.2 CP violation in decay

CP violation in decay corresponds to

$$|\overline{A}_{\overline{f}}/A_f| \neq 1. \tag{C.7}$$

In charged particle decays, this is the only possible contribution to the CP asymmetry:

$$\mathcal{A}_f \equiv \frac{\Gamma(B^- \to f^-) - \Gamma(B^+ \to f^+)}{\Gamma(B^- \to f^-) + \Gamma(B^+ \to f^+)} = \frac{|\overline{A}_{f^-}/A_{f^+}|^2 - 1}{|\overline{A}_{f^-}/A_{f^+}|^2 + 1}. \tag{C.8}$$

Using Eq. (C.1), we obtain, for $r_f \ll 1$,

$$\mathcal{A}_f = 2 r_f \sin\phi_f \sin\delta_f. \tag{C.9}$$

This result shows explicitly that we need two decay amplitudes, that is, $r_f \neq 0$, with different weak phases, $\phi_f \neq 0, \pi$ and different strong phases, $\delta_f \neq 0, \pi$.

A few comments are in order:

1. In order to have a large CP asymmetry, we need each of the three factors in (C.9) not to be small.
2. A similar expression holds for the contribution of CP violation in decay in neutral meson decays. In this case there are, however, additional contributions from mixing, as discussed below.
3. Another complication with regard to neutral meson decays is that it is not always possible to tell the flavor of the decaying meson, that is, if it is B^0 or $\overline{B}^0$. This can be a problem or a virtue.
4. In general, the strong phase is not calculable since it is related to QCD. This is not a problem if the aim is just to demonstrate CP violation, but it is if we want to extract the weak parameter ϕ_f. In some cases, however, the strong phase can be independently measured, eliminating this particular source of theoretical uncertainty.

C.3 CP violation in mixing

CP violation in mixing corresponds to

$$|q/p| \neq 1 \ . \tag{C.10}$$

In decays of neutral mesons into flavor specific final states ($\overline{A}_f = 0$ and, consequently, $\lambda_f = 0$), and, in particular, semileptonic neutral meson decays, this is the only source of CP violation:

$$\mathcal{A}_{\rm SL}(t) \equiv \frac{\hat\Gamma[\overline{B}^0(t) \to \ell^+ X] - \hat\Gamma[B^0(t) \to \ell^- X]}{\hat\Gamma[\overline{B}^0(t) \to \ell^+ X] + \hat\Gamma[B^0(t) \to \ell^- X]} = \frac{1 - |q/p|^4}{1 + |q/p|^4}. \tag{C.11}$$

Using Eq. (B.8), we obtain, for $|\Gamma_{B\overline{B}}/M_{B\overline{B}}| \ll 1$,

$$\mathcal{A}_{\rm SL} = -\left|\Gamma_{B\overline{B}}/M_{B\overline{B}}\right| \sin(\phi_M - \phi_\Gamma). \tag{C.12}$$

Two comments are in order:

1. Eq. (C.11) implies that $\mathcal{A}_{\rm SL}(t)$, which is an asymmetry of time-dependent decay rates, is actually time independent.
2. The calculation of $|\Gamma_{B\overline{B}}/M_{B\overline{B}}|$ is difficult, since it depends on low-energy QCD effects. Hence, the extraction of the value of the CP violating phase $\phi_M - \phi_\Gamma$ from a measurement of $\mathcal{A}_{\rm SL}$ involves, in general, large hadronic uncertainties.

CP violation in $K^0 - \overline{K}^0$ mixing is measured via a semileptonic asymmetry which is defined as follows:

$$\delta_L \equiv \frac{\Gamma(K_L \to \ell^+ \nu_\ell \pi^-) - \Gamma(K_L \to \ell^- \nu_\ell \pi^+)}{\Gamma(K_L \to \ell^+ \nu_\ell \pi^-) + \Gamma(K_L \to \ell^- \nu_\ell \pi^+)} = \frac{1 - |q/p|^2}{1 + |q/p|^2} \approx 2\mathcal{R}e(\epsilon_K), \tag{C.13}$$

where we use Eq. (C.6) and the fact that $|\epsilon_K| \ll 1$. This asymmetry is different from the one defined in Eq. (C.11) in that the decaying meson is the neutral mass eigenstate, rather than the flavor eigenstate, hence the different dependence on $|q/p|$.

C.4 CP violation in interference of decays with and without mixing

CP violation in interference of decays with and without mixing corresponds to

$$\mathcal{I}m(\lambda_f) \neq 0. \tag{C.14}$$

A particular simple case is the CP asymmetry in decays into final CP eigenstates. Moreover, a situation that is relevant in many cases is when one can neglect the effects of CP violation in decay and in mixing, that is when $|\overline{A}_{f_{CP}}/A_{f_{CP}}| \approx 1$ and $|q/p| \approx 1$. In this case, $\lambda_{f_{CP}}$ is, to a good approximation, a pure phase, $|\lambda_{f_{CP}}| = 1$. We further consider the case where we can neglect y ($|y| \ll 1$). Then,

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{\Gamma[\overline{B}^0(t) \to f_{CP}] - \Gamma[B^0(t) \to f_{CP}]}{\Gamma[\overline{B}^0(t) \to f_{CP}] + \Gamma[B^0(t) \to f_{CP}]} = \mathcal{I}m(\lambda_{f_{CP}}) \sin(\Delta m_B t). \tag{C.15}$$

The approximations made above are valid in cases that $|\Gamma_{B\overline{B}}/M_{B\overline{B}}| \ll 1$ and $a_2 \ll a_1$, which lead to

$$\frac{q}{p} = \frac{M^*_{B\overline{B}}}{|M_{B\overline{B}}|} = e^{-i\phi_M}, \qquad \frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} = e^{-2i\phi_A}, \tag{C.16}$$

where ϕ_M is defined in Eq. (C.3), and $\phi_A = \phi_1$ is defined in Eq. (C.1). We then get

$$\mathcal{I}m(\lambda_{f_{CP}}) = \mathcal{I}m\left(\frac{M^*_{B\overline{B}}}{|M_{B\overline{B}}|}\frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}}\right) = -\sin(\phi_M + 2\phi_A). \tag{C.17}$$

We learn that a measurement of a CP asymmetry in a process where these approximations are valid provides a direct probe of the weak phase between the mixing amplitude and the decay amplitude.

For the case where we measure decays of the K_L and K_S mass eigenstates into final CP-even eigenstates, one obtains

$$\mathcal{A}^{\rm mass}_{f_{CP}} \equiv \frac{\Gamma(K_L \to f_{CP})}{\Gamma(K_S \to f_{CP})} = \left|\frac{1 - \lambda_{f_{CP}}}{1 + \lambda_{f_{CP}}}\right|^2 = |\epsilon_{f_{CP}}|^2. \tag{C.18}$$

In particular, for $f_{CP} = \pi^+\pi^-$ we have

$$\mathcal{A}^{\rm mass}_{\pi^+\pi^-} = |\epsilon_K|^2. \tag{C.19}$$

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