Neutrino physics

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Surpassing all expectations, the Standard Model has predicted the outcomes of nearly every experiment conducted thus far. Neutrinos have no mass in it. However, we have gathered strong evidence in the last twenty years suggesting that the neutrinos have small but non-zero masses, These masses are a real delight because they allow neutrinos to oscillate and change flavor. I go over the characteristics of neutrinos both inside and outside the Standard Model in these lectures, as well as their incredible potential. I also revise the bits of data that defy the standard picture of the three neutrinos and discuss the possibilities of employing neutrinos to uncover any physics hidden outside the Standard Model.

1 Introduction

In the last twenty years, neutrino physics underwent a drastic change. Neutrinos have non-zero masses, which implies that leptons mix—this is an unequivocal finding. Neutrinos can change from one state, or flavor, to another, as experimental evidence has shown. All knowledge that we have about neutrinos is relatively recent, younger than thirty years of age. Since neutrino physics is still in its early stages as a solid science, it is in a wild and extremely exciting (and excited) state, similar to any young adult. But let's first discuss how and why neutrinos were born, before diving into the late "news" about them.

Several holy cows met their demise in the 1920s, and physics was no different. It appeared that the subatomic realm defies one of physics' most cherished principles: energy conservation. A non-negligible fraction of the energy of some radioactive nuclei seemed to just disappear, leaving no sign of its existence.

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"Dear radioactive Ladies and Gentlemen, ...," Pauli wrote in a quasi-apologetically worded letter to a meeting in 1920 (by now famous [1])."... as a desperate remedy to save the principle of energy conservation in beta decay, ... I propose the idea of a neutral particle of spin half". Pauli proposed that an additional particle—one that lacked an electric charge, mass, and was impossible to detect, and therefore invisible, because it was only very weakly interacting—was responsible for absorbing the missing energy.

It wasn't long before Fermi proposed the four-Fermi Hamiltonian to explain beta decay using the neutrino, electron, neutron, and proton. With these characteristics, the neutrino was thus introduced as one of the few components of the particle zoo. Weak interactions were then born, took center stage and never left, giving rise to a new field. To close the loop, Cowan and Reines obtained the experimental signature of anti-neutrinos emitted by a nuclear power plant twenty years after Pauli's letter.

Weak interactions gained credibility as a real new force of nature, with the neutrino being a fundamental component, as more particles involved in them were discovered in the years after the discovery of the neutrino.

Over the ensuing years, additional experimental testing revealed that there were, in fact, three different types, or "flavors," of neutrinos (named for the charged lepton they were produced in conjunction with: electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ)), and that, to the extent that we could test, had no mass (and no charge) at all.

A new test using neutrinos from the sun revealed that the neutrino saga was just getting started, even though it could have easily ended there.

In the original Standard Model, neutrinos were completely massless and as a consequence were naturally flavor eigenstates,

$$\begin{aligned} W^+ \longrightarrow e^+ \,+\, \nu_e \quad &; \quad Z \longrightarrow \nu_e \,+\, \bar{\nu}_e \;; \\ W^+ \longrightarrow \mu^+ \,+\, \nu_\mu \quad &; \quad Z \longrightarrow \nu_\mu \,+\, \bar{\nu}_\mu \;; \\ W^+ \longrightarrow \tau^+ \,+\, \nu_\tau \quad &; \quad Z \longrightarrow \nu_\tau \,+\, \bar{\nu}_\tau \;. \end{aligned} \tag{1}$$

In fact they would have been flavor eigenstates even if they were massive, if they had shared the same mass. In any case, they were supposed to move at the speed of light precisely because they were massless. However, their masslessness not only established their propagation speed but also fixed their flavor as they moved. It follows that, in terms of flavor, zero mass neutrinos were not a compelling subject for research, particularly when compared to quarks.

However, if neutrinos were massive, and these masses were not degenerate, as we have mentioned degenerate masses flavor-wise are identical to the zero mass case, would mean that neutrino mass eigenstates exist $\nu_i, i = 1, 2, \ldots$, each with a mass m_i. The effect of leptonic mixing is transparent when considering the leptonic decays of the charged vector boson W , $W^+ \longrightarrow \nu_i + \overline{\ell_\alpha}$. Where, $\alpha = e, \mu$, or τ, and ℓ_e refers to the electron, ℓ_μ the muon, or ℓ_τ the tau.

We denominate ℓ_α as the charged lepton of flavor α. Mixing basically implies that when the charged boson W^+ decays to a given kind of charged lepton $\overline{\ell_\alpha}$, the neutrino that goes along is not generally the same mass eigenstate ν_i. *Any* of the different ν_i can appear. Or all of them!!

The amplitude for the decay of a vector boson W^+ to a particular mix $\overline{\ell_\alpha} + \nu_i$ is given by $U^*_{\alpha i}$.

The neutrino that is emitted in this decay alongside the given charged lepton $\overline{\ell_\alpha}$ is then

$$|\nu_\alpha >= \sum_i U^*_{\alpha i} \, |\nu_i > \quad . \tag{2}$$

This specific mixture of mass eigenstates yields the neutrino of flavor α.

As there are nine of these elements, not independent, the different $U_{\alpha i}$ can be collected in a unitary matrix, in the same way they were collected in the CKM matrix in the quark sector. This matrix receives the name of the leptonic mixing matrix, or U_{PNMS} [2]. The unitarity of U ensures that every time a neutrino of flavor α produces a charged lepton through its interaction, the produced charged lepton will always be ℓ_α, the charged lepton of flavor α. That is, a ν_e produces exclusively an e, a ν_μ exclusively a μ, and similarly ν_τ can only make a τ.

Any mass eigenstate ν_i can be represented as an analogous linear combination of the three flavors by simply inverting the expression (2), which represents each neutrino of a given flavor as a linear combination of the three mass eigenstates:

$$|\nu_i >= \sum_\alpha U_{\alpha i} \, |\nu_\alpha > \quad . \tag{3}$$

The α-fraction or $|U_{\alpha i}|^2$ is clearly the amount of α-flavor in ν_i. This α-content, also known as the fraction, expresses the likelihood that a charged lepton produced by the interaction with a ν_i will have α flavor.

2 Neutrino oscillations basics

Let's begin with an explanation of what we understand for the phenomena known as neutrino flavor transitions, or oscillation for short: Together with a (positively) charged lepton $\overline{\ell_\alpha}$ of flavor α, a source produces or emits a neutrino. In this sense, the neutrino does have a distinct flavor at the emission site—a ν_α, that is. The neutrino then propagates, i.e. travels a distance L, until it is absorbed.

By now, the neutrino has (sometimes) reached the detector. These interactions produce another charged lepton, ℓ_β, of flavor β, which we can detect. This allows us to determine that the neutrino at the target is, once more a ν_β, a neutrino with a distinct flavor. Naturally, not every time both flavors are the same. Only sometimes, $\beta \neq \alpha$ For example, if ℓ_α is μ but ℓ_β is τ. Then, when traveling from the source to the detection point, the neutrino changes from ν_α to ν_β.

The transition from one flavor to another, $\nu_\alpha \longrightarrow \nu_\beta$, is not unique to neutrinos, but is just one example of the quantum mechanical effect known to exist in two level systems.

From Eq. (2), ν_α is the coherent superposition of the three mass eigenstates, ν_i, the actual neutrino traveling from the moment of its creation to its detection. So it can be any one of the three ν_is. That is why the contribution of each ν_i should be included coherently in the equation. As a result, the transfer energy Amp($\nu_\alpha \longrightarrow \nu_\beta$) receives a contribution from ν_i and appears as a product of three phases. The first component is the probability amplitude of the neutrino produced at the formation point with the $\overline{\ell_\alpha}$ lepton to be any of the three mass eigenstates. In particular: say a ν_i and is given by $U^*_{\alpha i}$.

The second part of our results is the amplitude of ν_i generated by the source to cover the distance to the detector. Now, let's name this element Prop(ν_i) and postpone the calculation of its value until later. The last (third) component is the probability amplitude of the traveling neutrino ν_i to produce a charged lepton ℓ_β .

As probabilities must be conserved, we know that the Hamiltonian that describes the interactions between neutrinos, charged leptons and charged bosons W bosons has to be hermitian, therefore Amp($W \longrightarrow \overline{\ell_\alpha}\nu_i) = U^*_{\alpha i}$, then Amp ($\nu_i \longrightarrow \ell_\beta W) = U_{\beta i}$. Thus, the last piece of the product, the ν_i contribution, is given by $U_{\beta i}$, and

$$\text{Amp}(\nu_\alpha \longrightarrow \nu_\beta) = \sum_i U^*_{\alpha i}\ \text{Prop}(\nu_i)\ U_{\beta i}\ . \tag{4}$$

The value of Prop(ν_i) is still up for determination. We'd best examine the ν_i in its rest frame in order to ascertain it. In such a framework, we shall designate the time as τ_i. In this frame of reference, ν_i's state vector fulfills the Schrödinger equation that, if it does have a rest mass m_i, can be written as

$$i\frac{\partial}{\partial\tau_i}|\nu_i(\tau_i)> = m_i|\nu_i(\tau_i)> \quad . \tag{5}$$

whose solution is given by

$$|\nu_i(\tau_i)> = e^{-im_i\tau_i}|\nu_i(0)> \quad . \tag{6}$$

After some time the initial ν_i, $|\nu_i(0)>$, has become the evolved state $|\nu_i(\tau_i)>$, specifically $\exp[-im_i\tau_i]$. Thus, the amplitude for a particular mass eigenstate ν_i to move freely throughout a time τ_i is just the amplitude $<\nu_i(0)|\nu_i(\tau_i)>$. Therefore, Prop(ν_i) only includes this amplitude in which we have utilized the fact that τ_i, the proper time, is the amount of time required for ν_i to travel the distance between the source and the detector.

However, if we want Prop(ν_i) to be useful to us, we must first re-write it in terms of variables that we can measure, which means expressing it in variables in the laboratory frame. A natural choice is the distance L that the neutrino travels between the source and the detector, shown in the laboratory frame and the time t, elapsed during the journey in the lab frame. The distance L is determined by the experiment by selecting the source and detector resolution points, which are unique for each test setup. Similarly, the value t is determined by the experiment by choosing when the neutrinos appear and when the neutrinos die, or are detected. Thus, L and t are determined by the test settings, which are the same for all ν_i in the beam. Different ν_i cover the same distance L in the same time t.

Two more lab frame variables, the energy E_i and three momentum p_i of the neutrino mass eigenstate ν_i, need to be found. The expression for the $m_i\tau_i$ appearing in the ν_i propagator Prop(ν_i) may be obtained in terms of the (simple to measure) lab frame variables by leveraging the Lorentz invariance of the four component internal product (scalar product),

$$m_i\tau_i = E_it - p_iL\ . \tag{7}$$

At this point, one could counter that the time t that elapses from the moment a neutrino is created until it dies in the detector is not truly observed because neutrino sources in real life are essentially constant in time. This is a perfectly valid argument. Actually, an experiment spreads out over the time t that the neutrino needs to travel through. But let's assume that the neutrino signal generated in the detector is composed coherently of two components of the neutrino beam: the first, with energy E_1, and the second, with energy E_2 (both measured in the lab frame). Let's now refer to the time t as the neutrino's travel over the distance between production and detection points.

The component with energy E_j $(j = 1, 2)$ has then picked up a phase factor $\exp[-iE_j t]$ by the time it reaches the detector. Consequently, there will be an interference with a phase factor of $\exp[-i(E_1 - E_2)t]$ between the constituents of the E_1 and E_2 beams. This factor vanishes when smeared throughout the non-observed journey time t, *except when* $E_2 = E_1$. Consequently, the neutrino oscillation signal is only coherently contributed to by components of the neutrino beam that have the same energy [3]. Particularly, only the beam's various mass eigenstate components with equivalent energy weigh in. Everything else is averaged out.

A mass eigenstate ν_i with mass m_i and energy E has three momentum p_i, whose absolute value is given, thanks to its dispersion relation, by

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \quad . \tag{8}$$

Since the neutrinos' masses are pitifully small and given the typical energies E that are involved in any experiment, we have used the fact that $m_i^2 \ll E^2$, i.e. the lowest energy neutrinos have MeV energies while its masses are sub-eV at most. It is simple to show that for a given energy E, the phase $m_i \tau_i$, appearing in Prop(ν_i), takes the value indicated by Eqs. (7) and (8),

$$m_i \tau_i \cong E(t - L) + \frac{m_i^2}{2E} L \quad . \tag{9}$$

When computing the transition amplitude, the phase $E(t - L)$ will finally vanish because it appears in all the interfering terms. The common phase factor, after all, has an absolute value of one. As a result, we can discard it right now and use

$$\mathrm{Prop}(\nu_i) = \exp[-im_i^2 \frac{L}{2E}] \quad . \tag{10}$$

Entering this into Eq. (4), we easily find that the amplitude for a neutrino that starts out as a ν_α and travels L with energy E to be detected as a ν_β is given by

$$\mathrm{Amp}(\nu_\alpha \longrightarrow \nu_\beta) = \sum_i U_{\alpha i}^* \, e^{-im_i^2 \frac{L}{2E}} U_{\beta i} \quad . \tag{11}$$

As long as the neutrinos pass through vacuum, the statement above holds true for any number of neutrino flavors and an equal number of mass eigenstates. Squaring it yields the probability $\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$ for $\nu_\alpha \longrightarrow \nu_\beta$.

$$\begin{aligned} &\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \longrightarrow \nu_\beta)|^2 = \delta_{\alpha\beta} - \\ &4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right), \end{aligned} \tag{12}$$

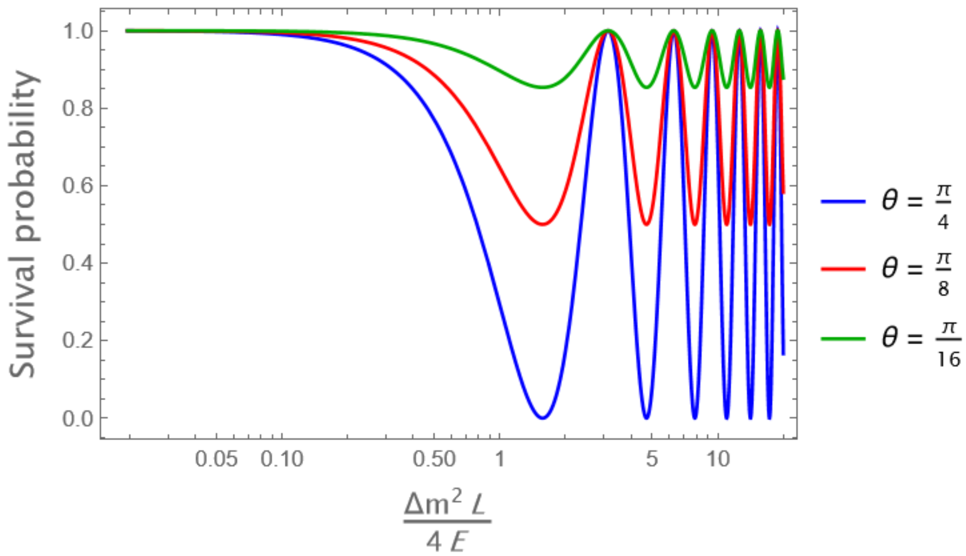

Fig. 1: Survival probability as a function of the kinematic phase. The amplitude of the oscillation is given by the mixing angle. The kinematic phase has to be order one for the oscillations to be seen.

with $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$. We have utilized the fact that the mixing matrix U is unitary to obtain Eq. (12).

Since the oscillating neutrino was created along with a charged *antilepton* $\bar{\ell}$ and gives birth to a charged *lepton* ℓ once it reaches the detector, the oscillation probability $\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$ that we have just obtained corresponds to that of a *neutrino* rather than a *antineutrino*. By utilizing the fact that the two transitions, $\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}$ and $\nu_\beta \longrightarrow \nu_\alpha$, are CPT conjugated processes, the corresponding probability $\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta})$ for an antineutrino oscillation can be found from $\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$. Thus, assuming that neutrino interactions respect CPT [4],

$$\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}) = \mathrm{P}(\nu_\beta \longrightarrow \nu_\alpha) \ . \tag{13}$$

Then, it is evident that $\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta})$ and $\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$ will not be equal in general if the mixing matrix U is complex. $\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}) \neq \mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$ would provide evidence of CP violation in neutrino oscillations, since $\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}$ and $\nu_\alpha \longrightarrow \nu_\beta$ are CP conjugated processes. Since CP violation has only been detected in the quark sector up to this point, measuring it in neutrino physics would be quite interesting.

We have been operating in natural units thus far. An observation made clear by examining the dispersion relation Eq. (9), where we have happily set both the c and $\hbar$ factors to one. If we return them to the oscillation probability, we discover that

$$\sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right) \longrightarrow \sin^2\left(\Delta m^2_{ij}c^4\frac{L}{4\hbar cE}\right) \tag{14}$$

After that, investigating the semi-classical limit, $\hbar \longrightarrow 0$, is simple and informative. The oscillation averages to 1/2 in this limit, and the oscillation length and phase both go to 0 and infinity, respectively. There is no longer any interference pattern. When we let the mass difference Δm^2 grow, we have a similar scenario. This is precisely the quark sector's behavior, and the reason why, despite our knowledge that mass eigenstates and flavor eigenstates do not coincide, we never investigate quark oscillations.

Given actual units, which are not "natural" units, the oscillation phase can be expressed as follows:

$$\Delta m_{ij}^2 \frac{L}{4E} = 1.27\, \Delta m_{ij}^2 (\mathrm{eV}^2) \frac{L\,(\mathrm{km})}{E\,(\mathrm{GeV})} \quad . \tag{15}$$

consequently, given that $\sin^2[1.27\, \Delta m_{ij}^2 (\mathrm{eV}^2) L\,(\mathrm{km})/E\,(\mathrm{GeV})]$ can only be experimentally detected, i.e. not smeared out, if its argument is roughly around one. Thus, an experimental setup that has a baseline of L (km) and an energy of E (GeV), is sensitive to the neutrino mass squared difference $\Delta m_{ij}^2 (\mathrm{eV}^2)$ of order $\sim [L\,(\mathrm{km})/E\,(\mathrm{GeV}]^{-1}$.

To investigate mass differences Δm_{ij}^2 down to $\sim 10^{-4}$ eV^2, for instance, an experiment with a baseline of $L \sim 10^4$ km, or nearly the size of Earth's diameter, and $E \sim 1$ GeV should be conducted. This fact demonstrates that even pitifully small neutrino mass variations can be tested by neutrino long-baseline experiments. It achieves this by taking advantage of the quantum mechanical interference between amplitudes whose relative phases are provided exactly by these minuscule neutrino mass discrepancies. By selecting L/E properly, these amplitudes can be converted into substantial impacts.

But let's analyze the oscillation probability further and see if we can learn more about neutrino oscillations by studying its mathematical formula. It's clear from $\mathrm{P}(\overset{(-)}{\nu}_\alpha \longrightarrow \overset{(-)}{\nu}_\beta)$ that if the mass of all the neutrinos is zero, or they are all mass degenerate, that is, if all $\Delta m_{ij}^2 = 0$, then, $\mathrm{P}(\overset{(-)}{\nu}_\alpha \longrightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$.

Neutrinos are therefore massive, and their masses are not degenerate, as evidenced by the experimental finding that they can change from one flavor to another. In actuality, it was this very evidence that established the mass of neutrinos beyond a shadow of a doubt.

All observed oscillations of neutrinos have at some point involved neutrinos passing through matter. However, the expression we deduced is limited to flavor change in vacuum and ignores any interaction that may occur between the neutrinos and the matter they pass through en route to their detector. Therefore, the question still stands whether the observed flavor transitions could actually be caused by some unidentified flavor-changing interaction between neutrinos and matter rather than neutrino masses.

In response to this inquiry, a few points ought to be made. Foremost, while it is true that the Standard Model of elementary particle physics only includes massless neutrinos, it also describes all the possible ways a neutrino can interact, and does so in an extraordinarily well-corroborated way. A description that however does not include flavor change.

Second, matter effects are predicted to be pitifully small for some of the experimentally observed processes where neutrinos do change flavor. In those cases, however, the evidence suggests that the flavor transition probability depends on L and E through the combination L/E, as predicted by the oscillation hypothesis. Besides, L/E is the exact proper time that passes in the neutrino's rest frame while it travels a distance L with energy E, modulo a constant. Hence, rather than being the outcome of a reaction with matter, these flavor transitions behave as though they were a real evolution of the neutrino over time.

Let's now investigate the scenario in which the leptonic mixing is negligible. This would suggest that the emerging charged antilepton $\overline{\ell_\alpha}$ of flavor α always follows the *same* neutrino mass eigenstate ν_i in the charged boson decay $W^+ \longrightarrow \overline{\ell_\alpha} + \nu_i$, which as we established has an amplitude $U_{\alpha i}^*$. In other words, if $U^*_{\alpha i} \neq 0$, then for all $j \neq i$, $U_{\alpha j}$ becomes zero because of unitarity. Consequently, it is evident from Eq. (12) that $\mathrm{P}(\overset{(-)}{\nu_\alpha} \longrightarrow \overset{(-)}{\nu_\beta}) = \delta_{\alpha\beta}$. As neutrinos are known to change flavor, this undoubtedly indicates the presence of a non-trivial mixing matrix.

Consequently, there are essentially two methods left for detecting neutrino flavor change. The first is to notice that some neutrinos of a new flavor β which differs from the original flavor α are present in a beam of neutrinos that were all formed with the same flavor, let's say α. We refer to this as appearance experiments. The second method involves beginning with a beam of identical ν_αs, whose flux is either known or measured, and seeing that this flux is exhausted after a certain distance. These kinds of investigations are known as disappearance studies.

The transition probability in vacuum depends on L/E and oscillates with it, as demonstrated by Eq. (12). This is the reason behind the term "neutrino oscillations" for neutrino flavor transitions. Note that the squared-mass *differences* determines the neutrino transition probabilities rather than the individual neutrino masses or masses squared. Thus, the neutrino mass squared spectrum is the only quantity that oscillation experiments can measure. not on an absolute scale. The pattern can be tested by experiments, but the distance above zero at which the entire spectrum lies cannot be found.

It is evident that neutrino transitions only change the distribution of flux among the many flavors in a neutrino beam, not its total flux. In fact, given the unitarity of the U matrix and Eq. (12), it is clear that

$$\sum_\beta \mathrm{P}(\overset{(-)}{\nu_\alpha} \longrightarrow \overset{(-)}{\nu_\beta}) = 1 \quad , \tag{16}$$

where the total includes the original flavor α as well as all other flavors β. It is evident from Eq. (16) that the likelihood of a neutrino changing its flavor, when paired with the probability of it being unchanged at birth, equals one. Thus, flavor changes don't affect the overall flow. However, some of the flavors that a neutrino can oscillate into, $\beta \neq \alpha$, might be *sterile* flavors—that is, flavors that avoid detection by not participating in weak interactions. An experiment that measures the total *active* neutrino flux—that is, the flux related to those neutrinos that couple to the weak gauge bosons: ν_e, ν_μ, and ν_τ—will note that it is smaller than the original flux if any of the original (active) neutrino flux becomes sterile. No flux has ever been overlooked in any experiment up until this point.

The various mass eigenstates ν_i that coherently contribute to a beam are typically assumed to share the same *momentum* in literature descriptions of neutrino oscillations, rather than the same *energy,* as we have proved they must have. Although the assumption of equal momentum is incorrect in theory, it is irrelevant (or not a mistake worth worrying about) because it leads to the same oscillation probability as the ones we have found, as can be readily demonstrated.

The case where just two flavors engage in the oscillation is a pertinent and intriguing example of the (not so simple) formula for $\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta})$. Numerous experiments are quite rigorously described by the only-two-neutrino scenario. Actually, a more complex (three neutrino description) was only required recently (and in a few experiments) in order to fit observations.

In order to ensure that only one squared-mass difference, $m_2^2 - m_1^2 \equiv \Delta m^2$, surfaces, let's assume that only two mass eigenstates, which we will name ν_1 and ν_2, and two reciprocal flavor states, which we will name ν_μ and ν_τ, are important. Furthermore, the mixing matrix U can be expressed as follows by ignoring phase variables that are demonstrably insignificant to oscillation probabilities:

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{17}$$

As a 2×2 rotation matrix, the unitary mixing matrix U of Eq. (17) is parameterized by a single rotation angle θ, which is known as the mixing angle in neutrino physics. We can easily demonstrate that, for $\beta \neq \alpha$, when only two neutrinos are important, by plugging the U of Eq. (17) and the unique Δm^2 into the general formula of the transition probability $\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta})$

$$\mathrm{P}(\overline{\nu_\alpha} \longrightarrow \overline{\nu_\beta}) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2\, L}{4E}\right) \quad . \tag{18}$$

Furthermore, as predicted, the survival probability, or disappearance probability is equal to one minus the chance that the neutrino will change from the flavor it was generated with.

3 Neutrino oscillations in a medium

A neutrino beam is created on Earth using an accelerator and sent thousands of kilometers to a detector; instead of traveling through vacuum, the beam travels through matter, or earthly matter. The neutrino beam then disperses from the particles it encounters on its journey and undergo a coherent forward scattering that can (and will) significantly impact the transition probabilities. For the time being, we will make the assumption that neutrino interactions with matter follow the Standard Model's description of flavor conservation and then discuss the potential of flavor-changing interactions. Then, there would only be two scenarios for this coherent forward scattering from matter particles to occur, since there are only two forms of weak interactions (mediated by charged and neutral currents). Weak interactions mediated by charged currents will only happen in the event that the neutrino entering the system is an electron neutrino, since only the ν_e is able to trade a charged boson W for an electron from Earth.

The W exchange mode of neutrino-electron coherent forward scattering thus provides an additional source of interaction energy V_W that is solely experienced by electron neutrinos. Given that it originates from weak interactions, the additional energy must obviously be proportional to the Fermi coupling constant, G_F. Furthermore, the interaction energy resulting from $\nu_e - e$ scattering increases in relation to the number of targets, N_e, which is the electron density per unit volume. When everything is considered, it is evident that

$$V_W = +\sqrt{2}\, G_F\, N_e \quad , \tag{19}$$

obviously, antineutrinos are likewise impacted by this interaction energy (although in the reverse way). If we swap ν_e with $\overline{\nu_e}$, the sign of the interaction energy changes.

Neutral current-mediated interactions are analogous to the situation in which a matter-interacting neutrino exchanges a neutral Z boson with an electron, proton, or neutron. The Standard Model states that weak interactions are independent of flavor. They are enjoyed by all flavors of neutrinos, and the Z exchange's amplitude is constant. It also informs us that protons and electrons bind to the Z boson with identical strength at zero momentum transfer. Nevertheless, the interaction has the opposite sign. Consequently, the contribution of protons and electrons to coherent forward neutrino scattering by Z exchange will add up to zero, presuming that the matter our neutrino travels through is electrically neutral (containing an equal number of protons and electrons). Thus, the influence of the Z exchange contribution to the interaction potential energy V_Z reduces exclusively to that with neutrons and will be proportional to N_n, the number density of neutrons. As a result, only interactions with neutrons will survive. It should go without saying that every flavor will be equal. In such a case we have,

$$V_Z = -\frac{\sqrt{2}}{2} G_F N_n \quad , \tag{20}$$

similar to the last instance for V_W, this contribution will reverse if anti-neutrinos are used in place of neutrinos.

However, if, as we previously stated, the Standard Model interactions do not alter the flavor of neutrinos, then even in the case of neutrinos moving through matter, flavor transitions or oscillations clearly indicate the mass and mixing of neutrinos. Unless flavor-changing interactions that aren't standard model-related come into play.

Analyzing neutrino propagation in matter using the lab frame's time-dependent Schrödinger equation makes it simple to understand.

$$i\frac{\partial}{\partial t}|\nu(t) > = \mathcal{H}|\nu(t) > \quad . \tag{21}$$

wherein each neutrino flavor in the (three component) neutrino vector state $|\nu(t) >$ corresponds to a single component. Similarly, in flavor space, the Hamiltonian $\mathcal{H}$ is a (tree $\times$ three) matrix. In order to simplify our analysis, let us consider the scenario in which ν_e and ν_μ, the only two neutrino flavors that are important. So that

$$|\nu(t) > = \begin{pmatrix} f_e(t) \\ f_\mu(t) \end{pmatrix} \quad , \tag{22}$$

The neutrino's amplitude at time t to be in, ν_i given by $\mid f_i(t) \mid^2$. This time, $\mathcal{H}$, the Hamiltonian, is a $2{\times}2$ matrix in $\nu_e - \nu_\mu$ space, which is the neutrino flavor space.

Working through the two flavor cases in vacuum first and adding matter effects later will prove to be illuminating. For the Hamiltonian in vacuum, $\mathcal{H}_{\rm Vac}$, we can write $|\nu_\alpha >$ as a linear combination of mass eigenstates using Eq. (2). This allows us to observe that the $\nu_\alpha - \nu_\beta$ matrix element may be expressed as,

$$< \nu_\alpha|\mathcal{H}_{\rm Vac}|\nu_\beta >=< \sum_i U^*_{\alpha i}\nu_i|\mathcal{H}_{\rm Vac}|\sum_j U^*_{\beta j}\nu_j >= \sum_j U_{\alpha j}U^*_{\beta j}\sqrt{p^2+m_j^2} \ . \tag{23}$$

where we assume that the neutrinos are part of a beam with the same definite momentum p shared by all of its mass components (the mass eigenstates). As we have already indicated, even if this hypothesis is incorrect in theory, it nevertheless results in the appropriate transition amplitude. The neutrinos ν_j with momentum p, the mass eigenstates, are the asymptotic states of the Hamiltonian, $\mathcal{H}_{\rm Vac}$ which provide an orthonormal basis, and are employed in the second line of Eq. (23),

$$\mathcal{H}_{\rm Vac}|\nu_j >= E_j|\nu_j > \tag{24}$$

and the canonical dispersion relation holds, $E_j = \sqrt{p^2 + m_j^2}$.

Neutrino oscillations, as we have already discussed, are the classic example of a quantum interference phenomena, where only the *relative* phases of the interfering states are involved. Consequently, the only values that matter are the *relative* energies of these states, which determine their relative phases. Therefore, we can gladly exclude any contribution proportionate to the identity matrix I from the Hamiltonian $\mathcal{H}$, if that turns out to be useful (which it will). As previously stated, the subtraction will not impact the variations among the eigenvalues of $\mathcal{H}$, and hence, it will not impact the estimation of $\mathcal{H}$ about flavor transitions. Naturally, since only two neutrinos are relevant in this instance, there are only two mass eigenstates, ν_1 and ν_2, and one mass splitting, $\Delta m^2 \equiv m_2^2 - m_1^2$. As a result, a unitary U matrix, given by Eq. (17), should exist as before to rotate from one basis to the other. After inserting it into Eq. (23), assuming that our neutrinos have low masses relative to their momenta, i.e. $(p^2+m_j^2)^{1/2} \cong p+m_j^2/2p$, and eliminating a term proportional to the identity matrix from $\mathcal{H}_{\text{Vac}}$ (a removal we know will be harmless), we obtain

$$\mathcal{H}_{\text{Vac}} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \; . \tag{25}$$

The ultra relativistic approximation, which states that $p \cong E$, is utilized to create this expression. where E is the mean energy of the ultra-high momentum p of the neutrino mass eigenstates in our neutrino beam.

The fact that this Hamiltonian $\mathcal{H}_{\text{Vac}}$ of Eq. (25) for the two neutrino scenario would yield the same oscillation probability, Eq. (18), as the one we have previously gotten differently, is easily corroborated. Analyzing the transition probability for the process $\nu_e \longrightarrow \nu_\mu$ is a simple method, for example. It is evident from Eq. (17) that the state describing the electron and muon neutrino is

$$|\nu_e> = \quad |\nu_1> \cos\theta + |\nu_2> \sin\theta \quad , \quad |\nu_\mu> = -|\nu_1> \sin\theta + |\nu_2> \cos\theta \; . \tag{26}$$

The eigenvalues of the vacuum Hamiltonian $\mathcal{H}_{\text{Vac}}$, Eq.25, can also be expressed in terms of the mass squared differences in this manner.

$$\lambda_1 = -\frac{\Delta m^2}{4E} \;\; , \; \lambda_2 = +\frac{\Delta m^2}{4E} \; . \tag{27}$$

Using Eqs. (26), the mass eigenbasis of this Hamiltonian, $|\nu_1>$ and $|\nu_2>$, may also be expressed in terms of the flavor eigenstates, $|\nu_e>$ and $|\nu_\mu>$. Consequently, the Schrödinger equation of Eq. (21), which identifies $\mathcal{H}$ in this instance with $\mathcal{H}_{\text{Vac}}$, indicates that if we start at a $|\nu_e>$ at time $t = 0$, then after a certain amount of time t passes, this $|\nu_e>$ will advance into the state provided by

$$|\nu(t)> = |\nu_1> e^{+i\frac{\Delta m^2}{4E}t} \cos\theta + |\nu_2> e^{-i\frac{\Delta m^2}{4E}t} \sin\theta \; . \tag{28}$$

Accordingly, from Eqs. (26) and (28), the probability $\mathrm{P}(\nu_e \longrightarrow \nu_\mu)$ that this evolved neutrino be discovered as a new flavor ν_μ is provided by,

$$\mathrm{P}(\nu_e \longrightarrow \nu_\mu) \quad = \quad |<\nu_\mu|\nu(t)>|^2 = |\sin\theta\cos\theta(-e^{i\frac{\Delta m^2}{4E}t} + e^{-i\frac{\Delta m^2}{4E}t})|^2$$

$$= \quad \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4E}\right) \quad . \tag{29}$$

Here, the distance L that our extremely relativistic state has traveled is used to replace the time t that it has traveled. As anticipated, the flavor transition or oscillation probability of Eq. (29) is precisely the same as the previous value we obtained, Eq. (18).

The analysis of neutrino propagation in matter can now be continued. In this instance, the two previously stated extra contributions are included into the $2{\times}2$ Hamiltonian that represents the propagation in vacuum, $\mathcal{H}_{\rm Vac}$, to provide $\mathcal{H}_M$, which is given by

$$\mathcal{H}_M = \mathcal{H}_{\rm Vac} + V_W \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + V_Z \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad . \tag{30}$$

The interaction potential resulting from the exchange of charged bosons is represented by the first additional contribution in the new Hamiltonian, Eq. (19). This contribution differs from zero only in the $\mathcal{H}_M(1,1)$ element or the $\nu_e - \nu_e$ element, since this interaction only affects ν_e. The Z boson exchange, Eq. (20), provides the second extra contribution, which is the final component of Eq. (30). It is safe to ignore this interaction as its contribution to $\mathcal{H}_M$ is proportional to the identity matrix, and it impacts all neutrino flavors in the same way because it is flavor blind. Consequently,

$$\mathcal{H}_M = \mathcal{H}_{\rm Vac} + \frac{V_W}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{V_W}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \tag{31}$$

wherein we have split the W-exchange contribution into two parts—one proportionate to the identity (which we will ignore in the following step) and the other, which is not proportional to the identity and which we will retain—for reasons that will become apparent later. As stated, we can disregard the first portion and use the results from Eqs. (25) and (31).

$$\mathcal{H}_M = \frac{\Delta m^2}{4E} \begin{pmatrix} -(\cos 2\theta - A) & \sin 2\theta \\ \sin 2\theta & (\cos 2\theta - A) \end{pmatrix} \quad , \tag{32}$$

where

$$A \equiv \frac{V_W/2}{\Delta m^2/4E} = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2} \quad . \tag{33}$$

The relative magnitude of the matter effects in relation to the vacuum contribution provided by the neutrino squared-mass splitting is clearly parameterized by A, which also indicates the circumstances in which these effects become significant.

Now, if we introduce a shorthand notation of physical significance

$$\Delta m_M^2 \equiv \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2} \tag{34}$$

and

$$\sin^2 2\theta^M \equiv \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \quad , \tag{35}$$

then the Hamiltonian describing the propagation of the neutrinos in matter $\mathcal{H}_M$ becomes

$$\mathcal{H}_M = \frac{\Delta m_M^2}{4E} \begin{pmatrix} -\cos 2\theta^M & \sin 2\theta^M \\ \sin 2\theta^M & \cos 2\theta^M \end{pmatrix} . \tag{36}$$

The Hamiltonian in a medium, $\mathcal{H}_M$, becomes formally identical from the vacuum one, $\mathcal{H}_{\text{Vac}}$, Eq. (25), as a result of our definitions. Then can be trivially diagonalized. The difference between the Hamiltonians resides in the fact that, the matter parameters, Δm_M^2 and θ^M, respectively, become now, what were previously the vacuum parameters, Δm^2 and θ.

It is evident that the mass eigenstates and eigenvalues of $\mathcal{H}_M$ differ from those in vacuum, but are still given by the mass differences and mixing angle in matter in the same way as the ones in vacuum before. Thus, the vacuum eigenvalues that make up the vacuum mixing matrix are distinct from the eigenstates in matter, i.e. the values of the unitary matrix that rotates from the flavor basis to the mass basis are not the same and obviously, θ_M is not equal to θ. However, just as $\mathcal{H}_{\text{Vac}}$ contains all the information regarding neutrino propagation in vacuum, the matter Hamiltonian $\mathcal{H}_M$ does in fact contain all the information regarding neutrino propagation in matter.

The functional dependence of $\mathcal{H}_M$ on the matter parameters Δm_M^2 and θ^M is the same as that of the vacuum Hamiltonian $\mathcal{H}_{\text{Vac}}$, Eq. (25), on the vacuum ones, Δm^2 and θ, according to Eq. (36). As a result, θ^M can be identified with an effective mixing angle in matter, and Δm_M^2 can be identified with an effective mass squared difference in matter.

In a standard experimental setup, the neutrino beam travels through earth matter just superficiall—it does not penetrate deeply—after being produced by an accelerator and directed toward a detector located hundreds or even thousands of kilometers away. Therefore, it may be assumed that the matter density this beam would meet is roughly constant during its journey. However, it is evident that this approximation is invalid for neutrinos traveling over the Earth. However, this also holds true for the electron density N_e and the A parameter in which it is included if the density of matter on Earth remains constant. Regarding the Hamiltonian $\mathcal{H}_M$, it is also accurate. Except for their specific parameter values, that as we have seen can be considered all almost constant, the situation is identical to the one we faced with the vacuum Hamiltonian $\mathcal{H}_{\text{Vac}}$. Therefore, we already know that, in the same way that $\mathcal{H}_{\text{Vac}}$ gives rise to vacuum oscillations with probability $\mathrm{P}(\nu_e \longrightarrow \nu_\mu)$ of Eq. (29), $\mathcal{H}_M$ must give rise to matter oscillations, which by comparing Eqs. (36) and (25), is given by

$$\mathrm{P}_M(\nu_e \longrightarrow \nu_\mu) = \sin^2 2\theta^M \sin^2 \left(\Delta m_M^2 \frac{L}{4E} \right) . \tag{37}$$

In other words, the survival and transition probabilities in matter are the same as in vacuum, with the exception that the parameters in vacuum, Δm^2 and θ, are now substituted by the corresponding values in matter, Δm_M^2 and θ^M.

Merely based on its potential, matter effects have the ability to significantly alter oscillation probabilities, at least in theory. Only until the specifics of the experiment's experimental setup are provided can the precise impact, if any, be calculated. Generally speaking, if the kinematic phase linked to the solar mass difference is still insignificant and neutrinos are traveling through the earth's mantle (no more than 200 km below the surface), then studying

$$A \cong \frac{E}{13 \ \text{GeV}} \tag{38}$$

will help us estimate the significance of matter effects. And we can easily see that matter effects are relevant only for beam intensities of few GeV.

And what effect do they have? They are really important! We can observe from Eq. (35) for the matter mixing angle, θ^M, that even in cases when the vacuum mixing angle θ is minuscule, for example, $\sin^2 2\theta = 10^{-4}$, in comparison to its vacuum value, $\sin^2 2\theta^M$ can be greatly boosted if we can obtain $A \cong \cos 2\theta$, i.e., for energies of a few tens of GeV. It can even reach maximal mixing, $\sin^2 2\theta^M = 1$.

This amazing enhancement, known as the Mikheyev-Smirnov-Wolfenstein effect [5, 6], is a "resonant" amplification of a small mixing angle in vacuum up to a significant one in matter, up to maximal. When solar neutrino physics first started, there was a theory that this violent amplification was really happening when neutrinos traveled across the sun. However, as we shortly afterward observed, the mixing angle linked to solar neutrinos is already relatively large ($\sim 34°$) in vacuum [7]. Consequently, while matter effects on the sun are indeed significant, they are sadly not as significant as we had originally anticipated. However, over long-baselines, they will—and already do—play a crucial part in deciding the ordering of the neutrino mass spectrum.

4 Evidence for neutrino oscillations

4.1 Atmospheric and accelerator neutrinos

Since we were first shown strong, persuasive evidence of neutrino masses and mixings more than two decades ago, the body of evidence has only increased. The first experiment to provide strong evidence of ν_μ disappearance in their atmospheric neutrino fluxes was SuperKamiokande (SK) [8]. The multi-GeV ν_μ sample's zenith angle dependency, or the angle subtended with the horizontal, and its disappearance as a function of L/E are displayed in Fig. 2. These findings fit the simple two-component neutrino theory with remarkable accuracy with

$$\Delta m^2_{\text{atm}} = 2 - 3 \times 10^{-3} \text{eV}^2 \quad \text{and} \quad \sin^2 \theta_{\text{atm}} = 0.50 \pm 0.13 \tag{39}$$

for oscillations of L/E of 500 km/GeV and nearly maximal mixing, suggesting that the mass eigenstates are nearly even admixtures of tau and muon neutrinos. Since the third flavor, ν_e, does not exhibit any evidence of involvement, it is assumed that atmospheric neutrino disappearance is essentially $\nu_\mu \longrightarrow \nu_\tau$. However, take note of the fact that more recent results strongly suggest a mixing angle that is not maximal.

Following the discovery of atmospheric neutrino oscillations, a new set of neutrino experiments was constructed, sending (man-made) beams of ν_μ neutrinos to detectors situated at great distances: the MINOS (NOvA) experiment [9, 10] sends its beam from Fermilab, near Chicago, to the Soudan mine (Ash river) in Minnesota, a baseline of 735 (810) km, while the K2K (T2K) experiment [11, 12] sends neutrinos from the KEK accelerator complex to the old SK mine, with a baseline of 120 (235) km. Evidence of ν_μ disappearance consistent with the one found by SK has been observed in all these

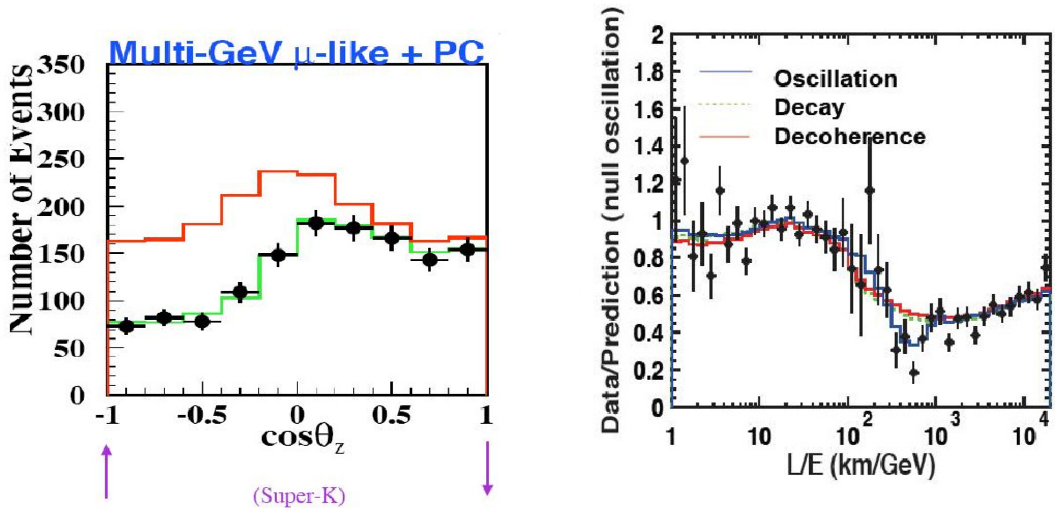

Fig. 2: Superkamiokande's evidence for neutrino oscillations both in the zenith angle and L/E plots.

studies. Figure 3 summarizes their findings.

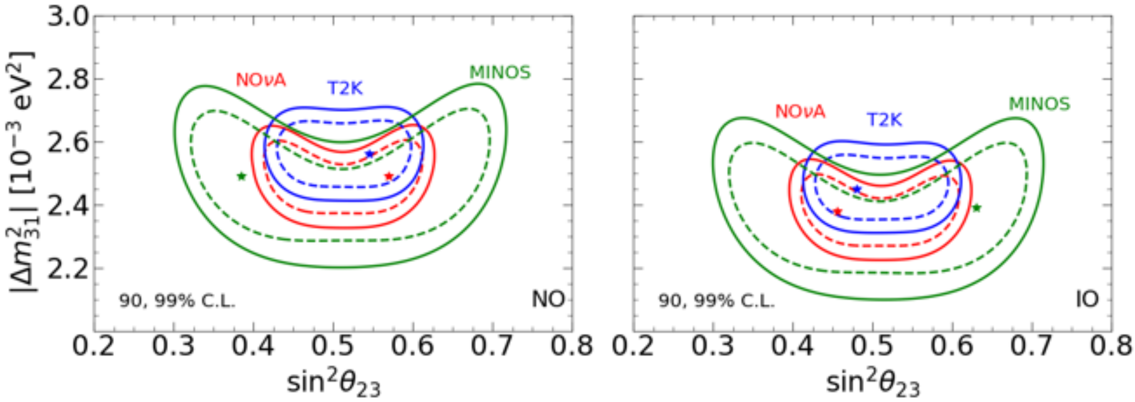

Fig. 3: Allowed regions in the Δm^2_{atm} vs $\sin^2\theta_{\text{atm}}$ plane for MINOS and NOVA data as well as for T2K data and two of the SK analyses. Results from https://globalfit.astroparticles.es/

4.2 Reactor and solar neutrinos

Evidence of neutrino oscillations has been observed in the KamLAND reactor experiment, an antineutrino disappearance experiment that receives neutrinos from sixteen different reactors at distances ranging from hundreds to thousands of kilometers, with an average baseline of 180 km and neutrinos of a few eV [13]. This proof was gathered not only at a distinct L/E compared to the atmosphere and accelerator studies, but also includes oscillations involving electron neutrinos, ν_e, which were not implicated before. Since the sun only creates electron neutrinos, these oscillations have also been observed in neutrinos that originate from it. However, we must make the assumption that neutrinos (from the sun) and antineutrinos (from the reactor) behave similarly in order to compare the two tests; this is known as CPT conservation.

For the KamLAND experiment, the best fit values in the two neutrino scenario are

$$\Delta m^2_\odot = 7.55 \pm 0.2 \times 10^{-5}\text{eV}^2 \quad \text{and} \quad \sin^2\theta_\odot = 0.32 \pm 0.03 \tag{40}$$

The mixing angle, while high, is obviously not maximal in this situation, and the L/E involved is 15 km/MeV.

The $\bar{\nu}_e$ disappearance probability for KamLAND and a few older reactor studies with shorter baselines are displayed in Fig. 4 as well as the best fit regions. The analysis of neutrinos originating

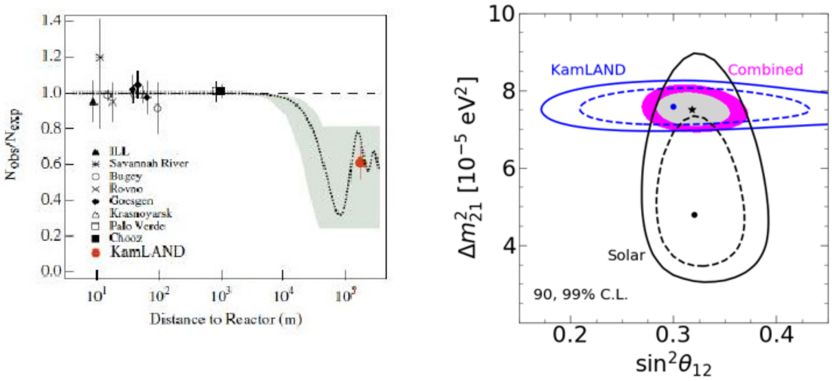

Fig. 4: Disappearance of the $\bar{\nu}_e$ observed by reactor experiments as a function of distance from the reactor. Favored region for all solar and reactor experiments. Results from https://globalfit.astroparticles.es/

from the sun is slightly more complex than the previous one we conducted. This is especially true for the ^{8}Boron neutrinos. Due to their lower energy, the pp and ^{7}Be neutrinos are not greatly affected by the existence of matter. This is not the case for ^{8}Boron neutrinos which depart the sun as ν_2, the second mass eigenstate, and do not oscillate, clearly indicating that they are affected by the existence of matter.

It is important to note, nevertheless, that solar neutrino oscillations are not really observed. We require a kinematic phase of order one in order to follow the oscillation pattern and test its unique shape; otherwise, the oscillations either do not emerge or average out to 1/2. For neutrinos that originate from the sun, the kinematic phase is

$$\Delta_\odot = \frac{\Delta m^2_\odot L}{4E} = 10^{7\pm1} \quad . \tag{41}$$

As a result, when solar neutrinos leave the sun, they behave as "effectively incoherent" mass eigenstates, and they stay that way when they get to Earth. As a result, the ν_e survival or disappearance probability is provided by

$$\langle P_{ee}\rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot \tag{42}$$

where f_1 (f_2) is the $\nu_1(\nu_2)$ content of ν_μ, and $f_1 + f_2 = 1$.

In the case of solar neutrinos originating from the pp and ^{7}Be chains, the fractions are $f_1 \approx \cos^2\theta_\odot = 0.69$ and $f_2 \approx \sin^2\theta_\odot = 0.31$ since they oscillate as in vacuum and are unaffected by solar matter. However, the impact of solar matter is significant, and the corresponding fractions are significantly changed in the ^{8}B neutrino case in which

$$f_2 = 0.91 \pm 0.02 \text{ at the 95 \% C.L.} \quad . \tag{43}$$

As a result, it is evident that the ^{8}B solar neutrinos are the cleanest mass eigenstate neutrino beam currently known.

Last but not least, the third mixing angle [14] was finally measured eleven years ago by the Daya Bay experiment, a reactor neutrino experiment in China. It was discovered to be

$$\sin^2(2\theta_{13}) = 0.092 \pm 0.017 \quad . \tag{44}$$

Subsequent to this discovery, other experiments validated the discovery, and in recent years, the last mixing angle measured emerged as the most accurate one. This angle's sizeable size, albeit being smaller than the other two, allows for the possibility of future neutrino studies that seek to address the unanswered concerns in the field.

5 ν Standard Model

After understanding the physics underlying neutrino oscillations and gaining insight from experimental data regarding the parameters causing these oscillations, we can proceed to build the Neutrino Standard Model, which consists of three light ($m_i < 1$ eV) neutrinos, or i.e. only two mass differences.

As of yet, there has been no conclusive, or even solid, experimental evidence supporting the need for more neutrinos, though it should be emphasized that there are a few weak signals. Since the invisible width of the Z boson has been measured for a long time and is found to be 3, within errors, any more neutrinos added to the model will have to be sterile since they will not be able to couple to the Z boson, i.e. they will not be able to experience weak interactions. Nevertheless, our Neutrino Standard Model will only include the three active flavors, e, μ, and τ, as sterile neutrinos have not been observed and are not required to explain any experimental data.

Three mixing angles, the so-called solar mixing angle: θ_{12}, the atmospheric mixing angle: θ_{23}, and the final one to be measured, the reactor mixing angle: θ_{13}, one Dirac phase, δ, and possibly two Majorana phases, $\alpha$and β, make up the unitary mixing matrix, also known as the PMNS matrix, which rotates from the flavor to the mass basis. It is given by

$$|\,\nu_\alpha\rangle = U_{\alpha i}\,|\,\nu_i\rangle \quad ,$$

$$U_{\alpha i} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. We can identify the (23) label in the three neutrino scenario as the atmospheric Δm^2_{atm} that we obtained in the two neutrino scenario thanks to the hierarchy in mass differences (and, to a lesser extent, the smallness of the reactor mixing angle). Similarly, the (12) label can be assimilated to the solar $\Delta m^2_\odot$. The ν_e flavor oscillations at the atmospheric scale are driven by the (13) sector.

As per current experimental results, the three sigma ranges associated with the neutrino parameters are

$0.271 < \sin^2\theta_{12} < 0.369$; $0.434 < \sin^2\theta_{23} < 0.610$; $0.0200 < \sin^2\theta_{13} < 0.0245$ $2.47\times10^{-3}\mathrm{eV}^2 < |\Delta m^2_{32}| < 2.63\times10^{-3}\mathrm{eV}^2$ and $6.94\times10^{-5}\mathrm{eV}^2 < \Delta m^2_{21} < 8.14\times10^{-5}\mathrm{eV}^2$.

Two orderings are possible because oscillation experiments only investigate the two mass differences. They are known as normal and inverted hierarchy, and indicate whether the lightest or heaviest mass eigenstate, respectively, is the one with the smallest electron neutrino content.

Although the mass of the lightest neutrino, or the absolute mass scale of neutrinos, is unknown, cosmological bounds already dictate that the heaviest neutrino must be lighter than roughly 2 eV.

No evidence of the Majorana phases could be seen in oscillation phenomena because transition or survival probabilities rely on the combination $U^*_{\alpha i}U_{\beta i}$. However, they will be noticeable in processes where the Majorana character of the neutrino is necessary for the process to occur, such as neutrino-less double beta decay.

6 Neutrino mass and character

6.1 Absolute neutrino mass

Although oscillation experiments are unable to yield the neutrino's absolute mass scale, or the mass of the lightest/heaviest neutrino, this does not mean that we are without means of obtaining it. Both direct experiments—such as tritium beta decay or neutrinoless double beta decay—and indirect ones—such as cosmic observations—have the capacity to provide us with the much-needed details on the precise scale of neutrino mass. The sensitivity of the Katrin tritium beta decay experiment, [15], given the "mass" of ν_e defined as

$$m_{\nu_e} = |U_{e1}|^2 m_1 + |U_{e2}|^2 m_2 + |U_{e3}|^2 m_3. \tag{45}$$

Neutrino-less double beta decay experiments assess a specific mixture of neutrino masses and mixings rather than the neutrino's absolute mass, see for example [16].

$$m_{\beta\beta} = |\sum m_i U^2_{ei}| = |m_a c^2_{13}c^2_{12} + m_2 c^2_{13}s^2_{12}e^{2i\alpha} + m_3 s^2_{13}e^{2i\beta}|, \tag{46}$$

wherein it is assumed that neutrinos are Majorana particles—that is, completely neutral particles. particles with zero quantum numbers throughout. In double beta decay, the goal of the latest studies is to lower the energy of $m_{\beta\beta}$ below 10 meV.

The total mass of neutrinos is measured by cosmological experiments [17] such as the Large Scale Structure experiments and the CMB. It is defined as

$$m_{\text{cosmo}} = \sum_i m_i \tag{47}$$

and allows to investigate additional features of neutrinos, such as neutrino asymmetries, and the mass ordering [18]. The present limit is ~ 0.2 eV. Although these bounds depend on the model, they all produce numbers with the same order of magnitude. As cosmological measurements are characterized by systematic errors, a definite limit of less than 200 meV appears much too aggressive.

6.2 Majorana vs Dirac

A coupling between a left-handed and a right-handed state is all that a fermion mass is. As a result, we can think of a massive fermion at rest as a linear combination of two massless particles, one left-handed and one right-handed. Both the left and right-handed particles must have the same charge if the particle under study is electrically charged, such as an electron or a muon[1]. We have then a Dirac mass.

Thus, a completely and totally neutral particle, which is inevitable to become its own antiparticle, has two options for obtaining a mass term: Majorana or Dirac. If neither option is prohibited, it will have both.

In the case of a neutrino, the left chiral field couples to $SU(2) \times U(1)$ indicating that a Majorana mass term is forbidden by gauge symmetry. However, the right chiral field carries no quantum numbers and is totally and absolutely neutral. Then, the Majorana mass term is unprotected by any symmetry, and it is expected to be very large, of the order of the largest scale in the theory. On the other hand, Dirac mass terms are expected to be of the order of the electroweak scale times a Yukawa coupling, giving a mass of the order of magnitude of the charged lepton or quark masses. Putting all the pieces together, the mass matrix for the neutrinos results in

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix} .$$

The neutrino mass matrix must be diagonalized in order to obtain the mass eigenstates. When this is done, one is left with two Majorana neutrinos: one super-light Majorana neutrino with mass m_D^2/M and one super-heavy Majorana neutrino with mass $\simeq M$. This is known as the seesaw mechanism because one mass rises while the other falls [19–21]. The simplest see-saw mechanisms can be divided into three groups based on their scalar content, depending on the high energy theory that is envisaged. While the heavy neutrinos are not accessible to current experiments, they may be responsible for explaining the baryon asymmetry of the universe by generating a lepton asymmetry at very high energy scales. This is because their decays have the potential to be CP violating because they depend on the two Majorana phases on the PNMS matrix, which are invisible for oscillations. The light neutrinos are those that are observed in current experiments. Due to their enormous masses, the very heavy Majorana neutrinos may contribute to inflation at very high energy [22].

Lepton number is no longer a valid quantum number if neutrinos are Majorana particles, and a variety of novel processes that are prohibited by lepton number conservation can occur—not just neutrino-less double beta decay. For instance, a positively charged muon can be created by a muon neutrino. Though mathematically permissible, these processes—and all others of their kind—would be suppressed by the minuscule 10^{-20} of $(m_\nu/E)^2$. As a result, they are experimentally unobservable. At

[1]We want the mass term to be electrically neutral

90% C.L., the most rigorous limit currently available for neutrino-less double beta decay comes from KamLAND-zen [23], which limits the half-life to $T^{0\nu}_{1/2} > 2.3 \times 10^{26}$ years. This sensitivity will increase by a factor of ten in the not so distant future.

Lately, low energy sew saw models [24] have come back into vogue and are being actively studied [25]. The LHC can be used to hunt for heavy states in these models that have energies of only a few tens of TeV. In these theories, the heavy right-handed states will be generated at the LHC by gauge coupling to right-handed gauge bosons or via Yukawa couplings.

7 Conclusions

After the Standard Model was established, there were still unanswered questions. These were resolved by the experimental observations of neutrino oscillations, which indicates that neutrinos have mass and mix. With the removal of those coverings, fresh inquiries arise that cast doubt on our comprehension.

What are neutrinos actually, Dirac or Majorana particles? Does a new scale related to the mass of neutrinos exist? Is it available at colliders? Is the spectrum inverted or normal? Which neutrino is lighter, the one with the fewest electron content on it, or is it the heaviest one? Is $\sin\delta \neq 0$? If yes, how is this phase connected to the universe's baryon asymmetry at all? Which is the neutrino's absolute mass scale? Do unanswered cosmological concerns like dark energy and/or dark matter have anything to do with neutrinos? Are (supposedly massive) neutrinos involved in the primordial inflation? Is CPT violated by neutrinos [26]? Lorentz invariance: what about it? How can we determine whether a non-standard neutrino interaction or a true CTP violation is the cause of a difference in the spectrum measured for neutrinos and antineutrinos if we ever measure it?

We would want to respond to these questions. We are already doing it, and we intend to do other studies. These experiments will undoubtedly raise important new questions and provide some solutions. There's only one thing that's obvious. Our exploration of the neutrino universe is only getting started.

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References

[1] *Symmetry* **4** (2007) ibc,
https://www.symmetrymagazine.org/article/march-2007/neutrino-invention.

[2] Z. Maki, M. Nakagawa and S. Sakata, *Prog. Theor. Phys.* **28**, (1962) 870–880, doi:10.1143/PTP.28.870.

[3] L. Stodolsky, *Phys. Rev.* **D58** (1998) 036006, doi:10.1103/PhysRevD.58.036006, arXiv:hep-ph/9802387; H.J. Lipkin, *Phys. Lett.* **B579** (2004) 355–360, doi:10.1016/j.physletb.2003.11.013, arXiv:hep-ph/0304187.

[4] An analysis of CPT violation in the neutrino sector can be found in: G. Barenboim and J.D. Lykken, *Phys. Lett.* **B554** (2003) 73–80, doi:10.1016/S0370-2693(02)03262-8, arXiv:hep-ph/0210411; G. Barenboim, J.F. Beacom, L. Borissov and B. Kayser, *Phys. Lett.* **B537** (2002) 227–232, doi:10.1016/S0370-2693(02)01947-0, arXiv:hep-ph/0203261; The world best bound in CPT conservation is found in G. Barenboim, C.A. Ternes and M. Tortola, *Phys. Lett.* **B780** (2018) 631–637, doi:10.1016/j.physletb.2018.03.060, arXiv:1712.01714 [hep-ph].

[5] L. Wolfenstein, *Phys. Rev.* **D17** (1978) 2369–2374, doi:10.1103/PhysRevD.17.2369.

[6] S.P. Mikheev and A.Y. Smirnov, *Sov. J. Nucl. Phys.* **42** (1985) 913 [*Yad. Fiz.* **42** (1985) 1441]; S.P. Mikheev and A.Y. Smirnov, *Sov. Phys. JETP* **64** (1986) 4 [*Zh. Eksp. Teor. Fiz.* **91** (1986) 7, arXiv:0706.0454 [hep-ph]; S.P. Mikheev and A.Y. Smirnov, *Nuovo Cim.* **C9** (1986) 17, doi:10.1007/BF02508049.

[7] N. Tolich [SNO Collaboration], *J. Phys. Conf. Ser.* **375** (2012) 042049, doi:10.1088/1742-6596/375/1/042049.

[8] V. Takhistov [Super-Kamiokande], *PoS* **ICHEP2020** (2021), 181, doi:10.22323/1.390.0181.

[9] J. Evans [MINOS and MINOS+], *J. Phys. Conf. Ser.* **888** (2017) 012017, doi:10.1088/1742-6596/888/1/012017.

[10] J.M. Carceller [NOvA], *PoS* **NOW2022** (2023) 015, doi:10.22323/1.421.0015.

[11] M.H. Ahn *et al.* [K2K], *Phys. Rev.,***D74** (2006) 072003, doi:10.1103/PhysRevD.74.072003, arXiv:hep-ex/0606032.

[12] L. Berns [T2K], *PoS* **NOW2022** (2023) 002, doi:10.22323/1.421.0002.

[13] M.P. Decowski [KamLAND], *Nucl. Phys.* **B908** (2016) 52–61, doi:10.1016/j.nuclphysb.2016.04.014.

[14] F.P. An *et al.* [Daya Bay], *Phys. Rev. Lett.* **129** (2022) 041801, doi:10.1103/PhysRevLett.129.041801.

[15] S. Mertens [KATRIN Collaboration], *Phys. Procedia* **61** (2015) 267, doi:10.1016/j.phpro.2014.12.043.

[16] S.R. Elliott and P. Vogel, *Ann. Rev. Nucl. Part. Sci.* **52** (2002) 115, doi:10.1146/annurev.nucl.52.050102.090641, arXiv:hep-ph/0202264.

[17] M. Lattanzi [Planck Collaboration], *J. Phys. Conf. Ser.* **718** (2016) 032008, doi:10.1088/1742-6596/718/3/032008.

[18] G. Barenboim, W.H. Kinney and W.I. Park, *Phys. Rev.* **D95** (2017) 043506, doi:10.1103/PhysRevD.95.043506, arXiv:1609.01584 [hep-ph]; G. Barenboim, W.H. Kinney and W.I. Park, *Eur. Phys. J.* **C77** (2017) 590, doi:10.1140/epjc/s10052-017-5147-4.

[19] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, edited by P.van Nieuwenhuizen and D. Freedman, (North-Holland,1979), p.315.

[20] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[21] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).

[22] G. Barenboim, JHEP **0903** (2009) 102 [arXiv:0811.2998]; G. Barenboim, Phys. Rev. D **82** (2010) 093014 [arXiv:1009.2504].

[23] K. Ichimura [KamLAND-Zen], PoS **NOW2022** (2023), 067 doi:10.22323/1.421.0067

[24] F. Borzumati and Y. Nomura, Phys. Rev. D **64** (2001) 053005 doi:10.1103/PhysRevD.64.053005 [hep-ph/0007018].

[25] C. G. Cely, A. Ibarra, E. Molinaro and S. T. Petcov, Phys. Lett. B **718** (2013) 957 doi:10.1016/j.physletb.2012.11.026 arXiv[1208.3654].

[26] G. Barenboim, L. Borissov, J. D. Lykken and A. Y. Smirnov, JHEP **0210** (2002) 001 [hep-ph/0108199]. G. Barenboim and J. D. Lykken, Phys. Rev. D **80** (2009) 113008 arXiv[0908.2993].