Flavour physics and CP violation

Alexander Lenz[a]
[a]University of Siegen, Siegen, Germany

We give a brief introduction into quark flavour physics and CP violation, starting in the first lecture with a review of the fundamental properties of the Standard Model of Particle Physics, a detailed discussion of the CKM matrix and a general classification of hadronic weak decays. The second lecture is devoted to describing the theoretical framework and in particular the concept of an effective Hamiltonian. In the third lecture we discuss mixing of neutral mesons and the effect of CP violation in hadron decays.

This chapter should be cited as: Flavour physics and CP violation, Alexander Lenz, DOI: 10.23730/CYRSP-2025-009.145, in: Proceedings of the 2023 European School of High-Energy Physics,
CERN Yellow Reports: School Proceedings, CERN-2025-009, DOI: 10.23730/CYRSP-2025-009, p.145.

1 Lecture 1: Standard Model, CKM matrix, weak decays

1.1 Motivation for Flavour Physics

The field of **Quark Flavour Physics** offers interesting opportunities in deepening our fundamental understanding of the world. In particular it sheds light into:

1. Matter-Antimatter asymmetry in the Universe: as we will see below the existence of matter in the Universe seems to be linked to the breaking of a symmetry, called CP. CP-violating effects were observed numerous times in the decays of hadrons containing heavy quarks.
2. Indirect searches for effects beyond the standard model (SM): finding discrepancies, when comparing precise measurements with precise SM predictions might give us hints how to extend the SM on a more fundamental level. This strategy benefits currently from the huge amount of data obtained by experiments like LHCb, Belle II, BES III, ATLAS and CMS. It is interesting to note that there are currently some anomalies, i.e. deviations from the SM expectations, observed in the decays of b- and c-hadrons.
3. Understanding of QCD: for the program of indirect new physics searches a rigorous control over hadronic effects is crucial. The theoretical description of these processes relies on different variations of effective theories.
4. Determination of SM parameter: the bread and butter physics goal of flavour physics is the determination of SM parameters like CKM-elements or quarks masses.

1.2 Lagrangian of the Standard Model

The Lagrangian of the Standard Model of Particle Physics (SM) [1–3] reads schematically

$$\begin{aligned} \mathcal{L} \quad = \quad & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i \bar{\Psi} \not{D} \Psi \\ & + |D_\mu \Phi|^2 - V(\Phi) \\ & + \bar{\Psi}_i Y_{ij} \Phi \Psi_j + h.c. \quad . \end{aligned} \tag{1}$$

The first line of Eq. (1) is the gauge kinetic term, which describes the propagation of the massless gauge fields of the strong, weak and electro-magnetic interactions, as well as, in the case of the latter two, their self-interaction. The second line is the fermionic kinetic term, which describes the propagation of massless fermions and their interactions with the gauge fields. The third line is known as the Higgs sector of the SM and contains the kinetic term for the complex Higgs doublet, including its interaction with the gauge fields, and the Higgs potential, see [4–7]. The specific form of the Higgs potential will give rise to mass terms for some of the gauge bosons. The last line is known as the Yukawa sector, which describes the interaction between the fermion fields Ψ and the complex scalar field Φ. When the Higgs field acquires the vacuum expectation value $(0, v/\sqrt{2})$, a mass term for the fermion fields is generated, with the mass being proportinal to $vY_{ij}/\sqrt{2}$.

The full Standard Model Lagrangian is invariant under Poincare transformations and local $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge transformations — $SU(3)_C$ describes the strong interaction, C stands for colour,

$SU(2)_L \times U(1)_Y$ describes the weak and the electromagnetic interaction, L stands for left-handed, and Y stands for hyper-charge. Looking at the $SU(2)_L \times U(1)_Y$-part in more detail, one gets in the case of one generation of fermions the following expressions:

$$\begin{aligned}
\mathcal{L}_{SU(2)_L \times U(1)_Y} \quad = \quad & -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu\, a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
& + \bar{\Psi}_L \gamma^\mu \left(i\partial_\mu - g_1 Y_L B_\mu - g_2 q_L \frac{\vec{\sigma} \cdot \vec{W}_\mu}{2} \right) \Psi_L \\
& + \bar{\Psi}_R \gamma^\mu \left(i\partial_\mu - g_1 Y_R B_\mu - g_2 q_R \frac{\vec{\sigma} \cdot \vec{W}_\mu}{2} \right) \Psi_R \\
& + \left| \left(i\partial_\mu - g_1 Y_\Phi B_\mu - g_2 q_\Phi \frac{\vec{\sigma} \cdot \vec{W}_\mu}{2} \right) \Phi \right|^2 - V(\Phi^\dagger \Phi) \\
& - \left(\bar{\Psi}_L \Phi^c Y_u u_R + \bar{u}_R \Phi^{c\dagger} Y_u^\dagger \Psi_L \right) - \left(\bar{\Psi}_L \Phi Y_d d_R + \bar{d}_R \Phi^\dagger Y_d^\dagger \Psi_L \right) . \qquad (2)
\end{aligned}$$

Let us first discuss the notation:

a) Ψ_L and Ψ_R denote left- and right-handed spinors describing the fermion fields

$$\Psi_{L,R} = \frac{1 \mp \gamma_5}{2} \Psi \, . \qquad (3)$$

The splitting into left- and right-handed components is motivated by the experimental observation of parity violation in weak decays. The violation of parity in the weak interaction was theoretically proposed in 1956 by Lee and Yang (Nobel Prize 1957) [8] and almost immediately verified in the experiment of Wu [9]. A way of implementing this fact into the theory is treating the right-handed fermions Ψ_R as $SU(2)_L$ singlets and the left-handed fermions Ψ_L as $SU(2)_L$ doublets, i.e.

$$\Psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} . \qquad (4)$$

Here, u_L is the four component Dirac spinor field of the up-quark, with weak isospin $+1/2$ and d_L is the four component Dirac spinor field of the down quark with weak isospin $-1/2$.

b) g_1 is the gauge coupling of the $U(1)_Y$ interaction mediated via the gauge field B_μ, with the corresponding field strength tensor defined as

$$B_{\mu\nu} \quad := \quad \partial_\mu B_\nu - \partial_\nu B_\mu \, . \qquad (5)$$

$Y_{L,R,\phi}$ are the hyper-charges of the left-handed fermions, right-handed fermions and of the Higgs field.

c) g_2 is the gauge coupling of the $SU(2)_L$ interaction mediated via the three gauge fields, $W^a_\mu (a = 1, 2, 3)$, with the corresponding field strength tensor, defined as

$$W^a_{\mu\nu} \quad := \quad \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu \, , \qquad (6)$$

where ϵ^{abc} are the structure constants of the $SU(2)$ algebra, i.e.

$$\left[\frac{\sigma^a}{2}, \frac{\sigma^b}{2}\right] = i\epsilon^{abc}\frac{\sigma^c}{2} \tag{7}$$

and $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ denote the Pauli matrices. Note the last term in Eq. (6) gives rise to self-interaction among the W_μ^a fields. The fact that only left-handed fermions take part in the weak interaction and right-handed do not, is fulfilled by the following choice of the charges: $q_R = 0$ and $q_L = q_\Phi = 1$. This describes correctly the experimentally found *maximal parity-violation* of the weak interaction.

d) Also the Higgs field is a $SU(2)_L$ doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{8}$$

with hyper-charge $Y = 1/2$. The complex Higgs doublet has four degrees of freedom and the quantum numbers,

	ϕ^+	ϕ^0
Q	$+1$	0
T_3	$+1/2$	$-1/2$
Y	$+1/2$	$+1/2$

Using the **unitary gauge** the Higgs field is expanded as

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}, \tag{9}$$

where $\langle\Phi\rangle = 1/\sqrt{2}(0, v)$ is the non-vanishing vacuum expectation value of the Higgs doublet Φ with $v \approx 246.22$ GeV,[1] and H describes the Higgs particle discovered at the LHC in 2012 [10,11]. To give both the up-type and the down-type quarks a mass we have to introduce a second Higgs field, which is not independent from the original one (in some extensions of the standard model, it will be independent, e.g. in the Two-Higgs Doublet Model (**2HDM**) or the Minimal Supersymmetric Standard Model (**MSSM**)), i.e.

$$\Phi^c = i\sigma_2\Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix}, \tag{10}$$

which can also be expanded as

$$\Phi^c = \frac{1}{\sqrt{2}}\begin{pmatrix} v+H \\ 0 \end{pmatrix}. \tag{11}$$

[1]Originally v is defined as the minimum of the Higgs potential, $v = \sqrt{-\mu^2/\lambda}$. Expressing the gauge boson masses in terms of v one gets $M_W = g_2 v/2$. Comparing this with the definition of the Fermi constant $G_F/\sqrt{2} = g_2^2/(8M_W^2)$ one sees that $v = \sqrt{1/(\sqrt{2}G_F)}$.

The potential of the Higgs field is given by

$$V(\Phi^\dagger\Phi) = \mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2\,, \;\; \mu^2 < 0\,. \tag{12}$$

e) Y^u and Y^d are the so-called Yukawa couplings. Since a naive fermion mass term of the form $m(\bar\Psi_L\Psi_R + \bar\Psi_R\Psi_L)$ is not gauge invariant under $SU(2)_L$, the gauge-invariant Yukawa interaction was introduced to generate fermion masses. This is discussed in detail below.

1.3 The Yukawa interaction

After spontaneous symmetry breaking, the Yukawa term, in the case of only one fermion generation, contains fermion masses:

$$\begin{aligned}\mathcal{L}_{Yukawa} &= -\left(\bar\Psi_L\Phi^c Y_u u_R + \bar u_R \Phi^{c\dagger} Y_u^* \Psi_L\right) - \left(\bar\Psi_L \Phi Y_d d_R + \bar d_R \Phi^\dagger Y_d^* \Psi_L\right)\\ &\to -\frac{vY_u}{\sqrt{2}}\left(\bar u_L u_R + \bar u_R u_L\right) - \frac{vY_d}{\sqrt{2}}\left(\bar d_L d_R + \bar d_R d_L\right) + \ldots\,.\end{aligned} \tag{13}$$

In the last line we assumed that the Yukawa couplings are real, which leads to a simple mass term for the up- and down quarks with $m_{u,d} = vY_{u,d}/\sqrt{2}$. The possibility of having complex values of the Yukawa coupling is studied below.

For three generations of quarks the situation gets still a little more involved. The Yukawa interaction reads now

$$\begin{aligned}&\mathcal{L}_{Yukawa} =\\ -\;&\left(\bar Q_{1,L}, \bar Q_{2,L}, \bar Q_{3,L}\right)\Phi^c \hat Y_u \begin{pmatrix} u_R\\ c_R\\ t_R\end{pmatrix} - \left(\bar u_R, \bar c_R, \bar t_R\right)\Phi^{c\dagger}\hat Y_u^\dagger \begin{pmatrix} Q_{1,L}\\ Q_{2,L}\\ Q_{3,L}\end{pmatrix}\\ -\;&\left(\bar Q_{1,L}, \bar Q_{2,L}, \bar Q_{3,L}\right)\Phi \hat Y_d \begin{pmatrix} d_R\\ s_R\\ b_R\end{pmatrix} - \left(\bar d_R, \bar s_R, \bar b_R\right)\Phi^{\dagger}\hat Y_d^\dagger \begin{pmatrix} Q_{1,L}\\ Q_{2,L}\\ Q_{3,L}\end{pmatrix}\,,\end{aligned} \tag{14}$$

with the three $SU(2)_L$ doublets

$$Q_{1,L} = \begin{pmatrix} u_L\\ d_L\end{pmatrix}, \quad Q_{2,L} = \begin{pmatrix} c_L\\ s_L\end{pmatrix}, \quad Q_{3,L} = \begin{pmatrix} t_L\\ b_L\end{pmatrix}. \tag{15}$$

Note, that now in general the Yukawa coupling matrices $\hat Y_{u,d}$ do not have to be diagonal! After spontaneous symmetry breaking one gets the following structure of the fermion mass terms:

$$-\bar\Psi_L^u \hat M_1 \Psi_R^u - \bar\Psi_R^u \hat M_1^\dagger \Psi_L^u - \bar\Psi_L^d \hat M_2 \Psi_R^d - \bar\Psi_R^d \hat M_2^\dagger \Psi_L^d\,, \tag{16}$$

with

$$\Psi^u = \begin{pmatrix} u \\ c \\ t \end{pmatrix} , \qquad \Psi^d = \begin{pmatrix} d \\ s \\ b \end{pmatrix} , \tag{17}$$

$$\hat{M}_1 = \frac{v}{\sqrt{2}}\hat{Y}_u , \qquad \hat{M}_2 = \frac{v}{\sqrt{2}}\hat{Y}_d . \tag{18}$$

Again, in general the mass matrices $\hat{M}_1$ and $\hat{M}_2$ do not have to be diagonal, but they can be diagonalised with unitary transformations

$$\Psi^u \rightarrow U_1\Psi^u \text{ with } U_1^\dagger U_1 = 1 , \tag{19}$$

$$\Psi^d \rightarrow U_2\Psi^d \text{ with } U_2^\dagger U_2 = 1 . \tag{20}$$

The transformed mass matrices read

$$U_1^\dagger \hat{M}_1 U_1 = \frac{v}{\sqrt{2}} U_1^\dagger \hat{Y}_u U_1 = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} , \tag{21}$$

$$U_2^\dagger \hat{M}_2 U_2 = \frac{v}{\sqrt{2}} U_2^\dagger \hat{Y}_d U_2 = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} . \tag{22}$$

The corresponding **mass eigenstates** describe the **physical eigenstates**, whereas the original fields that couple to the weak gauge bosons define **weak eigenstates**. In principle the mass matrices could also be diagonal from the beginning. We will start, however, with the most general case and let the experimental data show what is actually realised in nature.

1.4 The CKM matrix

The transformation between weak and mass eigenstates does not affect the electromagnetic interaction and also not the neutral weak currents. In this cases up-like quarks couple among each other and similarly for the down-like quarks, so that only the combinations $U_1^\dagger U_1$ and $U_2^\dagger U_2$ arise in the interaction terms. By definition these combinations give the unit matrix. Thus all neutral interactions are diagonal, in other words **there are no flavour changing neutral currents (FCNC) in the Standard Model at tree-level.** The originally diagonal charged weak interaction can, however, become non-diagonal after performing the unitary transformations above, namely

$$(\bar{u}, \bar{c}, \bar{t})\, \gamma_\mu \,(1-\gamma_5) \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\rightarrow \quad (\bar{u}, \bar{c}, \bar{t})\, \gamma_\mu \,(1-\gamma_5)\, U_1^\dagger U_2 \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\rightarrow \quad (\bar{u}, \bar{c}, \bar{t})\, \gamma_\mu\, (1 - \gamma_5) \begin{pmatrix} .. & .. & .. \\ .. & .. & .. \\ .. & .. & .. \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{23}$$

This defines the famous **Cabibbo-Kobayashi-Maskawa-Matrix** or **CKM-Matrix**

$$V_{CKM} := U_1^\dagger U_2 \, . \tag{24}$$

From a theory point of view it is not excluded that $U_1^\dagger U_2$ is diagonal (e.g. U_1 and U_2 are unit matrices or $U_1 = U_2$). In the end, it is the comparison with the experimental data that must indicate, as it has done, if the CKM-matrix is non-diagonal and thus transitions between different families are allowed. Historically this matrix was invented in two steps:

– 1963: 2×2 quark mixing matrix by Cabibbo [12]
– 1973: 3×3 quark mixing matrix by Kobayashi and Maskawa [13], who received the Nobel Prize in 2008.

Let us look a little closer at the properties of this matrix. By construction the CKM-Matrix is a unitary matrix, it connects the weak eigenstates q' with the mass eigenstates q. Note that instead of transforming both the up-type and down-type quark fields one can also solely transform the down-type fields, as

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} . \tag{25}$$

One can show, that a generic unitary $N \times N$-matrix has $N(N-1)/2$ real parameters and $(N-1)(N-2)/2$ phases, if unphysical phases are discarded. Specifically

N = 2	1 real parameter	0 phases
N = 3	3 real parameters	1 phase
N = 4	6 real parameters	3 phases

As will be discussed below, a complex coupling, e.g. a complex CKM-matrix element, leads to a phenomenon called **CP-violation**, which is strongly connected to the existence of matter in the Universe. Kobayashi and Maskawa found in 1973 that at least three families of quarks (i.e. six quarks) would be needed to implement CP-violation in the Standard Model. At that time only three quarks were known, the charm-quark was found in 1974 [14, 15].
The CKM-matrix allows for non-diagonal couplings in the charged currents, i.e. the u-quark does not only couple to the d-quark via a charged W boson, but it also couples to the s-quark and the b-quark, see Eq. (23). The entries of the CKM-matrix give the respective coupling strengths

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} , \tag{26}$$

so that e.g. the coupling of an u- and d-quark is given by

$$\frac{g_2}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)V_{ud}\,. \tag{27}$$

For a unitary 3×3 matrix with 3 real angles and 1 complex phase, different parameterisations are possible. The so-called **standard parameterisation** reads

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \tag{28}$$

with

$$s_{ij} := \sin(\theta_{ij}) \quad \text{and} \quad c_{ij} := \cos(\theta_{ij})\,. \tag{29}$$

The three angles are θ_{12}, θ_{23} and θ_{13}, the complex phase describing CP-violation is δ_{13}. This parameterisation is exact and it is typically used for numerical calculations. There is also a very ostensive parameterisation, the so-called **Wolfenstein parameterisation** [16]. This parameterisation follows from the experimentally found hierarchy $V_{ud} \approx 1 \approx V_{cs}$ and $V_{us} \approx 0.22498 =: \lambda$ and is based on performing a Taylor expansion in λ. It is expressed in terms of 4 parameters λ, A, ρ and η, where the latter leads to complex contributions, i.e.

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,. \tag{30}$$

In this form the hierarchies can be read off very nicely. Transitions within a family are strongly favoured, transitions between the first and second family are suppressed by one power of λ, transitions between the second and third family are suppressed by two powers of λ and transitions between the first and the third family by at least three powers. The most recent numerical values for the Wolfenstein parameter read (status August 2024 from the CKMfitter page [17])

$$\lambda = 0.22498^{+0.00023}_{-0.00021}\,, \qquad A = 0.8215^{+0.0047}_{-0.0082}\,, \tag{31}$$

$$\bar{\rho} = \rho\left(1-\frac{\lambda^2}{2}\right) = 0.1562^{+0.0112}_{-0.0040}\,, \qquad \bar{\eta} = \eta\left(1-\frac{\lambda^2}{2}\right) = 0.3551^{+0.0051}_{-0.0057}\,. \tag{32}$$

<u>**Remarks:**</u>

- The non-vanishing value of η describes CP-violation within the Standard Model, which is by now unambiguously established.
- Numerically one gets $|V_{ub}| = 0.003730 = \lambda^{3.7482}$, so V_{ub} is closer to λ^4 than to λ^3, as it was historically assumed by Wolfenstein.

Numerically the moduli of the CKM matrix elements reads (status August 2024 from CKMfitter [17])

$$V_{CKM} = \begin{pmatrix} 0.974358^{+0.000049}_{-0.000054} & 0.22498^{+0.00023}_{-0.00022} & 0.003730^{+0.000044}_{-0.000048} \\ 0.22484^{+0.00023}_{-0.00021} & 0.973509^{+0.000054}_{-0.000059} & 0.04160^{+0.00020}_{-0.00058} \\ 0.008573^{+0.000046}_{-0.000158} & 0.04088^{+0.00020}_{-0.00066} & 0.9991248^{+0.0000268}_{-0.0000074} \end{pmatrix} . \tag{33}$$

Remarks:

– From these experimental numbers we can see clearly, that the CKM-matrix is non-diagonal. So the initial ansatz of non-diagonal Yukawa interactions was necessary!

– We can also clearly see the hierarchy of the CKM-matrix. Transitions within a family are strongly favoured, whereas transitions between different families are disfavoured. Note, however, that in the lepton sector there is a very different hierarchy.

– The above numbers have very small uncertainties. This relies crucially on the assumption of having a unitary 3×3 CKM matrix. Giving up this assumption, e.g. in models with four fermion generations, would lead to considerably larger uncertainties [18, 19], albeit the simplest versions of such models have already been experimentally excluded [20, 21].

1.5 Baryogenesis

In this section we discuss the important phenomenon of CP violation, namely the violation of the discrete symmetries of parity and charge conjugation, and how this is related to the origin of matter in the Universe, hence triggering a huge interest in both the theoretical and experimental communities. For more details, see the cosmology lecture of Prof. Shaposhnikov.

The observed asymmetry between matter and antimatter in the Universe can be parameterised by the baryon to photon ratio η_B, which was measured by the PLANCK satellite [22] to be

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx (6.05 \pm 0.07) \cdot 10^{-10} , \tag{34}$$

n_B is the number of baryons in the Universe, $n_{\bar{B}}$ the number of anti-baryons and n_γ denotes the number of photons. The tiny[2] matter excess is responsible for the whole visible Universe! In the very early Universe the relative excess of matter over antimatter was much smaller, compared to now, namely

$$\eta_B(t \approx 0) = \frac{10000000001 - 10000000000}{n_\gamma} , \tag{35}$$

$$\eta_B(today) = \frac{1 - 0}{n_\gamma} . \tag{36}$$

Sakharov has shown in 1967 [23] that one can create a baryon asymmetry dynamically (**Baryogenesis**), if the laws of nature have certain properties. These basics properties are:

[2]The numerical value is obtained by investigating primordial nucleosynthesis and the cosmic microwave background, see e.g. the PLANCK homepage.

a) **C and CP-violation:** C is the charge conjugation, it changes the sign of the charges of the elementary particles; P is the usual parity, a reflection of the three-dimensional space axes. The violation of parity in the weak interaction was established in 1956 [9] and in 1964 a tiny CP violation effect was found in the neutral K-system — in an observable denoted by ϵ — by Christenson, Cronin, Fitch, Turlay [24] (NP 1980). As will be discussed below, by now CP violation has by been observed in many processes involving b-hadrons, yielding also large effects of the order of 50% and recently also in the decays of D^0 mesons, where, however the effect appears to be tiny.
CP violation can be implemented in the SM via complex Yukawa-couplings, as with the CKM matrix, but also via complex parameters in the Higgs potential, as for example in the case with 2 Higgs doublet models, see e.g. [25].

b) **Baryon number violation:** The necessity to violate the baryon number is obvious and in the Standard Model such effects are implemented via so-called sphaleron processes — non-perturbative tunneling effects that arise at finite temperatures [26, 27].

c) **Phase out of thermal equilibrium:** This could be e.g. a first order phase transitions during electro-weak baryogenesis.

A remark for students:

– Sakharov's paper was sent to the journal on 23.9.1966 and published on 1.1.1967; it was cited for the first time in 1976 by Okun and Zeldovich; in summer 2024 it had almost 5000 citations $\Rightarrow$ be patient with your papers!

All the three ingredients introduced above have to be part of the fundamental theory, not only in principal, but also to a sufficient extent.

a) In the Standard Model C and CP violation are implemented. A measure of the amount of CP violation is provided by the Jarlskog invariant J [28], which reads, in the Standard Model

$$J = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \cdot A \, . \tag{37}$$

Here, m_q denotes the mass of the quark q and A the area of the unitarity triangle, which will be discussed below. A is large if the CKM-matrix elements have large imaginary parts, i.e. in the presence of large CP violating contributions. Normalising J to the scale of the electroweak phase transition leads to the very small number, see e.g. [29]:

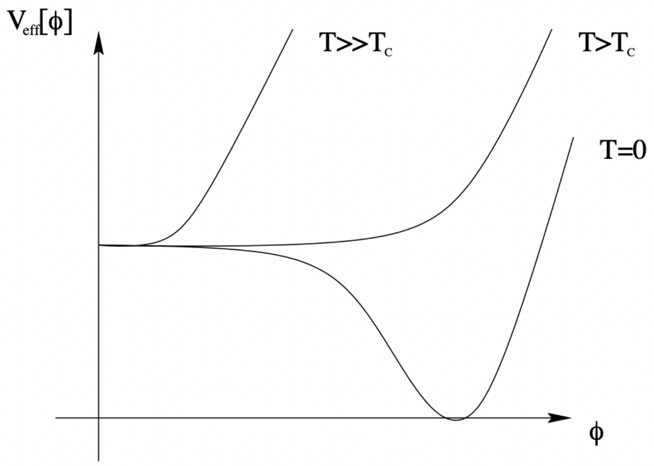
$$\frac{J}{(100\,\text{GeV})^{12}} \approx 10^{-20} \ll 6 \times 10^{-10} \approx \eta_B \, . \tag{38}$$

Hence, it seems that the amount of CP violation in the Standard Model is by far not sufficient to explain the observed baryon asymmetry. Note, however, that if one could let baryogenesis take place at a lower energy scale, e.g. 10 GeV, then the above ratio would be enhanced by a factor 10^{12} and the amount of CP violation in the Standard Model could be sufficient. Such a possibility is investigated e.g. in [30].

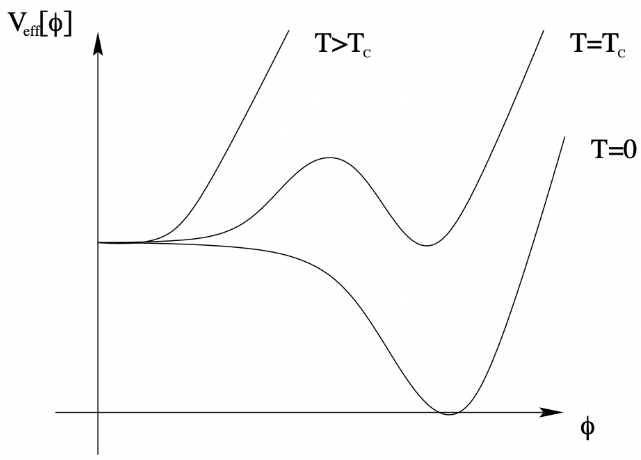
b) In the Standard Model the baryon number (B) and the lepton number (L) are conserved to leading order in perturbation theory. Considering also non-perturbative effects (there are no corresponding

Feynman diagrams!), in particular thermal effects, it is possible to create the needed amount of violation of B. These effects are called *sphalerons* (greek: weak, dangerous) [26, 27]. At temperatures T$<$ 100 GeV, this phenomenon is exponentially suppressed, while it grows very rapidly above 100 GeV. Sphalerons have not yet been detected experimentally.

c) Finally one needs to be out of thermal equilibrium at 100 GeV. During a second order phase transition the parameters change in a continuous way and there is no departure from thermal equilibrium:



In order to leave thermal equilibrium a first order transition is needed:



To answer the question about the nature of the electroweak phase transition one has to calculate the effective Higgs potential (classical potential plus quantum effects) as a function of the Higgs mass at finite temperature. In particular, one finds, that for masses $m_H < 72$ GeV a first order transition is possible, while the transition is continuous for higher masses, see e.g. [31–34]. Thus, the experimental value of the Higgs mass of 125 GeV clearly points towards a continuous phase transition within the Standard Model. We note again, that 2HDM models could also fulfill this Sakharov criteria and lead to a first order phase transition, see e.g. [35, 36].

For lecture notes on baryogenesis see e.g. [37].

1.6 Unitarity Triangle

Next we discuss in more detail the determination of the CKM-matrix and in particular the so-called unitarity triangle. By construction we have a unitary CKM matrix, i.e.

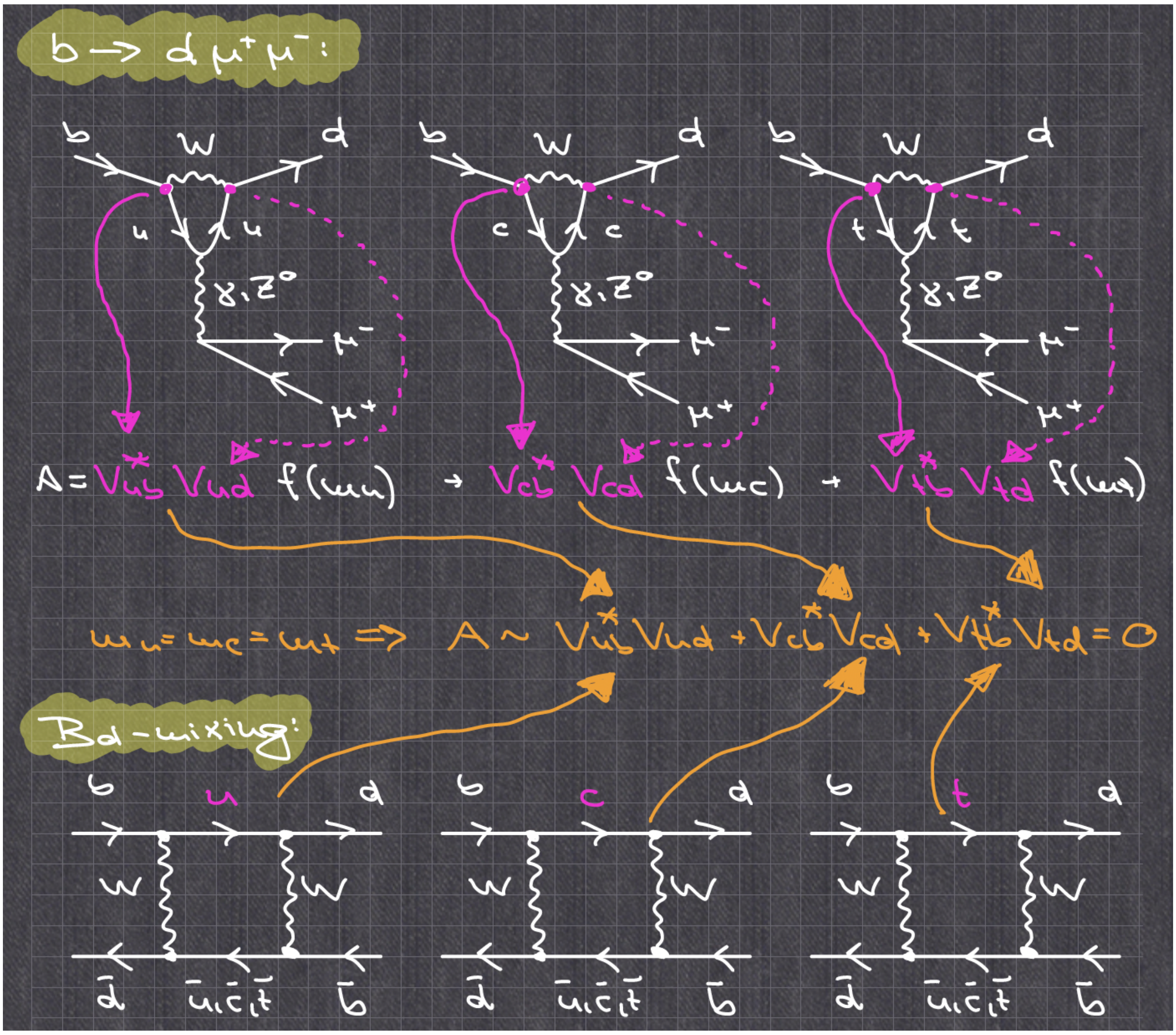
$$1 = V_{CKM}^{\dagger} V_{CKM} = \sum_{U=u,c,t} V_{Ud_1}^{*} V_{Ud_2} = \begin{pmatrix} 1 & 0_{K^0} & 0_{B_d} \\ 0 & 1 & 0_{B_s} \\ 0 & 0 & 1 \end{pmatrix} , \tag{39}$$

$$1 = V_{CKM} V_{CKM}^{\dagger} = \sum_{D=d,s,b} V_{u_1 D} V_{u_2 D}^{*} = \begin{pmatrix} 1 & 0_{D^0} & 0_{T^0} \\ 0 & 1 & 0_{T_c^0} \\ 0 & 0 & 1 \end{pmatrix} . \tag{40}$$

We will explain below, what the subscripts on the zero entries of the unit matrix mean.
Each of the unitarity conditions in Eq. (39) and Eq. (40) leads to nine equations, specifically three combinations of CKM matrix elements with sum equal to one and six combinations with sum equal to zero. In particular, the condition

$$V_{ud} V_{ub}^{*} + V_{cd} V_{cb}^{*} + V_{td} V_{tb}^{*} = 0_{B_d} , \tag{41}$$

arises in the penguin diagrams triggering the $b \to d\mu^+\mu^-$ decay or in B_d mixing (i.e. the transition of a B_d meson into its anti-particle, the $\bar{B}_d$ meson, via a so-called box diagrams), see the figure below, when it is assumed that the masses of the internal quarks (up, charm and top) are equal.



Since the masses of the up-, charm- and top-quark are in reality different from each other, the above observation means that finite contributions to the $b \to d\mu^+\mu^-$ penguin decay and B_d mixing are only arising due to the mass difference of the internal quarks. This is the essence of the so-called **GIM-mechanism** (Glashow-Iliopoulos-Maiani) [38].

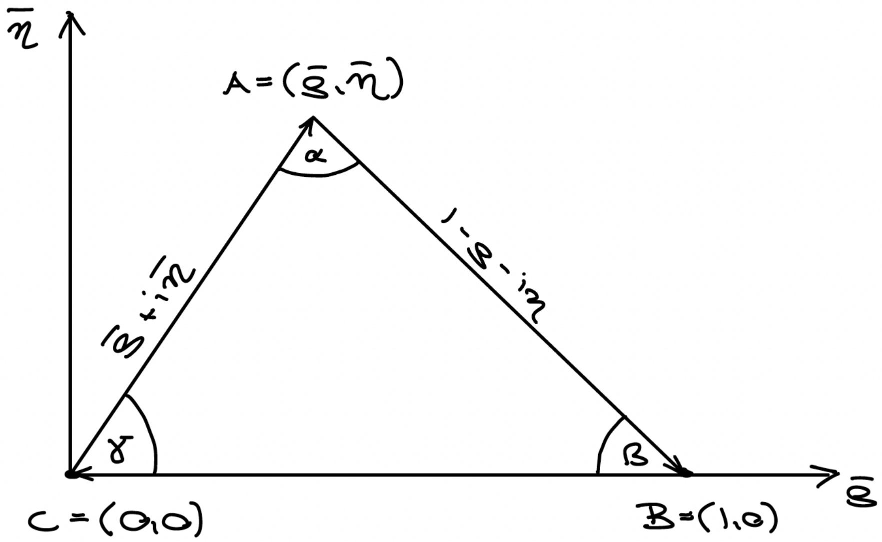
The remaining zero-sums in Eq. (39) and Eq. (40) correspond to $s \to d$ penguins (or K^0 mixing), $b \to s$ penguins (or B_s mixing), $c \to u$ penguins (or D^0 mixing), $t \to u$ penguins and $t \to c$ penguins.

Using the Wolfenstein parameterisation we get for the B_d zero sum

$$A\lambda^3 \left[(\bar{\rho} + i\bar{\eta}) - 1 + (1 - (\rho + i\eta)) \right] = 0 + \mathcal{O}(\lambda^4) \,. \tag{42}$$

As the values for A and λ are already quite precisely known, Eq. (42) can be used to determine ρ and η. The above sum of three complex numbers can be represented graphically as a triangle in the complex (ρ, η) plane, the so-called *unitarity triangle*[3], see the figure below

[3]In principle different unitarity triangles, apart from the B_d one, defined in Eq. 42, can be constructed, but they turn out to be very "flat", i.e. they have one very small angle, while in the B_d unitarity triangle all three lengths are of similar size.



The precise determination of the unitarity triangle given by Eq. 42 is of particular interest since a non-vanishing value of η is a measure of the size of CP-violation in the Standard Model.
In order to constrain the form of the unitarity triangle, the following strategy is used (for an early review see e.g. [39]): to compare the experimental value of some flavour observables with the corresponding theoretical expressions, where ρ and η are left as free parameters, plot the constraints on these two parameters in the complex (ρ, η) plane (in this case the values of λ and A are assumed to be known and fixed) e.g.:

- The amplitude describing the $b \to u$ decay is proportional to V_{ub}. Therefore the branching fraction for the semileptonic decay of a $\bar{B}$-meson (containing a b-quark) into a meson containing a u-quark, i.e. X_u, is proportional to $|V_{ub}|^2$:

$$B^{\text{exp.}}(B \to X_u e\overline{\nu}) = \tilde{a}^{\text{th.}} \cdot |V_{ub}|^2 = a^{\text{th.}} \cdot \left(\rho^2 + \eta^2\right) ,$$

$$\Rightarrow \rho^2 + \eta^2 = \frac{B^{\text{exp.}}(B \to X_u e\overline{\nu})}{a^{\text{th.}}} , \tag{43}$$

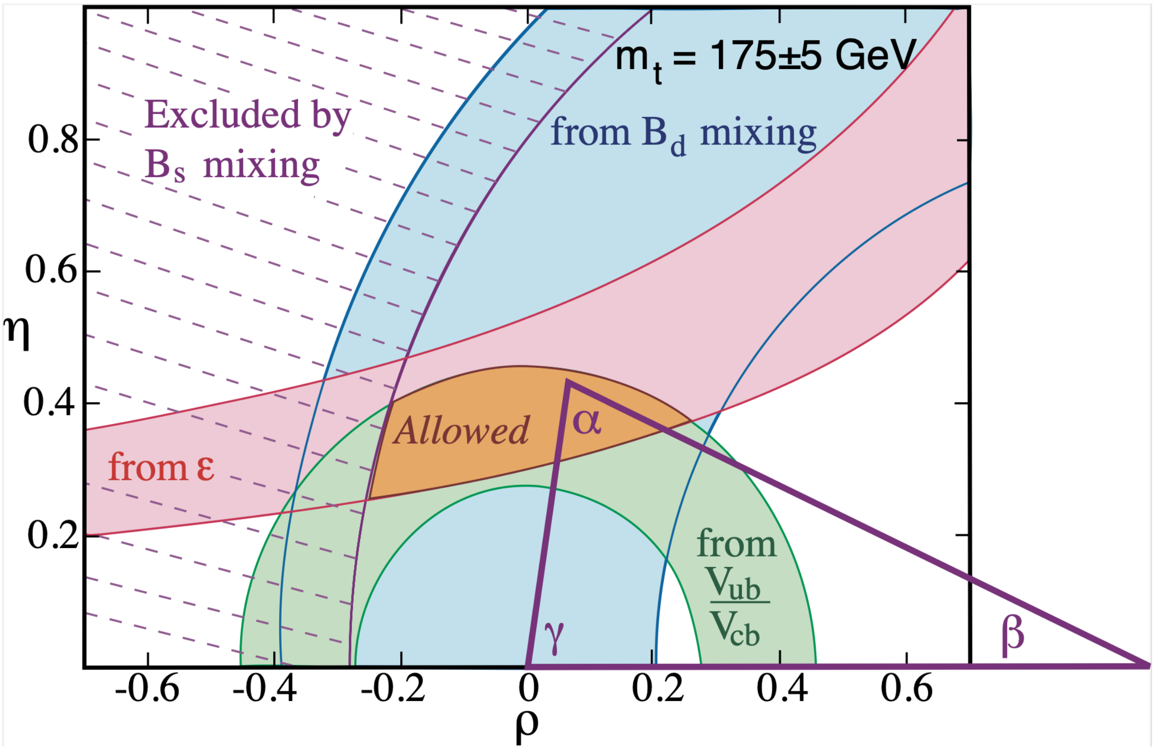
  where $a^{\text{th.}}$ contains the result of the theoretical calculation. Comparing experiment and theory for this observable and leaving ρ and η as free parameters, leads to a constraint in the (ρ, η)-plane in the form of a circle around $(0, 0)$ with the radius $B^{\text{exp.}}(B \to X_u e\overline{\nu})/a_{\text{th.}}$.
- Investigating the dynamics of neutral B_d-mesons, one finds that the physical eigenstates (i.e. the states that are propagating with a definite mass) are a mixture of the flavour eigenstates (defined by the quark content of the neutral mesons), as it will be discussed in more detail below. As a result of this mixing, the two physical eigenstates have different masses and their difference is denoted by ΔM_{B_d}. Theoretically one has to determine the above shown box-diagrams and one finds to a very good approximation that only the CKM structure stemming from the internal top-quark is contributing. Therefore the following relation holds:

$$\Delta M_{B_d}^{\text{exp.}} \propto |V_{td}|^2 \propto (\rho - 1)^2 + \eta^2 , \tag{44}$$

which leads to a circle in the (ρ, η)-plane around $(1, 0)$, when comparing experiment and theory.

– Comparing theory and experiment for CP-violation in the neutral K-meosn system, denoted by ϵ, gives an hyperbola in the (ρ, η)-plane.

The overlap of these three regions constrains the values of ρ and η, as schematically shown in the figure below, with the bound from the semileptonic B decay denoted in green, the constraint from B-mixing in blue and the hyperbolic constraint from ϵ in pink.



This figure is just meant to visualise the method in principle, below we show a plot with the latest experimental numbers.

Remarks:

– The angles of the unitarity triangle can be expressed as

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \qquad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \qquad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \tag{45}$$

The length of the sides of the unitarity triangle can be related to the angles α, β, γ, as

$$\overline{AC} = R_b = \frac{\sin\beta}{\sin(\beta+\gamma)}, \qquad \overline{AB} = R_t = \frac{\sin\gamma}{\sin(\beta+\gamma)}. \tag{46}$$

To a good approximation we can also write

$$V_{ub} = |V_{ub}|e^{-i\gamma}, \qquad V_{td} = |V_{td}|e^{-i\beta}. \tag{47}$$

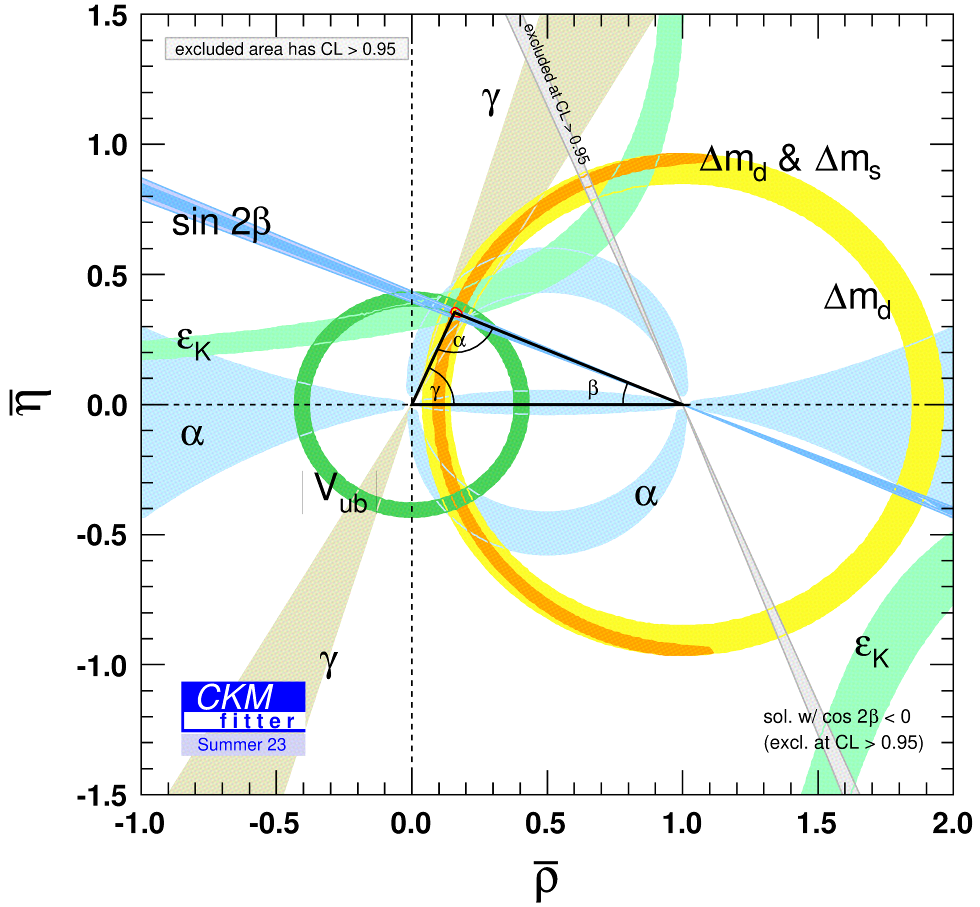
Bigi and Sanda have shown that the angle β can be extracted directly, with almost no theoretical uncertainty — thus it is called *gold-plated mode* — from the following CP-asymmetry in exclusive

B-meson decays [40].

$$a_{CP} \quad := \quad \frac{\Gamma(B \to J/\Psi + K_S) - \Gamma(\bar{B} \to J/\Psi + K_S)}{\Gamma(B \to J/\Psi + K_S) + \Gamma(\bar{B} \to J/\Psi + K_S)} \propto \sin 2\beta \tag{48}$$

The fact that the size of $\sin 2\beta$ was expected to be of order one was a strong reason for building the **B-factories** in SLAC (with the detector BaBar) and at KEK (with the detector Belle(II)) to measure for the first time CP violation outside the Kaon sector, which was achieved in 1999 [41, 42]. Currently a huge experimental effort is put into the direct determination of the CKM angle γ. Assuming no BSM effects in hadronic tree-level decays, this extraction can be done with essentially no (of the order of 10^{-6}) uncertainties [43].[4] The determination of the CKM angle α suffers from more pronounced theory uncertainties.

– The above programme was performed in the last years with great success and it confirmed the CKM picture, see below the most recent contraints for the unitarity triangle from the CKMfitter group [17]. Similar results have been obtained by the UTfit collaboration [46]. As a result of these efforts Kobayashi and Maskawa were awarded the Nobel Prize in 2008.

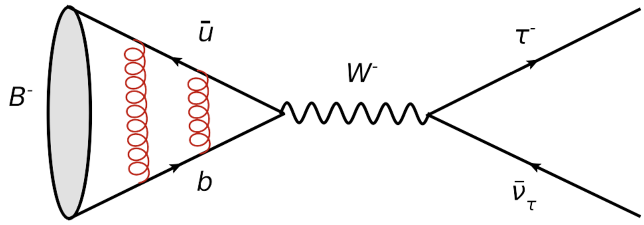


[4]Giving up the assumption of having no BSM effects in tree-level non-leptonic decays can lead to very large shifts in the determination of the CKM angle γ [44, 45].

1.7 Classification of hadronic decays

1.7.1 Leptonic decays

Leptonic decays have only leptons in the final state, e.g. the tree-level decay $B^- \to \tau^- \, \bar{\nu}_\tau$.



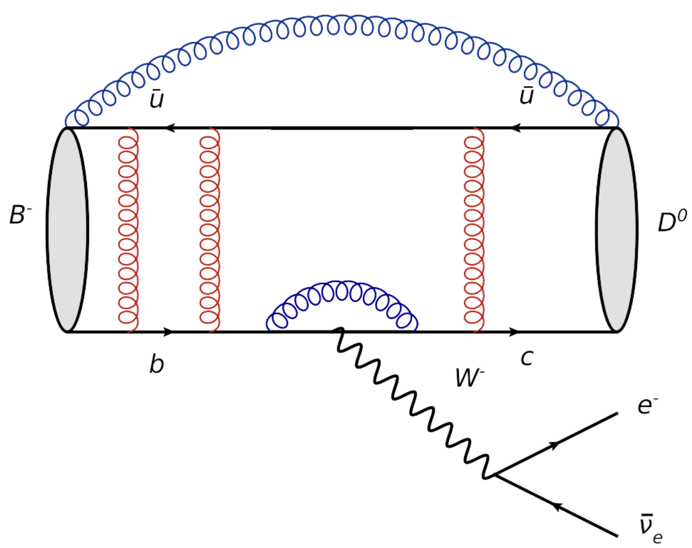
Such decays have the simplest hadronic structure. Gluons bind the quarks of the initial state into a hadron. All non-perturbative effects are described by one parameter: the **decay constant**, f_{B^-}, which is defined for generic B mesons as

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|B_q(p)\rangle \;\; = \;\; i f_{B_q} p^\mu \, , \tag{49}$$

where b and u are the spinors of the bottom and up quark and p^μ is the B_q-meson four-momentum. Decay constants can nowadays be precisely determined by lattice QCD simulations. Note that leptonic decays can also proceed via loop-level contributions in the SM, an example is the decay $B_s \to \mu^+\mu^-$.

1.7.2 Semi-leptonic decays

Semi-leptonic decays have leptons and hadrons in the final state, e.g. the tree-level decay $B^- \to D^0 \, e^- \, \bar{\nu}_e$.



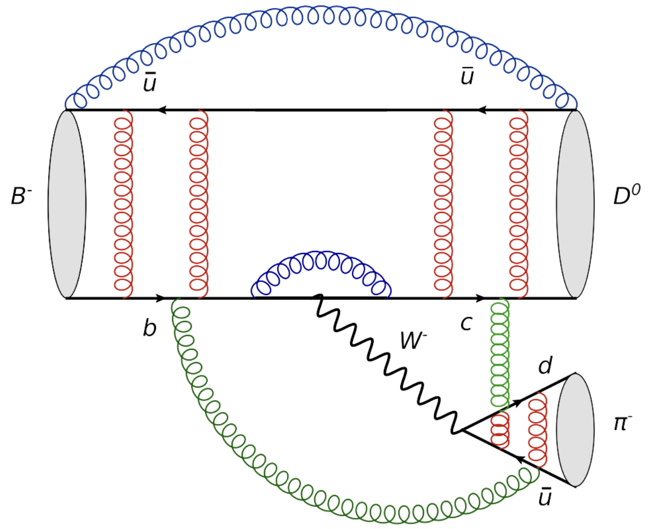
Now the hadronic structure is more complicated, as the non-perturbative QCD-effects are not only responsible for the binding of the hadrons in the initial and final states, but also for the strong interaction between the initial and the final state. The non-perturbative dynamics, in this case, is described in terms of two functions, the **form factors** $f_+^{B^- \to D^0}(q^2)$ and $f_0^{B^- \to D^0}(q^2)$, that depend on the momentum transfer q^2. They are defined as

$$\langle D^0(p_D)|\bar{c}\gamma^\mu b|B^-(p_B)\rangle = f_+^{B^- \to D^0}(q^2)\left(p_B^\mu + p_D^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu\right) + f_0^{B^- \to D^0}(q^2)\frac{m_B^2 - m_D^2}{q^2} q^\mu \, . \tag{50}$$

Form factors can be determined by sum rules or Lattice QCD calculations. There are again semi-leptonic decays that can only proceed via loop-contributions in the SM, e.g. $B_d \to K^{0*}\mu^+\mu^-$, which belong to the class of decays, where currently deviations between experiment and theory are observed.

1.7.3 Non-leptonic decays

Non-leptonic decays have only hadrons in the final state, e.g. the tree-level decay $B^- \to D^0\,\pi^-$.



These are the most complicated decays and they can only be described theoretically by making additional assumptions that then allow for a factorisation, e.g.

$$\begin{aligned} &\langle D^0\pi^-|\bar{c}\gamma_\mu(1-\gamma_5)b \cdot \bar{d}\gamma^\mu(1-\gamma_5)u|B^-\rangle \\ &\approx \langle D^0|\bar{c}\gamma_\mu(1-\gamma_5)b|B^-\rangle \cdot \langle \pi^-|\bar{d}\gamma^\mu(1-\gamma_5)u|0\rangle \\ &\propto f^{B^- \to D^0}(q^2 = m_\pi^2) \cdot f_\pi \, . \end{aligned} \tag{51}$$

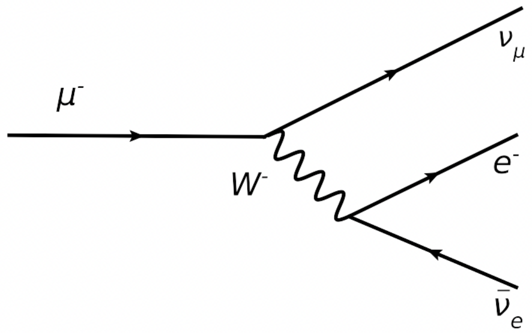
Theoretical investigations, of when the factorisation assumption is justified and when not, are a hot topic of current research, see e.g. [47].

2 Lecture 2: Theoretical framework

In comparison to the first and third lecture, this lecture will be considerably more technical, in particular the main part, which is devoted to the effective Hamiltonian.

2.1 The Muon Decay

The muon decay $\mu^- \to \nu_\mu \, e^- \, \bar{\nu}_e$ represents the most simple type of weak decay, because there are no QCD effects involved.[5] This process is given by the following Feynman diagram.



The total decay rate of the muon reads (see e.g. [48] for an early reference)

$$\Gamma_{\mu \to \nu_\mu + e + \bar{\nu}_e} = \frac{G_F^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e}{m_\mu}\right) = \frac{G_F^2 m_\mu^5}{192\pi^3} c_{3,\mu} \, . \tag{52}$$

$G_F = g_2^2/(4\sqrt{2}M_W^2)$ denotes the Fermi constant and f the phase space factor for one massive particle in the final state. It is given by

$$f(x) \quad = \quad 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln(x) = c_{3,\mu} \, . \tag{53}$$

The coefficient $c_{3,\mu}$ is introduced here to be consistent with the later notation. The result in Eq. (52) is already very instructive, since from that we obtain that the measurable lifetime of the muon which reads

$$\tau = \frac{1}{\Gamma} = \frac{192\pi^3}{G_F^2 m_\mu^5 f\left(\frac{m_e}{m_\mu}\right)} \, . \tag{54}$$

Thus the lifetime of a weakly decaying particle is proportional to the inverse of the fifth power of its mass. Using the measured values [49] for $G_F = 1.1663788(6) \cdot 10^{-5}$ GeV^{-2}, $m_e = 0.51099895000(15)$ MeV and $m_\mu = 0.1056583755(23)$ GeV, we can predict[6] the lifetime of the muon to be

$$\tau_\mu^{\text{th.}} = 2.18776 \cdot 10^{-6}\,\text{s} \, , \tag{55}$$

[5]This statements holds to a high accuracy. QCD effects arise for the first time at the two loop order.

[6]This is of course not really correct, because the measured muon lifetime was used to determine the Fermi constant, but for pedagogical reasons we assume that the Fermi constant is already known.

which is in excellent agreement with the measured value [49] of

$$\tau_\mu^{\text{exp.}} = 2.1969811(22) \cdot 10^{-6}\,\text{s} \,. \tag{56}$$

The remaining tiny difference (the prediction is about 0.4% smaller than the experimental value) is due to higher order electro-weak corrections. These are crucial for a highly precise determination of the Fermi constant. The dominant contribution is given by the 1-loop QED corrections, calculated already in the 1950s [50, 51]:

$$c_{3,\mu} = f\left(\frac{m_e}{m_\mu}\right)\left[1 + \frac{\alpha}{4\pi} 2 \left(\frac{25}{4} - \pi^2\right)\right] \,. \tag{57}$$

Taking this effect into account ($\alpha = 1/137.035999074(44)$ [49]) we then obtain

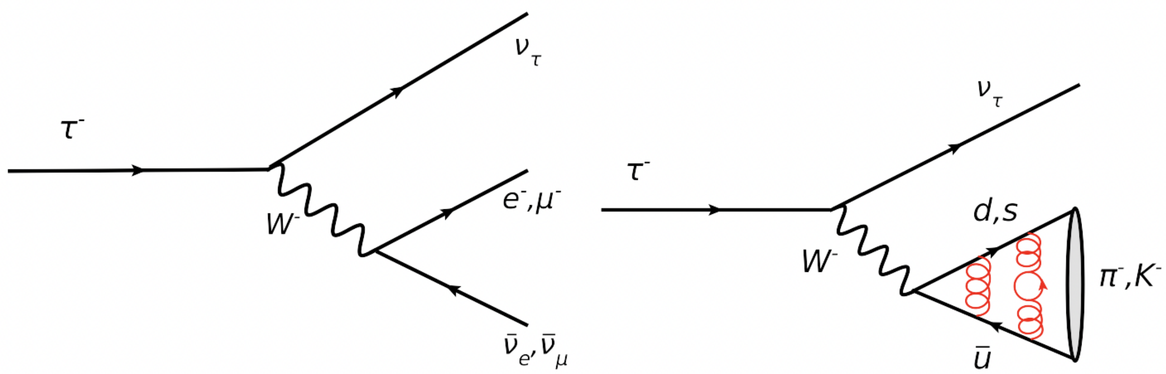
$$\tau_\mu^{\text{th.}} = 2.19699 \cdot 10^{-6}\,\text{s} \,, \tag{58}$$

which almost exactly reproduces the measured value given in Eq. (56). The complete 2-loop QED contributions have been determined in [52], while a review of loop-corrections to the muon decay is given in [53].
Note that the phase space factor gives an almost negligible suppression in the specific case of the muon decay - namely $f(m_e/m_\mu) = 0.999813 = 1 - 0.000187051$. However, phase space effects will turn out to be quite sizable, e.g. for the decay of a b-quark into a c-quark.

2.2 The tau decay

Moving to the tau lepton, both decays into leptons as well as into quarks and leptons are possible, specifically

$$\tau \quad \rightarrow \quad \nu_\tau + \begin{cases} e^- + \bar{\nu}_e \\ \mu^- + \bar{\nu}_\mu \\ d + \bar{u} \\ s + \bar{u} \end{cases} \,.$$



Note that heavier quarks, like the charm- or bottom-quark cannot be created, since in this case the lightest meson possible, i.e. $D^0 = c\bar{u}$ with $M_{D^0} \approx 1.86\,\text{GeV}$ is heavier than the tau lepton

($m_\tau = 1.77682(16)$ GeV). Thus the total decay rate of the tau lepton reads

$$\Gamma_\tau = \frac{G_F^2 m_\tau^5}{192\pi^3}\left[f\left(\frac{m_e}{m_\tau}\right) + f\left(\frac{m_\mu}{m_\tau}\right) + N_c\,|V_{ud}|^2\, g\left(\frac{m_u}{m_\tau},\frac{m_d}{m_\tau}\right) + N_c\,|V_{us}|^2\, g\left(\frac{m_u}{m_\tau},\frac{m_s}{m_\tau}\right)\right]$$
$$=: \frac{G_F^2 m_\tau^5}{192\pi^3} c_{3,\tau}\,. \tag{59}$$

Here, $N_c = 3$ is a colour factor and we have introduced the new phase space function g, which accounts for two massive particles in the final state. If we neglect the phase-space effects due to the lepton and quark masses ($f(m_e/m_\tau) = 1 - 7\cdot 10^{-7}$; $f(m_\mu/m_\tau) = 1 - 0.027$; ...) and if we use $V_{ud}^2 + V_{us}^2 \approx 1$, we obtain $c_{3,\tau} = 5$ and thus the simple approximate relation

$$\frac{\tau_\tau}{\tau_\mu} = \left(\frac{m_\mu}{m_\tau}\right)^5 \frac{1}{5}\,. \tag{60}$$

Using the experimental values for τ_μ, m_μ and m_τ we can predict

$$\tau_\tau^{\text{th.}} = 3.26707\cdot 10^{-13}\ \text{s}\,, \tag{61}$$

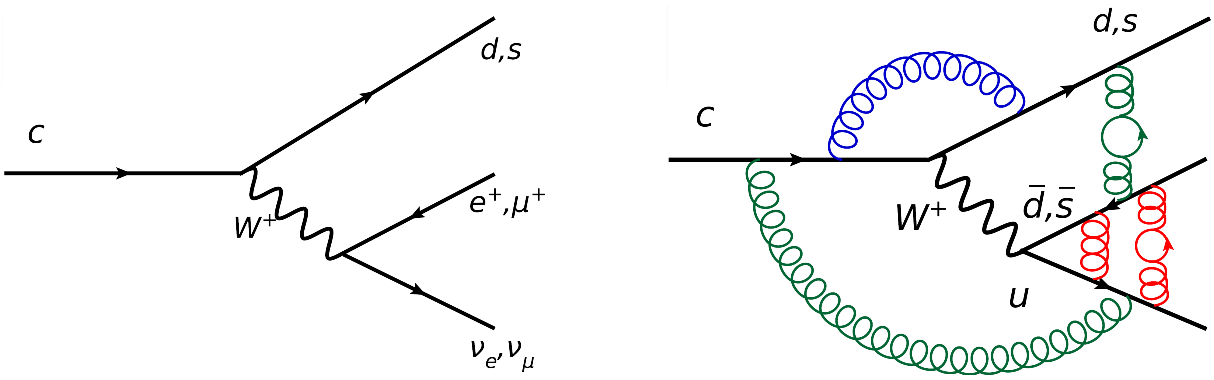
which is quite close to the experimental value of

$$\tau_\tau^{\text{exp.}} = 2.906(1)\cdot 10^{-13}\ \text{s}\,. \tag{62}$$

Note that the theory prediction is about 12% larger than the measured value. This is mostly due to the effect of sizable QCD corrections, which must be taken into account since now there are quarks in the final state - contrary to the case of the muon decay. QCD contributions currently have been calculated up to five loop accuracy [54], a review of higher order corrections can be found in [55].
Because of the pronounced and clean dependence on the strong coupling, tau decays can also be used for precision determinations of α_s, see, e.g., the review [56].
This example already shows, that a proper treatment of QCD effects is mandatory for precise lifetime studies. In the case of meson decays this will be even more important.

2.3 Charm-quark decay

Before trying to investigate the complicated structure of meson decays, let us have a look at the decay of free c- and b-quarks. Within the framework of the **Heavy Quark Expansion (HQE)** one can show that the free quark decay is the leading term in a systematic expansion in the inverse of the heavy (decaying) quark mass. For a both pedagogical and technical introduction into the HQE, see [57], for a review with many historic details, see [58] and for a review covering the state of the art of HQE predictions and comparisons to experiment, see [59].
A charm quark can decay weakly into a strange- or a down-quark and a W^+-boson, which can then further decay either into leptons (semi-leptonic decay) or into quarks (non-leptonic decay).



Calculating the inclusive total decay rate of a charm-quark we obtain

$$\Gamma_c = \frac{G_F^2 m_c^5}{192\pi^3} |V_{cs}|^2 c_{3,c} \,, \tag{63}$$

with

$$\begin{aligned} c_{3,c} = \quad & g\left(\frac{m_s}{m_c}, \frac{m_e}{m_c}\right) + g\left(\frac{m_s}{m_c}, \frac{m_\mu}{m_c}\right) + N_c |V_{ud}|^2 h\left(\frac{m_s}{m_c}, \frac{m_u}{m_c}, \frac{m_d}{m_c}\right) \\ & + N_c |V_{us}|^2 h\left(\frac{m_s}{m_c}, \frac{m_u}{m_c}, \frac{m_s}{m_c}\right) \\ & + \left|\frac{V_{cd}}{V_{cs}}\right|^2 \left\{ g\left(\frac{m_d}{m_c}, \frac{m_e}{m_c}\right) + g\left(\frac{m_d}{m_c}, \frac{m_\mu}{m_c}\right) + N_c |V_{ud}|^2 h\left(\frac{m_d}{m_c}, \frac{m_u}{m_c}, \frac{m_d}{m_c}\right) \right. \\ & \left. + N_c |V_{us}|^2 h\left(\frac{m_d}{m_c}, \frac{m_u}{m_c}, \frac{m_s}{m_c}\right) \right\} . \end{aligned} \tag{64}$$

Here h denotes a new phase space function, for the case of three massive particles in the final state. If we set all phase space factors to one ($f(m_s/m_c) = f(0.0935/1.471) = 1 - 0.03, \ldots$ with $m_s = 93.5(2.5)$ MeV [49]) and use $|V_{ud}|^2 + |V_{us}|^2 \approx 1 \approx |V_{cd}|^2 + |V_{cs}|^2$, then we get $c_{3,c} = 5$, similar to the τ decay. In that case we can predict a charm lifetime of

$$\tau_c = \left\{ \begin{array}{l} 0.84 \text{ ps} \\ 1.70 \text{ ps} \end{array} \right. \quad \text{for } m_c = \left\{ \begin{array}{lll} 1.471 & \text{GeV} & (\text{Pole-scheme}) \\ 1.277(26) & \text{GeV} & (\overline{MS} - \text{scheme}) \end{array} \right. . \tag{65}$$

These predictions lie roughly in the ball-bark of the experimental numbers for D-meson lifetimes, but at this stage some comments are mandatory:

- Predictions of the lifetimes of free quarks have a huge parametric dependence on the definition of the quark mass ($\propto m_q^5$). This is the reason, why for a long time only lifetime ratios (the dominant m_q^5 dependence as well as CKM factors and some sub-leading non-perturbative corrections cancel) were determined theoretically. In our case the value obtained in the $\overline{MS}$ − scheme for the charm quark mass is about a factor of 2 larger than the one obtained in the pole-scheme. At LO-QCD the definition of the quark mass is completely arbitrary, which leads to huge uncertainties. Performing the computation at NLO-QCD a consistent treatment of the quark masses has to be defined within the calculation, leading to a considerably weaker dependence of the final result on the quark mass definition.

– Taking only the decay of the free c-quark into account, one obtains the same lifetimes for all charm-mesons, which is clearly a very bad approximation, taking the large spread of lifetimes of different D-mesons into account.

$$\frac{\tau(D^+)}{\tau(D^0)} = 2.54(2)\,, \qquad \frac{\tau(D_s^+)}{\tau(D^0)} = 1.20(1)\,. \tag{66}$$

Within the framework of the HQE one will find that in the case of charmed mesons a very sizable contribution to the total decay rate stems from spectator effects where also the valence quark of the D-meson is involved in the decay.

– Perturbative QCD corrections to the free charm quark decay turn out to be very important, because $\alpha_s(m_c) \approx 0.35$ is quite large.

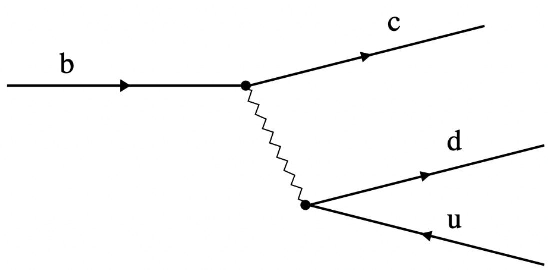
In the framework of the HQE the spectator effects will turn out to be suppressed by $1/m_c^3$ and since m_c is not very large, the suppression is also not expected to be very pronounced. This will change in the case of B-mesons. Because of the larger value of the b-quark mass, one expects a better description of the meson decay in terms of the simple b-quark decay, which is confirmed by experiment

$$\frac{\tau(B^+)}{\tau(B_d^0)} = 1.076(4)\,, \qquad \frac{\tau(B_s^0)}{\tau(B_d^0)} = 1.002(4)\,. \tag{67}$$

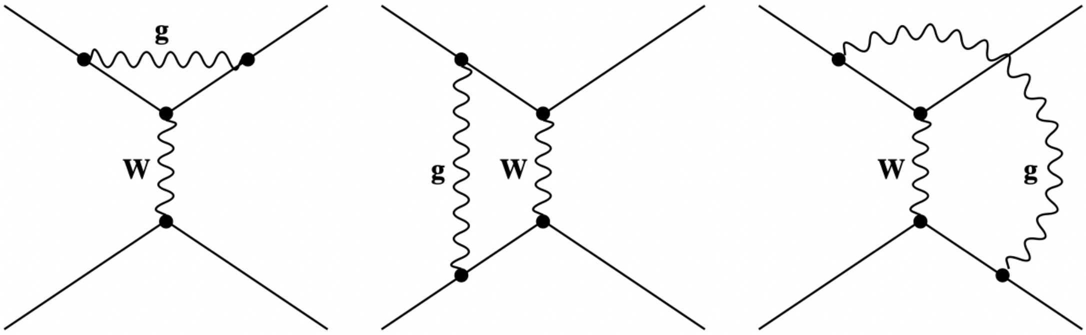
In the next lecture we will have a closer look into how one can determine perturbative QCD contributions to the quark and meson decay in a consistent and effective way - this will lead to the concept of the **effective Hamiltonian**. A nice pedagogical introduction to this topic is given in [60].

2.4 The effective Hamiltonian

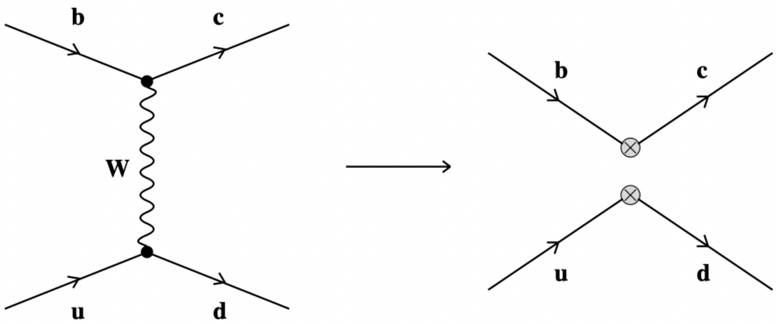
Most weak decays proceed via the exchange of heavy W-bosons, e.g. the decay $b \to c + W^- \to c + \bar{u} + d$ is described by the following Feynman diagram,



Neglecting the masses of the final state quarks, two very different scales arise in this decay: the mass of the W-boson ($\approx$ 80 GeV) and the mass of the b-quark ($\approx$ 4.5 GeV). Since the strong coupling is sizeable at the b quark scale, $\alpha_s(m_b) \approx 0.2$, perturbative QCD corrections are expected to be important. Specifically, 1-loop diagrams will give α_s corrections, 2-loop diagrams $\alpha_s^2 \approx 0.04$ corrections, and so on. Calculating the 1-loop QCD corrections one finds, however, that besides terms of order α_s, also **large logarithmic terms** of the form $\alpha_s \ln\left(m_b^2/M_W^2\right)$ appear.



As a result we do not obtain a Taylor expansion in α_s but an expansion in $\alpha_s \ln\left(m_b^2/M_W^2\right) \approx 6\alpha_s$ which clearly spoils the perturbative approach.
The solution to this problem lies in the introduction of the **effective Hamiltonian**, described in some detail below. The basic idea is to derive an effective theory valid at scales of the order of m_b, in which the heavy W-boson ($m_W \gg m_b$), that triggers the weak decay, is **integrated out** by performing an operator product expansion (OPE), see, e.g., [60] for a nice introduction, as well as [61]. Schematically in the resulting effective theory, the propagator of the W-boson is contracted to a point and new effective four-fermion operators are generated, see figure below.



The Feynman rules for the diagram on the l.h.s. give the following expression, which can be Taylor-expanded in k^2/M_W^2.

$$\bar{c}\frac{ig_2 V_{cb}^*}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)b\frac{1}{k^2-M_W^2}\bar{d}\frac{ig_2 V_{ud}}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)u\,. \tag{68}$$

Note, that the momentum transfer in the virtual W-boson is of the order of the b-quark mass, $k \approx m_b$. The leading term of the Taylor-expansion of this expression in $k^2/M_W^2 \approx 3.6 \cdot 10^{-3}$, gives the effective Hamiltonian in zeroth order in the strong coupling.[7]

$$\begin{aligned} \Rightarrow \mathcal{H}_{eff}(x) &= \left(\frac{g_2}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} V_{cb}^* V_{ud}\bar{c}\gamma^\mu(1-\gamma_5)b \cdot \bar{d}\gamma_\mu(1-\gamma_5)u \\ &=: \frac{G_F}{\sqrt{2}} V_{CKM} C_2 Q_2\,. \end{aligned} \tag{69}$$

[7] Expanding the W-propagator in coordinate space, leads to a delta function with the difference of the coordinates of the two arising quark currents as an argument. Thus this approximation yields an effective local operator.

We have introduced the following notation: the four quark operator is denoted by Q_2

$$\begin{aligned} Q_2 &= (\bar{c}_\alpha \gamma_\mu (1-\gamma_5) b_\alpha) \cdot \left(\bar{d}_\beta \gamma^\mu (1-\gamma_5) u_\beta\right) , \\ &=: (\bar{c}_\alpha b_\alpha)_{V-A} \cdot \left(\bar{d}_\beta u_\beta\right)_{V-A} , \end{aligned} \tag{70}$$

where α and β denote colour indices, this operator is a colour-singlet. as well as the Fermi constant G_F with

$$G_F = \frac{g_2^2}{4\sqrt{2} M_W^2} = 1.1663787(6) \cdot 10^{-5} \frac{1}{\text{GeV}^2} , \tag{71}$$

thus leading to the **Fermi-theory** of the weak interaction. We stress that so far the **Wilson coefficient** $C_2 = 1$, however, including also QCD corrections the above description is generalised as follows:

- The value of $C_2 = 1 + \mathcal{O}(\alpha_s)$ deviates from one and it depends on the renormalisation scale μ. For b and c-decays its value is slightly larger than one.
- A second colour-rearranged operator Q_1 arises.

$$Q_1(x) =: (\bar{c}_\alpha b_\beta)_{V-A} \cdot \left(\bar{d}_\beta u_\alpha\right)_{V-A} . \tag{72}$$

The value of $C_1 = \mathcal{O}(\alpha_s)$ is negative for b and c-decays and of the order of -20% for b-decays.

- The generic effective Hamiltonian for tree-level decays reads thus

$$\mathcal{H}_{eff}(x) = \frac{G_F}{\sqrt{2}} V_{CKM} \left[C_1(\mu) Q_1(x) + C_2(\mu) Q_2(x)\right] . \tag{73}$$

- As mentioned above in the Standard Model large logarithms arise in the perturbative expansion and in the end one will not have an expansion in the strong coupling, $\alpha_s(m_b) \approx 0.2$, but an expansion in $\ln(m_b/M_W)^2 \alpha_s \approx 1.2$. So the convergence of the expansion is not ensured. In particular, the general structure of the perturbative expansion reads

	LL	NLL	NNLL	NNNLL
Tree	1	–	–	–
1-loop	$\alpha_s \ln$	α_s	–	–
2-Loop	$\alpha_s^2 \ln^2$	$\alpha_s^2 \ln$	α_s^2	–
3-loop	$\alpha_s^3 \ln^3$	$\alpha_s^3 \ln^2$	$\alpha_s^3 \ln$	α_s^3
	...	...	...	...

(74)

Computations within the full Standard Model correspond to calculating row by row, whereas using the effective Hamiltonian corresponds to calculating column by column. Importantly, the latter framework also allows to **sum up the large logarithms to all orders**.[8] An example for such a summation is given by the solution of the renormalisation group equations for the strong coupling, which is discussed e.g. in the lecture notes in [62].

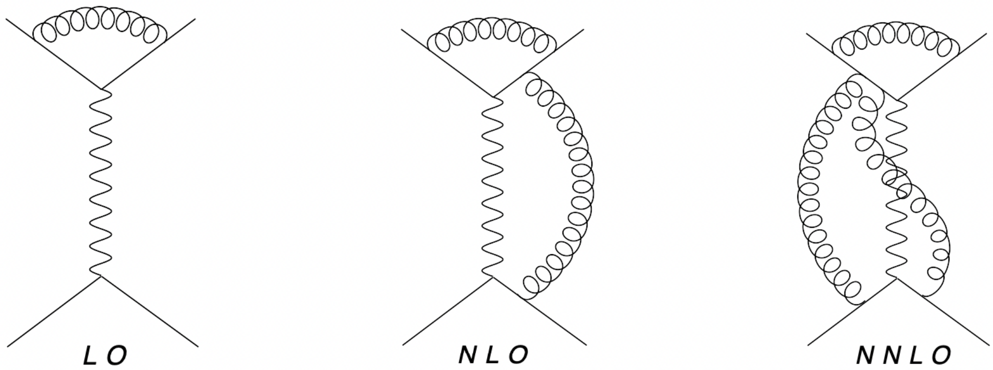
- In meson decays besides perturbatively calculable short-distance (SD) QCD effects (e.g. those at the scale M_W), also long-distance (LD) effects arise (at the scale Λ_{QCD}), the latter being of

[8]LL = leading logarithmic approximation, NLL = next-to-leading logarithmic approximation,...

non-perturbative origin. The construction of an OPE leads to a well-defined **separation of scales**. Namely the high energy physics is described by the Wilson coefficients, which can be calculated in perturbation theory, whereas the hadronic effects are parametrised by the matrix element of the operators Q_1, Q_2. In this case non-perturbative methods like lattice QCD or QCD sum rules must be employed. The renormalisation scale μ acts as the factorisation scale.

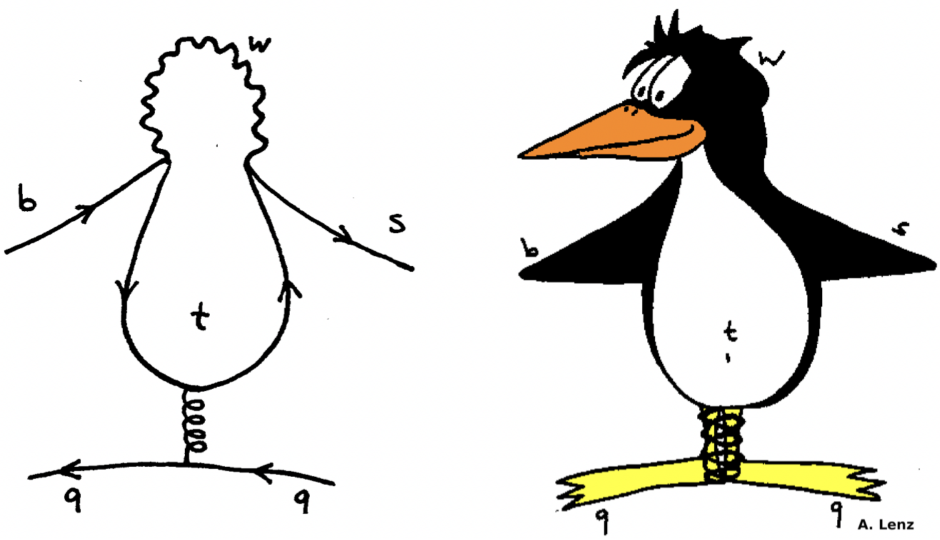
– Calculations within the framework of the effective Hamiltonian are technically simpler, because fewer propagators appear in the expressions.

Historic remark: The effective Hamiltonian in Eq. (73) was already obtained in 1974 in LL-QCD [63], a nice review of the NLL-results is given in [61]. Currently, also NNLL results are available [64]. To obtain the expression of the Wilson coefficients at LO-QCD 1-loop diagrams have to be calculated, at NLO-QCD 2-loop corrections and for NNLO-QCD 3-loop corrections, i.e.



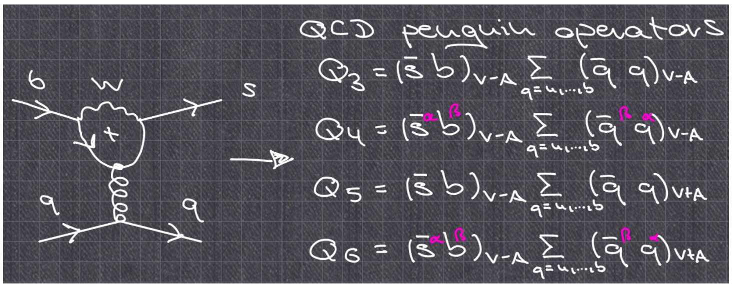
2.5 Penguins and friends

So far we have only considered tree-level decays, however there are also loop-induced decays, like those described by the penguins diagram[9].

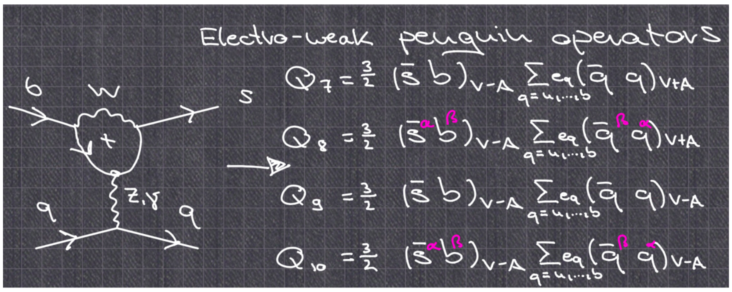


[9]For the origin of the name penguin diagram have a look at page 5 of [65].

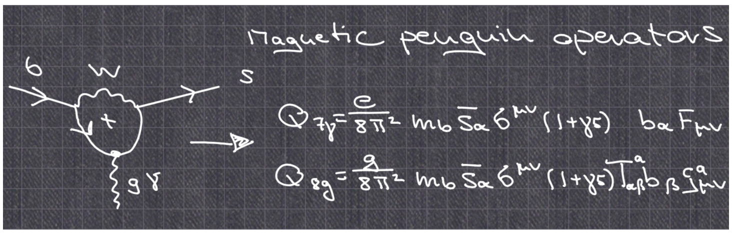
Here again, the particles heavier than the bottom quark — now in addition to the W, also the Z and the top-quark can appear — are integrated out, yielding new, effective operators, such as the **QCD-penguin operators** $Q_{3,...,6}$, explicitly given in the figure below.



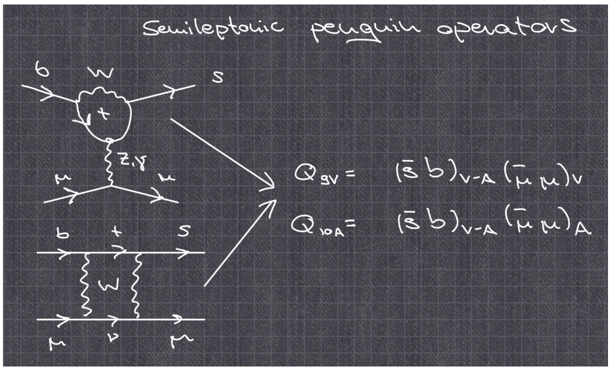
The QCD-penguin operators contribute e.g. to the decays $b \to c\bar{c}s$, where they give sizable corrections and to $b \to u\bar{u}s$, where they can even dominate. **Electro-weak penguin operators** (in which the gluon is replaced by a photon or a Z-boson) are denoted by $Q_{7,...,10}$, see below.



Note that the latter operators typically give tiny corrections, however, they give the dominant contribution to the direct CP violation in the Kaon system, namely to ϵ'/ϵ. **Magnetic penguin operators**, see below,



correspond to penguin diagrams in which the photon ($Q_{7\gamma}$) or the gluon is on-shell (Q_{8g}), i.e. its four momentum squared is zero. $Q_{7\gamma}$ gives the leading contribution to the radiative decay $b \to s\gamma$. Finally we have **semi-leptonic penguin operators**, that are generated either by penguin or by box diagrams, see below.



Q_{10A} gives the main contribution to the decay $B_s \to \mu\mu$, while in semi-leptonic decays like $B \to K^*\mu^+\mu^-$ also Q_{9V} contributes. Note that often the same notation is used for these operators as for the electro-weak penguins.

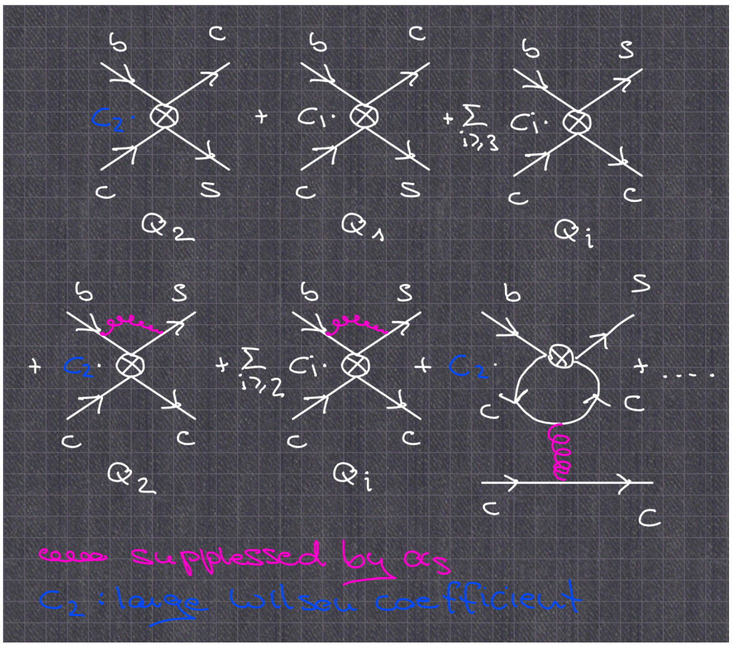
In the end we arrive at the complete expression of the **effective weak Hamiltonian**:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{q=u,c} V_c^q \left\{C_1(\mu)Q_1^q + C_2(\mu)Q_2^q\right\} - V_p \sum_{j\geq 3} C_j(\mu)Q_j\right] + \text{h.c.}\,. \tag{75}$$

The Vs denote different combinations of CKM elements, the operators $Q_3, ..$ denote all the different penguin operators discussed above. The values of the numerically leading tree-level Wilson coefficients C_1 and C_2 have been determined above, the QCD penguin Wilson coefficients are below 5%, with the exception of C_{8g}, the coefficient of the chromomagnetic operator and electro-weak penguins are even smaller.

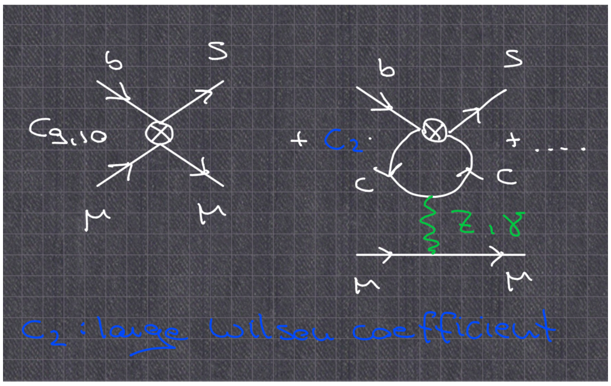
Having the effective Hamiltonian at hand, we can now calculate different processes — forgetting about the underlying weak structure of the SM — using the Wilson coefficients as basic couplings of our theory and the four quark operators as basic vertices.

For a calculation of the rate $\Gamma(b \to c\bar{c}s)$, we have to consider the following contributions:



The leading contribution is of course the tree-level insertion of the operator Q_2, the tree-level insertion of Q_1 and Q_{3-6} leads to smaller corrections. QCD effects are also sizable. The dominant one originates from QCD-corrections to the tree-level insertion of color-singlet operator Q_2. Note that at this order, also the penguin insertion of Q_2 is expected to be sizable.

Finally, to compute the decay rate $\Gamma(b \to s\mu^+\mu^-)$, where currently some anomalies are observed, we have to consider the following contributions:



Here the dominant one comes from the tree-level insertion of the operators $Q_{9,10}$. Due to the very large Wilson coefficient, we also expect, however, sizable corrections from the penguin insertion of the operator Q_2. These are the so-called charm-loop effects, often discussed in the context of the flavour anomalies, see e.g. [66].

3 Lecture 3: Mixing and CP violation

After having introduced the concept of the effective Hamiltonian, which is the starting point for calculating decay rates of exclusive or inclusive decays, we will switch now the topic and give a brief introduction into the concepts of mixing and CP violation. More detailed discussions of mixing and CP violation can be found e.g. in the reviews [67–69].

3.1 General introduction

In these lecture, we mostly discuss the case of B_d mesons; changes in formulae in order to describe B_s mesons or D^0 mesons follow straightforwardly. However, when important differences arise, we will discuss each system separately.

Neutral mesons like the B_d^0 and its anti particle $\bar{B}_d^0$ form a two-states system, which can be described by a Schrödinger-like equation[10], as

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} B_d^0 \\ \bar{B}_d^0 \end{pmatrix} = \hat{H} \begin{pmatrix} B_d^0 \\ \bar{B}_d^0 \end{pmatrix} = \begin{pmatrix} M_{11}^d - \frac{i}{2}\Gamma_{11}^d & 0 \\ 0 & M_{22}^d - \frac{i}{2}\Gamma_{22}^d \end{pmatrix} \begin{pmatrix} B_d^0 \\ \bar{B}_d^0 \end{pmatrix} . \tag{76}$$

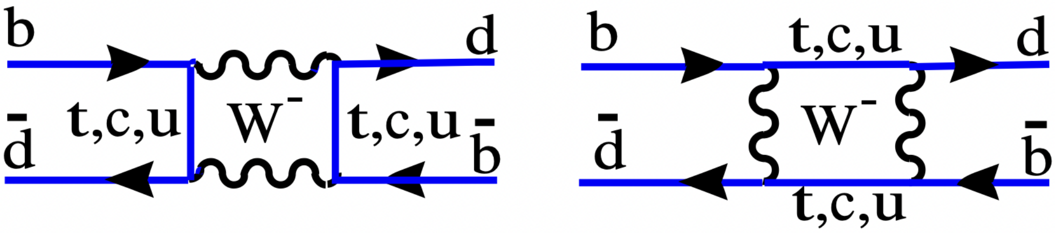
[10] Since in this case the Hamiltonian is not hermitian, we do not call it Schrödinger-equation, even if the mathematical structure of the differential equation is identical to the usual Schrödinger-equation.

This leads to the following time evolution of B mesons:

$$\Rightarrow B_i(t) \quad = \quad e^{\frac{1}{i\hbar}\left(M_{ii}^d - \frac{i}{2}\Gamma_{ii}^d\right)t} = e^{\frac{1}{i\,\hbar}M_{ii}^d t} e^{-\frac{1}{2\,\hbar}\Gamma_{ii}^d t} \,. \tag{77}$$

- $M_{11}^d (M_{22}^d)$ is the mass of the $B_d^0(\bar{B}_d^0)$-meson.
- $\Gamma_{11}^d(\Gamma_{22}^d)$ is the total decay rate of the $B_d^0(\bar{B}_d^0)$-meson.
- CPT invariance implies $M_{11}^d = M_{22}^d$ and $\Gamma_{11}^d = \Gamma_{22}^d$.

Due to the weak interaction, however, transitions of a B_d^0-meson into a $\bar{B}_d^0$ (and vice versa) are possible via the so-called **box diagrams**, as shown below.



The box diagrams generate off-diagonal entries in the Hamiltonian, e.g.

$$\hat{H} \quad = \quad \begin{pmatrix} M_{11}^d - \frac{i}{2}\Gamma_{11}^d & M_{12}^d - \frac{i}{2}\Gamma_{12}^d \\ M_{21}^d - \frac{i}{2}\Gamma_{21}^d & M_{22}^d - \frac{i}{2}\Gamma_{22}^d \end{pmatrix} . \tag{78}$$

Γ_{12}^d corresponds to intermediate on-shell states, like $(c\bar{c})$, in the box-diagrams, while M_{12}^d corresponds to virtual, i.e. off-shell, intermediate states. Therefore the top quark as well as other potential heavy new physics particles contribute only to M_{12}^d. Thus both the mass and decay rate matrices are non-diagonal, simply meaning that the flavour eigenstates (defined by the quark content) of the two-meson system are not mass eigenstates (defined by the mass of the physical meson).
CPT invariance implies again that $M_{11}^d = M_{22}^d$ and $\Gamma_{11}^d = \Gamma_{22}^d$, while hermiticity gives $M_{21}^d = \left(M_{12}^d\right)^*$ and $\Gamma_{21}^d = \left(\Gamma_{12}^d\right)^*$.
By diagonalising the 2×2 Hamiltonian matrix of Eq. (78), we obtain mass eigenstates, labelled respectively as "heavy" (H) and "light" (L), that is

$$\begin{aligned} B_{d,H} &= pB_d^0 - q\bar{B}_d^0 \,, \\ B_{d,L} &= pB_d^0 + q\bar{B}_d^0 \,, \end{aligned} \tag{79}$$

with $p = p(M_{12}^d, \Gamma_{12}^d)$ and $q = q(M_{12}^d, \Gamma_{12}^d)$. The new eigenstates $B_{d,H}$ and $B_{d,L}$ have now definite masses M_H^d, M_L^d and definite decay rates Γ_H^d and Γ_L^d, leading to a decay rate difference $\Delta\Gamma_d$ and a mass difference ΔM_d.

$$\begin{aligned} \Delta\Gamma_d &= \Gamma_L^d - \Gamma_H^d = \Delta\Gamma_d(M_{12}^d, \Gamma_{12}^d)\,, \\ \Delta M_d &= M_H^d - M_L^d = \Delta M_d(M_{12}^d, \Gamma_{12}^d)\,, \end{aligned} \tag{80}$$

which are observable in experiment. The following relations hold exactly

$$(\Delta M_d)^2 - \frac{1}{4}(\Delta\Gamma_d)^2 = 4\left|M_{12}^d\right|^2 - \left|\Gamma_{12}^d\right|^2 , \tag{81}$$

$$\Delta M_d \cdot \Delta\Gamma_d = -4\mathrm{Re}\left(M_{12}^d \Gamma_{12}^{d*}\right) , \tag{82}$$

$$\frac{q}{p} = -\frac{\Delta M_d + \frac{i}{2}\Delta\Gamma_d}{2M_{12}^d - i\Gamma_{12}^d} . \tag{83}$$

Solving for the mass and decay rate difference gives

$$\begin{aligned} 2\Delta M_d^2 &= \sqrt{\left(4\left|M_{12}^d\right|^2 - \left|\Gamma_{12}^d\right|^2\right)^2 + 16\left|M_{12}^d\right|^2\left|\Gamma_{12}^d\right|^2\cos^2\phi_{12}^d} + 4\left|M_{12}^d\right|^2 - \left|\Gamma_{12}^d\right|^2 , \\ \frac{1}{2}\Delta\Gamma_d^2 &= \sqrt{\left(4\left|M_{12}^d\right|^2 - \left|\Gamma_{12}^d\right|^2\right)^2 + 16\left|M_{12}^d\right|^2\left|\Gamma_{12}^d\right|^2\cos^2\phi_{12}^d} - 4\left|M_{12}^d\right|^2 + \left|\Gamma_{12}^d\right|^2 , \end{aligned} \tag{84}$$

with the mixing phase $\phi_{12}^d = \arg(-M_{12}^d/\Gamma_{12}^d)$.[11] Hence, in general, both M_{12}^d and Γ_{12}^d need to be known in order to determine either the decay rate difference or the mass difference.
One finds, however, that one can make the approximations $|\Gamma_{12}^d/M_{12}^d| \ll 1$ in the B_d-system (as well as in the B_s-system) and $|\phi_{12}^D| \ll 1$ in the D-system, leading to the simplified relations:

$$\Delta M_d = 2|M_{12}^d| , \quad \Delta\Gamma_d = 2|\Gamma_{12}^d|\cos(\phi_{12}^d) \qquad \text{for } B_d \text{ mixing} , \tag{85}$$

$$\Delta M_s = 2|M_{12}^s| , \quad \Delta\Gamma_s = 2|\Gamma_{12}^s|\cos(\phi_{12}^s) \qquad \text{for } B_s \text{ mixing} , \tag{86}$$

$$\Delta M_D = 2|M_{12}^D| , \quad \Delta\Gamma_D = 2|\Gamma_{12}^D| \qquad \text{for } D \text{ mixing} . \tag{87}$$

3.2 Time evolution

Now we can derive in a similar way, as in the well-known example of neutrino oscillations (see also the lectures of Prof. Barenboim), the time evolution of the B_d mesons. For the mass eigenstates, the time evolution is trivial, i.e.

$$|B_{d,H/L}(t)\rangle = e^{-\left(iM_{H/L}^d + \Gamma_{H/L}^d/2\right)t}|B_{d,H/L}(0)\rangle . \tag{88}$$

For the flavour eigenstates it reads

$$|B_d^0(t)\rangle = g_+(t)|B_d^0\rangle + \frac{q}{p}g_-(t)|\bar{B}_d^0\rangle , \tag{89}$$

$$|\bar{B}_d^0(t)\rangle = \frac{p}{q}g_-(t)|B_d^0\rangle + g_+(t)|\bar{B}_d^0\rangle , \tag{90}$$

with the coefficients

$$g_+(t) = e^{-i\cdot M_{B_d}t}e^{-\Gamma_{B_d}/2t}\left[\cosh\frac{\Delta\Gamma_d t}{4}\cos\frac{\Delta M_d t}{2} - i\sinh\frac{\Delta\Gamma_d t}{4}\sin\frac{\Delta M_d t}{2}\right] , \tag{91}$$

$$g_-(t) = e^{-iM_{B_d}\cdot t}e^{-\Gamma_{B_d}/2t}\left[-\sinh\frac{\Delta\Gamma_d t}{4}\cos\frac{\Delta M_d t}{2} + i\cosh\frac{\Delta\Gamma_d t}{4}\sin\frac{\Delta M_d t}{2}\right] . \tag{92}$$

[11] With $M_{12}^d = |M_{12}^d|e^{i\phi_{M^d}}$ and $\Gamma_{12}^d = |\Gamma_{12}^d|e^{i\phi_{\Gamma^d}}$ we get $\phi_{12}^d = \pi + \phi_{M^d} - \phi_{\Gamma^d}$.

Here we introduced the averaged masses $M_{B_d^0}$ and decay rates Γ:

$$M_{B_d} = \frac{M_H^d + M_L^d}{2}\,, \qquad \Gamma_{B_d} = \frac{\Gamma_H^d + \Gamma_L^d}{2}\,. \tag{93}$$

The coefficient functions $g_+(t)$ and $g_-(t)$ give the probability for mixing to occur, namely

$$\left|\langle B_d^0|B_d^0(t)\rangle\right|^2 = |g_+(t)|^2 = \left|\langle \bar{B}_d^0|\bar{B}_d^0(t)\rangle\right|^2\,, \tag{94}$$

$$\left|\langle \bar{B}_d^0|B_d^0(t)\rangle\right|^2 = \left|\frac{q}{p}\right|^2 |g_-(t)|^2\,. \tag{95}$$

The arguments of the trigonometric and hyperbolic functions in Eq. (91) and Eq. (92) can be rewritten as

$$\frac{\Delta M_d \cdot t}{2} = \frac{1}{2} x(B_d^0) \frac{t}{\tau(B_d^0)} \quad \text{with} \quad x(B_d^0) := \frac{\Delta M_d}{\Gamma_d}\,, \tag{96}$$

$$\frac{\Delta \Gamma_d \cdot t}{4} = \frac{1}{2} y(B_d^0) \frac{t}{\tau(B_d^0)} \quad \text{with} \quad y(B_d^0) := \frac{\Delta \Gamma_d}{2\Gamma_d}\,, \tag{97}$$

where the lifetime $\tau(B_d)$ is related to the total decay rate Γ_d via $\tau(B_d) = 1/\Gamma_{B_d}$. The oscillation length of the trigonometric functions can be determined via

$$\frac{\Delta M_d \cdot t}{2} = \pi \Rightarrow t = \frac{2\pi}{\Delta M_d} \tag{98}$$

$$\Rightarrow x = vt' = \beta\gamma ct = \beta\gamma \frac{2\pi c}{\Delta M_d}\,. \tag{99}$$

Next we can also write down the time dependent decay rate of a B_d meson, that was initially (at time $t = 0$) tagged as a B_d flavour eigenstate into an arbitrary final state f:

$$\Gamma\left[B_d(t) \to f\right] = N_f \left|\mathcal{A}_f\right|^2 \left(1 + |\lambda_f|^2\right) e^{-\Gamma t} \left\{ \frac{\cosh\left(\frac{\Delta\Gamma_d}{2} t\right)}{2} + \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \frac{\cos\left(\Delta M_d t\right)}{2} - \frac{2\Re(\lambda_f)}{1 + |\lambda_f|^2} \frac{\sinh\left(\frac{\Delta\Gamma_d}{2} t\right)}{2} - \frac{2\Im(\lambda_f)}{1 + |\lambda_f|^2} \frac{\sin\left(\Delta M_d t\right)}{2} \right\}\,. \tag{100}$$

Here N_f denotes a time-independent normalisation factor, which includes e.g. phase space effects. The decay amplitude describing the transition of the flavour eigenstate B_d in the final state f is denoted by $\mathcal{A}_f$; for the decay of a $\bar{B}_d$ state into f we use the notation $\bar{\mathcal{A}}_f$:

$$\mathcal{A}_f = \langle f|\mathcal{H}_{eff}|B_d\rangle\,, \qquad \bar{\mathcal{A}}_f = \langle f|\mathcal{H}_{eff}|\bar{B}_d\rangle\,. \tag{101}$$

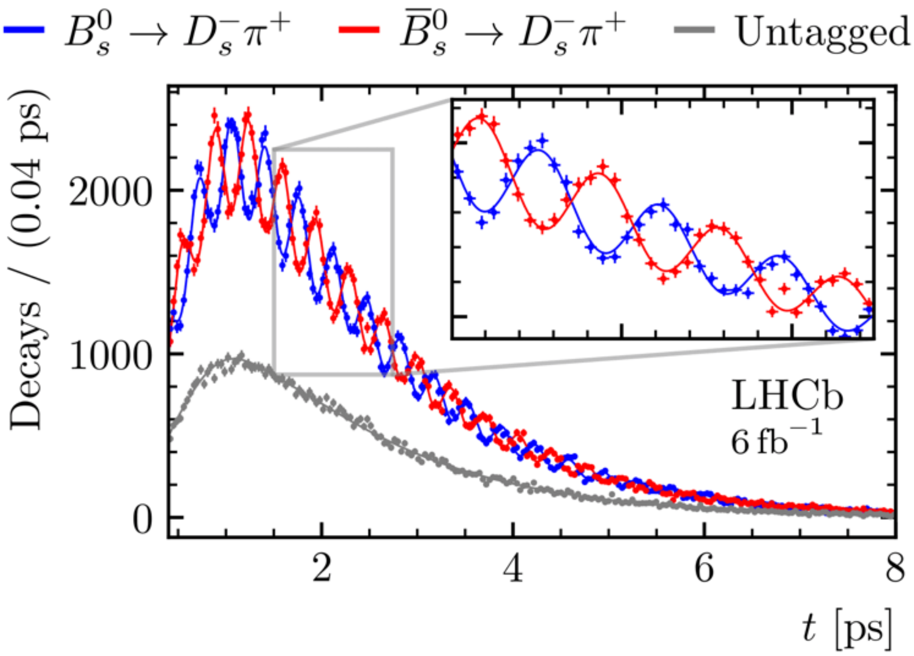
The flavour changing weak quark transitions are described by the effective Hamiltonian described in the previous lecture. The amplitudes $\mathcal{A}_f$ and $\bar{\mathcal{A}}_f$ are typically governed by hadronic effects and they are very difficult to be calculated reliably in theory. Below we will see that CP-symmetries are governed by a single quantity λ_f, which is given by

$$\lambda_f = \frac{q}{p} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}\,. \tag{102}$$

Following the derivation of Eq. (100), the time dependent decay rates of $\Gamma\left[\bar{B}_d(t) \to f\right]$, $\Gamma\left[B_d(t) \to \bar{f}\right]$ and $\Gamma\left[\bar{B}_d(t) \to \bar{f}\right]$ can be derived in a similar way, see e.g. [68,69] for the analytic expressions.

Remarks:

– The formulae for the time dependent decay rates can be used to extract the observables ΔM_d and $\Delta\Gamma_d$ from experiment, which can then be compared with the theory predictions, see the B_s oscillation plot obtained by LHCb [70].



According to Eq. (84) these observables are related to the matrix elements Γ_{12}^d and M_{12}^d, thus a Standard Model calculation of the mixing observables requires a calculation of the box-diagrams shown above.

In the B_s-system one finds [59] e.g.

$$\Delta M_s^{\text{exp.}} = 17.765(6)\text{ps}^{-1}, \qquad \Delta\Gamma_s^{\text{exp.}} = 0.083(5)\text{ps}^{-1}, \tag{103}$$

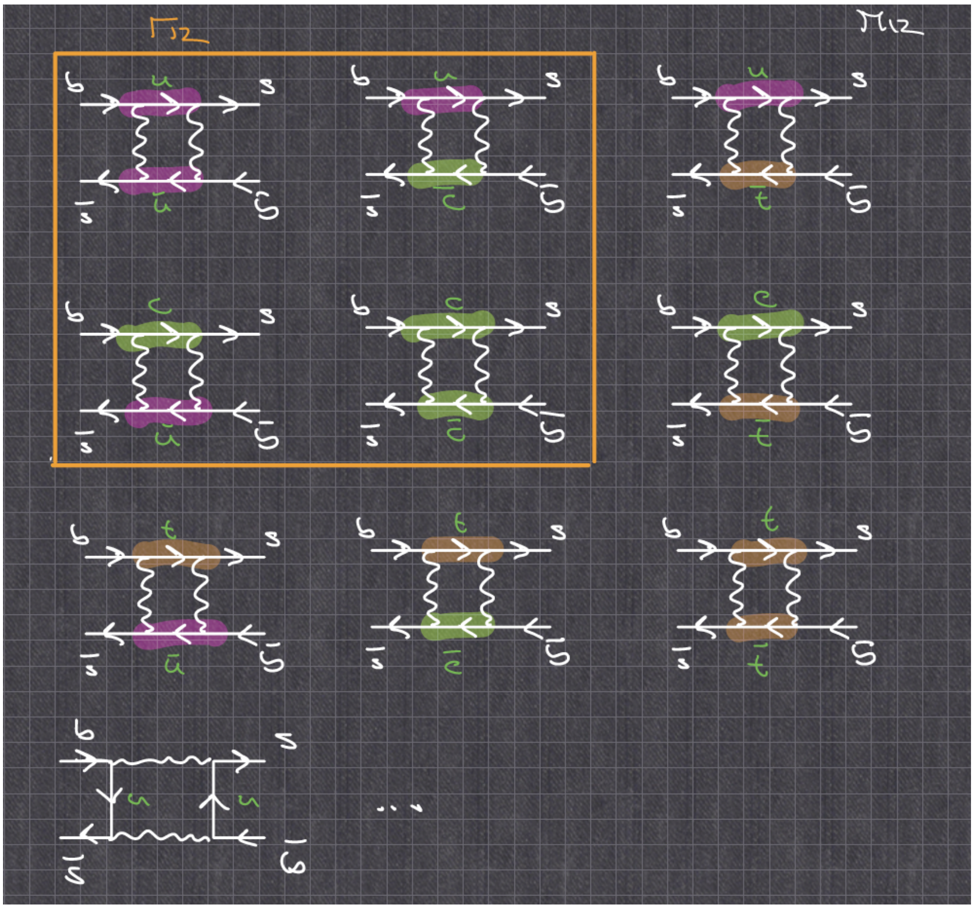
$$\Delta M_s^{\text{th.}} = 18.23(63)\text{ps}^{-1}, \qquad \Delta\Gamma_s^{\text{th.}} = 0.091(15)\text{ps}^{-1}. \tag{104}$$

– The common pre-factors in the time-dependent decay rates, i.e. N_f and $|\mathcal{A}_f|^2\left(1 + |\lambda_f|^2\right)$, typically cancel in CP asymmetries and we do not need to know their value. This is very advantageous because the hadronic quantity $\mathcal{A}_f$ is notoriously difficult to calculate. For the remaining unknown parameter λ_f in some cases additional assumptions can be made, e.g.

1. In the case of flavour-specific decays, we have $\bar{\mathcal{A}}_f = 0$ and thus $\lambda_f = 0$.
2. For gold-plated modes we have $f = \bar{f}$ and in addition we consider only one contributing CKM structure in the decay amplitude (in many case this is equivalent to neglect penguin contributions). In that case we arrive at $|\lambda_f| = 1$.

3.3 Theory determination of mixing observables

Mixing arises due to the box diagrams and in leading order there are actually 18 box diagrams contribution to M_{12} and four diagrams contributing to Γ_{12}, see below the B_s-mixing case.



For M_{12}^d we get the following general structure

$$\begin{aligned} M_{12}^d &= \lambda_u^2 F(u,u) + \lambda_u \lambda_c F(u,c) + \lambda_u \lambda_t F(u,t) + \\ &\quad \lambda_c \lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c \lambda_t F(c,t) + \\ &\quad \lambda_t \lambda_u F(t,u) + \lambda_t \lambda_c F(t,c) + \lambda_t^2 F(t,t)\,, \end{aligned} \tag{105}$$

with the CKM structures $\lambda_q = V_{qd}^* V_{qb}$ for $(q = u, c, t)$ and the loop function $F(q_1, q_2)$ describing internal q_1 and q_2 quarks. For M_{12} we take the off-shell part of the loop diagrams and for Γ_{12} we take the on-shell parts. For B_s mixing we have to use instead the CKM structure $\lambda_q = V_{qs}^* V_{qb}$. In the case of D mixing the internal quarks (u, c, t) are replaced by (d, s, b) and the CKM factor read $\lambda_q = V_{cq}^* V_{uq}$ for $(q = d, s, b)$. In these three systems we get the following CKM hierarchies.

D^0	B_d	B_s
$\lambda_d = V_{cd} V_{ud}^* \propto \lambda$	$\lambda_u = V_{ub} V_{ud}^* \propto \lambda^{3.75}$	$\lambda_u = V_{ub} V_{ud}^* \propto \lambda^{4.75}$
$\lambda_s = V_{cs} V_{us}^* \propto \lambda$	$\lambda_c = V_{cb} V_{cd}^* \propto \lambda^3$	$\lambda_c = V_{cb} V_{cd}^* \propto \lambda^2$
$\lambda_b = V_{cb} V_{ub}^* \propto \lambda^{5.75}$	$\lambda_t = V_{tb} V_{td}^* \propto \lambda^3$	$\lambda_c = V_{tb} V_{td}^* \propto \lambda^2$

(106)

We see a pronounced hierarchy of the CKM elements for D and B_s mixing and a slight one for B_d

mixing. Within the SM we can further use the unitarity of the CKM matrix ($\lambda_u + \lambda_c + \lambda_t = 0 = \lambda_d + \lambda_s + \lambda_b$) to eliminate one of the CKM structure and to make use of numerical hierarchies. For the B_d-system we find

$$\begin{aligned} M_{12}^d &= \lambda_u^2 \left[F(c,c) - 2F(u,c) + F(u,u)\right] \\ &\quad +2\lambda_u\lambda_t \left[F(c,c) - F(u,c) + F(u,t) - F(c,t)\right] \\ &\quad +\lambda_t^2 \left[F(c,c) - 2F(c,t) + F(t,t)\right] . \end{aligned} \tag{107}$$

Remarks:

1. In the case of D-mixing Eq. (107) reads

$$\begin{aligned} M_{12}^d &= \lambda_s^2 \left[F(s,s) - 2F(d,s) + F(d,d)\right] \\ &\quad +2\lambda_s\lambda_b \left[F(s,s) - F(d,s) + F(d,b) - F(d,b)\right] \\ &\quad +\lambda_b^2 \left[F(s,s) - 2F(s,b) + F(b,b)\right] . \end{aligned} \tag{108}$$

2. GIM cancellations [38]: the general result of a loop calculation looks like

$$F(p,q) = f_0 + f(x_q, x_p) , \tag{109}$$

with a constant value f_0 and a mass dependent term $f(x_q, x_p)$ with $x_y = m_y^2/M_W^2$. Thus one finds that f_0 cancels in Eq. (107) due to GIM cancellation - therefore also no renormalisation is necessary if the individual loop diagrams were divergent. If all internal masses would be equal (or zero), M_{12}^q would vanish. Looking at the values of quark masses we find

$$x_u = 7.2 \cdot 10^{-10} , \qquad x_d = 3.4 \cdot 10^{-9} , \tag{110}$$

$$x_c = 2.5 \cdot 10^{-4} , \qquad x_s = 1.3 \cdot 10^{-6} , \tag{111}$$

$$x_t = 4 , \qquad x_b = 2.7 \cdot 10^{-3} . \tag{112}$$

Thus only the top quark has a sizable value of the mass - all other masses are close to be negligible. Thus in the case of B mixing only the last line of Eq. (107) is important, which is also the CKM leading term. In D mixing the same approximation would infer a vanishing result - taking only the effects of the b-quark into account will probably also be a bad approximation, as the last line of Eq. (108) is heavily CKM suppressed. Hence in D-mixing all contributions have to be considered and the result is affected by severe GIM cancellations.

3. Γ_{12} can also be read of Eq. (107) by deleting all terms with a heavy top quark (or b quark in case of D mixing) and by replacing the off-shell function $F(p,q)$ with the on-shell function $F^{OS}(p,q)$

4. In B mixing we find to a very good approximation

$$M_{12}^d = \lambda_t^2 \left[f(t,t) - 2f(c,t) + f(c,c)\right] \propto \lambda_t^2 S(m_t^2/M_W^2) , \tag{113}$$

with the Inami-Lim function $S(x)$ [71]:

$$S(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x \ln x}{2(1-x)^2} \,. \tag{114}$$

Hence we expect

$$\frac{\Delta M_d}{\Delta M_s} = \frac{|V_{td}|^2}{|V_{ts}|^2} = 0.044\,, \tag{115}$$

which fits already quite well with the experimental value of 0.03.

Performing the complete calculating of the box diagrams contributing to the last line of Eq. (107) (with internal top and charm quarks) one obtains

$$M_{12}^d = \frac{G_F^2}{12\pi^2} (V_{td}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_d} f_{B_d}^2 M_{B_d} \hat{\eta}_B \,. \tag{116}$$

The Inami-Lim function $S_0(x_t = \bar{m}_t^2/M_W^2)$ was discussed above. It results from the box diagram without any gluon corrections. The NLO QCD correction is parameterised by $\hat{\eta}_B \approx 0.84$ [72]. The non-perturbative matrix element of the $\Delta B = 2$ operator

$$O_1 = (\bar{d}b)_{V-A} (\bar{d}b)_{V-A} \,. \tag{117}$$

is parameterised by the bag parameter B_{B_d} and the decay constant f_{B_d}

$$\langle \bar{B}_d | O_1 | B_d \rangle = \frac{8}{3} f_{B_d}^2 B_{B_d} M_{B_d}^2 \,. \tag{118}$$

These non-perturbative parameters can be determined with lattice QCD [73] or with the help of QCD sum rules, see e.g. [74]. Current state-of-the-art SM predictions for mixing observables are shown in Eq. (104) and reviewed in [59].

3.4 CP asymmetries

As our last topic we discuss CP violation in hadron decays, where we can distinguish three different origins of CP violation:

1. **CP violation in mixing:** here we consider flavour-specific decays $B \to f$, which means that the decays $\bar{B} \to f$ and $B \to \bar{f}$ are not allowed. Examples for such decays are semi-leptonic B-decays or e.g. $B_s \to D_s^- \pi^+$, which was used for the precision determination of ΔM_s, shown above. We further assume that in these decays no direct CP violation arises, i.e. $A_f = \bar{A}_{\bar{f}}$. Below we will see that this is equivalent to only one contributing CKM structure in the decay amplitude. For such decays we immediately obtain $\lambda_f = 0$, which considerably simplifies the expressions for the time-dependent decay rates in Eq. (100).
   We define the asymmetry for CP violation in mixing as

$$a_{CP}^{\rm mix} = \frac{\Gamma(\bar{B}_d(t) \to f) - \Gamma(B_d(t) \to \bar{f})}{\Gamma(\bar{B}_d(t) \to f) + \Gamma(B_d(t) \to \bar{f})} \,. \tag{119}$$

Inserting the expressions for the time-dependent rates (a la Eq. (100)) one finds

$$a_{CP}^{\rm mix} \;=\; a_{fs}^{d} = \Im\left(\frac{\Gamma_{12}^{d}}{M_{12}^{d}}\right) . \tag{120}$$

The flavour-specific CP asymmetry, a_{fs}^{d} has not been measured yet, the current experimental bound reads $a_{fs}^{d,\rm exp.} = -21(14)\cdot 10^{-4}$, while the SM expectation is $a_{fs}^{d,\rm th.} = -5.1(0.5)\cdot 10^{-4}$, see [59]. In case of B_s mesons one gets $a_{fs}^{s,\rm exp.} = -60(280)\cdot 10^{-5}$, while the SM expectation is $a_{fs}^{s,\rm th.} = 2.2(0.2)\cdot 10^{-5}$. Due to their smallness, the flavour-specific CP asymmetries present excellent opportunities for so-called Null-tests of the SM.

2. **Indirect CP violation or CP violatoin in the interference of mixing and decay:** CP violation was first observed in the decays of Kaons in 1964 as a small effect of the order of several per mille [24], denoted by the quantity ϵ, mentioned in the first lecture. Based on the CKM-mechanism for CP violation, Bigi and Sanda [40] expected in 1981 large (of the order of 50%) CP violating effects in certain B-decays - this was the main motivation for building the B-factories at KEK and SLAC in the 1990s.

   Here one investigates the following CP violating asymmetry

$$a_{CP}^{\rm ind} \;=\; \frac{\Gamma(B_d(t)\to f) - \Gamma(B_d(t)\to \bar f)}{\Gamma(B_d(t)\to f) + \Gamma(B_d(t)\to \bar f)} . \tag{121}$$

   Considering further decays, like $B_d \to J/\psi K_S$, for which $f = \bar f$ and which are dominated by a single decay amplitude with the general form

$$\mathcal{A}_f \;=\; a\cdot e^{i\phi}\cdot e^{i\theta} , \tag{122}$$

   with the modulus of the amplitude a, the strong phase ϕ and the weak phase θ one finds that the parameter λ_f can be simplified as

$$\lambda_f = \frac{q}{p}\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} = \frac{q}{p}\frac{\bar{\mathcal{A}}_{\bar f}}{\mathcal{A}_f} = \frac{q}{p}\frac{a\cdot e^{i\phi}\cdot e^{-i\theta}}{a\cdot e^{i\phi}\cdot e^{i\theta}} = \frac{q}{p}e^{-2i\theta} . \tag{123}$$

   Here all hadronic parameters have canceled! In this case the CP asymmetry $a_{CP}^{\rm ind}$ turns out to be proportional to the sine of twice the angle β of the unitarity triangle. The measurement of a large value of $a_{CP}^{\rm ind}(B_d \to J/\psi K_S)$ [41,42] confirmed the CKM mechanism and lead to the Nobel Prize for Kobayashi and Maskawa in 2008.

   By now the experimental precision in this asymmetry has dramatically improved and currently an uncertainty of β of around $\pm 0.6°$ is quoted [75]. Hence we have to revisit the simplifications made in the theoretical derivation of the asymmetry. In particular, it is well-known that the decay $B_d \to J/\psi K_S$ can also proceed besides the tree-level $b \to c\bar c s$-decay via a penguin contribution, leading to two contributing amplitudes

$$\mathcal{A}_f \;=\; a\cdot e^{i\phi}\cdot e^{i\theta} + b\cdot e^{i\tilde\phi}\cdot e^{i\tilde\theta}$$

$$= \quad a \cdot e^{i\phi} \cdot e^{i\theta} \left[1 + \frac{b}{a} \cdot e^{i(\tilde{\phi}-\phi)} \cdot e^{i(\tilde{\theta}-\theta)}\right] , \tag{124}$$

with the modulus of the penguin amplitude b and the strong penguin phase $\tilde{\phi}$ and the weak penguin phase $\tilde{\theta}$. Now the parameter λ_f and the asymmetry get corrections due to the ratio of the penguin and tree amplitude.

$$\lambda_f = \frac{q}{p} e^{-2i\theta} [1 + r....] , \tag{125}$$

$$a_{CP}^{\text{ind}} \propto \sin 2\beta [1 + r....] , \quad \text{with } r = \frac{b}{a} . \tag{126}$$

This so-called **penguin pollution** is a purely hadronic quantity and therefore very hard to be estimated reliably, but it is expected to be of the order of $\pm 1°$. Hence progress in theory is urgently needed to cope with the increasing experimental precision.

3. **Direct CP violation :** Finally we define the direct CP asymmetry as

$$a_{CP}^{\text{dir}} = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})} . \tag{127}$$

Now B can be a neutral or charged B meson. Writing a general decomposition of the decay amplitude for the decay $B \to f$ again as

$$\begin{aligned} \mathcal{A}_f &= a \cdot e^{i\phi} \cdot e^{i\theta} + b \cdot e^{i\tilde{\phi}} \cdot e^{i\tilde{\theta}} \\ &= a \cdot e^{i\phi} \cdot e^{i\theta} \left[1 + r \cdot e^{i(\tilde{\phi}-\phi)} \cdot e^{i(\tilde{\theta}-\theta)}\right] , \end{aligned} \tag{128}$$

with strong phases $\phi, \tilde{\phi}$ and weak phases $\theta, \tilde{\theta}$, we derive the following expression for the direct CP asymmetry

$$a_{CP}^{\text{dir}} \propto r \sin(\tilde{\phi} - \phi) \sin(\tilde{\theta} - \theta) . \tag{129}$$

We find that direct CP violation requires at least two different contributions to the decay amplitude, with different strong and weak phases. Note, that $B_s \to D_s^- \pi^+$ has only one contributing CKM structure, hence no direct CP violation arises, which was used above. Moreover we see, that now the asymmetry is directly proportional to the hadronic ratio r (in the case of indirect CP violation, r was a correction), which is very hard to be determined theoretically. Therefore reliable theory predictions for direct CP asymmetries are very hard to be made, while the theory status of CP violation in mixing and for indirect CP violation is much better under control.
It is interesting to note that recently a large direct CP violation in the decays $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ has been measured by the LHCb collaboration [76, 77], which required values of r being an order of magnitude larger than naively expected. Recent theory progress based on the use of light cone sum rules [47, 78] seems to give some evidence for the smallness of r in the Standard Model.

Acknowledgment

I would like to thank the organisers for the invitation to this amazing school, the students for showing intensive interest in both the GIM- and the Gym-mechanism and Maria Laura Piscopo for a careful reading of the notes.

References

[1] S. L. Glashow, Partial Symmetries of Weak Interactions, Nucl. Phys. **22** (1961), 579-588 doi:10.1016/0029-5582(61)90469-2

[2] S. Weinberg, A Model of Leptons, Phys. Rev. Lett. **19** (1967), 1264-1266 doi:10.1103/PhysRevLett.19.1264

[3] A. Salam, Weak and Electromagnetic Interactions Conf. Proc. C **680519** (1968), 367-377 doi:10.1142/9789812795915_0034

[4] P. W. Higgs, Broken Symmetries and the Masses of Gauge Bosons, Phys. Rev. Lett. **13** (1964), 508-509 doi:10.1103/PhysRevLett.13.508

[5] P. W. Higgs, Broken symmetries, massless particles and gauge fields, Phys. Lett. **12** (1964), 132-133 doi:10.1016/0031-9163(64)91136-9

[6] F. Englert and R. Brout, Broken Symmetry and the Mass of Gauge Vector Mesons, Phys. Rev. Lett. **13** (1964), 321-323 doi:10.1103/PhysRevLett.13.321

[7] G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Global Conservation Laws and Massless Particles, Phys. Rev. Lett. **13** (1964), 585-587 doi:10.1103/PhysRevLett.13.585

[8] T. D. Lee and C. N. Yang, Question of Parity Conservation in Weak Interactions, Phys. Rev. **104** (1956), 254-258 doi:10.1103/PhysRev.104.254

[9] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes and R. P. Hudson, Experimental Test of Parity Conservation in β Decay, Phys. Rev. **105** (1957), 1413-1414 doi:10.1103/PhysRev.105.1413

[10] G. Aad *et al.* [ATLAS], Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B **716** (2012), 1-29 doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]].

[11] S. Chatrchyan *et al.* [CMS], Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC, Phys. Lett. B **716** (2012), 30-61 doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]].

[12] N. Cabibbo, Unitary Symmetry and Leptonic Decays, Phys. Rev. Lett. **10** (1963), 531-533 doi:10.1103/PhysRevLett.10.531

[13] M. Kobayashi and T. Maskawa, CP Violation in the Renormalizable Theory of Weak Interaction, Prog. Theor. Phys. **49** (1973), 652-657 doi:10.1143/PTP.49.652

[14] J. J. Aubert *et al.* [E598], Experimental Observation of a Heavy Particle J, Phys. Rev. Lett. **33** (1974), 1404-1406 doi:10.1103/PhysRevLett.33.1404

[15] J. E. Augustin *et al.* [SLAC-SP-017], Discovery of a Narrow Resonance in e^+e^- Annihilation, Phys. Rev. Lett. **33** (1974), 1406-1408 doi:10.1103/PhysRevLett.33.1406

[16] L. Wolfenstein, Parametrization of the Kobayashi-Maskawa Matrix, Phys. Rev. Lett. **51** (1983), 1945 doi:10.1103/PhysRevLett.51.1945

[17] J. Charles *et al.* [CKMfitter Group], CP violation and the CKM matrix: Assessing the impact of the asymmetric B factories, Eur. Phys. J. C **41** (2005) no.1, 1-131 doi:10.1140/epjc/s2005-02169-1 [arXiv:hep-ph/0406184 [hep-ph]].

[18] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, How much space is left for a new family of fermions?, Phys. Rev. D **79** (2009), 113006 doi:10.1103/PhysRevD.79.113006 [arXiv:0902.4883 [hep-ph]].

[19] O. Eberhardt, A. Lenz and J. Rohrwild, Less space for a new family of fermions, Phys. Rev. D **82** (2010), 095006 doi:10.1103/PhysRevD.82.095006 [arXiv:1005.3505 [hep-ph]].

[20] A. Djouadi and A. Lenz, Sealing the fate of a fourth generation of fermions, Phys. Lett. B **715** (2012), 310-314 doi:10.1016/j.physletb.2012.07.060 [arXiv:1204.1252 [hep-ph]].

[21] O. Eberhardt, G. Herbert, H. Lacker, A. Lenz, A. Menzel, U. Nierste and M. Wiebusch, Impact of a Higgs boson at a mass of 126 GeV on the standard model with three and four fermion generations, Phys. Rev. Lett. **109** (2012), 241802 doi:10.1103/PhysRevLett.109.241802 [arXiv:1209.1101 [hep-ph]].

[22] P. A. R. Ade *et al.* [Planck], Planck 2013 results. VI. High Frequency Instrument data processing, Astron. Astrophys. **571** (2014), A6 doi:10.1051/0004-6361/201321570 [arXiv:1303.5067 [astro-ph.CO]].

[23] A. D. Sakharov, Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe, Pisma Zh. Eksp. Teor. Fiz. **5** (1967), 32-35 doi:10.1070/PU1991v034n05ABEH002497

[24] J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, Evidence for the 2π Decay of the K_2^0 Meson, Phys. Rev. Lett. **13** (1964), 138-140 doi:10.1103/PhysRevLett.13.138

[25] O. Atkinson, M. Black, A. Lenz, A. Rusov and J. Wynne, Cornering the Two Higgs Doublet Model Type II, JHEP **04** (2022), 172 doi:10.1007/JHEP04(2022)172 [arXiv:2107.05650 [hep-ph]].

[26] N. S. Manton, Topology in the Weinberg-Salam Theory, Phys. Rev. D **28** (1983), 2019 doi:10.1103/PhysRevD.28.2019

[27] F. R. Klinkhamer and N. S. Manton, A Saddle Point Solution in the Weinberg-Salam Theory, Phys. Rev. D **30** (1984), 2212 doi:10.1103/PhysRevD.30.2212

[28] C. Jarlskog, Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Nonconservation, Phys. Rev. Lett. **55** (1985), 1039 doi:10.1103/PhysRevLett.55.1039

[29] M. E. Shaposhnikov, Possible Appearance of the Baryon Asymmetry of the Universe in an Electroweak Theory, JETP Lett. **44** (1986), 465-468

[30] G. Alonso-Álvarez, G. Elor and M. Escudero, Collider signals of baryogenesis and dark matter from B mesons: A roadmap to discovery, Phys. Rev. D **104** (2021) no.3, 035028 doi:10.1103/PhysRevD.104.035028 [arXiv:2101.02706 [hep-ph]].

[31] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Is there a hot electroweak phase transition at $m_H \gtrsim m_W$?, Phys. Rev. Lett. **77** (1996), 2887-2890 doi:10.1103/PhysRevLett.77.2887 [arXiv:hep-ph/9605288 [hep-ph]].

[32] K. Rummukainen, M. Tsypin, K. Kajantie, M. Laine and M. E. Shaposhnikov, The Universality class of the electroweak theory, Nucl. Phys. B **532** (1998), 283-314 doi:10.1016/S0550-3213(98)00494-5 [arXiv:hep-lat/9805013 [hep-lat]].

[33] F. Csikor, Z. Fodor and J. Heitger, Endpoint of the hot electroweak phase transition, Phys. Rev. Lett. **82** (1999), 21-24 doi:10.1103/PhysRevLett.82.21 [arXiv:hep-ph/9809291 [hep-ph]]. [34]

[34] Y. Aoki, F. Csikor, Z. Fodor and A. Ukawa, The Endpoint of the first order phase transition of the SU(2) gauge Higgs model on a four-dimensional isotropic lattice, Phys. Rev. D **60** (1999), 013001 doi:10.1103/PhysRevD.60.013001 [arXiv:hep-lat/9901021 [hep-lat]].

[35] O. Atkinson, M. Black, C. Englert, A. Lenz, A. Rusov and J. Wynne, The flavourful present and future of 2HDMs at the collider energy frontier, JHEP **11** (2022), 139 doi:10.1007/JHEP11(2022)139 [arXiv:2202.08807 [hep-ph]].

[36] O. Atkinson, M. Black, C. Englert, A. Lenz and A. Rusov, MUonE, muon g-2 and electroweak precision constraints within 2HDMs, Phys. Rev. D **106** (2022) no.11, 115031 doi:10.1103/PhysRevD.106.115031 [arXiv:2207.02789 [hep-ph]].

[37] D. E. Morrissey and M. J. Ramsey-Musolf, Electroweak baryogenesis, New J. Phys. **14** (2012), 125003 doi:10.1088/1367-2630/14/12/125003 [arXiv:1206.2942 [hep-ph]].

[38] S. L. Glashow, J. Iliopoulos and L. Maiani, Weak Interactions with Lepton-Hadron Symmetry, Phys. Rev. D **2** (1970), 1285-1292 doi:10.1103/PhysRevD.2.1285

[39] D. Abbaneo, A. Ali, P. Amaral, V. Andreev, M. Artuso, E. Barberio, M. Battaglia, C. Bauer, D. Becirevic and M. Beneke, *et al.* The CKM matrix and the unitarity triangle. Workshop, CERN, Geneva, Switzerland, 13-16 Feb 2002: Proceedings, doi:10.5170/CERN-2003-002-corr [arXiv:hep-ph/0304132 [hep-ph]].

[40] I. I. Y. Bigi and A. I. Sanda, Notes on the Observability of CP Violations in B Decays, Nucl. Phys. B **193** (1981), 85-108 doi:10.1016/0550-3213(81)90519-8

[41] K. Abe *et al.* [Belle], Observation of large CP violation in the neutral B meson system, Phys. Rev. Lett. **87** (2001), 091802 doi:10.1103/PhysRevLett.87.091802 [arXiv:hep-ex/0107061 [hep-ex]].

[42] B. Aubert *et al.* [BaBar], Observation of CP violation in the B^0 meson system, Phys. Rev. Lett. **87** (2001), 091801 doi:10.1103/PhysRevLett.87.091801 [arXiv:hep-ex/0107013 [hep-ex]]. Copy to ClipboardDownload

[43] J. Brod and J. Zupan, The ultimate theoretical error on γ from $B \to DK$ decays, JHEP **01** (2014), 051 doi:10.1007/JHEP01(2014)051 [arXiv:1308.5663 [hep-ph]].

[44] J. Brod, A. Lenz, G. Tetlalmatzi-Xolocotzi and M. Wiebusch, New physics effects in tree-level decays and the precision in the determination of the quark mixing angle γ, Phys. Rev. D **92** (2015) no.3, 033002 doi:10.1103/PhysRevD.92.033002 [arXiv:1412.1446 [hep-ph]].

[45] A. Lenz and G. Tetlalmatzi-Xolocotzi, Model-independent bounds on new physics effects in non-leptonic tree-level decays of B-mesons, JHEP **07** (2020), 177 doi:10.1007/JHEP07(2020)177 [arXiv:1912.07621 [hep-ph]].

[46] M. Bona *et al.* [UTfit], The Unitarity Triangle Fit in the Standard Model and Hadronic Parameters from Lattice QCD: A Reappraisal after the Measurements of ΔM_s and $BR(B \to \tau\nu_\tau)$, JHEP **10** (2006), 081 doi:10.1088/1126-6708/2006/10/081 [arXiv:hep-ph/0606167 [hep-ph]].

[47] A. Lenz, M. L. Piscopo and A. V. Rusov, Two body non-leptonic D^0 decays from LCSR and implications for$\Delta a_{\rm CP}^{\rm dir}$, JHEP **03** (2024), 151 doi:10.1007/JHEP03(2024)151 [arXiv:2312.13245 [hep-ph]].

[48] L. Michel, Interaction between four half spin particles and the decay of the μ meson, Proc. Phys. Soc. A **63** (1950), 514-531 doi:10.1088/0370-1298/63/5/311

[49] R. L. Workman *et al.* [Particle Data Group], Review of Particle Physics, PTEP **2022** (2022), 083C01 doi:10.1093/ptep/ptac097

[50] R. E. Behrends, R. J. Finkelstein and A. Sirlin, Radiative corrections to decay processes, Phys. Rev. **101** (1956), 866-873 doi:10.1103/PhysRev.101.866

[51] T. Kinoshita and A. Sirlin, Radiative corrections to Fermi interactions, Phys. Rev. **113** (1959), 1652-1660 doi:10.1103/PhysRev.113.1652

[52] T. van Ritbergen and R. G. Stuart, Complete two loop quantum electrodynamic contributions to the muon lifetime in the Fermi model, Phys. Rev. Lett. **82** (1999), 488-491 doi:10.1103/PhysRevLett.82.488 [arXiv:hep-ph/9808283 [hep-ph]].

[53] A. Sirlin and A. Ferroglia, Radiative Corrections in Precision Electroweak Physics: a Historical Perspective, Rev. Mod. Phys. **85** (2013) no.1, 263-297 doi:10.1103/RevModPhys.85.263 [arXiv:1210.5296 [hep-ph]].

[54] P. A. Baikov, K. G. Chetyrkin and J. H. Kuhn, Order α_s^4 QCD Corrections to Z and tau Decays, Phys. Rev. Lett. **101** (2008), 012002 doi:10.1103/PhysRevLett.101.012002 [arXiv:0801.1821 [hep-ph]].

[55] A. Pich, Precision Tau Physics, Prog. Part. Nucl. Phys. **75** (2014), 41-85 doi:10.1016/j.ppnp.2013.11.002 [arXiv:1310.7922 [hep-ph]].

[56] S. Bethke, Look how it runs!, [arXiv:2310.01111 [hep-ph]].

[57] M. L. Piscopo, Higher order corrections to the lifetime of heavy hadrons, doi:10.25819/ubsi/10024 [arXiv:2112.03137 [hep-ph]].

[58] A. Lenz, Lifetimes and heavy quark expansion, Int. J. Mod. Phys. A **30** (2015) no.10, 1543005 doi:10.1142/S0217751X15430058 [arXiv:1405.3601 [hep-ph]].

[59] J. Albrecht, F. Bernlochner, A. Lenz and A. Rusov, Lifetimes of b-hadrons and mixing of neutral B-mesons: theoretical and experimental status, Eur. Phys. J. ST **233** (2024) no.2, 359-390 doi:10.1140/epjs/s11734-024-01124-3 [arXiv:2402.04224 [hep-ph]].

[60] A. J. Buras, Weak Hamiltonian, CP violation and rare decays," [arXiv:hep-ph/9806471 [hep-ph]].

[61] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Weak decays beyond leading logarithms, Rev. Mod. Phys. **68** (1996), 1125-1144 doi:10.1103/RevModPhys.68.1125 [arXiv:hep-ph/9512380 [hep-ph]].

[62] A. Lenz, Heavy Flavour Physics and Effective Field Theories, Lecture notes, weblink: https://tp.nt.uni-siegen.de/wp-content/uploads/2024/02/Lecture_Flav_2021.pdf.

[63] G. Altarelli and L. Maiani, Phys. Lett. B **52** (1974), 351-354 doi:10.1016/0370-2693(74)90060-4

[64] M. Gorbahn and U. Haisch, Effective Hamiltonian for non-leptonic $|\Delta F| = 1$ decays at NNLO in QCD, Nucl. Phys. B **713** (2005), 291-332 doi:10.1016/j.nuclphysb.2005.01.047 [arXiv:hep-ph/0411071 [hep-ph]].

[65] M. A. Shifman, Foreword to ITEP lectures in particle physics, [arXiv:hep-ph/9510397 [hep-ph]].

[66] J. Albrecht, D. van Dyk and C. Langenbruch, Flavour anomalies in heavy quark decays, Prog. Part. Nucl. Phys. **120** (2021), 103885 doi:10.1016/j.ppnp.2021.103885 [arXiv:2107.04822 [hep-ex]].

[67] A. Lenz and G. Wilkinson, Mixing and CP Violation in the Charm System, Ann. Rev. Nucl. Part. Sci. **71** (2021), 59-85 doi:10.1146/annurev-nucl-102419-124613 [arXiv:2011.04443 [hep-ph]].

[68] M. Artuso, G. Borissov and A. Lenz, CP violation in the B_s^0 system, Rev. Mod. Phys. **88** (2016) no.4, 045002 doi:10.1103/RevModPhys.88.045002 [arXiv:1511.09466 [hep-ph]].

[69] K. Anikeev, D. Atwood, F. Azfar, S. Bailey, C. W. Bauer, W. Bell, G. Bodwin, E. Braaten, G. Burdman and J. N. Butler, *et al.* B physics at the Tevatron: Run II and beyond, [arXiv:hep-ph/0201071 [hep-ph]].

[70] R. Aaij *et al.* [LHCb], Precise determination of the $B^0_{\rm s}$–$\overline{B}^0_{\rm s}$ oscillation frequency, Nature Phys. **18** (2022) no.1, 1-5 doi:10.1038/s41567-021-01394-x [arXiv:2104.04421 [hep-ex]].

[71] T. Inami and C. S. Lim, Effects of Superheavy Quarks and Leptons in Low-Energy Weak Processes $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+\nu\bar{\nu}$, and $K^0 \leftrightarrow \bar{K}^0$, Prog. Theor. Phys. **65** (1981), 297 [erratum: Prog. Theor. Phys. **65** (1981), 1772] doi:10.1143/PTP.65.297

[72] A. J. Buras, M. Jamin and P. H. Weisz, Leading and Next-to-leading QCD Corrections to ϵ Parameter and $B^0 - \bar{B}^0$ Mixing in the Presence of a Heavy Top Quark, Nucl. Phys. B **347** (1990), 491-536 doi:10.1016/0550-3213(90)90373-L

[73] Y. Aoki *et al.* [Flavour Lattice Averaging Group (FLAG)], FLAG Review 2021, Eur. Phys. J. C **82** (2022) no.10, 869 doi:10.1140/epjc/s10052-022-10536-1 [arXiv:2111.09849 [hep-lat]].

[74] D. King, A. Lenz and T. Rauh, B_s mixing observables and $|V_{td}/V_{ts}|$ from sum rules, JHEP **05** (2019), 034 doi:10.1007/JHEP05(2019)034 [arXiv:1904.00940 [hep-ph]].

[75] R. Aaij *et al.* [LHCb], Measurement of CP Violation in B0→ψ(→ℓ+ℓ-)KS0(→π+π-) Decays, Phys. Rev. Lett. **132** (2024) no.2, 021801 doi:10.1103/PhysRevLett.132.021801 [arXiv:2309.09728 [hep-ex]].

[76] R. Aaij *et al.* [LHCb], Observation of CP Violation in Charm Decays, Phys. Rev. Lett. **122** (2019) no.21, 211803 doi:10.1103/PhysRevLett.122.211803 [arXiv:1903.08726 [hep-ex]].

[77] R. Aaij *et al.* [LHCb], Measurement of the Time-Integrated CP Asymmetry in D0→K-K+ Decays, Phys. Rev. Lett. **131** (2023) no.9, 091802 doi:10.1103/PhysRevLett.131.091802 [arXiv:2209.03179 [hep-ex]].

[78] A. Lenz, M. L. Piscopo and A. V. Rusov, Towards a SM prediction for CP violation in charm, [arXiv:2403.02267 [hep-ph]].