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Neutrino Physics

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Abstract
This is summary of three lectures on neutrino physics at the CERN school in Evora, Portugal, for experimental PhD students. There is a brief review of neutrino interactions in the Standard Model, Majorana and Dirac mass terms, oscillations in vaccuum and matter for 2 generations, the leptonic unitarity triangle and 3 generation mixing, and bounds on the the absolute neutrino mass scale. Follows a few topics going beyond the physics of three light active neutrinos: an introduction to a few seesaw models for Majorana masses, and leptogenesis in the type I seesaw.

Keywords
Lectures; neutrinos; leptogenesis, neutrino oscillations, mass, flavor, Majorana

1 Introduction

Neutrinos are shy particles in the laboratory, but make several relevant contributions in cosmology and astrophysics. They could be responsable for the observed matter excess of the Universe [1], and possibly also the dark matter [2]. We know that there were three species of relativistic neutrinos in thermal equilibrium in the plasma when the Univers was a few minutes old at the moment of Big Band Nuceosynthesis [3], because the observed primordial ratios of light elements depend on the energy density at the time. Additional constraints on the summed-mass, and number of light neutrinos in equilibrium are obtained from the observed anisotropies in the Cosmic Microwave Background (CMB) [4]. In the following 10^{10} years of the life of our Universe, stars were born, radiated photons and neutrinos, and died — the massive ones in supernova explosions [5] (whose explosion probably required assistance from neutrinos), thereby spreading heavy elements through the Universe and making our life possible. Humanity became acquainted with neutrinos only in the previous century, and in the last decades, they have given us laboratory evidence [14, 15] of New Physics beyond the Standard Model (SM). This has generated significant interest in the community — so many excellent review articles are available. Some review articles that I have read (much more complete than this introduction), can be found in reference [6], and useful websites in reference [8].

1.1 Notation

I use chiral (2-component) spinors, but 4-component spinor notation, where a 4-component spinor χ has 4 degrees of freedom labelled by $\{\pm E, \pm s\}$, and can be written in the chiral decomposition

$$\chi = \left(\begin{array}{c} \psi_L \\ \psi_R \end{array} \right) \ , \ \{\gamma^\alpha\} = \left\{ \left[\begin{array}{cc} 0 & I \\ I & 0 \end{array} \right], \left[\begin{array}{cc} 0 & \sigma_i \\ -\sigma_i & 0 \end{array} \right] \right\} \ , \ \{\sigma_i\} = \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right], \left[\begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right], \left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right] \tag{1}$$

where $\psi_L = P_L \chi, \psi_R = P_R \chi$ with $P_L = \frac{(1-\gamma_5)}{2}$. Recall that chirality is not an observable, but becomes helicity (the projection of spin along the direction of motion $\pm \hat{s} \cdot \hat{k} = \pm 1/2$) in the relativistic limit, and is simpler to calculate with than helicity.

In later sections of these notes, the chiral subscript on the fermions may suppressed (for instance, in the leptogenesis section, I write N for N_R).

The Higgs vev $v = 174$ GeV.

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2 Neutrino interactions

2.1 Weak neutrino Interactions in the Standard Model

The Standard Model (SM) contains 3 generations of lepton doublets, and charged singlets:

$$\ell_{\alpha L} \in \left\{ \left(\begin{array}{c} \nu_{eL} \\ e_L \end{array} \right) , \left(\begin{array}{c} \nu_{\mu L} \\ \mu_L \end{array} \right) , \left(\begin{array}{c} \nu_{\tau L} \\ \tau_L \end{array} \right) \right\} \quad e_{\alpha R} \in \{e_R, \ \mu_R, \ \tau_R\}$$

which here are listed in the charged lepton mass eigenstate basis, to which a greek index is commonly attributed. I do not include a ν_R in the SM because data did not require m_ν when the SM was written down, and because ν_R has no gauge interactions, so a ν_R is not required in each generation for anomaly cancellation. However, some authors consider that the SM can be defined including three ν_R and neutrino Dirac masses, in which case they do not count neutrino masses as evidence for "New Physics".

A Lagrangian which reproduces all the observed interactions of neutrinos and charged leptons is:

$$\mathcal{L} \;=\; i\overline{\ell_{L\alpha}}\gamma^\mu \mathbf{D}_\mu \ell_{L\alpha} + i\overline{e_{R\alpha}}\gamma^\mu D_\mu e_{R\alpha} - \left[(\overline{\nu_{\alpha L}}, \overline{e_{\alpha L}}) y_\alpha \left(\begin{array}{c} -H^+ \\ H^{0*} \end{array} \right) e_{\alpha R} + \text{h.c.} \right] \tag{2}$$

where $\alpha \in \{e, \mu, \tau\}$ is a sum over generations, $\mathcal{L}$ is in the charged lepton mass basis, the covariant derivatives are

$$\mathbf{D}_\mu = \partial_\mu + i\frac{g}{2}\sigma^a W^a_\mu + ig' Y(\ell_L) B_\mu \; , \; D_\mu = \partial_\mu + ig' Y(e_R) B_\mu \quad , \tag{3}$$

B^μ is the hypercharge gauge boson, the fermion hypercharge is $Y(f) = T_3 + Q_{em}$, and $\widetilde{H}^T = (-H^+, H^{0*})$ gives masses $m_\alpha = y_\alpha \langle H^0 \rangle$ to the charged leptons.

The first term of eqn (2) , $\overline{\ell_{L\alpha}}^T \gamma^\mu \mathbf{D}_\mu \ell_{L\alpha}$ gives:

$$\left(\begin{array}{cc} \overline{\nu_L} & \overline{e_L} \end{array} \right) \gamma^\mu \left(\begin{array}{cc} \frac{g}{2\cos\theta_W} Z_\mu & \frac{g}{\sqrt{2}} W^+_\mu \\ \frac{g}{\sqrt{2}} W^-_\mu & eA_\mu - ...Z_\mu \end{array} \right) \left(\begin{array}{c} \nu_L \\ e_L \end{array} \right) \tag{4}$$

where $s_W = \sin\theta_W$, $\tan\theta_W = g'/g$, and the photon and Z fields are defined as $A_\mu \equiv c_W B_\mu + s_W W^3_\mu$, $Z_\mu \equiv -s_W B_\mu + c_W W^3_\mu$. This gives the familiar Feynman rules illustrated in figure 1. These

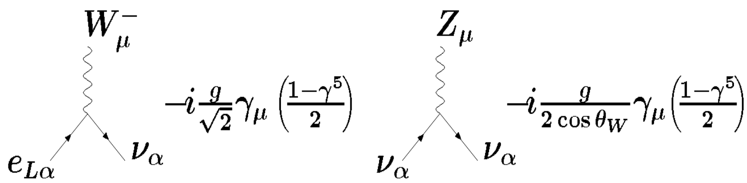


Fig. 1: W,Z Feynman rules in the SM with massless neutrinos

Feynman rules illustrate that, in the SM, there is no flavour change in the lepton sector — lepton flavour is conserved.

The Lagrangian of eqn (2) does not only reproduce all lepton interactions (except neutrino oscillations); it is also the most general renormalisable, $SU(2) \times U(1)$-invariant $\mathcal{L}$ for those particles. In order to see that, one has to show how to get rid of flavour-changing kinetic or Yukawa terms such as:

$$\left(\overline{\nu_{eL}}, \overline{e_L} \right) \not{D} \left(\begin{array}{c} \nu_{\mu L} \\ \mu_L \end{array} \right) \quad , \quad \left(\overline{\nu_{eL}}, \overline{e_L} \right) \widetilde{H} \tau_R \ .$$

Since such terms are gauge invariant, the most general Lagrangian can be written as

$$i\overline{\ell_L}^{b'} Z_{bc}\gamma^\mu \mathbf{D}_\mu \ell_L^{c'} + i\overline{e_R}^f Z^{(e)}_{fg}\gamma^\mu D_\mu e_R^g - \overline{\ell}_L'^b [\tilde{Y}_e]_{bd}\widetilde{H} e_R^d + h.c. \tag{5}$$

and the next few paragraphs aim to show that eqn (5) can be transformed into the canonical version give in eqn (2).

The first step is to diagonalise Z, which must be hermitian because $\mathcal{L}$ should be real. It can therefore be diagonalised as $\boldsymbol{VZV^\dagger = D_Z}$ where V is unitary and $\boldsymbol{D_Z}$ diagonal, as

$$\overline{\ell_L'^b}\mathbf{Z}_{bc}\not{D}\,\ell_L'^c = \overline{\ell_L'^b}[V_Z^\dagger \mathbf{D_Z} V_Z]_{bc}\not{D}\,\ell_L'^c = \overline{\ell_L^{b''}}\mathbf{D_Z}_{bb}\not{D}\,\ell_L^{b''} = \overline{\ell_L}^b\not{D}\,\ell_L^b$$

where at the last equality, the eigenvalues of Z were absorbed into the definition of the fields. This is allowed, because the magnitude of a fermion field cannot be measured.

The basis transformation and field rescaling that removed the $\boldsymbol{Z}$ matrix affect the definition of the Yukawa matrix. Defining $\mathbf{Y_e} = \mathbf{D_Z^{-1/2}V_Z\widetilde{Y}_e}$ (and implicitly performing similar operations to remove $\boldsymbol{Z^{(e)}}$) , the Lagrangian now can be written

$$\mathcal{L} \;\; = \;\; i\,\overline{\ell_L^b}^T \not{D}\,\ell_L^b + i\,\overline{e_R^a}\not{D}\,e_R^a - \{\,(\overline{\ell_L^b}\,[\mathbf{Y_e}]_{bc}\widetilde{H})e_R^c + \text{h.c.}\}$$

where $[\boldsymbol{Y_e}]$ is in principle an arbitrary 3×3 matrix. A diagonal charged-lepton mass matrix can be obtained by different unitary transformations on left and right:

$$V_L[\mathbf{Y_e}]V_R^\dagger = D_e \quad .$$

(Notice that the Yukawa index order is LR in these notes). The matrices V_L, V_R can be obtained by diagonalising the hermitian matrices $[\mathbf{Y_e}][\mathbf{Y_e}]^\dagger = V_L^\dagger D_e^2 V_L$ and $[\mathbf{Y_e}]^\dagger[\mathbf{Y_e}] = V_R^\dagger D_e^2 V_R$.

2.2 Gravitational interactions

Neutrinos also have gravitational interactions, as is expected form the equivalence principle, since they carry 4-momentum. We know this because light elements (such as H, D, ${}^4He$, and ${}^7Li$) were produced in the first few minutes of the life of the Universe ("Big Bang Nucleosynthesis" [3]), and their primordial abundances can be infered from observation. They depend on the age of the Universe at the time, which depends on the energy density (dominated at the time by relativistic species), and allows to conclude that three or four species of neutrino were in thermal equilibrium in the Universe at that time. Current Cosmic Microwave Background data can constrain neutrino parameters [4], which also confirms that neutrinos have gravitational interactions.

2.3 Historical problems

Since a long time, neutrinos have disappeared... The solar neutrino problem is the most long-standing: the sun produces energy by a network of nuclear reactions, which should produce ν_e, which escape the sun without interacting. The photons diffuse slowly to the surface. However, the observed ν_e flux from the sun is $\sim .3 \to .5$ that expected from the solar energy output. This problem was resolved by the SNO experiment [14], who showed that the flux in all flavours was as expected from the photon output, and as predicted by solar models.

There was also an "atmospheric neutrino problem", which was a deficit in the neutrinos produced in cosmic ray interactions in the earth atmosphere: such interactions produce many pions, who generically decay ($\pi^- \to \mu\bar{\nu}_\mu \to e\bar{\nu}_e\nu_\mu\bar{\nu}_\mu$) to twice as many $\nu_\mu + \bar{\nu}_\mu$ as $\nu_e + \bar{\nu}_e$. However, there was a deficit of $\nu_\mu + \bar{\nu}_\mu$, and the community became convinced that neutrinos had mass, when the SuperKamiokande Collaboration [15] showed that there was a deficit of $\nu_\mu, \bar{\nu}_\mu$ from below, that could nicely be fit by $\nu_\mu \to \nu_\tau$ oscillations.

3 Neutrino masses

Before discussing oscillations and the kinematics of m_ν, let us first think about how to write a mass term for neutrinos in $\mathcal{L}$. Since it is known from cosmology that neutrino masses $\lesssim$ eV, we start in the effective QED and QCD invariant theory that is relevant below m_W, and neglect the SU(2) invariance of the Lagrangian. Then the only constraint on the neutrino mass is that it must be a Lorentz-scalar. The only possibility that can be constructed with two chiral fermion fields is

$$m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R \;+ m\overline{\psi_R}\,\psi_L \tag{6}$$

3.1 Dirac masses

The first way to construct such a mass term for an active ν_L of the SM, is to introduce a chiral gauge singlet fermion ν_R for each SM generation. Then one can construct a fermion number conserving mass term, as for other SM fermions: $m\overline{\nu_L}\,\nu_R \;+ m\overline{\nu_R}\,\nu_L$. In the full SU(2)-invariant SM, this can be written as :

$$\lambda(\overline{\nu_L}, \overline{e_L}) \left(\begin{array}{c} H_0 \\ H_- \end{array} \right) \nu_R + h.c \equiv \lambda(\overline{\ell} H)\nu_R + hc \rightarrow \;\; m = \lambda\langle H_0\rangle$$

In three generations, the neutrino Yukawa coupling λ generalises to an arbitrary 3×3 matrix $[\boldsymbol{\lambda}]_{\sigma I}$, which can be diagonalised like other Yukawa matrices with different unitary transformations on left and right: $U[\boldsymbol{\lambda}]U_{R\nu}^\dagger = D_\nu$. If this diagonalisation is performed in the charged lepton mass eigenstate basis, the matrix U is the leptonic version of CKM sometimes called the PMNS matrix (Pontecorvo, Maki, Nakagawa and Sakata).

3.2 Majorana masses

There is a second way to write a Lorentz-invariant mass term for ν_L, in our low-energy not-SU(2)-invariant theory. This is called a Majorana mass term. It uses the fact that the charge conjugate of ν_L is right-handed: charge conjugation on a Dirac fermion is defined as

$$\psi^c \;\; = \;\; -i\gamma_0\gamma_2\overline{\psi}^T = -i\gamma_0\gamma_2\gamma_0\psi^* = i\gamma_2^*\psi^* = \left[\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right] \left(\begin{array}{c} \left(\psi_L^* \right) \\ \left(\psi_R^* \right) \end{array} \right)$$

so applied to the ν_L, this gives

$$(\nu_L)^c = \left(\begin{array}{c} \left(\begin{array}{c} 0 \\ 0 \end{array} \right) \\ \left[\begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right] \left(\nu_L^* \right) \end{array} \right) = \left(\begin{array}{c} \left(0 \right) \\ \left(-i\sigma_2\nu_L^* \right) \end{array} \right) \tag{7}$$

This allows to write a mass term with only ν_L (no new fields are required):

$$\begin{aligned} \frac{m}{2}[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] \;\; &= \;\; \frac{m}{2}[(\nu_L)^\dagger\gamma_0(\nu_L)^c + ((\nu_L)^c)^\dagger\gamma_0\nu_L] \\ &= \;\; -i\frac{m}{2}[\nu_L^\dagger\sigma_2\nu_L^* + \nu_L^T\sigma_2\nu_L] \equiv \frac{m}{2}\nu_L\nu_L + h.c. \end{aligned}$$

(where the second line is in 2 component notation for fermions, reviewed in appendix 12, which has the attraction of being less cluttered).

Notice that the mass term involves either the field twice[1], or its complex conjugate$\times$charge conjugate, so this mass violates fermion number by two units, and cannot be written in this way for a fermion

[1] The factor of $\frac{1}{2}$ in $\mathcal{L}$ is to avoid 2s in Feynman rules and physical parameters, because I work in conventions where $\overline{\nu^c}$ and ν are considered identical. Recall that Feynman rules are obtained as $\delta^n\mathcal{L}/\delta\nu^n$, so $\delta(\overline{\nu^c}\nu)/\delta\nu = 2\nu$.

with gauge interactions (= with a conserved charge). So the simplest way to write this mass term in the full SU(2) invariant SM is to write the dimension five operator (often called Weinberg operator)

$$\mathcal{L} = ... + \frac{K}{2\Lambda}(\bar{\ell}H)(\ell^c H) + h.c. \rightarrow \frac{m}{2}\overline{\nu_L}\nu_L^c + h.c. \quad , \quad m = \frac{K}{\Lambda}\langle H_0 \rangle^2 \tag{8}$$

Since this operator is non-renormalisable, we assume it is induced by heavy new particles at the scale M, whose interactions with active neutrinos are parametrised in K.

With multiple generations, the Majorana mass matrix $\frac{1}{2}\overline{\nu_L}^\alpha [m]_{\alpha\beta} (\nu_L)^c_\beta$ is *symmetric*[2], so can be diagonalised as:

$$U^T m U = D_m \,. \tag{9}$$

If the eigenvalues of m are non-degenerate, the matrix U can be obtained by diagonalising $U^\dagger m^\dagger m U = D_m^2$.

The diagonalisation recipe of eqn (9) implies that the eigenvector equation for Majorana matrices is modified with respect to the familiar case of a hermitian matrix H with eigenvalues h_i and eigenvectors $\vec{v}_i$: $H\vec{v}_i = h_i\vec{v}_i$. In the Majorana case, eqn (9) implies $mU = U^* D_m$, or $m\vec{u}_i = m_i \vec{u}_i^*$, where the eigenvectors $\vec{u}_i$ are the colomns of U.

3.3 U

The leptonic mixing matrix, which lives in 3 generation space and rotates from the charged lepton mass basis (index α) to the neutrino mass basis (index i), has three angles and at least one phase:

$$\begin{aligned} U_{\alpha i} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} P \\ &= \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{bmatrix} P \end{aligned} \tag{10}$$

where P is a diagonal matrix discussed in section 3.3.1, two of the angles are large, and the CP-violating phase δ is placed on the smallest one:

$$\theta_{23} \simeq \pi/4 \quad \theta_{12} \simeq \pi/6 \quad \theta_{13} \simeq 0.15, 8^o \quad \delta \sim 1.4\pi$$

The current experimental determinations of the angles can, for instance, be found in [10, 11]. For comparaison, the magnitudes of off-diagonal CKM matrix elements [11] are much smaller

$$V_{cb} \simeq 0.04 \quad V_{us} \simeq 0.225 \quad V_{ub} \simeq 0.004$$

One of the reasons that the PDG quotes ranges for CKM matrix elements, and for leptonic mixing angles, is that CKM matrix elements are probed in meson decays, whereas the dynamics of neutrino oscillations makes it convenient to measure the angles of the leptonic mixing matrix.

3.3.1 Majorana phases

The diagonal matrix of phases P is the identity for Dirac neutrinos, and diag$\{e^{-i\phi_1/2}, e^{-i\phi_2/2}, 1\}$ for Majorana. The origin of these phases can be understood as follows:

1. suppose that all parameters in $\mathcal{L}$ that can be complex (U and $m_{\nu i}$), are complex

[2]Fermion operators anti-commute, but the spinor contraction for the Majorana mass is also antisymmetric. This is easiest seen in 2-component spinor notation: $\hat{\nu}^\rho_{Li}\varepsilon_{\rho\sigma}\hat{\nu}^\sigma_{Lj} = -\hat{\nu}^\sigma_{Lj}\varepsilon_{\rho\sigma}\hat{\nu}^\rho_{Li} = \hat{\nu}^\sigma_{Lj}\varepsilon_{\sigma\rho}\hat{\nu}^\rho_{Li}$, where α, ρ are spinor indices, and $\hat{\nu}_j$ is the operator for mass eigenstare j.

2. There are 3 angles and 6 phases in a generic unitary matrix U (There are 18 real parameters in an arbitrary 3×3 complex matrix; then the Unitarity condition $UU^\dagger = 1$ reduces this to 9.)
3. There are five relative phases between the fields $e_L, \mu_L, \tau_L, \nu_1, \nu_2$, ν_3 ...so they can chosen to remove all but one phase in the mixing matrix.
4. now check that the masses can be made real: for dirac masses, the phase of the mass can be absorbed with ν_{RI}. If ν_{L3} has a Majorana mass, between itself and anti-self, the absolute phase of ν_{L3} can be chosen to make the mass real. This fixes all LH fermion phases, so the phases from $m_{\nu 1}, m_{\nu 2}$ cannot be removed. They contribute extra CP Violation in processes where Majorana masses appear linearly (not as mm^*, so not in kinematics = not in oscillations). These phases can be left on the masses, or rotated into the diagonal P given in eqn (10).

3.3.2 *Where do mixing matrices appear?*

As in the quark sector, the mixing matrix will appear at W vertices. This can be seen by writing the $\{e_R^\alpha\}$, and $\{\nu_R^I\}$ in the mass eigenstate basis (means $U_{R\nu}$ is unphysical), and the ℓ^a in the mass basis of charged leptons:

$$\ell_L^e \equiv \left(\begin{array}{c} U_{ei}\nu_L^i \\ e_L \end{array} \right) \ , \ell_L^\mu \equiv \left(\begin{array}{c} U_{\mu j}\nu_L^j \\ \mu_L \end{array} \right) \ , \ell_L^\tau \equiv \left(\begin{array}{c} U_{\tau k}\nu_L^k \\ \tau_L \end{array} \right)$$

so the Lagrangian becomes

$$i\,(U_{ej}^*\overline{\nu_L^j} \ \ \overline{e_L})\,\gamma^\mu \mathbf{D}_\mu \left(\begin{array}{c} U_{ek}\nu_L^k \\ e_L \end{array} \right) + i\,(U_{\mu j}^*\overline{\nu_L^j} \ \ \overline{\mu_L})\,\gamma^\mu \mathbf{D}_\mu \left(\begin{array}{c} U_{\mu k}\nu_L^k \\ \mu_L \end{array} \right) + ...$$

The 3×3 mixing matrix $U_{\alpha,i}$ appears at $W^\pm$ vertices

$$\rightarrow \quad -\, i\,\frac{gU_{ej}^*}{\sqrt{2}}\overline{\nu_L^j}\gamma^\mu W_\mu^+ e_L + ...$$

while the Z vertex remains flavour-diagonal:

$$\propto \sum_\alpha -i\,\frac{g}{2}U_{\alpha j}^*\overline{\nu_L^j}\gamma^\mu Z_\mu^+ U_{\alpha k}\nu_L^k = \delta_{jk}\frac{g}{2}\overline{\nu_L^j}\gamma^\mu Z_\mu^+\nu_L^k \ \ .$$

3.4 Dirac vs Majorana

There is a *discrete* difference in the number of light degrees of freedom required for Dirac or Majorana masses: a ν_L with a Majorana (Dirac) mass requires one(two) light chiral fermions. However this distinction is not currently observable. There is also a *continuous* difference, that Majorana masses are Lepton Number Violating(LNV) so give rise to ΔL = 2 processes *e.g.* $0\nu2\beta$. There is also more CP violation in the Majorana case (all but one of the Majorana ν masses are complex), but this is only detectable in LNV processes.

In the community, it is common to present Majorana vs Dirac as a "either–or" question. Which it is, as a "model discrimination" question: are there three light majorana ν with LNV masses, or three light dirac ν with LN conserving masses. However, if its neither of those models, it seems to the author that the question is continous, not discrete, because the phenomenological question is the LNV rate (one can't measure number of light chiral fermions). For instance, if ones adds an undetectably small LNV mass to a Dirac mass matrix; does that make the neutrinos Majorana? (There would be 6 chiral fermions as for Dirac, and no observed LNV. This case has been studied recently in [16].)

4 Two generation vaccuum oscillations

This section gives three derivations of the 2-neutrino oscillation probability in vaccuum. The first is a relativistic quantum mechanics version, which in my opinion gives the right intuition and physics, but contains several twiddles so the normalisation of the result is doubtful. The second is a quantum mechanical derivation using the Schrodinger equation, which is easy to rederive and gives the correct answer, but is a doubtable formalism for studying neutrinos (One can wonder if the Schrodinger equation is appropriate for relativistic neutrinos, whether the ν propagates with fixed $\vec{k}$ and variable energy, and whether the notion of neutrino flavour eigenstate is useful, since we usually quantise mass eigenstates.). The last is a quantum field theory justification for the Schrodinger equation version, whose purpose is to justify the Schrodinger approach used for matter oscillations in a later section.

An insightful discussion clarifying many questions about neutrino oscillations can be found in [17].

4.1 Relativistic Quantum Mechanics

We are interested in a physical process, where a muon decays at the production point, then later a muon is produced in the detector. We do not know what happened between these two events, so we should sum all the possibilities at the amplitude level.

We suppose a relativistic neutrino is produced in muon decay at $t = 0$. We know how to quantise and do perturbation theory with mass eigenstate particles, so we suppose that neutrinos propagate as mass eigenstates. The amplitude to produce a mass eigenstate i is

$$\propto U_{\mu i} \quad .$$

The propagator for a scalar particle of mass m_i to travel a distance L in time t to the detector is

$$G[(0,0);(L,t)] \propto \int \frac{d^3p}{(2\pi)^3} e^{i(Et-pL)} \theta(t)$$

This position-space formula (which can be found in chapter 6 of Bjorken and Drell volume II) looks unfamiliar, because propagators are usually given in momentum space. I suppose that including spin would be a straightforward complication. The amplitude to produce a charged lepton e_α at detector is then:

$$\mathcal{A}_{\mu\alpha} \propto \sum_j U_{\mu j} \times e^{-i(E_j t - k_j L)} \times U^*_{\alpha j} \quad .$$

In the relativistic limit where $m_j \ll E, p$, one can take $L \simeq t$ so $-i(E_j t - p_j L) \simeq -i(E_j - p_j)L$ $= -i\frac{E_j^2 - p_j^2}{E_j + p_j} L \simeq -i\frac{m_j^2}{2E} L$ which gives

$$\mathcal{P}_{\mu\alpha} = |\mathcal{A}_{\mu\alpha}|^2 = |\sum_j U_{\mu j} e^{-im_j^2 L/(2E)} U^*_{\alpha j}|^2$$

In the 2 generation case, where the mixing matrix U is

$$U = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

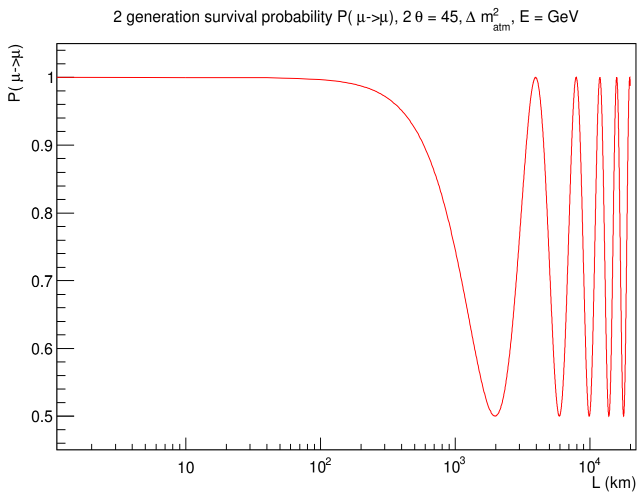
one obtains

$$\mathcal{P}_{\mu\to\tau}(t) = \left|\sin\theta\,\cos\theta\left(e^{-im_2^2 L/2E} - e^{-im_3^2 L/2E}\right)\right|^2$$

$$= \quad \sin^2(2\theta)\sin^2\left(L\frac{\Delta^2_{32}}{4E}\right) \quad \Delta^2_{32} \equiv m^2_3 - m^2_2 \tag{11}$$

$$\mathcal{P}_{\mu\to\mu}(\tau) \quad = \quad 1 - \sin^2(2\theta)\sin^2\left(L\frac{\Delta^2}{4E}\right) = 1 - \sin^2(2\theta)\sin^2\left(1.27\frac{L}{km}\frac{\Delta^2}{\text{eV}^2}\frac{\text{GeV}}{4E}\right) \tag{12}$$

where E is the ν energy and L is the source-detector distance (for atmospheric νs: $E \sim 10$ GeV and $L : 20km \to 10000km$. For reactor neutrinos, $E \sim$ MeV and $L \sim km$). The probability for a muon to decay in production and reappear in the detector (sometimes called the ν_μ survival probability) is illustrated in figure 4.1.



4.2 neutrino oscillations in quantum mechanics(easy to rederive)

A relativistic neutrino, with momentum $\vec{k}$, is produced in muon decay at $t = 0$ (at Tokai/edge atmosphere). It can be described as a quantum mechanical state: $|\nu(t = 0)\rangle = |\nu_\mu\rangle$. After it travels a distance L in time t to the detector, it can be written $|\nu(t)\rangle$. We wish to calculate the probability with which it produces an μ in CC scattering at the detector:

$$\mathcal{P}_{\mu\to\mu}(t) = |\langle\nu_\mu|\nu(t)\rangle|^2 \quad = \quad ?$$

For two generations of massive neutrinos the flavour and mass eigenstates are related as $\nu_\alpha = U_{\alpha i}\nu_i$:

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}.$$

If time evolution in the mass basis is described by a Schrodinger-like equation

$$i\frac{d}{dt}\begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} = \begin{bmatrix} E_2 & 0 \\ 0 & E_3 \end{bmatrix}\begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} \qquad , \quad E^2_i = k^2 + m^2_i$$

then one obtains

$$|\nu(t)\rangle = \sum_j U_{\mu j}|\nu_j(t)\rangle = \sum_j U_{\mu j}e^{-iE_j t}|\nu_j\rangle$$

so the amplitude for the neutrino to produce a charged lepton α in CC scattering in detector after time t is:

$$|\langle\nu_\alpha|\nu(t)\rangle| = \left|\sum_j U_{\mu j}e^{-iE_j t}U^*_{\alpha j}\right|$$

So in the 2 generation case, using $t = L$, $E_3 - E_2 \simeq \frac{m_3^2 - m_2^2}{2E} \equiv \frac{\Delta_{32}^2}{2E}$, one obtains the probabilities already given in equations (12) and (11).

One can anticipate that if the neutrino propagates distances $L \gg E/\Delta^2$, some sort of decoherence should occur, and one should sum the probabilities to propagate the various mass eigenstates. Issues of quantum coherence and decoherence have been discussed in [17]. Here are just some simple estimates about the overlaps of wavepackets:

1. at production, the neutrino energy and momentum are not perfectly known (otherwise one could compute the masses as $\sqrt{E^2 - p^2}$), so one should sum the amplitudes for a given ν_2 and ν_3 to have various energies and momenta: this gives two wavepackets of masses m_2, m_3.
2. The group velocity of the packets is $v_i = \frac{\partial E}{\partial p} = \frac{p}{E} \simeq 1 - \frac{m_i^2}{2E^2}$, so after a distance L, the packets have separated by
$$(v_2 - v_3)L \simeq \frac{m_3^2 - m_2^2}{E^2} L \simeq \frac{L}{\ell_{osc}} \frac{1}{E}$$
3. one could expect the packets to not interfere, if they are separated by more than their size, which by the uncertainty principle should by $\sim 1/(\delta|p|)$, where $\delta|p| \sim \delta E \sim$ the energy uncertainty of the packet. So one expects the oscillating $\sin^2(\Delta^2 L/4E)$ to average to 1/2 when
$$\frac{L}{\ell_{osc}} \gtrsim \frac{E}{\delta E} \quad .$$

4.3 A skeletal QFT derivation of oscillations

One can think that since neutrinos are relativistic, one should do oscillations in Quantum Field Theory. The aim of this skeletal derivation, is to show that QFT is equivalent to the Schrodinger equation of the previous subsection.

In second quantised field theory, in the Heisenberg representation where operators are time-dependent, the equations of motion for the number operator $\hat{n}$ are

$$\frac{d}{dt}\hat{n} = +i[\hat{H}, \hat{n}] \tag{13}$$

where the Hamiltonian $\hat{H}$ for vaccuum oscillations can be taken as free $= \hat{H}_0 \sim \sum \omega \hat{n}_\omega$. Recall the free hamiltonian is the sum over all states of the number of particles $\times$ their energy. It is the integral of hamiltonian density, with which it should not be confused (the dimensions are different).

In second quantised formalism, in the conventions of Peskin and Schroeder, the neutrino field can be written:

$$\hat{\psi}^I(x) = \sum_{s=+,-} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E}} \left(e^{-ip\cdot x} \hat{a}_s^I(\vec{p}) u_s(p) + e^{ip\cdot x} \hat{b}_s^{I\dagger}(\vec{p}) v_s(p) \right)$$

where s is helicity, I is generation, $\hat{a}^\dagger$ creates particles, et $\hat{b}^\dagger$ creates anti-particules. The creation/annihilation operators $\hat{a}$ are defined here for energy= mass eigenstates, but the formalism is covariant.

We want to know the time/space evolution of a beam neutrinos (no $\overline{\nu}$), of positive helicity, and, to simplify the notation, the momentum is fixed to $\vec{p}$. The number operator for such modes is

$$\hat{n}_{sr}^{IJ}(\vec{p}) = \hat{a}_+^{I\dagger}(\vec{p}) \hat{a}_+^J(\vec{p})$$

which is covariant in generation space (indices I, J).

The equation of motion for the number operator $\hat{n}$ is given in (13), where

$$H_0 = \sum_I \int \frac{d^3p}{(2\pi)^3} \omega_{II}(|\vec{p}|) \left(\hat{n}^{II}_{++}(\vec{p}) + \hat{n}^{II}_{--}(\vec{p})\right) \;,\; \omega_{II} = \sqrt{\vec{p}^2 + m_I^2} \;.$$

The commutator can then be calculated as

$$\begin{aligned}
\frac{d}{dt}\hat{n}^{IJ}_{++}(\vec{p}) &= i\int \frac{d^3k}{(2\pi)^3}\Big(\omega_2(\vec{k})\hat{a}^{2\dagger}_{+}(\vec{k})\hat{a}^{2}_{+}(\vec{k}) + \omega_1(\vec{k})\hat{a}^{1\dagger}_{+}(\vec{k})\hat{a}^{1}_{+}(\vec{k})\Big)\hat{a}^{I\dagger}_{+}(\vec{p})\hat{a}^{J}_{+}(\vec{p}) \\
&\quad -\hat{a}^{I\dagger}_{+}(\vec{p})\hat{a}^{J}_{+}(\vec{p})\Big(\omega_2(\vec{k})\hat{a}^{2\dagger}_{+}(\vec{k})\hat{a}^{2}_{+}(\vec{k}) + \omega_1(\vec{k})\hat{a}^{1\dagger}_{+}(\vec{k})\hat{a}^{1}_{+}(\vec{k})\Big) \\
&= i\Big\langle \begin{bmatrix} 0 & (\omega_1-\omega_2)\hat{a}^{1\dagger}_{+}(\vec{p})\hat{a}^{2}_{+}(\vec{p}) \\ (\omega_2-\omega_1)\hat{a}^{2\dagger}_{+}(\vec{p})\hat{a}^{1}_{+}(\vec{p}) & 0 \end{bmatrix}\Big\rangle
\end{aligned} \tag{14}$$

which turns out to be the equation one would obtain for the neutrino density matrix in the quantum mechanical formulation.

To see this connection, identify the vacuum-expectation value $\langle \hat{n}^{II}_{++}(\vec{p})\rangle \equiv [f_{++}]^{IJ}(\vec{p})$ with the density matrix for the 2-state neutrino system. The QM density matrix for $|\nu(t)\rangle = s|\nu_1(t)\rangle + c|\nu_2(t)\rangle$ can also be constructed as

$$[f_{++}] = \begin{bmatrix} s^2|\nu_1(t)\rangle\langle\nu_1(t)| & sc|\nu_1(t)\rangle\langle\nu_2(t)| \\ sc|\nu_2(t)\rangle\langle\nu_1(t)| & c^2|\nu_2(t)\rangle\langle\nu_2(t)| \end{bmatrix}$$

One can then check that the evolution of $[f_{++}]$ given by eqn (14) and by QM Hamiltonian

$$\begin{bmatrix} -\frac{m_2^2-m_1^2}{4\omega} & 0 \\ 0 & \frac{m_2^2-m_1^2}{4\omega} \end{bmatrix}$$

is identical.

5 Two generation matter oscillations

Neutrinos have weak cross sections which are very small:$\sigma \sim G_F^2 E_\nu^2$. Nonetheless, when the propagate in matter, they have an amplitude to notice the matter, which is $\propto G_F$ and can contribute an effective mass. This effect is described by *coherent* forward scattering of ν in matter, as illustrated in figure 2, which will give an extra contribution to the Hamiltonian. This effect can be relevant for neutrinos propagating in the earth, the sun or supernovae, here only the sun is discussed.

Fig. 2: Forward scattering interactions(the neutrino momentum is unchanged) of neutrinos with matter. The Z exchange diagram affects all neutrinos in the same way, so gives a contribution to the Hamiltonian $\propto$ identity. Therefore it does not induce phase differences between the propagation amplitudes of different ν_i, and can be neglected from oscillation studies.

To see how forward scattering on matter can give rise to an effective mass, one can use the Hamiltonian $H_{\rm mat} = H_0 + H_{int}$ in the QFT derivation of oscillations, with

$$H_{int} \simeq 2\sqrt{2}G_F \int d^4x (\overline{\hat{\nu}_e}(x)\gamma^\alpha P_L \hat{\nu}_e)(\bar{\hat{e}}\gamma_\alpha P_L \hat{e}(x)) \tag{15}$$

evaluated in a medium with electrons. Only the charged current interaction of ν_e with e need be included, because the NC interaction is the same for all the ν_L, so could only induce a universal a contribution to H proportional to the unit matrix. One can show that

$$\langle {\rm medium}|\bar{\hat{e}}\gamma_\alpha P_L \hat{e}(x)|{\rm medium}\rangle \to \delta_{\alpha 0}\frac{n_e}{2} ,$$

so that $H_{\rm mat}$ in the flavour basis $(\nu_e, (\nu_\tau - \nu_\mu)/\sqrt{2})$, is

$$\begin{aligned} H_{\rm mat} &= ... + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & \Delta^2/(2E) \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} V_e & 0 \\ 0 & 0 \end{bmatrix} \\ &= ... + \begin{bmatrix} -\frac{\Delta^2}{4E}\cos 2\theta + V & \frac{\Delta^2}{4E}\sin 2\theta \\ \frac{\Delta^2}{4E}\sin 2\theta & \frac{\Delta^2}{4E}\cos 2\theta \end{bmatrix} \end{aligned}$$

where $V_e = \sqrt{2}G_F n_e$.

This matter hamiltonian can be diagonalised by a rotation through the angle $\theta_{\rm mat}$, where

$$\begin{aligned} \tan(2\theta_{\rm mat}) &= \frac{\Delta^2 \sin(2\theta_{12})}{2EV_e - \Delta^2\cos(2\theta_{12})} \\ \Delta^2_{\rm mat} &= \sqrt{(\Delta^2 c2\theta - 2EV)^2 + (\Delta^2 s2\theta)^2} \end{aligned} \tag{16}$$

so we see that for $V_e \ll \frac{\Delta^2}{2E}\cos(2\theta_{12})$, matter effects are negligeable. However the matter mixing angle becomes maximal ($\theta_{mat} \to \pi/4$) when $V_e \simeq \frac{\Delta^2}{2E}\cos(2\theta_{12})$, corresponding to the MSW resonance. And for $V \gg \frac{\Delta^2}{2E}\cos(2\theta_{12})$, ν_e propagates as a mass eigenstate. A useful expression for V_e, which allows to estimate where matter is relevant for which energy neutrinos, is

$$\begin{aligned} V_e &= \sqrt{2}G_F n_e \simeq 8\,{\rm eV}\frac{\rho Y_e}{10^{14} g/cm^3} \\ Y_e &= \frac{n_e}{n_n + n_p} \ , \ \rho = \begin{cases} 10 g/cm^3 & \text{earth} \\ 100 g/cm^3 & \text{sun} \\ 10^{14} g/cm^3 & \text{supernova} \end{cases} \end{aligned} \tag{17}$$

Finally, it is important to notice that V_e is of opposite sign[3] for $\bar{\nu}$, which allows to determine the sign of the vaccuum mass difference Δ^2_{12}. In particular, since matter effects are observed in solar neutrinos, one concludes that $m_2^2 > m_1^2$.

5.1 matter of varying density

In order to understand the effect of solar matter on the neutrinos exiting the sun, one should consider matter of varying density. For varying $\rho(r)$, the matter Hamiltonian becomes time-dependent:

$$\begin{bmatrix} -\frac{\Delta^2}{4E}\cos 2\theta + V_e(t) & \frac{\Delta^2}{4E}\sin 2\theta \\ \frac{\Delta^2}{4E}\sin 2\theta & \frac{\Delta^2}{4E}\cos 2\theta \end{bmatrix}$$

[3] The sign arises because the interaction Hamiltonian of eqn (15) contains $\bar{\hat{\nu}}\gamma_0\hat{\nu} \supset \hat{a}^\dagger\hat{a}, \hat{b}\hat{b}^\dagger$. The negative sign arises in anti-commuting the $\hat{b}\hat{b}^\dagger$, in order to annihilate the incident $\bar{\nu}$ before creating the outgoing one.

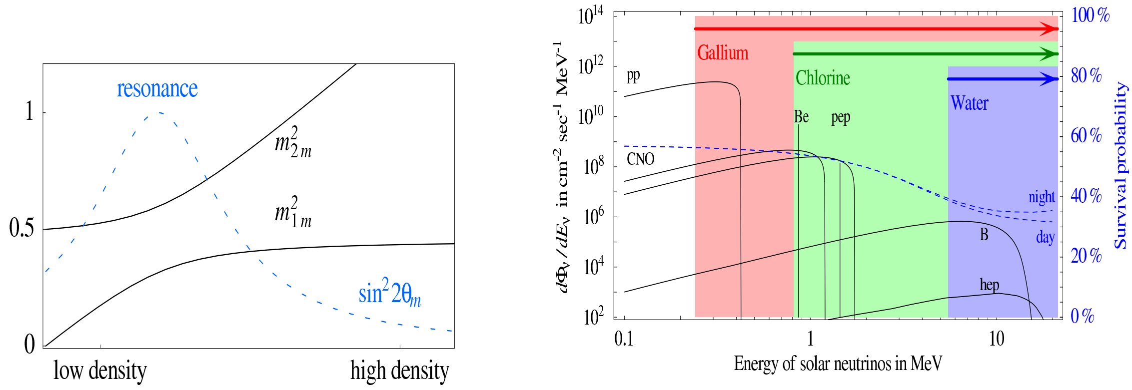


Fig. 3: On the right, solar neutrino fluxes and sensitivities of various experiments. On the left, the effective neutrino mass^2 as a function of density, normalised to $2m^2_{sol}$. Figures from hep-ph/0606054.

so the mixing angle θ_{mat} becomes time dependent. This is simple to account for in the adiabatic limit, where the time variation $\dot{\theta}_{mat}$ can be neglected compared to the oscillation timescale. Then one can imagine that if oscillations occur, they are between the instantaneous mass eigenstates. In the case of the sun, the adiabatic condition is satisfied, and it turns out that the matter effect can suppress oscillations.

The core of the sun produces ν_e in various nuclear reactions, with energies from 0.4 to 10 MeV (see [13] for a review). The principle fluxes, as well as the sensitivities of different detectors, are illustrated in figure 3.

1. From equations (16) and (17), one sees that $V_e > \Delta^2_{21}/2E$ for the $E \sim 8MeV$ Boron neutrinos, observed in SNO and SK. So these ν_es are mass eigenstates when they are produced, and remain mass eigenstates as they exit the sun, despite that their mass is adiabatically changing. They have no amplitude to be any other state, so there no oscillations, and they exit the sun as the heavier mass eigenstate ν_2. This is illustrated in the left in figure 3 : the neutrino just tracks the mass eigenstate (upper line). The probability to produce an electron in a detector on earth is therefore $|U_{e2}|^2$:
$$P_{ee} \simeq \sin^2\theta_{12}$$
2. On the other hand, the matter potential V_e is negligeable for the ν_e with energies $\sim$ MeV, who therefore oscillate as in vaccuum. However the vacuum oscillation length $\sim \frac{E}{\Delta^2} \ll R_{sun}$, so the oscillations decohere ($\sin^2\frac{\Delta^2 L}{E} \to 1/2$) and the probability of producing an electron in a detector on earth is
$$P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta_{12}$$

This explains why the ν_e survival probability was higher at the Davis experiment [12], than in the water cherenkov detectors [14].

6 Three generations

It is well-known that the SM has three generations. Nonetheless, in neutrino oscillations, some observables can be approximately calculated in the much simpler 2-generation formalism, because the dynamics of oscillations selects a particular mass difference and allows to measure a particular angle [4]. This is the first topic of this section.

[4] So in neutrino physics, one quotes experimental constraints on the angles θ_{ij}, rather than the matrix elements as is quark flavour physics.

Secondly, the CP-violating part of the 3-generation oscillation probability is introduced, and the current preference of T2K for $\delta \sim -\pi/2$ is discussed.

6.1 The drunken Unitarity triangle

The unitarity triangle is less discussed in lepton flavour physics than in quark flavour. This is perhaps because one discusses the angles rather than the matrix elements in the lepton sector, however, I use it here to illustrate why 2-flavour oscillations can be a good approximation to some observables.

The amplitude to oscillate from flavour α to β over distance L is:

$$\mathcal{A}_{\alpha\beta}(L) = U_{\alpha 1}U^*_{\beta 1} + U_{\alpha 2}U^*_{\beta 2}e^{-i(m_2^2-m_1^2)L/(2E)} + U_{\alpha 3}U^*_{\beta 3}e^{-i(m_3^2-m_1^2)L/(2E)} \quad . \tag{18}$$

At $L = 0$, this is just the unitarity relations $\mathcal{A}_{\alpha\beta} = 1$ for $\alpha = \beta$, $\mathcal{A}_{\alpha\beta} = 0$ for $\alpha \neq \beta$, which just say the rows of U are orthonormal. The three terms (complex numbers) can be represented as vectors adding to zero in the complex plane, as in figure 4. At $L = t \neq 0$, eqn (18) implies that two of the vectors rotate in

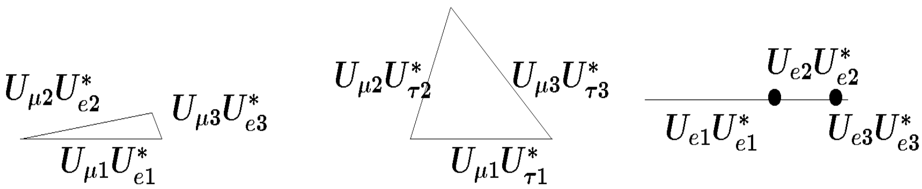


Fig. 4: Example unitarity triangles: for $\alpha = \mu$, $\beta = e$ the triangle is flattened, because $U_{e3} \sim \sin\theta_{13} \sim 0.015$ is small. For $\alpha = \mu$, $\beta = \tau$, the triangle is more equilateral.

the complex plane, with frequencies $(m_j^2 - m_1^2)/2E$, so oscillations can be visualised as time-dependent non-unitarity.

As a first example of why the two-flavour approximation works, consider the amplitude to oscillate from e to e at an energy and baseline combination such that $4E/L \simeq m_2^2 - m_1^2$. This corresponds, for instance, to reactor anti-neutrinos travelling to the Kamland detector. The amplitude is

$$\mathcal{A}_{ee}(L) = U_{e1}U^*_{e1} + U_{e2}U^*_{e2}e^{-i(m_2^2-m_1^2)L/(2E)} + U_{e3}U^*_{e3}e^{-i(m_3^2-m_1^2)L/(2E)} \tag{19}$$

and is illustrated on the right in figure 4. At $L \sim (m_2^2 - m_1^2)/2E$, vector 2 rotates, at a frequency $(m_2^2 - m_1^2)/2E$, whereas vector 3 spins rapidly at a frequency $(m_3^2 - m_1^2)/2E$. The two-flavour approximation works because $U_{e3} = sin\theta_{13}$ is small, so the rapid spinning of vector 3 can be neglected.

As a second example, consider the determination of θ_{13} at reactors. The amplitude is again given in eqn (19), and the diagram is again on the right in figure (4).

However, in this case, the energy-baseline is chosen such that $4E/L \sim (m_3^2 - m_1^2)$, so only the third vector rotates. The first and second are stationary, and $U_{e3} \sim \theta_{13}$ is obtained by measuring the small $\overline{\nu_e}$ disappearance, corresponding to the decreased length of the vector in figure 4, resulting from the rotation of the short vector "3".

6.2 What is left?

Adding three generations of massive neutrinos to the SM introduces new parameters: 3 masses and a mixing matrix containing three angles and at least one phase. The three angles are measured, as are two mass-squared differences: $(m_2^2 - m_1^2)$, $|m_3^2 - m_j^2|$. From matter effects in the sun, it is also known that $(m_2^2 - m_1^2)$ is positive. Remaining to be determined are the sign of $(m_3^2 - m_j^2)$ (refered to as the "hierarchy": $m_3 > m_2 > m_1$ is the normal hierarchy, $m_2 \gtrsim m_1 > m_3$ is inverse hierarchy), the absolute mass scale, and the phase.

6.3 The phase δ

The CP-violating phase δ would be absent in 2 generations, so all three generations must contribute to the oscillation amplitude, in order to have sensitivity to δ.

To compactify the $\nu_\alpha \to \nu_\beta$ oscillation amplitude (eqn (18)), it is convenient to define[5] $x_{ji} \equiv (m_j^2 - m_i^2)L/(2E)$, and the mixing matrix combination [6] $\lambda_i = U_{\alpha i}U^*_{\beta i}$. Then

$$\begin{aligned}\mathcal{P}_{\alpha\beta}(L) &= |\lambda_1+\lambda_2+\lambda_3|^2 - \lambda_1\lambda_2^* - \lambda_1^*\lambda_2 - \lambda_1\lambda_3^* - \lambda_1^*\lambda_3 - \lambda_3\lambda_2^* - \lambda_3^*\lambda_2 \\ &\quad +\lambda_1\lambda_2^* e^{+ix_{21}} + \lambda_1^*\lambda_2 e^{-ix_{21}} + \lambda_1\lambda_3^* e^{+ix_{31}} + \lambda_1^*\lambda_3 e^{-ix_{31}} \\ &\quad +\lambda_2\lambda_3^* e^{+ix_{32}} + \lambda_2^*\lambda_3 e^{-ix_{32}} \\ &= \delta_{\alpha\beta} - 4\sum_{i<j}\mathrm{Re}\{U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\}\sin^2\frac{x_{ji}}{2} \\ &\qquad + 2\sum_{i<j}\mathrm{Im}\{U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\}\sin x_{ji} \end{aligned} \tag{20}$$

To make a first acquaintance with the real part of this formula, one can take the 2 generation limit ($\alpha = \mu$, $\beta = \tau$, $i = 2, j = 3$) and see that the formula (11) is recovered:

$$\begin{aligned}\mathcal{P}_{\mu\tau}(L) &= 0 - 4\mathrm{Re}\{-\cos\theta\sin\theta\sin\theta\cos\theta\}\sin^2\frac{x_{32}}{2} \\ &\quad +0 \\ &= \sin^2(2\theta)\sin^2\frac{(m_3^2-m_2^2)L}{4E}\end{aligned}$$

The imaginary part of the three-flavour oscillation probability (the last sum in eqn (20)) represents CP Violation, and it will give a dependence on the phase δ. To see that this term is CP Violating, one can check that it has opposite sign in the transition probabilities for $\nu_\alpha \to \nu_\beta$ vs $\overline{\nu_\alpha} \to \overline{\nu_\beta}$. The amplitude $\mathcal{A}(\nu_\alpha \to \nu_\beta)$ was obtained in section 4.1; following the same steps, but using that the Feynman rule for $\overline{e_\alpha} \to \overline{\nu_i}$ is $\sim U^*_{\alpha i}$, one sees that the Imagainary term in eqn (20) is of opposite sign in the two cases.

It can be checked that $\{U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\}$ is invariant under changes in the choices of phases of fields. made in order to remove phases from U. Recall that in section 3.3.1, the 5 relative phases of $\{e_\alpha, \nu_j\}$ were chosen in order to remove 5 phases from U. If the field phases were chosen differently, for instance $e_\alpha \to e^{-i\phi_\alpha}, \nu_j \to \nu_j e^{-i\phi_j}$, then $U_{\alpha j} \to e^{-i\phi_\alpha}U_{\alpha j}e^{i\phi_j}$, but the combination $\{U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\}$ is invariant, because $e^{\pm i\phi_\alpha}$ cancels between $U_{\alpha i}$ and $U^*_{\alpha j}$, etc.

Indeed, the combination $\{U_{\alpha i}U^*_{\beta i}U_{\alpha j}U^*_{\beta j}\}$ is proportional to the area of the unitarity triangle area, and to the Jarlskog invariant. Suppose the phases are chosen such that the base $= U_{\mu 1}U^*_{\tau 1}$ of the central triangle in figure (4) is real. Then base $\times$ height $\propto \mathrm{Im}\{U_{\mu 1}U^*_{\tau 1}U^*_{\mu j}U_{\tau j}\}$ for both $j = 2$ and $j = 3$ (its less simple to demonstrate that the third term of the imaginary sum is also $\propto$ the triangle area). To compactify the notation, it is convenient to define

$$\widetilde{J} = 8c_{13}^2 s_{13}c_{23}s_{23}c_{12}s_{12} \tag{21}$$

where the area of the triangle and the Jarlskog invariant computed from the neutrino and charged lepton mass matrices are propotional to $\widetilde{J}\sin\delta$.

6.3.1 θ_{13}, δ at T2K

The current T2K data [19] has some sensitivity to the CP violating phase δ, and favours a maximal value $\delta \sim -3\pi/2$. The aim of this section is to understand this preference.

[5] From the review [7].

[6] The dependence on the indices α, β is suppressed because they are fixed by the physical process under consideration.

At JPARC, a beam of muons, [or anti-muons], hits a target, and produces neutrinos of energy $\simeq 0.6$ GeV, which travel 295 km underground to SuperK. In order to be sensitive to θ_{13} and δ, SuperK then searches for electrons, [or positrons]. The baseline and energy are chosen to maximise the appearance probability of electrons via the angle $\sin\theta_{13}$. So at leading order, the vaccuum probability is

$$\mathcal{P}_{\mu e} \simeq \sin^2(\theta_{23})\sin^2(2\theta_{13})\sin^2(\frac{x_{31}}{2}) \quad x_{31} = \frac{(m_3^2 - m_1^2)L}{2E} \tag{22}$$

However, $s_{13}^2 \sim \Delta_{21}^2/\Delta_{31}^2$, so the x_{21} oscillations could give some detectable contribution, in which case a dependence on δ becomes possible in the three generation oscillation probability. But if CP violation from the mixing matrix is allowed in the calculation, matter effects should also be included, because, like the Imaginary term in the oscillation probability of eqn (20), they are of opposite sign for neutrinos and anti-neutrinos.

So in principle, the relevant amplitude is

$$\mathcal{A}_{\mu e} = \widetilde{U}_{\mu 1}\widetilde{U}_{e1}^* + \widetilde{U}_{\mu 2}\widetilde{U}_{e2}^* e^{-i\widetilde{x}_{21}} + \widetilde{U}_{\mu 3}\widetilde{U}_{e3}^* e^{-i\widetilde{x}_{31}}$$

where $\widetilde{x} = \Delta_{mat}^2 L/2E$, and $\widetilde{U}$ are the mass differences and mixing matrix in matter.

Matter effects in three generations are discussed, for instance, in [18]; here, the small matter effects are only included in the leading two-generation mixing term, following the discussion of section 5, so eqn (22) becomes

$$\mathcal{P}_{\mu e} \simeq \sin^2(\theta_{23})\sin^2(2\widetilde{\theta}_{13})\sin^2(\frac{\widetilde{x}_{31}}{2}) \tag{23}$$

where $2EV_e/\Delta_{31}^2 \lesssim .1$, and the mass difference and mixing angles in matter are given in eqn (16).

Then the three-generation mixing term that depends on δ can be included in perturbation theory by writing $e^{-ix_{21}} \simeq 1 - i\Delta_{21}^2 L/2E$:

$$\mathcal{P}_{\mu e} \simeq \sin^2(\theta_{23})\sin^2(2\widetilde{\theta}_{13})\sin^2(\frac{\widetilde{x}_{31}}{2}) + \widetilde{J}\,\frac{\Delta_{21}^2 L}{2E}\sin(\widetilde{x}_{31})\cos(\pm\delta + \frac{x_{31}}{2})$$

where $\widetilde{J}$ is defined in eqn (21).

The current T2K data contains a larger ratio of ν_e to $\overline{\nu_e}$ (electrons to positrons) than expected for any value of δ. So there is a preference for δ that flips the sign of the second term. Since T2K is on the oscillation peak $x_{31} \simeq \pi/2$, this suggests that $\delta \simeq 3\pi/2$.

If this observation is confirmed with more data, it is doubly interesting: first because it indicates that CP violation is generic, and not just a property of the CKM matrix. Secondly, leptogenesis scenarios require CP violation in the leptonic sector, so its presence supports them.

7 Mass pattern/hierarchy

Two possible mass patterns, or hierarchies, are consistent with current oscillation data:
normal hierarchy: $m_1 < m_2 \ll m_3$
inverted hierarchy: $m_3 \ll m_1 \lesssim m_2$
It is known that $m_2^2 > m_1^2$ because there are matter effects for Boron neutrinos exiting from the sun.

The sign of the big mass difference $m_3^2 - m_j^2$ appears in oscillation probability:

1. when ν and $\overline{\nu}$ travel through matter, because the matter contribution to the Hamiltonian is of opposite sign for ν and $\overline{\nu}$...
2. in 3-neutrino oscillations, where interference between Δ_{21}^2 and Δ_{31}^2 occurs, suppressed by $\sin^2\theta_{13}$

So there are proposals [20] to determine the hierarchy by studying atmospheric ν_e and $\overline{\nu}_e$ in high statistics detectors such as PINGU or ORCA [21]. Determining the hierarchy is also among the aims of the DUNE experiment [22].

8 mass scale

8.1 cosmology —a probe of the neutrino mass scale

The mass of neutrinos can have effects on the growth of Large Scale Structure, and also on the Cosmic Microwave Background (principally its evolution from recombination until today).

The participation of neutrinos in Structure formation is intuitive: they are "hot dark matter", that is, in the early Universe after matter-radiation equality, neutrinos still have non-trivial velocities. They can therefore free-stream out of over-densities, rather than collapsing with the overdensity as cold dark matter would do. However, since the neutrinos progressively slow down due to the expansion of the Universe, they only succeed in escaping from small overdensities, which suppresses the power spectrum of Large Scale Structure on small scales. The scale below which the power spectrum is suppressed allows to identify the neutrino mass. However, if neutrino masses are small, the suppression factor is small.

The effect of neutrino masses on the CMB is more subtle, because neutrinos become non-relativistic after recombination (= the moment when the CMB is born), so their masses affect the propagation of CMB photons from recombination until today. This is pedagogically explained in [4]. One of the subtleties of the CMB dependence on neutrino parameters, is that other physical processes, encoded in other parameters of the cosmological ΛCDM model, can have some of the same effects. This was explored in [23], who obtained bounds on the sum of neutrino masses $\Sigma \equiv \sum_i m_{\nu_i}$ [23]

$$
\begin{aligned}
\Sigma \quad & \lesssim \quad 0.1 \to .6\, eV \quad \text{now : PLANCK, +LSS/Ly}\alpha \text{ (in } \Lambda\text{CDM)} \\
& \lesssim \quad 0.6 \to 1\, eV \quad \text{now : PLANCK + ... (in 12 param } \Lambda\text{CDM)} \\
& \to \quad \lesssim 2m_{atm} \quad \text{cosmo.indep. (Planck + EUCLID...)} \\
& \quad\quad \sim m_{atm} \quad \Lambda\text{CDM}
\end{aligned}
$$

8.2 Beta decay

β decay provides a direct kinematic probe of the neutrino mass, because m_ν^2 distorts the e spectrum in $n \to p + e + \bar{\nu}$. The KATRIN experiment [24], which is running now, uses Tritium, so consider Tritium β decay:

$$
{}^3H \to {}^3He + e + \bar{\nu}_e \ , \quad Q = E_e + E_\nu = 18.6\text{eV}
$$

where the high-energy tail of the electron distribution turns down due to the neutrino mass $E_e = Q - E_\nu \leq Q -$ "m_{e_ν}". The endpoint of e spectrum can be described as :

$$
\frac{dN_e}{dE_e} \propto \sum_i |U_{ei}|^2 \sqrt{(18.6 \text{ keV} - E_e)^2 - m_{\nu_i}^2} \tag{24}
$$

The current β-decay bound is $m_{\nu_e} \lesssim 2$ eV; the Katrin sensitivity [24] is expected to be ~ 0.3 eV.

8.2.1 *ν-capture and the Cosmic Neutrino Background*

It is known that in the early Universe at the moment of of Nucleosynthesis (BBN), there was a thermal density of SM neutrinos, comparable to the density of photons. So today there should be a Cosmic Neutrino Background, comparable to the CMB, consisting of $\sim 100\nu/\text{cm}^3$ [25]. Detecting these non-relativistic neutrinos would be interesting, and also difficult since their energy $\sim m_\nu$.

A not unpromising detection possibility [26] could be neutrino capture βdecay: $n + \nu_{CNB} \to p + e$. One can compare the ν capture rate on a nucleus N, to the usual β decay rate as the ratio of the incident CNB number density to the outgoing phase space density in β decay:

$$
\frac{n_{\nu CNB}}{\nu \text{ phase space}} \simeq \frac{T_{CNB}^3}{\pi^2} \frac{1}{Q^3} \sim \left(\frac{10^{-4}\text{eV}}{20\text{keV}} \right)^3 \sim 10^{-24}
$$

However, in the capture case, the electron energy is $E_e = Q + m_\nu$, so is $2m_\nu$ larger than the upper bound of the β decay spectrum. So with improved resolution, perhaps the CNB could be measured by observing electrons beyond the end-point of the β decay spectrum.

8.3 Neutrinoless double beta decay

Neutrinoless double beta decay ($0\nu2\beta$) is a Lepton *Number* Violating (LNV) process, to which Majorana neutrino masses can contribute. Other lepton number violating scenarios (such as sparticles in R-parity violating supersymmetry) can also contribute, but here only the Majorana mass contribution will be discussed.

For some nuclei, single β decay is kinematically forbidden: for instance, ${}^{76}_{32}Ge$ is lighter than ${}^{76}_{33}As$, so ${}^{76}_{32}Ge$ has a double beta decay to ${}^{76}_{34}Se + ee\bar{\nu}_e\bar{\nu}_e$, with a lifetime $\sim 10^{21}$ yrs. The lifetime is long because the matrix element is suppressed by $\sim G_F^2$ (two W are exchanged), as illustrated on the left in figure 5. In the presence of Majorana masses, double beta decay can be *neutrinoless*, as illustrated on

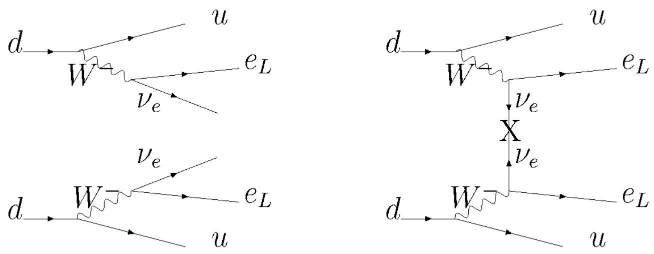


Fig. 5: Double beta decay (left) and neutrinoless double beta decay(right)

the right in figure 5. This diagram can only occur for Majorana neutrino masses, which violate lepton number, because two units of lepton number disappear into the mass insertion x on the neutrino line. As a result, the electrons emerge back-to-back, with opposite momenta and half the available energy each. So the signature of $0\nu2\beta$ is a line just beyond the end of the electron spectrum of 2ν double beta decay.

8.3.1 The $0\nu2\beta$ matrix element

The matrix element for $0\nu2\beta$ can schematically be written

$$|\mathcal{M}|^2 = \left| \begin{array}{c} \text{nuclear} \\ \text{matrix} \\ \text{element} \end{array} \right|^2 \times \left| \sum_i U_{ei}^2 m_i \right|^2$$

for $m_\nu \ll Q \sim 100$ MeV (the mass of heavier Majorana neutrinos would appear downstairs in the propagator). The calculation of the nuclear matrix elements is involved; experts obtain results in different models that can differ by factors of a few [27, 28].

It is interesting to focus on the neutrino part of $|\mathcal{M}|^2$:

$$|\mathcal{M}|^2 \propto \left| c_{13}^2 c_{12}^2 e^{-i\phi_1} m_1 + c_{13}^2 s_{12}^2 e^{-i\phi_2} m_2 + s_{13}^2 e^{-i2\delta} m_3 \right|^2$$

where the majorana masses appear linearly, so accompagnied by their phases (or equivalently, by the Majorana phases from P of eqn (10). So this Lepton Number Violating process is sensitive to the Majorana phases.

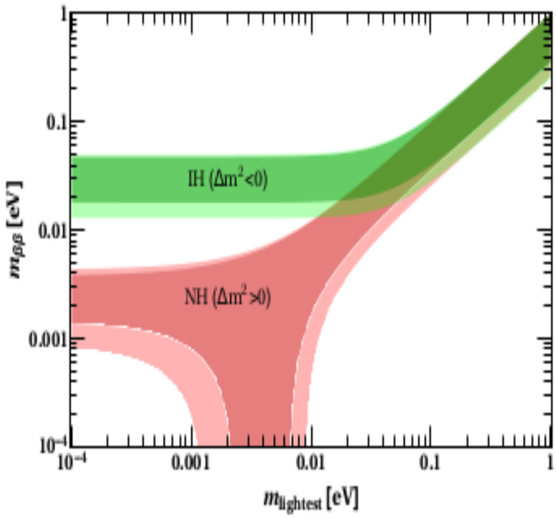


Fig. 6: $|\mathcal{M}|$ for $0\nu 2\beta$ mediated by majorana neutrino masses, plotted as a function of the lightest neutrino mass. The green region is allowed for the inverse hierarchy, and the red region corresponds to the normal hierarchy.. The plot is taken from 1601.07512 by Dell'Oro *etal* [28].

8.4 What can we learn/confirm?

Suppose that the only source of lepton number violation is the majorana masses of the SM neutrinos, the largest of which is $\sim \sqrt{|m_3^2 - m_2^2|}$. Then the rate is larger for the Inverse Hierarchy $m_1 \sim m_2 > m_3$:

$$\begin{aligned} |\mathcal{M}|^2 &\propto |\frac{3}{4}e^{-i2\phi}m_1 + \frac{1}{4}e^{-i2\phi'}m_2 + s_{13}^2 e^{-i2\delta}m_3|^2 \\ &\to m_{atm}^2|3 + e^{-i2(\phi'-\phi)}|^2 \end{aligned}$$

For this hierarchy, which corresponds to the green band in figure 6, either $0\nu 2\beta$ is observed, or neutrino masses are Dirac.

On the other hand, in the case of the Normal Hierarchy, ($m_1 < m_2 < m_3$), the contribution of the atmospheric mass is suppressed by s_{13}^2:

$$\begin{aligned} |\mathcal{M}|^2 &\to |\frac{3}{4}e^{-i2\phi}m_1 + \frac{1}{4}e^{-i2\phi'}m_{sol} + (.15)^2 e^{-i3\pi}m_{atm}|^2 \\ &\simeq m_{sol}^2|\frac{3m_1}{m_{sol}} + e^{-i2(\phi-\phi')}|^2 \end{aligned}$$

so the rate is lower, and for $m_1 \sim m_2/3$ and suitably chosen Majorana phases, the matrix element can vanish, despite that the neutrinos are Majorana. This case corresponds to the red region of figure 6.

8.4.1 *A curious example of EFT*

From an Effective Field Theory point of view, it may seem curious to set restrictive bounds on the coefficient of of the dimension 5 operator $\frac{K}{\Lambda_{NP}}\bar{\ell}H\ell^c H$, from upper bound on coefficients of dimension 9 or 11 operators such as

$$\begin{array}{rl} (\bar{u}\gamma^\mu P_R d)(\bar{u}\gamma_\mu P_R d)(\bar{\ell}H)(\ell^c H) & (\bar{q}\tau_i\gamma^\mu P_L q)(\bar{q}\tau_j\gamma_\mu P_L q)(\bar{\ell}\tau_i H)(\ell^c\tau_j H) \\ dim9 & (\bar{u}\gamma^\mu P_R d)(\bar{u}\gamma_\mu P_R d)\bar{e}e^c \end{array}$$

because New Physics is expected to appear in lower dimensional operators (those of higher dimension would be suppressed by additional powers of $1/\Lambda_{NP}$).

However, the naive expectation does not take into account the matching of the SMEFT onto a QED×QCD invariant EFT at m_W, where the Higgs gets a vev. Factors of G_F and v can change operator dimensions in this process, for instance the coefficient of the dimension 11 operator is only suppressed by one power of Λ_{NP}:

$$\sim \frac{K}{v^4\Lambda}(\bar{u}\gamma^\mu P_R d)(\bar{u}\gamma_\mu P_R d)(\bar{\ell}H)(\ell^c H)$$

Furthermore, Avogadro's number($\simeq 6\times 10^{23}$) is large, allowing a great sensitivity to rare decays of otherwise stable particles: $0\nu 2\beta$ may occur 10^{-16} times in the age of the Universe, but it still be observed by watching a tonne of material for a year.

9 Mechanisms and models for small neutrino masses

This section outlines a few models involving heavy New Particles, that are renormalisable and can generate Majorana masses for the SM neutrinos at tree level. They are refered to as "seesaw models", because the light SM neutrino masses are obtained as the ratio of larger scales. These models are attractive because they involve a minimal number of new particles and couplings.

There are three models that generate the Majorana mass operator

$$\frac{K}{2\Lambda}[\bar{\ell}H][\ell^c H] \to \overline{\nu}\nu^c \frac{K\langle H_0\rangle^2}{2\Lambda}$$

by tree-level exchange of at most one new particle per generation. The new particle can be an SU(2) singlet fermion (the Type I seesaw [29]), an SU(2) triplet fermion(the Type III seesaw [32]), or a scalar triplet (the Type II seesaw [30, 31]).

9.1 The Type I seesaw, one generation

Consider the type I seesaw [29] in one generation, where a singlet fermion N_R, with all its allowed renormalisable interactions, is added to the SM Lagrangian. It is allowed a Yukawa coupling with the doublet lepton and the Higgs, and a Majorana mass, which is not bounded above by the weak scale because the mass of N is not generated by the Higgs vev. The leptonic Lagrangian, written in terms of chiral fermions, is then:

$$\begin{aligned}\mathcal{L}_{lep}^{Yuk} = h_e(\overline{\nu_L},\overline{e_L})\begin{pmatrix}-H^+\\ H^{0*}\end{pmatrix} e_R + \ & \lambda(\overline{\nu_L},\overline{e_L})\begin{pmatrix}H^0\\ H^-\end{pmatrix} N_R \ + \frac{M}{2}\overline{N_R^c}N_R + h.c.\\ m_e\overline{e_L}e_R \qquad\qquad & + m_D\overline{\nu_L}N_R \qquad\qquad + \frac{M}{2}\overline{N_R^c}N_R + h.c.\end{aligned}$$

where the second line gives the masses after the Higgs gets a vev. The neutrino mass matrix can be written (in notation where $\nu_L^c \equiv (\nu_L)^c$)

$$\begin{pmatrix}\overline{\nu_L} & \overline{N_R^c}\end{pmatrix}\begin{bmatrix}0 & m_D\\ m_D & M\end{bmatrix}\begin{pmatrix}\nu_L^c\\ N_R\end{pmatrix}$$

Unlike a Dirac mass matrix, where different fields appear on either side, in this "Majorana" mass matrix, the same chiral degrees of freedom appear on either side: to the left are all the chiral fermions in barred-left-handed form, and to the right, the same fermions appear in unbarred right-handed form.

The eigenvectors/values are approximately ν_L with $m_\nu \sim \frac{m_D^2}{M}$ and N_R with mass $\sim M$.

9.1.1 *Factors of 2 in the seesaw*

A Majorana mass m_ν appears in the low-energy $\mathcal{L}$ as $\frac{m_\nu}{2}\overline{\nu_L^c}\nu_L + h.c.$. The previous section showed that in the low-energy effective theory after electroweak symmetry breaking, the type-1 seesaw gives $m_\nu = m_D^2/M$. Here we want to check that the same result is obtained with the SU(2) invariant model and operator.

The SU(2)-invariant Majorana mass operator is of dimension 5, and can be written

$$\mathcal{L} \supset \frac{K}{2\Lambda}(\overline{\ell^c}H)(\ell H) + h.c. = \frac{K}{2\Lambda}(\overline{\ell^c}_n\delta^{nN}H_N)(\ell_m\delta^{mM}H_M) + h.c.$$

where n, m, N, M are SU(2) indices that run from 1 to 2. In the high energy SM, with dynamical Higgs, this interaction has Feynman rule:

$$i\frac{\delta^4\mathcal{L}}{\delta\bar{\ell}_i\delta\ell^c{}_j\delta H_I\delta H_J} = i\frac{K}{2\Lambda}\frac{\delta^4\mathcal{L}}{\delta\bar{\ell}_i\delta H_I\delta H_J}((\bar{\ell}H)\delta_{jN}H^N + \delta_{jM}H^M(\bar{\ell}H)) \quad (25)$$

$$= i\frac{K}{2\Lambda}\frac{\delta^4\mathcal{L}}{\delta H_I\delta H_J}(\delta_{iN}\delta_{jM} + \delta_{jN}\delta_{iM})H^N H^M$$

$$= -i\frac{K}{\Lambda}(\delta_{iI}\delta_{jJ} + \delta_{jI}\delta_{iJ}) \quad (26)$$

Now to match this operator onto the seesaw model, in the seesaw model there is an s and a t channel

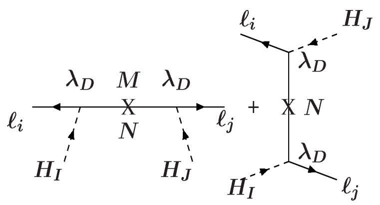


Fig. 7: Seesaw diagrams matching onto the Feynman rule for the one-generation neutrino mass operator, given in eqn (26). i, j, I, J are SU(2) doublet indices that run over 1,2.

diagram, as illustrated in figure 7, so one obtains $\frac{K}{\Lambda} = \frac{\lambda^2}{M}$ in agreement with the result from diagonalising the mass matrix.

9.2 The type I seesaw in three generations

Add 3 singlet N_Rs to the SM (the chiral projection subscript will be dropped in the following to streamline the notation). One can always choose to work in the mass eigenstate basis of the charged leptons and N_Rs, where the Lagrangian can be written

$$\mathcal{L} = \mathcal{L}_{SM} + \boldsymbol{\lambda}_{\alpha J}\overline{\ell_\alpha}\cdot HN_J - \frac{1}{2}\overline{N_J}M_J N_J^c + h.c. \quad (27)$$

The three generation type 1 seesaw adds 18 parameters to the Lagrangian: three (real) singlet masses, and 18 real parameters in the Yukawa matrix λ, from which three phases can be removed by phase choices on the doublets ℓ_α.

In the presence of electroweak symmetry breaking, for $M \gg m_D = \lambda v$, the mass matrix for SM neutrinos is

$$[m_\nu] = \boldsymbol{\lambda}M^{-1}\boldsymbol{\lambda}^T v^2 \quad , \quad v = \langle H^0\rangle \ .$$

In the effective low-energy Lagrangian, where the SM neutrinos have this Majorana mass matrix, there are nine new parameters in the Lagrangian: the three neutrino masses m_1, m_2, m_3 , and the 3 angles and 3 phases of U_{MNS}. There are therefore 9 free parameters of the high-scale model which are inaccessible at low energy, so in a later section of these lectures, it will not come as a surprise that they can be chosen to reproduce the Baryon Asymmetry of the Universe.

An attractive feature of the seesaw, is that one can easily obtain the observed neutrino masses for reasonable choices of the singlet masses M and Yukawas λ. For instance, if the neutrino Yukawa matrix ressembles that of the up-type quarks, with $\lambda \sim h_t$, then $m_\nu \sim .1$ eV is obtained for $M \sim 10^{15}$ GeV. Or if one prefers an electroweak scale $M \sim$ TeV, then $\lambda \sim 10^{-6}$ ($\sim$ the electron Yukawa coupling) generates $m_\nu \sim .1$ eV.

However, a disadvantage of the type I seesaw, is that even if the singlets are kinematically accessible at the LHC, their only coupling to the SM is their Yukawa, which is small and suppresses the production rate.

Another drawback of the non-supersymmetric seesaw is that the neutrino loop contributions to the Higgs mass can be uncomfortably large. A one-loop diagram is illustrated in figure 8. The loop is finite

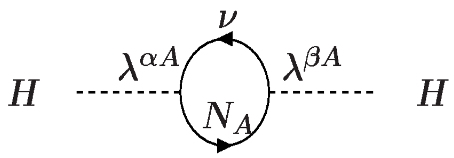


Fig. 8: Loop contribution to the Higgs mass in the seesaw model .

and calculable [33]:

$$\delta m_H^2 \quad \simeq \quad -\sum_I \frac{[\boldsymbol{\lambda}^\dagger \boldsymbol{\lambda}]_{II}}{8\pi^2} M_I^2 \sim \frac{m_\nu M_I^3}{8\pi^2 v^4} v^2 \tag{28}$$

so for $M \gtrsim 10^7$ GeV, this loop gives a larger contribution to the Higgs mass-squared than its observed value. Of course, the Higgs mass sitting in the Lagrangian is unknown, so a cancellation is possible but requires a "tuning" for which no justification is known. Alternatively, the loop contribution can be cancelled by another loop contribution, as arises in, for instance, supersymmetric models.

9.3 A low-scale tree model detectable at the LHC: the inverse seesaw

The "inverse seesaw" [34] is a model that gives Majorana masses of the observed magnitude to the SM neutrinos, and contains heavy singlets that could be found at the LHC. There are more new particles than in the type 1 seesaw.

For each generation, add two gauge-singlet, chiral fermions N, S, which share a $\sim$ TeV-scale Dirac mass. In the one generation case:

$$\mathcal{L} = \mathcal{L}_{SM} + \boldsymbol{\lambda} \overline{N} \ell \cdot H - \overline{N} M S - \frac{1}{2} \overline{S} \mu S^c \tag{29}$$

where N is the usual "right-handed neutrino" who interacts with the SM doublet leptons via the Yukawa coupling, and N and S share a TeV-scale Dirac mass M. Then a small ($\sim$ keV) Majorana mass μ is added for S. In the limit $\mu = 0$, lepton number conserved, and L=1 for ℓ, N, S^c. However $m_\nu = 0$ in this limit, as can by writing the 1 generation mass matrix in Majorana form

$$\begin{pmatrix} \overline{\nu_L} & \overline{N^c} & \overline{S} \end{pmatrix} \begin{bmatrix} 0 & m_D & 0 \\ m_D & 0 & M \\ 0 & M & \mu \end{bmatrix} \begin{pmatrix} \nu_L^c \\ N \\ S^c \end{pmatrix} .$$

For $\mu = 0$ the determinant vanishes, but S and N share a Dirac mass M, so ν_L must be massless. For $\mu \neq 0$, the determinant is μm_D^2, so for $M > m_D \gg \mu$, the three masses are $M, M, m_D^2\mu/M^2$. It is straightforward to check that for $\lambda \sim 0.1$, $M \sim$ TeV, and $\mu \sim 0.1$ keV, $m_\nu \sim$ .1eV is obtained. So as advertised, this model gives naturally small m_ν, and singlets with TeV masses and with $\mathcal{O}(1)$ yukawa couplings.

The three generation light neutrino mass matrix in this model:

$$[m_\nu] \quad = \quad [\boldsymbol{\lambda}][M]^{-1}[\mu][M]^{-1}[\boldsymbol{\lambda}]^T v^2 \sim .05 \text{ eV}$$

(in square brackets are matrices) can be obtained diagrammatically from figure 9, where one distributes the various mass insertions upstairs in the fraction if they are small, and downstairs if they are large.

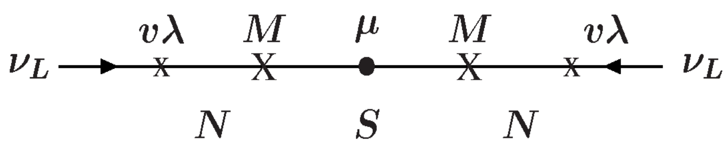


Fig. 9: Diagram for the SM neutrino mass in the inverse seesaw model

10 Leptogenesis

Leptogenesis [1, 41] is a class of recipes, that use majorana neutrino mass models to generate the matter excess of the Universe. The model generates a lepton asymmetry (before the Electroweak Phase Transition), and the non-perturbative SM B+L violation reprocesses it to a baryon excess.

10.1 The Matter Excess of the Universe

If you step out the door in the countryside at night, the sky is decorated with stars. Like us, they are all made of matter, and the puzzle is to understand the origin of this excess of matter over anti-matter in our Universe.

Stars are mostly made of Hydrogen, containing a proton(baryon) and an electron. So the matter excess is equivalent to an excess of baryons over anti-baryons. Leptons are neglected in this discussion, because (despite that every Hydrogen contains an electron), there should be a Cosmic Background of Neutrinos whose density is far higher than that of electrons, which could contain a significant (and difficult-to-observable) lepton asymmetry today.

We define the nucleons that we are made of to be baryons (as opposed to anti-baryons), then by touching objects around us, we observe that they are also made of baryons (as opposed to anti-baryons) because matter combined with anti-matter becomes a puff of photons. This argument can be extended to the solar system (bathed in the solar wind), and to the scale of galaxy clusters, because if a cluster of matter brushing against a cluster of anti-matter, photons would be produced by proton-antiproton annihilation, and these are not observed in the cosmological spectrum. So we assume that all the Universe *we see* is made of matter (dark matter, of course, can be matter-antimatter symmetric).

The matter density of the Universe ($\sim 5\%$ of the energy budget today) can be quantified [35] as

$$Y_B \equiv \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = 3.86 \times 10^{-9} \Omega_B h^2 \simeq (8.53 \pm 0.11) \times 10^{-11}$$

where s is the entropy density (conserved during most the Universe history) whose value today s_0 is about 7$\times$ the number density of CMB photons, and n_B, $(n_{\bar{B}})$ is the number density of (anti)baryons. So in practise, there are 6 baryons for every 10^{10} photons in our Universe today.

A first question about the baryon asymmetry, is "where did it come from?"

1. For instance, maybe the Universe is matter-anti-matter symmetric, but composed of islands of matter and anti-matter? The islands would need to be larger than galaxy clusters, in order to agree with the photon background, and it appears more difficult to make a model that spatially separates baryons from anti-baryons than to make a model that generates an asymmetry. So this idea is not pursued.
2. Putting the baryon excess as an initial condition at the birth of the Universe does not work to well either, because a period of inflation is required to explain the large-scale coherent temperature fluctuations in the CMB. After "60 e-folds" of inflation, the volume of the Universe has grown by $\sim (10^{30})^3$, so any pre-existing density of baryons is decreased by $\sim 10^{-90}$...and the energy density that drives inflation usually appears as entropy after inflation, so it seems difficult to obtain $Y_B \sim 10^{-10}$ this way.

So it seems that the baryon asymmetry needs to be generated in the early Universe after inflation.

10.2 Required Ingredients

There are many recipes for making the baryon asymmetry, but they all share three required ingredients, initially given by Sakharov [36] and sometimes called Sakharov conditions:

1. Baryon number violation : if the Universe starts in a state of $n_B - n_{\bar{B}} = 0$, then $\not{B}$ is required to evolve to $n_B - n_{\bar{B}} \neq 0$.
2. C and CP violation : it is clear that particles need to behave differently from anti-particles. Otherwise the particles would make a baryon asymmetry, the anti-particles would make an anti-baryon asymmetry of the same magnitude, and no net asymmetry would be created.
   C is maximally violated in the SM, and CP violation is present in the SM quarks, observed in Kaons and Bs, and current neutrino oscillation data favours a non-zero phase in the leptonic mixing matrix.
3. departure from thermal equilibrium: the generation of the baryon asymmetry is a dynamical process, so cannot occur in thermal equilibrium, which is static. An alternate way to see this, is that there are no asymmetries in un-conserved quantum numbers in equilibrium (and B is not conserved, by condition 1).
   In the standard cosmological model, departures from equilibrium can be obtained by interactions that occur on timescales of order or longer than the age of the Universe, or at phase transitions.

10.2.1 B non-conservation in the SM

The second and third Sakharov conditions are realised in the Standard Model (of particle physics and cosmology). And contrary to superficial expectations, it turns out that B+L violation is also present in the SM, and rapid at temperatures above m_W.

B and L are global symmetries of the SM Lagrangian, in which appear terms of the form

$$\mathcal{L}_{SM} \supset \overline{q}\not{D}q \, , \, \overline{\ell}\not{D}\ell \ , \ \overline{\ell}He \ , \ \overline{q}\widetilde{H}u \ , \ \overline{q}Hd$$

where q, ℓ are the quark and lepton SU(2) doublets, e, u, and d are the SU(2) singlets. So it is clear that there are symmetries under phase rotations of all the leptons, or all the quarks, or equivalently, that all the Feynman rules conserve B and L. This is reassuring, because the lower bound on the proton lifetime, for decays such as $p \to e^+\pi_0$ is $\gtrsim 10^{33}$ years (to be compared with the age of the Universe $\gtrsim 10^{10}$ yrs).

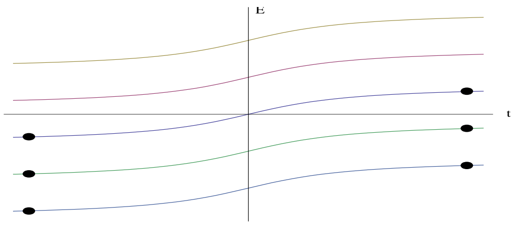
Nonetheless, the SM *does not conserve* $B + L$. This is a consequence of the axial anomaly [37] in QFT, which says that axial currents (containing a γ_5, which count the number of left minus right fermions) which appear conserved at the classical level, are not conserved at one loop.

We are interested in $B + L$. This is not a pure axial current (left - right), but it is classically conserved and has an axial component because the SM is a chiral theory. As a result of the anomaly, one obtains for one generation(α is colour)

$$\sum_{\frac{SU(2)}{singlets}} \partial^\mu(\overline{\psi}\gamma_\mu\psi) + \partial^\mu(\overline{\ell}\gamma_\mu\ell) + \partial^\mu(\overline{q}^\alpha\gamma_\mu q_\alpha) \propto \frac{1}{64\pi^2} W^A_{\mu\nu}\widetilde{W}^{\mu\nu A}.$$

where integrating the RHS over space-time counts the "winding number" of the SU(2) gauge field configuration. As a result, W field configurations of non-zero winding number are sources of a doublet lepton and three (for colour) doublet quarks for each generation (no singlets because they do not have SU(2) interactions). These field configurations therefore change baryon and lepton number by three units (one for each generation).

It is curious that this (non-perturbative) effect does not appear in the Feynman rules. Some intuition for what is happening can be obtained in the Dirac sea picture of the fermion vacuum, illustrated in the following figure(for which I think V Rubakov). At $t \to -\infty$, on the left of the figure, is a vacuum. Then at $t = 0$, is a W field configuration of finite winding number — for each doublet field of the SM, one of the negative energy states from the sea becomes a positive energy state.



For baryogenesis, it is important to know the rate for this SM non-perturbative B+L violation. At zero temperature, it is tunneling process (from a vaccuum with one winding number to the next), and exponentially suppressed [38] $\Gamma \propto e^{-8\pi/g^2}$(this is usually negligeable). At finite temperature, $0 < T < m_W$, the fields can climb over the barrier and the rate is only Boltzmann suppressed [39]: $\Gamma_{\not B+\not L} \sim e^{-m_W/T}$, and finally most interestingly, $\Gamma_{\not B+\not L} \sim \alpha^5 T$ for $T > m_W$ so SM $\not B+\not L$ is "in equilibrium (=fast) for $m_W < T < 10^{12}$ GeV. This SM $\not B+\not L$ is sometimes called "sphalerons", and in the presence of a lepton asymmetry, they partially transform it to a baryon asymmetry.

10.2.2 Summary of preliminaries:

There are three required ingredients to generate the Baryon asymmetry of the Universe :$\not B$, $\not{CP}$, and $\not{TE}$. They are all present in the Standard Models of particle physics and cosmology, but to my knowledge noone has succeeded[7] to combine them so as to obtain a big enough asymmetry Y_B. So the baryon asymmetry is usually taken to be evidence for New Physics from Beyond the Standard Model. However,

[7] The cold electroweak baryogenesis mechanism of Tranberg *etal* [40] is interesting.

since there is only one number to fit, and NP models have many parameters, it is motivated to try to make the baryon asymmetry in models that are introduced for some other reason... such as the type 1 seesaw, which can fit the observed neutrino masses and mixings, and will be discussed next.

10.3 Leptogenesis in the type I seesaw

This section sketches how leptogenesis occurs in seesaw models with mediators of mass $M \gtrsim, \gg$ TeV. To be concrete, the type 1 seesaw is discussed ; the details are different in the type 2 and 3 seesaws, but the general picture is similar.

The type one seesaw Lagrangian is given in eqn (27). Recall first that for $\lambda \sim 1$, the singlets should have masses $\lesssim 10^{15}$ GeV. So here, suppose that the lightest singlet, N_1, has a mass $M_1 \sim 10^9$ GeV, and that the reheat temperature of the Universe after inflation $T_{reheat} \gtrsim M_1$. Different mass spectra for the singlets will be discussed afterwards. Recall also that the 3 generation type-1 seesaw has 18 parameters in the high-scale Lagrangian, to be compared with the 3 masses, 3 mixing angles and 3 phases of the low-energy majorana mass matrix for SM neutrinos. This implies that there are numerous parameters in the high-energy Lagrangian that can be ajusted to obtain the correct baryon asymmetry, without observable consequences. Leptogenesis in the type 1 seesaw (originally proposed by [41]) with heavy singlets is therefore something of a fairy tale for physicists, and its as a fairy tale that it is presented here.

10.3.1 The Fairy Tale

Once upon a time, a Universe was born. So all the fairies came to the christening of the Universe...

... and gave to the Universe the Standard Model and the Seesaw (heavy sterile N_j with $\not{L}$ masses and $\not{CP}$ interactions).

The adventure begins after inflationary expansion of the Universe:

1. If its hot enough, a population of Ns appear(they like heat).
2. The temperature drops below M, and the N population decays away.
3. In the $\not{CP}$ and $\not{L}$ interactions of the N, an asymmetry in SM leptons is created.
4. If this asymmetry can escape the big bad wolf of thermal equilibrium...
5. ...the lepton asym gets partially reprocessed to a baryon asymmetry by non-perturbative $B + L$ -violating SM processes ("sphalerons").

And the Universe lived happily ever after, containing many photons. And for every 10^{10} photons, there were 6 extra baryons (wrt anti-baryons).

10.3.2 *To calculate something?*

There are a very large number of bayogenesis scenarios, and in some cases the state of the art in calculating the dynamics is advanced (quantum field theory of oscillations at finite temperature in curved space-time). The aim here is simpler: given a baryogenesis scenario at the "fairy tale" level, how does one estimate whether it could work? For this, it is helpful to focus on the Sakharov conditions, calculate a suppression factor for each Sakharov condition, then multiply them together to get Y_B:

$$\frac{n_B - n_{\bar{B}}}{s} \sim \frac{1}{3g_*}\epsilon\eta \sim 10^{-3}\epsilon\eta \qquad (\text{want } 10^{-10}) \tag{30}$$

where the entropy density in the early Universe at temperature T is $s \sim g_* n_\gamma$ (g_* counts the number of light modes, the definition can be found in table A.1 of [1]), ϵ is the lepton asymmetry generated in the CP and L violating interactions, and η is some measure of the departure from thermal equilibrium.

As an illustration, we estimate η and ϵ for the fairy tale. Suppose at $T \gtrsim M_1$, an N_1 density $\sim T^3$ is produced. Later, at the temperature drops below M_1 the N_1 population starts to decay away. We assume that a lepton asymmetry is always generated in these decays; however, it can only *survive* if it is not washed out by inverse decays $H\ell \to N_1$. Or equivalently, the asymmetry can only survive after the inverse decays go out of equilibrium

$$\Gamma_{ID}(H\ell \to N) \simeq \Gamma(N \to H\ell)e^{-M_1/T} = \frac{[\lambda\lambda^\dagger]_{11}M_1}{8\pi}e^{-M_1/T} < \mathcal{H} \sim \frac{10T^2}{m_{pl}} \tag{31}$$

where the out-of-equilibrium condition is that the rate is small compared to $\mathcal{H}$, the expansion rate of the Universe[8].

Since the interactions of the Ns are in equilbrium, they should follow a thermal Boltzmann distribution, so the fraction of N_1 remaining at T_{ID} (=when the inverse decays turn off), is

$$\frac{n_N}{n_\gamma}(T_{ID}) \simeq e^{-M_1/T_\alpha} \simeq \frac{\mathcal{H}}{\Gamma(N \to \ell_\alpha H)} \equiv \eta \tag{32}$$

where $\mathcal{H}$ is the Hubble expansion rate $\sim 10T^2/m_{pl}$. This is the density of N_1 whose decays can contribute to the baryon asymmetry of the Universe.

Now estimate ϵ, the CP asymmetry in decays. The constraints from unitarity and CPT and general results of about CP violation are clearly presented in the Appendix of [42]. Recall that the CP transformation is defined, in the **S**-matrix as

$$CP : \langle H\ell|\boldsymbol{S}|N\rangle \to \langle\overline{H\ell}|\boldsymbol{S}|\overline{N}\rangle = \langle\overline{H\ell}|\boldsymbol{S}|N\rangle \tag{33}$$

[8] equivalently, one can say that the interaction timescale is long compared to the age of the Universe

where overline means the CP-conjugate particle (anti-particle). N is its own anti-particle because it is Majorana.

In leptogenesis, we are interested in the $\not{CP}$,$\not{L}$ interactions of N_I. In the fairy tale, we only included decays, so consider the asymmetry:

$$\epsilon_1^\alpha = \frac{\Gamma(N_1 \to H\ell_\alpha) - \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell}_\alpha)}{\Gamma(N_1 \to H\ell) + \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell})} \qquad (\text{recall } N_1 = \bar{N}_1) \tag{34}$$

which represents the fraction of N_1 decays producing excess leptons. It is labelled by lepton flavour, because the flavour is relevant in detailed calculations, but from now on here, we sum on α and drop the α index.

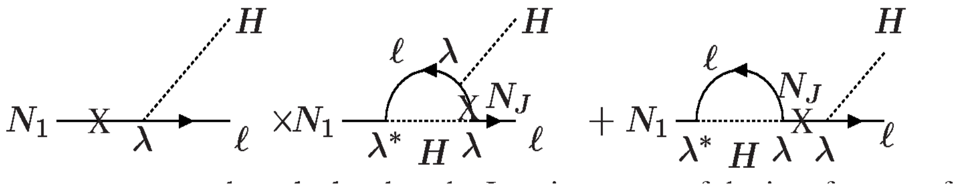


Fig. 10: Tree×loop diagrams generating a CP asymmetry ϵ in the decay of the heavy singlet N_1.

The asymmetry ϵ_1 can be calculated as the Imaginary part of the interference of tree $\times$ loop diagrams illustrated in figure 10 [43]. The $\not{CP}$ arises from complex coupling, and must be multiplied by an imaginary part of the amplitude (sometimes refered to as a “strong phase”) arising from some particles in the loop being on-shell. So in practise, it arises from the Imaginary part of the Feynman parameter integration that one performs in evaluating a loop integral in dimensional regularisation. This is not very intuitive, so lets try to estimate ϵ without doing a loop calculation. This is possible because some of the loop particles need to be on-shell in order to give the strong phase.

The unitarity and CPT invariance of S-matrix elements can be used to calculate ϵ from tree amplitudes. However, here we just estimate diagrammatically. Consider $M_1 \ll M_{2,3}$, so in the loop diagrams contributing to ϵ, the internal N_J can be replaced by the SM neutrino mass matrix $\frac{[K]_{\alpha\beta}}{\Lambda} \equiv \frac{[m_\nu]_{\alpha\beta}}{v^2}$. This works because the momentum in the loop is of order M_1 (so can be neglected compared to $M_{2,3}$), and N_1 on the internal line does not contribute because the coupling constant combination must be Imaginary. Then can estimate

$$\epsilon_1 \sim \frac{1}{16\pi^2}\frac{\lambda^2 K}{\lambda^2 \Lambda}M_1 \quad < \quad \frac{3}{8\pi}\frac{m_\nu^{max} M_1}{v^2} \quad \sim 10^{-6}\frac{M_1}{10^9\text{GeV}}$$

where in the first estimate, the $1/16\pi^2$ is for the loop (but for the Im part of the loop, one should really take $1/8\pi$), the $|\lambda|^2$ downstairs is because ϵ is normalised to Γ, and the mass factor M_1 is to make the dimensions work. The second inequality is a $\sim$ in our approach here, it gives an idea of the magnitude of ϵ, assuming that the phases cooperate. However, in a more careful derivation, the inequality is an upper bound, which combined with eqn (30):

$$\frac{n_B - n_{\bar{B}}}{s} \quad \sim \quad 10^{-3}\epsilon\eta \sim 10^{-3}\frac{\mathcal{H}}{\Gamma}10^{-6}\frac{M_1}{10^9\text{GeV}}$$

implies that one needs $M_1 \gtrsim 10^9$ GeV to get a sufficient asymmetry $Y_B \sim 10^{-10}$ for hierarchical singlets in the type 1 seesaw.

10.3.3 Leptogenesis for $M_1 < 10^9$ GeV?

Singlets with $M \gtrsim 10^9$ GeV have some undesirable features: they are not kinematically accessible at upcoming colliders, and overcontribute to the Higgs mass (see eqn 28). The contribution to the Higgs mass can be cancelled by considering the SUSY seesaw, but in some low-scale ($<$ 10 TeV) SUSY models, gravitinos are over-produced in the early Universe if the reheat temperature is above $\sim 10^5$ GeV (so heavy supersymmetric Ns may have troubles too). Fortunately it is simple to do leptogenesis with TeV $< M_K < 10^7$ GeV: for $M_I \sim M_J$ the second loop diagram of figure 10 resonantly enhances ϵ, allowing to reach $\epsilon \lesssim 1/8\pi$.

In the leptogenesis scenario discussed here, where the asymmetry is produced in N decay, there is a lower bound on the singlet mass from requiring that the asymmetry be produced before the Electroweak Phase Transition (in order to profit from sphalerons):

$$\Gamma_{ID} \sim e^{-M/T}\Gamma(N \to \phi\ell) < H \quad \Rightarrow \quad M \gtrsim 10T_c$$

where $T_c \sim 100$ GeV is the critical temperature of the electroweak phase transition (a cross-over in the SM). So in summary, the fairy tale can work for N_I with $M_I \gtrsim$ TeV.

Leptogenesis can also work with lighter singlets (who decay after the electroweak phase transition), provided that the asymmetry is generated as the singlets are produced. The scenario outlined here relies in oscillations among the singlets, so they must be sufficiently degenerate ($m_{N_2} \simeq m_{N_3}$). This was initially explored in [44], then revisited in the context of the νM(inimal)SM — see for instance [45, 46]. Here is presented only a superficial summary.

The singlets N are "light" (suppose 1 GeV), so the Yukawas λ are necessarily small: $\lambda \sim \sqrt{\frac{m_\nu * \text{GeV}}{v^2}} \lesssim 10^{-7}$. The N_2, N_3 start being produced at temperatures $T \lesssim$ TeV, via their Yukawa interactions. Then they oscillate, among themselves and can transform back to doublet leptons via the Yukawa interactions. These three processes occur coherently, so CP violation in $\lambda \Delta M^2 \lambda^T$ generates lepton flavour asymmetries in the $\nu_{L\alpha}$ (Notice that lepton number in $\ell_L + N_R$, defined as $L|_{SM}$ + helicity of N, is conserved in these processes.). The sphalerons only see the lepton number in the SM doublets, and partially transform it to a baryon asymmetry. From the time oscillations start, until the sphalerons turn off at the electroweak phase transition, the asymmetries in the $\nu_{L\alpha}$ seed asymmetries in the N which give larger asymmetries in the $\nu_{L\alpha}$. Recent calculations (see *e.g.* [47]) show that a sufficiently large baryon asymmetry can be obtained.

11 The End

Most of the students at the school work at hadron colliders, where neutrinos are missing energy. However, neutrino physics might by interesting for two reasons: the observed neutrino masses are evidence for Beyond-the Standard Model Physics (and its encouraging to know that BSM exists somewhere, despite being shy at the LHC), and secondly, the Standard Model neutrino lives in an SU(2) doublet, so there should be BSM physics involving charged leptons — we just need to find it.

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12 Appendix: To convert from 4- to 2-component fermion notation

A 4-component fermion ψ_D can be written as two chiral 2-comp fermions (LeftHanded = χ, and RightHanded = $\bar{\eta}$):

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\eta}^{\bar{\beta}} \end{pmatrix}$$

where usual dotted indices of the right-handed fermion are here written barred. The 2-comp indices α and $\bar{\beta}$ run from 1..2, and are contracted with the anti-symmetric epsilon tensor

$$\varepsilon_{\bar{\alpha}\bar{\beta}} = \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \varepsilon^{\bar{\alpha}\bar{\beta}} = \varepsilon_{\alpha\beta} = -\varepsilon^{\alpha\beta}$$

Notice the sign flip in going from dotted to undotted indices.

Undotted indices are always contracted up-down:

$$\chi\rho = \chi^\alpha\rho_\alpha = \varepsilon^{\alpha\beta}\chi_\beta\rho_\alpha = -\rho_\alpha\chi^\alpha = \rho^\alpha\chi_\alpha$$

and dotted indices down-up, and the ε flips sign in getting bars (sign flip because of up-down vs down-up summing conventions: $\bar{\rho}^{\bar{\beta}} = \bar{\rho}_{\bar{\alpha}}\varepsilon^{\bar{\alpha}\bar{\beta}}$, but $\bar{\rho}^{\bar{\beta}} = (\rho^\beta)^* = (\rho_\alpha\varepsilon^{\beta\alpha})^*$. This perverse set of conventions is so that one can copy Wess+Bagger(W+B) 2-component spinor results, and also Peskin and Schroeder. W+B define

$$\begin{aligned} (\eta\rho)^* &= (\eta\rho)^\dagger = (\varepsilon^{\alpha\beta}\eta_\alpha\rho_\beta)^* = (-\varepsilon^{\bar{\alpha}\bar{\beta}})\bar{\rho}_{\bar{\beta}}\bar{\eta}_{\bar{\alpha}} \\ &= \bar{\rho}_{\bar{\alpha}}\bar{\eta}^{\bar{\alpha}} \end{aligned}$$

So, eg

$$\bar{\psi}_D = \begin{pmatrix} \bar{\chi}_{\bar{\alpha}}\eta^\beta \end{pmatrix} \begin{pmatrix} 0 & \delta^{\bar{\alpha}}_{\bar{\rho}} \\ \delta^\omega_\beta 0 & \end{pmatrix} = \begin{pmatrix} \eta^\omega & \bar{\chi}_{\bar{\rho}} \end{pmatrix} \tag{35}$$

In practice, there is a -ve sign from interchanging fermion fields in an operator, but not when you take cc of the op.

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