Proceedings of the CAS–CERN Accelerator School: Free Electron Lasers and Energy Recovery Linacs, Hamburg, Germany, 31 May–10 June 2016, edited by R. Bailey, CERN Yellow Reports: School Proceedings, Vol. 1/2018, CERN-2018-001-SP (CERN, Geneva, 2018)

High-Gain Regime: 3D[1]

K.-J. Kim[1], L. Ryan[2] and Z. Huang[3]
[1]The University of Chicago, Illinois, USA
[2]Argonne National Lab, Illinois, USA
[3]Stanford University, California, USA

Abstract
Although the FEL interaction is predominantly longitudinal in nature, transverse physics cannot be neglected if one wants to have a complete picture of the FEL. Specifically, we must understand the roles of radiation diffraction and how the electron's betatron motion in the undulator affects performance. We discuss these effects emphasizing the underlying physical picture. A high-gain FEL has a set of transverse modes, of which the fundamental mode has the largest growth rate and thus become dominant as the radiation-electron beam system travels along the undulator. To maximize the growth rate, the electron beam phase space distribution should be matched to the guided optical beam, leading to criteria on electron beam parameters. The FEL gain length is presented near the end of this chapter.

Keywords
High-Gain Regime; FELs.

1 Diffraction and guiding

A remarkable feature of a self-amplified spontaneous emission [SASE] FEL is its transverse coherence. The spontaneous undulator radiation has a transverse phase space area that is determined by the electron beam emittance $(2\pi\varepsilon_x)^2$. This area is typically much larger than the diffraction-limited phase space area $(\lambda/2)^2$, especially at X-ray wavelengths, so that undulator radiation is composed of many transverse modes. Thus, in a SASE FEL, the initial transverse phase space of the spontaneous emission also consists of an incoherent sum of many spatial modes. However, since the FEL interaction is localized within the electron beam near the peak electron density, there is one 'dominant' mode whose transverse size σ_r is dictated by the beam area, and whose natural divergence satisfies $\sigma_r\sigma_{r'} = \lambda/4\pi$. Higher-order spatial modes either diffract more, which results in greater effective losses, or are of larger spatial extent and couple less efficiently to the particles. Thus, the fundamental mode has the highest effective gain, so that it eventually becomes the preferred spatial distribution for the SASE radiation. This surviving fundamental mode appears to be guided after a sufficient undulator distance, a phenomenon commonly referred to as 'optical guiding' or 'gain guiding' [1, 2].

We illustrate the general idea of gain guiding schematically in Fig. 1. Since gain is only effective within the central area, one 'matched' transverse mode shape is selected over all others, and this mode then appears to be guided over many vacuum Rayleigh lengths due to the gain. The transverse mode selection is also clearly evident in Fig. 2, which was obtained from a three-dimensional (3D) GENESIS simulation of SASE. Initially, the radiation power is randomly distributed in the transverse plane, but after a sufficient propagation length only one localized coherent mode survives. For one Gaussian-like transverse mode to completely dominate in this way, there must be enough propagation distance for the competing modes to communicate transversely via diffraction.

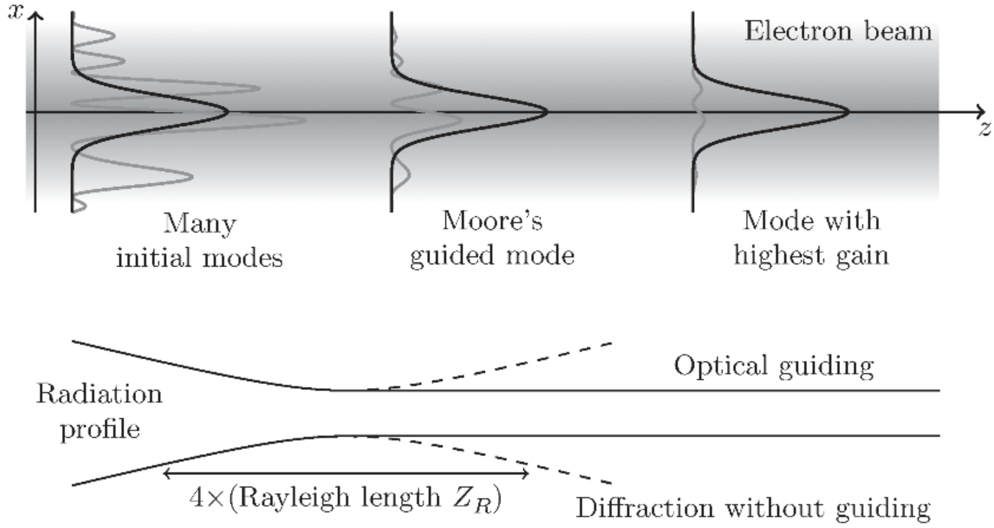

Fig. 1: Illustration of Moore's guided mode. In the top panel the preferentially guided mode is plotted in black, while the higher-order modes are in grey. The intensity at each z location is scaled to keep the height of the guided (black) mode invariant, so that what appears to be a decrease in the power in the higher-order (grey) modes is actually the larger gain of the Gaussian guided profile outstripping the smaller gain associated with all other modes. The bottom panel compares the natural diffraction of the radiation with that of the guided mode generated by FEL gain.

In the one-dimensional (1D) analysis, we introduced the important FEL scaling or Pierce parameter ρ, defined through the relation $n_e\kappa_1\chi_1 = 4k_u^2\rho^3$ which is equivalent to

$$\rho = \left(\frac{e^2K^2[\mathrm{JJ}]^2n_e}{32\epsilon_0\gamma_r^3mc^2k_u^2}\right)^{1/3} = \left[\frac{1}{8\pi}\frac{I}{I_\mathrm{A}}\left(\frac{K[\mathrm{JJ}]}{1+K^2/2}\right)^2\frac{\gamma_r\lambda_1^2}{2\pi\sigma_x^2}\right]^{1/3}, \tag{1}$$

where $I_\mathrm{A} = ec/r_e \approx 17045$ A is the Alfvén current. Many important characteristics of the FEL scale with ρ: the gain length and saturation length scale inversely with ρ, while the bandwidth is proportional to ρ. As shown in the paper titled "High-Gain Regime: 1D" in these proceedings, for vanishing e-beam energy spread the ideal gain length is given by

$$L_{G0} = \frac{\lambda_u}{4\sqrt{3}\pi\rho}\,. \tag{2}$$

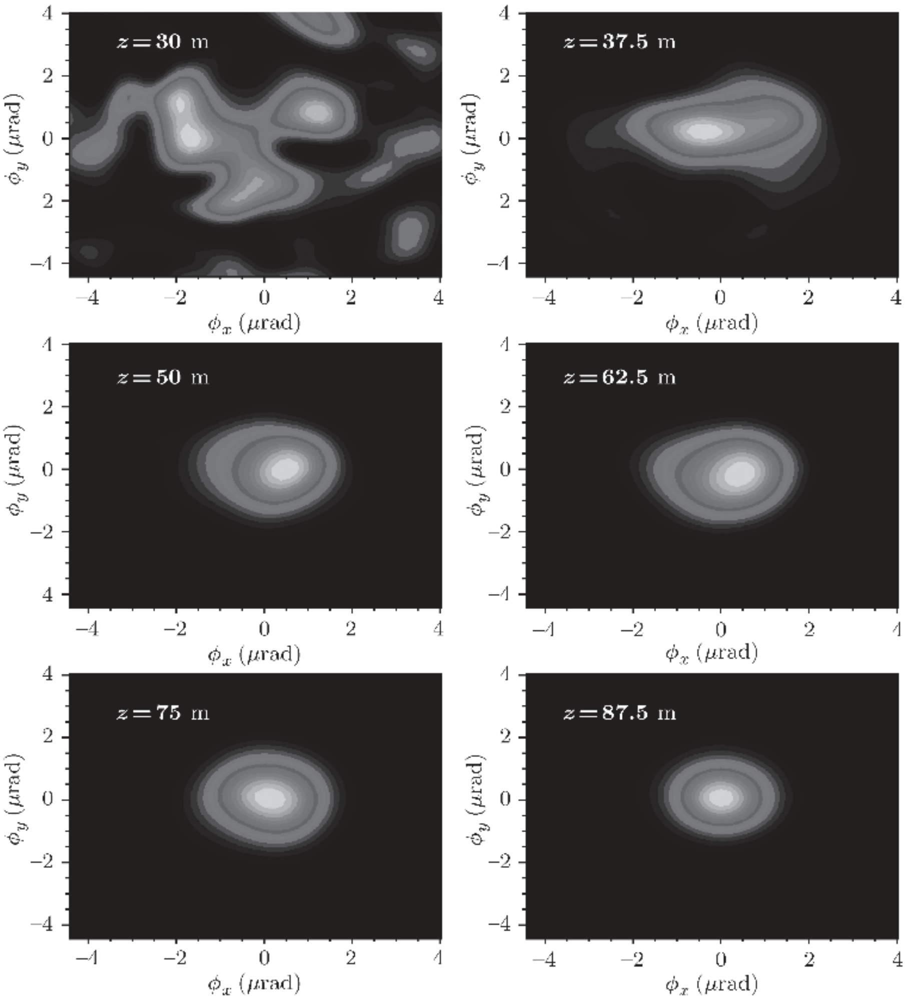

Fig. 2: Evolution of the LCLS radiation angular distribution at different z location. Courtesy of S. Reiche

When 3D effects are included, a different dimensionless combination of parameters may govern the gain characteristics of the FEL. To see this, consider the extreme case where the effect of diffraction is 'large', meaning that the radiation mode size is significantly larger than the electron beam size. To better describe the interaction between the electrons and the radiation in this 3D limit, the beam area $\mathcal{A}_{\text{tr}} = 2\pi\sigma_x^2$ in Eq. (1) should be replaced by the diffraction-limited cross-section which is as follows:

$$2\pi\sigma_x^2 \to 2\pi\frac{\lambda_1}{4\pi}Z_R \, . \tag{3}$$

Here Z_R is the Rayleigh length of the radiation, which from our discussion on gain guiding ought to be of order a few gain lengths. Thus, by inserting $2\pi\sigma_x^2 \to \lambda_1 L_G$ into Eq. (1) and then the resulting expression for ρ into Eq. (2), one can solve the resulting algebraic equation for the gain length L_G to find

$$L_G^{-1} = \frac{4\pi}{\lambda_u}\frac{3^{3/4}}{2}\sqrt{\frac{I}{\gamma I_{\text{A}}}\frac{K^2[\text{JJ}]^2}{(1+K^2/2)}} \, . \tag{4}$$

This equation gives an approximate formula for the growth rate when the 3D effect of diffraction dominates, specifically, when the optical mode is larger than the electron beam cross-sectional area. Thus, it may be convenient to introduce the diffraction D-scaling for certain FEL applications as was done in Ref. [3]. Notice that L_G^{-1} scales as $I^{1/2}$ in the 3D diffractive limit, which is in contrast to the $I^{1/3}$ behaviour that characterizes the 1D limit when the electron beam size is larger than that of the optical mode. Additionally, the D-scaling shows that shrinking the electron beam cross-section much below that of the radiation mode does not further reduce the gain length. In fact, reducing the beam size beyond a certain point actually tends to increase the gain length, since decreasing the physical beam size necessarily increases the angular spread of an electron beam with non-zero emittance. It then follows from this discussion that the optimal electron beam size should roughly match the size of the radiation beam:

$$\sigma_x \sim \sigma_r = \sqrt{\varepsilon_r Z_R} \sim \sqrt{\varepsilon_r L_G}\ , \tag{5}$$

where $\varepsilon_r = \lambda_1/4\pi$ is the radiation emittance.

The above qualitative arguments are useful for understanding the effect of diffraction and for estimating the gain length of certain high-gain FEL projects operating in the infrared and visible wavelengths, where the optical mode size is larger than the e-beam size. Nevertheless, we will continue to scale quantities by the dimensionless parameter ρ for two reasons. First, ρ-scaling is more relevant for X-ray FELs because the typical optical mode size is smaller than the RMS beam size. Second, ρ does not require introducing the (formally undetermined) Rayleigh range, and instead relies on the electron beam cross-sectional area as shown in Eq. (1).

2 Beam emittance and focusing

An electron beam with finite emittance ε_x has a RMS angular spread $\sigma_{x'} = \varepsilon_x/\sigma_x$, so that its size will expand in free space. Hence, to keep a nearly constant e-beam size and maximize the FEL interaction in a long undulator channel requires proper electron focusing. The undulator magnetic field does provide a 'natural' focusing effect. The natural focusing strength, however, is typically too weak, so that external focusing by quadrupole magnets is often required. This focusing is used to decrease the beam size, thereby increasing the ρ parameter and decreasing the gain length. As mentioned in the previous section, decreasing the beam size below that of the optical mode may actually degrade the FEL performance, because the increasing angular spread introduces a spread in the resonant wavelength. This effect is similar to that of energy spread, and can be understood by considering the FEL resonance condition

$$\lambda_1(\psi) = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\psi^2\right), \tag{6}$$

where ψ is the angle the particle trajectory[1] makes with respect to the z axis. From Eq. (6), we see that the spread in particle angles given by $\psi = \sigma_{x'}$ causes a spread in the resonant wavelength

$$\frac{\Delta\lambda}{\lambda_1} = \sigma_{x'}^2 \frac{\lambda_u}{\lambda_1} = \frac{\varepsilon_x}{\beta_x}\frac{\lambda_u}{\lambda_1}\ . \tag{7}$$

To not adversely affect the FEL gain, we demand that the induced wavelength variation due to the angular spread be less than the FEL bandwidth $\sim \rho$, namely that

$$\frac{\Delta\lambda}{\lambda_1} = \sigma_{x'}^2 \frac{\lambda_u}{\lambda_1} \lesssim \rho \approx \frac{\lambda_u}{4\pi L_G}\ . \tag{8}$$

Due to optical guiding, the radiation Rayleigh range is of order the gain length, $Z_R \sim L_G$, so that Eq. (8) implies that the electron beam angular divergence should be no more than that of the radiation:

$$\sigma_{x'} = \sqrt{\frac{\varepsilon_x}{\beta_x}} \leq \sqrt{\frac{\varepsilon_r}{L_G}} \sim \sigma_{r'}\ . \tag{9}$$

The inequalities regarding the beam size (see Eq. (5)), and angular divergence (see Eq. (9)), together require

$$\varepsilon_{x,y} \lesssim \varepsilon_r = \frac{\lambda_1}{4\pi}\,, \tag{10}$$

while the optimal focusing beta function for a given emittance saturates the inequality seen in Eq. (9):

$$\beta_x \sim L_G \frac{\varepsilon_x}{\varepsilon_r}\,. \tag{11}$$

A smaller beam emittance allows for a tighter focused beam size and hence a smaller gain length.

References

[1] G.T. Moore, *Nucl. Instrum. Methods Phys. Res., Sect. A* **239** (1985) 19. https://doi.org/10.1016/0168-9002(85)90693-x

[2] E.T. Scharlemann, A.M. Sessler and J.S. Wurtele, *Phys. Rev. Lett.* **54** (1985) 1925. https://doi.org/10.1103/physrevlett.54.1925

[3] L.-H. Yu, S. Krinsky and R.L. Gluckstern, *Phys. Rev. Lett.* **64** (1990) 3011. https://doi.org/10.1103/physrevlett.64.3011