Temporal Coherence of Radiation from a Collection of Electrons[1]

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Abstract

We review [1] the temporal characteristics of the radiation produced by an electron beam, in time-domain as well as in the frequency domain. For synchrotron radiation, the radiation is chaotic, while it is coherent when the beam is micro-bunched as in a free-electron laser.

Keywords

Radiation; electron beam; time domain; frequency domain.

1. Time-domain picture

Temporal coherence of a radiation specifies the extent to which the radiation maintains a definite phase relationship at two different times. Temporal coherence is characterized by the coherence time, which can be experimentally determined by measuring the path length difference over which fringes can be observed in a Michelson interferometer. A simple representation of a coherent wave in time is given by

$$E_0(t) = e_0 \exp\left(-\frac{t^2}{4\sigma_\tau^2} - i\omega_1 t\right). \tag{1}$$

Here σ_τ is the root mean square (RMS) temporal width of the intensity profile $|E_0(t)^2|$. The coherence time t_{coh} can be defined as

$$t_{\text{coh}} \equiv \int d\tau |C(\tau)|^2 , \tag{2}$$

where $C(\tau)$ is the normalized, first-order correlation function (or complex degree of temporal coherence) given by

$$C(\tau) \equiv \frac{\langle \int dt\, E(t)E^*(t+\tau)\rangle}{\langle \int dt\, |E(t)|^2\rangle} , \tag{3}$$

and the brackets denote ensemble averaging. In the simple Gaussian model of Eq. (1), the coherence time $t_{\text{coh}} = 2\sqrt{\pi}\sigma_\tau$.

In the frequency domain, we have

$$E_\omega^0 = \int dt\, e^{i\omega t}E_0(t) = \frac{e_0\sqrt{\pi}}{\sigma_\omega}\exp\left[-\frac{(\omega-\omega_1)^2}{4\sigma_\omega^2}\right], \tag{4}$$

where $\sigma_\omega = (2\sigma_\tau)^{-1}$ is the RMS width of the frequency profile $|E_\omega|^2$. Let us introduce the temporal (longitudinal) phase space variables ct and $(\omega-\omega_1)/\omega_1 = \Delta\omega/\omega_1$. The Gaussian wave packet then satisfies

$$c\sigma_\tau \cdot \frac{\sigma_\omega}{\omega_1} = \frac{\lambda_1}{4\pi}. \tag{5}$$

Most radiation observed in nature, however, is temporally incoherent. Sunlight, fluorescent light bulbs, black-body radiation, and undulator radiation are all temporally incoherent, and are often referred to as chaotic light or as a partially coherent wave. As a mathematical model of such chaotic light, we consider a collection of coherent Gaussian pulses that are displaced randomly in time with respect to each other:

$$E(t) = \sum_{j=1}^{N_e} E_0(t - t_j) = e_0 \sum_{j=1}^{N_e} \exp\left[-\frac{(t - t_j)^2}{4\sigma_\tau^2} - i\omega_1(t - t_j)\right]. \tag{6}$$

In Eq. (6), t_j is a random number, and the sum extends to N_e to suggest that these wave packets have been created by electrons. We illustrate this partially coherent wave (chaotic light) in Fig. 1, which we obtained by using $N_e = 100$ wave packets with $\lambda_1 = 2\pi/\omega_1 = 1$ and $\sigma_\tau = 2$ ($\sigma_\omega = 0.25$), assuming that the t_j are randomly distributed with equal probability over the bunch length duration $T = 100$. Panel (a) shows 10 randomly chosen such wave packets; plotting many more than this results in a jumbled disarray. Figure 1(b) shows the $E(t)$ that results by summing over all 100 waves.
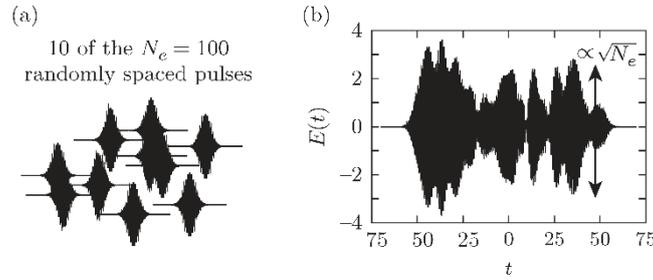

Fig. 1: (a) Representation of the randomly phased wave packets that chooses 10 out of the 100 total waves. The individual waves are shown transversely displaced for illustrative purposes only. (b) Total electric field, given by the incoherent sum of the 100 wave packets. The field consists of order $T/4\sigma_\tau \approx 10$ regular regions (i.e., $M_L \approx 10$ longitudinal modes).

The remarkable feature of this plot is that the resultant wave is a relatively regular oscillation that is interrupted only a few times, much fewer than one might have naively guessed based on the fact that it is a random superposition of 100 wave packets. In fact, the duration of each regular region is independent of the number of wave packets, and is instead governed by the time over which the wave maintains a definite phase relationship, namely, the coherence time. Note that the coherence time of a random collection of Gaussian waves Eq. (6) equals that of the single mode Eq. (1). Thus, each regular region can be identified with a coherent mode whose temporal width is of order the coherence time t_{coh}. The number of regular regions equals the number of coherent longitudinal modes M_L, which is roughly the ratio of the bunch length to the coherence length. Approximately, we have

$$M_L \approx \frac{T}{t_{coh}} = \frac{T}{2\sqrt{\pi}\sigma_\tau} \approx \frac{T}{4\sigma_\tau}. \tag{7}$$

The average field intensity scales linearly with the number of sources, while the instantaneous intensity fluctuates as a function of time. Associated with this intensity variation will be a fluctuation in the observed number of photons $\mathcal{N}_{ph}$ over a given time. Denoting the average photon number by $\langle\mathcal{N}_{ph}\rangle$, the RMS squared fluctuation in the number of photons observed is

$$\sigma^2_{\mathcal{N}_{\text{ph}}} = \frac{\langle \mathcal{N}_{\text{ph}} \rangle^2}{M_{\text{L}}} , \tag{8}$$

where M_{L} is the number of longitudinal modes in the observation time T.

The formula Eq. (8) for the photon number variation can be generalized in two respects. First, the mode counting must include the number of transverse modes M_{T} in both the x and y directions, so that the total number of modes

$$M = M_{\text{L}} M_{\text{T}}^2. \tag{9}$$

Second, there are inherent intensity fluctuations arising from quantum mechanical uncertainty in the form of photon shot noise. This number uncertainty is attributable to the discrete quantum nature of electromagnetic radiation, and, like any shot noise, it adds a contribution to $\sigma^2_{N_{\text{ph}}}$ equal to the average number $\mathcal{N}_{\text{ph}}$. Thus, the RMS squared photon number fluctuation is

$$\sigma^2_{\mathcal{N}_{\text{ph}}} = \frac{\langle \mathcal{N}_{\text{ph}} \rangle^2}{M} + \langle \mathcal{N}_{\text{ph}} \rangle = \frac{\langle \mathcal{N}_{\text{ph}} \rangle^2}{M} \left(1 + \frac{1}{\delta_{\text{degen}}} \right) . \tag{10}$$

The second term in parentheses is the inverse of the number of photons per mode, which is also known as the degeneracy parameter. In the classical devices that we consider there are many photons per mode, $\langle \mathcal{N}_{\text{ph}} \rangle / M \equiv \delta_{\text{degen}} \gg 1$, and the fluctuations due to quantum uncertainty are negligible. In this classical limit the length of the radiation pulse can be determined by measuring its intensity fluctuations, from which the source electron beam length may be deduced, see Ref. [2].
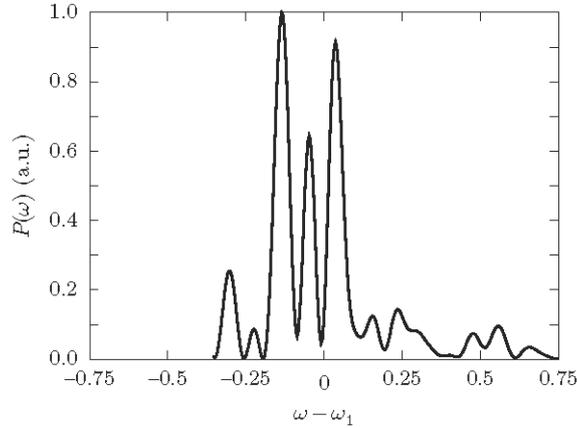

Fig. 2: Intensity spectrum of Eq. (11) using identical parameters as Fig. 1(b). The spectrum consists of $M \sim 10$ sharp frequency spikes of approximate width $2/T \approx 0.02$, which are distributed within a Gaussian envelope of RMS width $\sigma_\omega \sim 0.25$. The height and placement of the spectral peaks fluctuate by 100 per cent for different sets of random numbers.

2. Frequency-domain picture

It is interesting to note that the mode counting we performed in the time domain can also be done in the frequency domain. Figure 2 shows the intensity spectrum $P(\omega) \propto |E_\omega|^2$, where

$$E_\omega = \frac{e_0 \sqrt{\pi}}{\sigma_\omega} \sum_{j=1}^{N_e} \exp\left[-\frac{(\omega - \omega_1)^2}{4\sigma_\omega^2} + i\omega t_j \right] \tag{11}$$

using the same wave parameters as in Fig. 1. The spectrum consists of sharp peaks of width $\Delta\omega \sim 2/T$ that are randomly distributed within the radiation bandwidth $\sigma_\omega = (2\sigma_\tau)^{-1}$. In other words, the frequency bandwidth $\Delta\omega$ of each mode is set by the duration of the entire radiation pulse T, while the frequency range over which the modes exist is given by the inverse coherence time. Thus, the total number of spectral peaks is the same as the number of the coherent modes in the time domain.

References

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