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Introduction to Beam Transfer

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Abstract
Beam transfer refers to the process of moving a collection of particles between accelerators or onto external targets, dumps or measurement devices. A recall of the salient features of transverse beam dynamics is given, and some common manipulations portrayed in normalized phase space. Different beam transfer processes and techniques are introduced, including key generic requirements, constraints, and performance indicators

Keywords
Injection; extraction; kickers; septa; losses; emittance.

1 Introduction

There are, by some counts, over 18 000 particle accelerators worldwide, almost all of which require the injection and/or extraction of the particles for beam transfer. High-energy particles have been widely used over the past 100 years since Rutherford's seminal scattering experiments, to extend the knowledge of the fundamental nature of matter and forces at smaller and smaller scales. Today, accelerators have become a common tool in society, and alongside the rare behemoths like the Large Hadron Collider (LHC), there are linacs, cyclotrons, and synchrotrons used in materials, medical, nuclear, high-energy physics, and other domains.

Beam transfer can be from a linear to a circular accelerator, or between circular accelerators. Extraction is the process of removal of a beam from an accelerator, which can either be for the transfer to another accelerator or for use on a target, dump, or measurement system. Injection is the insertion of a beam into another accelerator, usually (but certainly not always) to accelerate to higher energy. The specific equipment systems used for injection and extraction need to be designed for several constraints: to transfer the beam with minimum beam loss; to achieve the desired beam parameters; and frequently to minimize dilution of emittance.

The different injection and extraction techniques described in this paper share many similarities and often use the same hardware concepts. One important difference is that extraction is often at higher beam rigidity, which reduces the effects of space charge but requires stronger and normally longer deflecting systems, with a consequent need for a larger footprint in the lattice and insertions.

1.1 Single-turn injection and extraction

Single-turn injection and extraction methods are used for both lepton and hadron machines, see e.g. [1–5], and generally involve a septum system to deflect the beam into (or out of) the accelerator aperture and a kicker system to deflect the beam onto (or away from) the closed orbit. A closed orbit bump is sometimes used to reduce the kicker strength required. For these methods, the beam losses can be very low. The emittance dilution is defined by the delivery precision, the kicker flat top ripple, and septum stability. For both injection and extraction, the circulating beam quality can be degraded by septum stray fields and by the kicker field rise time. Injecting a bunched beam into another accelerator also generally requires that the momentum spread and phase be matched to the radiofrequency (RF) bucket.

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1.2 Multiple-turn injection and extraction

Injection can also take place over multiple turns [6–8] to fill the circumference of a receiving accelerator, in order to increased the obtained bunch intensity. Multiple-turn extraction [9–11] is used to produce a spill length longer than the circumference of the accelerator.

A variety of multiple-turn injection and extraction schemes exist, and these can be very different between lepton and hadron machines. Lepton injection can take advantage of synchrotron radiation damping to achieve high beam brightness, while for hadron machines space charge effects dominate, especially at low energy. High brightness proton injection can make use of phase-space painting, particularly with H^- charge exchange injection, while resonant multiple turn extraction can provide quasi-continuous particle fluxes for periods which range from milliseconds to hours. The additional systems required for these more advanced injection and extraction techniques include multiple RF systems, programmed fast closed orbit bumps, stripping foils, and non-linear lattice elements.

2 Recall of transverse phase space representation

A particle accelerator beam typically consists of 10^6–10^{13} particles. We can apply the methods of statistical mechanics and treat properties of the statistical ensemble, not of individual particles. This is a useful concept for representation of many common beam manipulations and hence for helping the understanding of injection and extraction processes.

2.1 Phase space

In relativistic classical mechanics, the motion of a single particle is fully defined at any instant t, if the position r and momentum p of the particle are given with the forces (fields) F acting on the ith particle:

$$\begin{aligned}\vec{r}_i &= x_i\hat{x} + y_i\hat{y} + z_i\hat{z},\\ \vec{p}_i &= p_{xi}\hat{x}_i + p_{yi}\hat{y} + p_{zi}\hat{z},\\ \vec{F}_i &= F_{xi}\hat{x}_i + F_{yi}\hat{y} + F_{zi}\hat{z}.\end{aligned} \tag{1}$$

It is convenient to use a six-dimensional phase space representation, where the ith particle has coordinates

$$P_i \equiv \{x_i, p_{xi}, y_i, p_{yi}, z_i, p_{zi}\}. \tag{2}$$

Generally, the three planes can be considered decoupled (with some important exceptions), and it is very common to study particle evolution separately in each plane:

$$\{x_i, p_{xi}\}, \{y_i, p_{yi}\}, \{z_i, p_{zi}\}. \tag{3}$$

For the two transverse planes, we generally use a modified phase space, see Fig. 1, where the transverse momentum coordinates p_{xi} and p_{yi} are replaced by angles x' and y':

$$\begin{aligned}p_x \rightarrow x' &= \frac{\mathrm{d}x}{\mathrm{d}s} = \tan\theta_x,\\ p_y \rightarrow y' &= \frac{\mathrm{d}y}{\mathrm{d}s} = \tan\theta_y.\end{aligned} \tag{4}$$

The angle is related to the momentum by

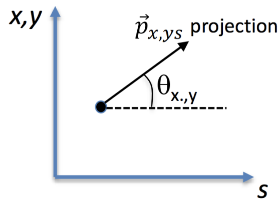


Fig. 1: x' and y' from projection on the $x(y)$–s plane

$$p_x = \gamma_r m_0 \frac{\mathrm{d}x}{\mathrm{d}t} = \gamma_r m_0 v_s \frac{\mathrm{d}x}{\mathrm{d}s} = \beta_r \gamma_r m_0 c x',$$
$$p_y = \gamma_r m_0 \frac{\mathrm{d}y}{\mathrm{d}t} = \gamma_r m_0 v_s \frac{\mathrm{d}y}{\mathrm{d}s} = \beta_r \gamma_r m_0 c y'. \quad (5)$$

One important note is that (unlike x and p_x) x and x' are not canonical conjugate variables unless γ_r and β_r remain constant (i.e. no acceleration).

2.2 Poincaré map

A first recurrence map, or Poincaré map, is the intersection of a periodic orbit in the state space of a continuous dynamical system with a certain lower-dimensional subspace, called the Poincaré section, transversal to the flow of the system [12]. More simply, to generate a Poincaré map, we plot the evolution of the phase space coordinates at one physical location over time (turns), see Fig. 2.

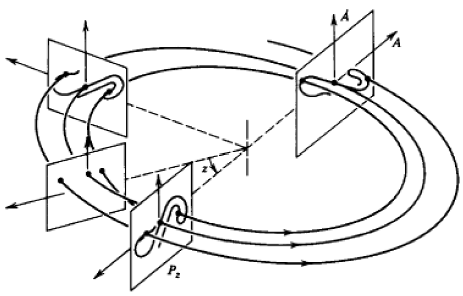


Fig. 2: A Poincaré map is generated by plotting the phase space coordinates at one physical location over time

The Poincaré map is a common representation in non-linear systems, where characteristic behaviour can become apparent. Some examples are shown in Fig. 3.

2.3 Phase space ellipses and emittance

For a circular accelerator, if we make the $\{x, x'\}$ or $\{y, y'\}$ Poincaré map for any single particle at one location over many turns, it traces an ellipse, see Fig. 4. This ellipse at this location always has the same shape and orientation for all particles—the local ellipse form is a property of the focussing lattice. The ellipse changes shape and orientation as it propagates along the accelerator, but the area is preserved.

For many calculations and applications, the canonical conjugate variable pair called the *action-angle variables* (J_x, ϕ_x) are a practical concept. The relation between Cartesian and action-angle variables is given in Fig. 4. The figure shows the well-known elliptic particle trajectory in horizontal phase space. The area A of the phase space ellipse is proportional to action J_x via $A = 2\pi J_x$.

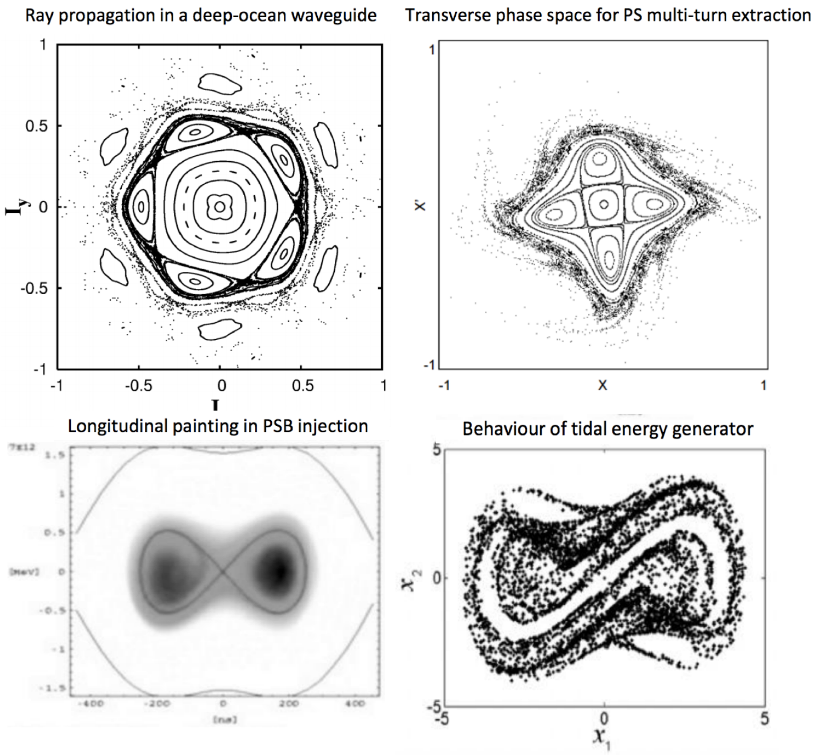


Fig. 3: Example Poincaré maps from different non-linear physical systems, including CERN's Proton Synchrotron (PS) and Proton Synchrotron Booster (PSB).

The transformation between action-angle variables and Cartesian variables is summarized in equations (6) and (7). The action J_x is a measure of the particle amplitude of motion, see Eq. (7):

$$\begin{aligned} 2J_x &= \gamma_x x^2 + 2\alpha_x x' x + \beta_x x'^2, \\ \tan\phi_x &= -\beta_x x \tfrac{x}{x'} - \alpha_x, \end{aligned} \tag{6}$$

$$\begin{aligned} x &= \sqrt{2\beta_x J_x}\cos\phi_x, \\ x' &= -\sqrt{\tfrac{2J_x}{\beta_x}}(\sin\phi_x + \alpha_x\cos\phi_x). \end{aligned} \tag{7}$$

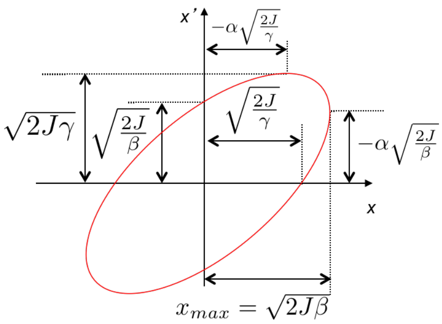


Fig. 4: Phase space in the horizontal plane. The area of the ellipse defines the action J_x, with $A = 2\pi J_x$

The characteristics of the ellipse at a certain location are parameterized by the Twiss parameters, which describe the aspect ratio and orientation. The size of the ellipse (area) is defined by the distribution of angles and positions in the beam. It is very important to recall the following relation, which defines the Courant–Synder invariant as the area of the ellipse $A(s)$, a conserved quantity:

$$A(s) = 2\pi J_x = \pi(\gamma x^2 + 2\alpha x x' + \beta x'^2 = x^2_{\max}/\beta). \tag{8}$$

Very importantly, the emittance is defined as the mean action of all particles in the beam:

$$\varepsilon_x = \langle J_x \rangle. \tag{9}$$

2.4 Normalized phase space

Ellipses are mathematically more difficult to manipulate than circles, and the concept of normalized phase space is used to simplify the representation and highlight the key physical processes. The normalized phase space coordinates are obtained from the real phase space coordinates by a simple linear transformation:

$$\begin{bmatrix} \overline{\mathbf{X}} \\ \overline{\mathbf{X}'} \end{bmatrix} = \mathbf{N} \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta_s}} \begin{bmatrix} 1 & 0 \\ \alpha_s & \beta_s \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix} \tag{10}$$

From this the transformations for the individual coordinates are easily written down as

$$\overline{\mathbf{X}} = \sqrt{\frac{1}{\beta_s}} x, \qquad \overline{\mathbf{X}'} = \sqrt{\frac{1}{\beta_s}} \alpha_s x + \sqrt{\beta_s} x'. \tag{11}$$

In normalized phase space the elliptical trajectories from real phase space become circles, see Fig. 5.

While the equation of the ellipse for real phase space is given by Eq. (6), in normalized phase space it becomes the equation of a circle:

$$2J_x = \overline{x}^2 + \overline{x}'^2. \tag{12}$$

The area enclosed by the trajectory in real as well as normalized phase space is $A = 2\pi J_x$.

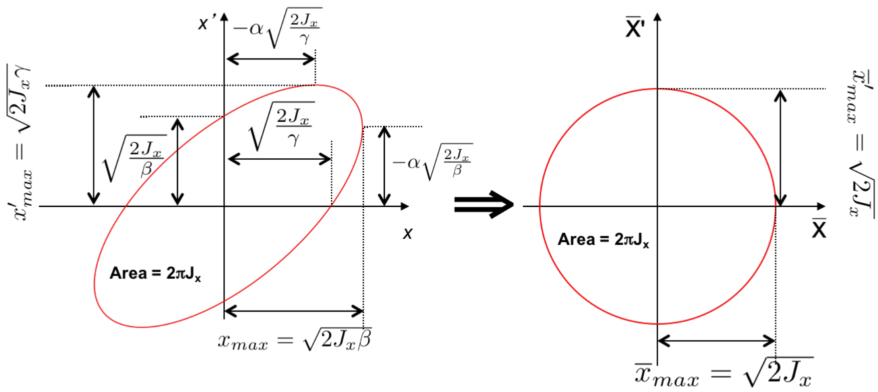


Fig. 5: In normalized phase space the trajectories are circles. The area stays the same and is linked to the action of the particle.

2.5 Common phase space manipulations

Some commonly occurring phase space transformations are illustrated as a precursor to the discussions of the different injection and extraction techniques.

In normalized phase space this is just a clockwise rotation of the distribution around the origin by a phase advance θ, Fig. 6, maintaining the radius of each particle:

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ x_0' \end{bmatrix} . \tag{13}$$

A dipole deflection (kick) k is a vertical displacement in normalized phase space. The two are combined together to described a kick followed by a transport.

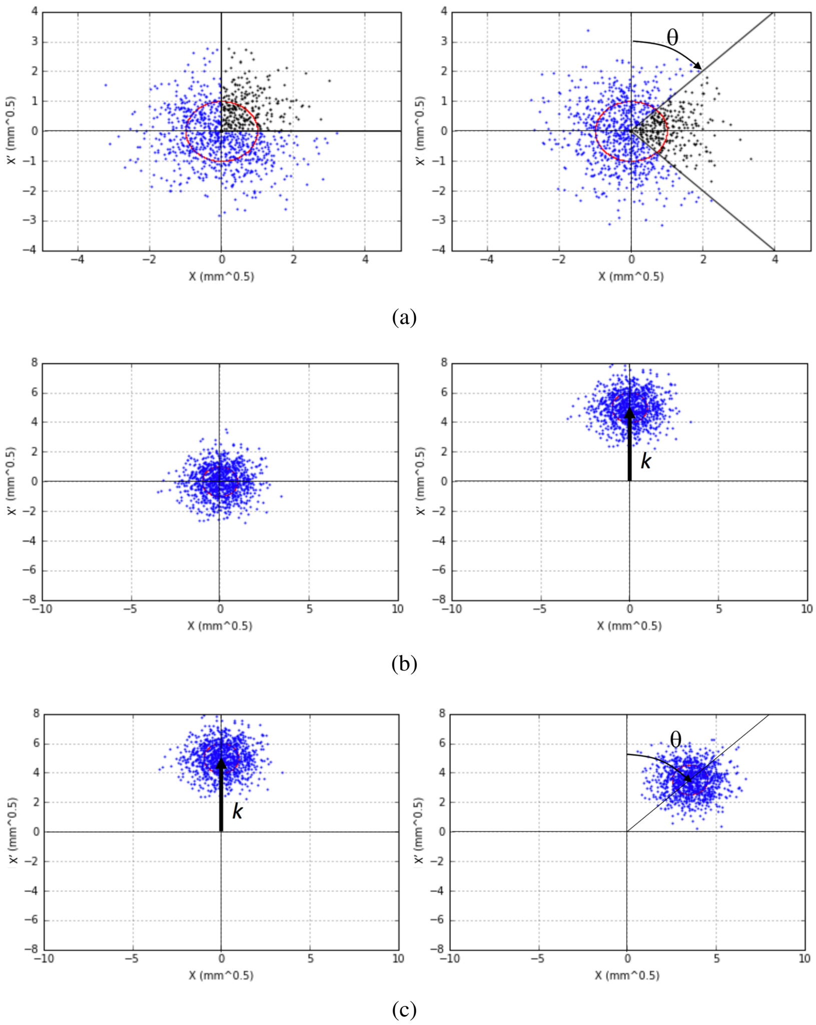
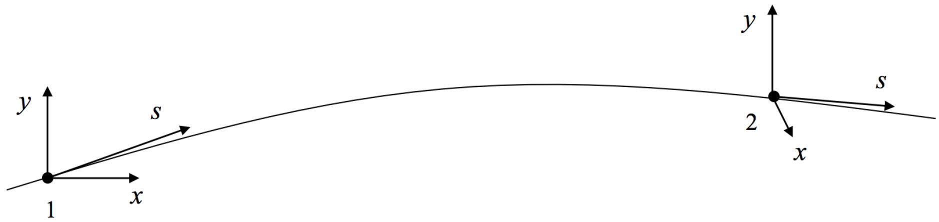


Fig. 6: Manipulations of particle distributions in normalized phase space: (a) rotation of the particle distribution corresponding to transport along a lattice by phase advance θ; (b) displacement of the particle distribution corresponding to a dipole deflection k; (c) a dipole kick followed by transport.

3 Beam transport and difference between rings and transfer lines

As will be already firmly ingrained in the attentive student's mind, each accelerator element has an associated matrix (with the cos-like and sin-like terms) which describes how the input particle coordinates are transferred from the entrance to the exit of the element. The generalized beam transport through a number of n elements moving from s_1 to s_2, each with transfer matrix M_i, is then simply the product of all the individual transfer matrices. This is illustrated for transport in a beam line in Fig. 7.



$$\begin{bmatrix} x_2 \\ x'_2 \end{bmatrix} = \mathbf{M}_{1\to 2}\cdot\begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix}\cdot\begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} \qquad \mathbf{M}_{1\to 2} = \prod_{i=1}^{n}\mathbf{M}_n$$

Twiss parameterisation

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\,\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[(\alpha_1-\alpha_2)\cos\Delta\mu - (1+\alpha_1\alpha_2)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2\sin\Delta\mu\right) \end{bmatrix}$$

Fig. 7: Generalized beam transport in a *beam line* using the product of individual transfer matrices

In a circular accelerator, the transport is similarly generalized, with the important difference that the one-turn matrix must result in a unique solution for the different Twiss parameters at any s location, since a periodic boundary condition exists. This periodicity condition for one turn in a closed ring imposes $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, and $D_1 = D_2$, which uniquely determines $\alpha(s)$, $\beta(s)$, $\mu(s)$, and $D(s)$ around the whole ring, see Fig. 8. The one-turn phase advance $\mu_2 - \mu_1$ reduces to the machine tune Q.

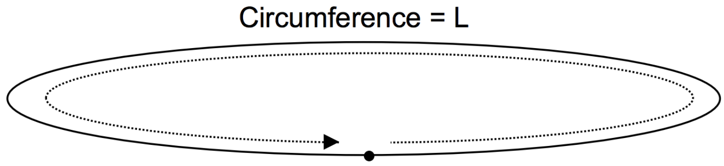
In a transfer line this periodicity condition is absent: the Twiss parameters are still of great use in the description of the beam transport, but the parameters are propagated from the initial conditions at the start of the line. In a sense, they are therefore dependent on the beam distribution, since for non-Gaussian distributions (as obtained in slow extraction, for example), the best-fitting ellipse at the extraction point for the extracted beam distribution may differ greatly from the periodic conditions defined by the circular lattice.

3.1 Effect of dipole kick in a transfer line

In a transfer line or over one turn in a ring we can calculate the effect of a dipole kick k at one location. Using the Twiss parameters, we calculate the downstream position x (or y) and angle $x'_0 = k$ (or $y'_0 = k$) as a function of initial coordinates and lattice functions. We can often, for beam transfer, assume an initial x (or y) $= 0$, as we are deflecting from or onto the closed orbit, which gives

$$x_s = x'_0\sqrt{\beta_0\beta_s}\sin 2\pi(\mu_s - \mu_0). \tag{14}$$

Two points are immediately obvious by inspection of (14). First, a 90 degree phase advance $\Delta\mu$ and large β at the deflecting element and observation point give maximum deflection x_s. This is important in designing an injection or extraction system, where the optimization of the system will depend on the location of the different elements in the lattice. Second, the beam as a whole performs



$$\text{One turn} \qquad \mathbf{M}_{1\to 2} = \mathbf{M}_{0\to L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -\frac{1}{\beta}\left(1+\alpha^2\right)\sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$$

Fig. 8: Beam transport in a *ring* using the product of individual transfer matrices, where the periodicity condition fixes the Twiss parameters around the whole ring.

transverse betatron oscillations after a kick. These oscillations can be a short fraction of one wavelength, or can stretch for many wavelengths over a large distance, see Fig. 9.

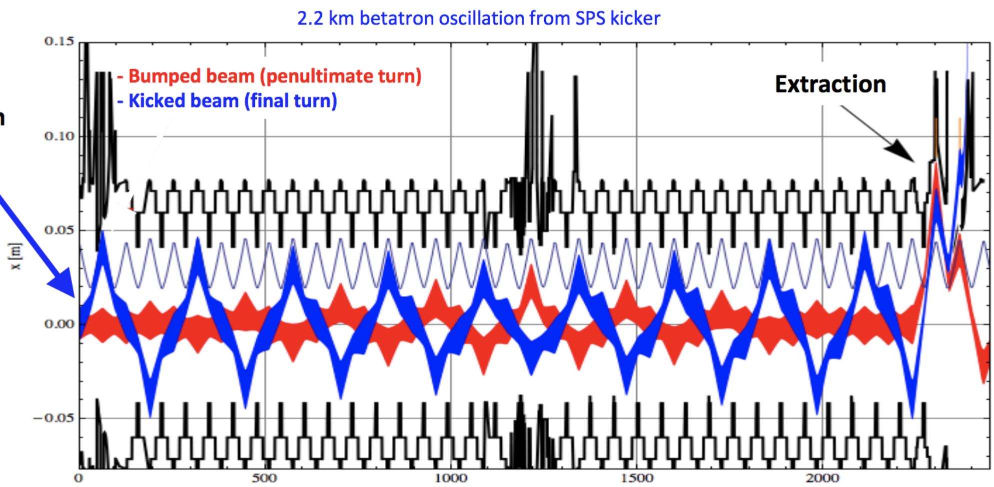


Fig. 9: Betatron oscillation (blue) over several kilometres, from a kicker in one part of the SPS used to extract the beam in a different straight section.

4 The beam transfer process

Almost by definition, beam transfer is a pulsed or transient process, although this can be stretched to several hours for the longest slow extraction techniques. It is often a critical event (or series of events) in the accelerator cycle, requiring specialized equipment not used at other times in the cycle. If badly designed or not optimized it can have a large negative impact on subsequent beam performance, in terms of emittance, intensity, and stability. The beam transfer between machines or onto a target often needs regular operational adjustment to optimize these performance criteria.

For high-energy/high-intensity beams in particular, control of beam losses becomes critical, to avoid excessive irradiation and activation of the hardware, but also to avoid direct damage from beam impact. For both injection and extraction there are a number of desiderata common to most techniques and applications:

– a low fraction of beam lost (for injected/extracted beam and any already circulating beam);

- the need to maintain a large filling factor for injection (i.e. the injection process does not cause too much empty ring circumference);
- precise beam delivery onto the reference orbit;
- minimum perturbation to any circulating beam;
- for injection, as high an energy as possible (sometimes full ring energy);
- a minimum effect on the overall ring performance (e.g. impedance, aperture, etc.);
- a simple setup and operation;
- the need for reliability, reproducibility, and stability.

For transfer lines, some additional design factors need to be taken into account:

- a precise delivery onto the target, dump, or the next injection system;
- precise optical matching to the ring optics (and minimum optical perturbation);
- easy steering and trajectory control;
- filtering of different beam energies or ion species;
- achromatic transport;
- accurate beam characterization.

5 Types of injection and extraction

A wide variety of injection and extraction techniques are deployed in accelerators. A brief summary of the main features of a few of the most common is given below, as an introduction to the concepts explored in detail in the other contributions to this CAS.

5.1 Fast injection and extraction

Fast injection and extraction are typically used to fill or empty a machine, which is often with bunch-to-bucket transfer, with one or many injections (boxcar stacking) [13, 14]. This is typically accomplished with fast kickers and septa. Very fast kicker rise times are required to maximize the amount of beam which can be injected, especially in machines with small circumferences, since the kicker rise and fall times must be significantly shorter than the revolution time. Orbit bumpers can move the beam closer to the septum to reduce the kicker strength needed. Schematic layouts for fast injection and extraction are shown in Fig. 10.

Injection of leptons can take advantage of the strong damping, present from the emission of synchrotron radiation, to accumulate intensity [15]. This is very commonly used at synchrotron radiation light sources, where the top-up operation consists of frequently injecting small amounts of beam to replace losses [16].

5.2 H^- charge-exchange injection

High-brightness proton machines frequently make use of H^- charge exchange injection [17–19]. A linac accelerates negative hydrogen ions which are then merged with a circulating proton beam in a dipole magnet, Fig. 11, before the two electrons are stripped. This technique allows the accumulation of high-brightness beams, since the method allows injection into the already occupied phase space area. Transverse particle distributions can be controlled using phase space painting, to ameliorate space charge effects, reduce beam losses, and increase accumulated intensity. The stripping is achieved with thin foils of carbon or other materials, with a thickness typical in the micrometre range to minimize losses from scattering.

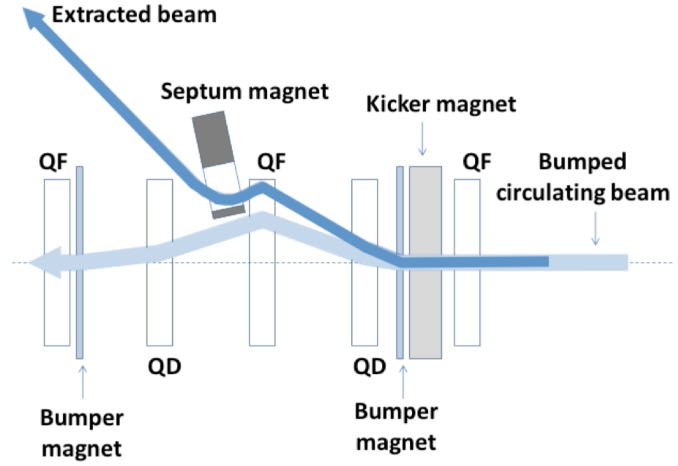


Fig. 10: Schematic of a fast extraction with kicker, septum, and orbit bumpers

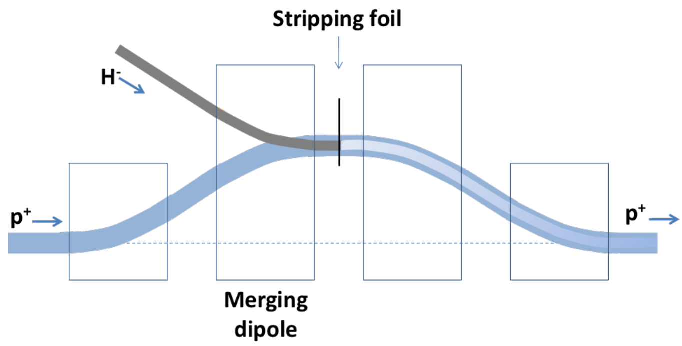


Fig. 11: Merging H^- and p^+ beams in charge exchange injection

5.3 Resonant extraction

Resonant extraction using the third integer is the most common method of providing such uniform spills. A triangular stable area in phase space (usually horizontal) is defined by exciting sextupole elements, and by moving the machine tune close to the third integer resonance. The beam is extracted by driving some particles unstable in a controlled way. The unstable particle amplitudes increase rapidly, following the outward-going separatrix every three turns, and the particles eventually move into the high-field region of a very thin electrostatic septum and are extracted, Fig. 12. Spills can be very long if stochastic excitation is used [20].

6 Slip-stacking injection

In slip-stacking injection, two trains of bunches are merged to increase the bunch intensity, using three separate RF systems (or a single RF system that can operate in a sufficiently flexible manner at two frequencies). A first train of bunches is injected on the closed orbit and captured by the first RF system. This train of bunches is then decelerated, and circulates on a different orbit. A second batch is then injected on the closed orbit and captured by the second RF system. The two trains of bunches have slightly different energies and move relative to each other in phase. When the phase difference reaches zero, both sets of bunches are captured together by the third RF system (or a fast frequency change of one of the two initial systems).

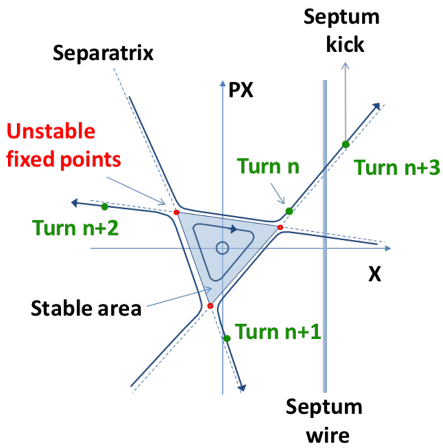


Fig. 12: Resonant extraction in normalized phase space. The amplitudes of particles outside the stable area grow rapidly, following the outward-going separatrix lines every three turns until they reach the electrostatic septum.

7 Conclusion

Beam transfer is ubiquitous to accelerators and complexes of accelerators. It is a pulsed/transient process which, because the equipment is often pulsed, requires specific attention on the timing, controls, and machine protection. The process requires specialized equipment, such as kickers and septa, and has many associated constraints associated with the performance requirements and the space limitations in the accelerators. Ultimately it is a challenging domain which gives scope for very creative and elegant solutions [22–27], some of which will be explored in the companion lecture on Exotic Injection and Extraction Techniques at this CAS.

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