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Physics beyond the Standard Model (Mostly Supersymmetry)

S. K. Vempati
Centre for High Energy Physics, Indian Institute of Science, Bangalore, India.
Institut de Physique Théorique, CEA/Saclay, Gif-sur-Yvette, France

Abstract
We motivate various reasons for going beyond Standard Model. A detailed introduction of Supersymmetry is presented and the Supersymmetric Standard Model is introduced. The present status of supersymmetry is reviewed. A brief introduction to extra dimensions is presented. These are notes for lectures presented at AEPSHEP 2016, Beijing, China.

Keywords
Standard Model; Hierarchy Problem; Supersymmetric Standard Model; Phenomenology.

1 Prerequisites

In the course of these lectures, I will summarise the various reasons we need to extend the Standard Model of Particle Physics. This includes theoretical issues like hierarchy problem and phenomenological issues like dark matter, neutrino masses etc. Of all the models which have been proposed to be extensions of the Standard Model, we focus on supersymmetry. To discuss how supersymmetry solves the hierarchy problem, we will use a simple toy model of SUSY QED. In this model, we show that scalar masses are protected by supersymmetry. We then discuss the construction of the Minimal Supersymmetric Standard Model (MSSM) including soft supersymmetry breaking and electroweak symmetry breaking. The physical mass spectrum of the supersymmetric particles is presented.

In the second part, I discuss the phenomenological status of the MSSM, reviewing all the probes of supersymmetric particles in various direct and indirect experiments. I review the status in flavour violating rare decays, dark matter and of course the LHC results. I then discuss the implications of various results on various supersymmetry breaking models, like minimal Supergravity/Constrained Minimal Supersymmetric Standard Model (mSUGRA/cMSSM) and Gauge Mediation. I finally close with with a small discussion on extra-dimensional models. An is provided on the Standard Model, which is to make these lecture notes self-consistent. We will refer to some of the equations in the appendix while discussing MSSM.

A small list of textbooks and review articles on BSM physics is given below: (a) P. Ramond, Journeys Beyond Standard Model [1], (b)R. N. Mohapatra, Unification and Supersymmetry [2], (c) G. G. Ross, Grand Unified Theories [3], (d) T. T. Yanagida, Physics of Neutrino Physics and Applications to Astrophysics [4] (e) C. Csaki and F. Tanedo, Beyond Standard Model, [5], which contains a modern review of most of the ideas of extensions of physics beyond Standard Model. I also recommend a concise and a pedagogical review by (f) Gautam Bhattacharyya [6].

2 Why BSM ?

The Standard Model provides a coherent successful explanation of electroweak and strong interactions in terms of a (non-abelian) gauged quantum field theory. A lightning review of the structure of the Standard Model lagrangian is presented in Appendix A. The Standard Model has been well tested at various colliders starting from the CERN SPS where the W and Z bosons have been discovered. All the particles in the SM fermion spectrum have been discovered with the top quark discovered in 1995 and the tau neutrino in 2000. Precision measurements conducted at CERN LEP experiment confirm that the quantum

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Table 1: A summary of some of the main reasons why we consider Standard Model to be incomplete.

Phenomenological	*Theoretical*
neutrino masses	hierarchy problem
dark matter	Grand Unification, Quantum Gravity
leptogenesis/baryogenesis	Strong CP

perturbative theory of the SM works very well and matches with the experiment. Measurements of the rare decays at various B-factories like Babar and Belle studied the flavour mixing and CP violation of the quark sector confirming the Cabibbo-Kobayashi-Masakawa (CKM) mixing of the Standard Model. In addition, other measurements include the renormalisation group running of α_s, qauntum chromodynamic (QCD) corrections to electroweak and strong production cross-sections, electric and magnetic dipole moments etc,. The crowning glory being the recent discovery of the Higgs boson which confirms the symmetry breaking mechanism of the Standard Model. While this success of the Standard Model is amazing, there are still several theoretical and phenomenological issues with the Standard Model which give us strong hints that the Standard Model is not the complete picture of the Nature. In the following we list some of the reasons why we need to go beyond the Standard Model.

2.1 Hierarchy Problem

Quantum field theories with a fundamental scalar have a technical problem called hierarchy problem. We will illustrate this problem by comparing two well known theories, namely, QED (Quantum Electrodynamics) and Yukawa theory [7, 8].

Consider QED, the lagrangian is given by

$$\mathcal{L}_{\text{QED}} = \bar{\Psi}\left(i\not{D} - m_e\right)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{1}$$

Here the electron mass is protected by a symmetry. In the limit $m_e \to 0$, there is an enhanced symmetry in the theory, which is the chiral symmetry of the electron. Using cut-off regularisation, we find at 1-loop the correction to the electron mass to be

$$\Delta m_e \sim e^2 m_e \ln \Lambda \tag{2}$$

This correction is proportional to electron mass itself, and thus, in the limit $m_e \to 0$, $\Delta m_e \to 0$. This holds true at all orders in perturbation theory, with the chiral symmetry protecting the left handed and right handed states separately. Theories such as these are called 'natural' theories based on G. 't Hooft principle of naturalness [9]. u In these theories, there is an enhanced symmetry in the limit when a parameter of the theory is set to zero.

Consider now a Yukawa theory with the following lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) - \frac{1}{2}m_S^2\phi^2 + \psi(i\not{\partial} - m_F)\psi + y\bar{\psi}\psi\phi \tag{3}$$

In this case, the mass of the scalar particle is not protected by any symmetry. In the limit $m_S \to 0$ there is no enhanced symmetry in the theory. Furthermore, the corrections to the scalar mass at 1-loop are not proportional to scalar mass itself. Thus even if we set the tree level scalar mass term to zero, it can be generated at higher order in perturbation theory. The correction from the fermion Yukawa coupling to the scalar mass at 1-loop (using dimensional regularisation) is given by

$$\delta m_S^2 = -y^2 m_F^2 \ln\left(\frac{m_F^2}{\mu^2}\right), \tag{4}$$

where μ is the parameter which sets the renormalisation scale. The fermion corrections to the scalar mass do not decouple in the limit $m_F \to \infty$ but instead drive mass to infinity. Theories such as these are technically unnatural theories. We now consider the case of the Standard Model.

In the Standard Model, the Higgs boson is introduced as a fundamental scale, making it a technically unnatural theory. The mass of the Higgs boson is not protected by any symmetry. In the limit $m_H^2 \to 0$ there is no enhanced symmetry in the Standard Model. There are several ways in which this unnaturalness manifests itself in the Standard Model which forms the crux of the *hierarchy problem.*

(a) Within a framework of Grand Unification
Consider a Grand Unified theory (GUT) which is spontaneously broken at the GUT scale. All the particles which are so far protected by the GUT symmetry, like the GUT gauge bosons are no longer protected by it. They attain masses at the GUT scale $\sim \mathcal{O}(g_{GUT}^2 M_{GUT}^2)$. However, the particles which are protected by the residual symmetry of the theory do not attain masses. The Standard Model gauge bosons remain massless as they are protected by the Standard Model gauge symmetry, which remains unbroken at the GUT scale. The SM fermions remain massless as they are protected by the chiral symmetries. The Higgs boson, however as we have discussed does not posses any symmetry which protects it's mass. Thus when the GUT symmetry is broken, one would expect that the SM Higgs boson also attains mass of the order of GUT scale [10]. To avoid this, one can fine tune the parameters of the GUT scalar potential so that the SM Higgs can be light ≈ 125 GeV. However, such a fine tuned mass may not be stable under radiative corrections as discussed earlier. This gets related to the doublet triplet splitting problem in GUT models like SU(5) [11].

(b) SM as an effective theory
Irrespective of the existence of a Grand Unified Theory at the high scale, the Standard Model is by no means a complete theory of the Universe. This is because gravitational interactions are not incorporated in the Standard Model. The gravitational interactions become important (quantum mechanically) at a scale around $M_{Pl} \sim 10^{19}$ GeV. The Standard Model can be viewed as an effective lagrangian of the full theory describing all the interactions including quantum gravity. Assuming Standard Model is valid all the way up to the Planck scale and remains perturbative, one can derive the one loop effective lagrangian of the SM [68]. Again since the Higgs mass is not protected by any symmetry, it gets corrected by the highest momentum cut-off of the effective theory which is the M_{Pl}.

$$\begin{aligned} \delta m_h^2 &\approx \frac{1}{16\pi^2}\Lambda^2 \approx \frac{1}{16\pi^2} M_{Pl}^2, \\ (m_h^2)_{phys} &\approx (m_h^2)_{bare} + \delta m_h^2 \end{aligned} \tag{5}$$

where δm_h^2 represents the radiative correction to the Higgs mass and $(m_h^2)_{bare}$ and $(m_h^2)_{phys}$ represent the bare and the physical Higgs masses. It would mean that to generate the Higgs mass of the right order $(m_h^2)_{phys} \sim (125 \text{ GeV})^2$, one needs fine tuning of one part in $\mathcal{O}(10^{38})$ between the bare mass term and radiative corrections, which is highly unnatural [12]. In the next section, we will discuss ways to make the Standard Model natural.

In the rest of this section, we will summarise the other theoretical and phenomenological reasons to go beyond the Standard Model.

2.1.1 Evolution of Standard Model gauge couplings

Most Grand Unified theories are described in terms of a single (also simple) gauge group for all three interactions, namely, strong and electroweak interactions, and also a single coupling constant. Examples are SU(5) guage group and SO(10) gauge group. They predict that all gauge couplings of the Standard Model unify at the GUT scale. Using the renormalisation group equations for the gauge couplings of the Standard Model, we can run these couplings from the weak scale, where they are well known, all the way up to GUT scale. With the particle content of the Standard Model, as we can see from Fig.(1), the gauge couplings do not exactly unify at the high scale. This figure assumes an SU(5) GUT at the

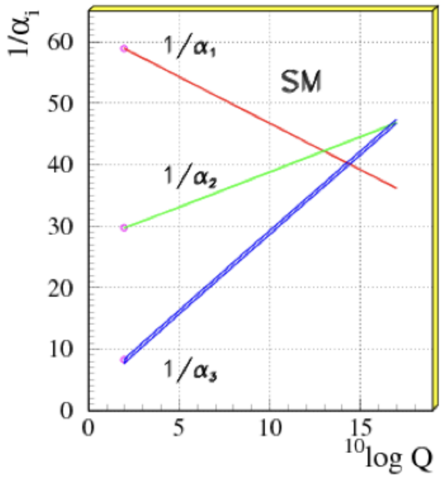

Fig. 1: Evolution of gauge couplings in the Standard Model. It can be clearly seen that they do not unify at high scale.

high scale. The non-unification can be seen as an indication for existence of new particles at the weak scale, which in turn modify the renormalisation group equations such that the gauge couplings unify at the GUT scale.

2.1.2 Dark Matter

There is strong phenomenological evidence for the existence of dark matter at all length scales in Nature. The (virial velocity) rotational curves of (spherical) galaxies, collision of bullet cluster, structure formation and measurement of cosmological energy density of the universe all point out to the existence of dark matter. More details on this can be found in lectures by Moroi san [13]. The Standard Model does not have a particle which can be the dark matter. The dark particle should be chargeless, (most probably) colourless and long lived (stable). Quarks (in addtion to be being coloured) and charged leptons are charged. The gauge and Higgs bosons are short lived. Neutrinos are too light (and thus fast) to form large scale structure which requires significant amount of cold dark matter. One possibility is that the dark matter is made up of a stand-alone particle independent of the Standard Model. A more interesting possibility would be to consider that dark matter is a part of the physics beyond standard model and could have weak interactions (Weakly Interacting Massive Particle). Frameworks like supersymmetry and extra-dimensions typically have such a particle, which thus could be produced at a collider.

2.1.3 Baryon Asymmetry

The standard big bang cosmology predicts equal amount of matter and anti-matter to be present at the beginning of the Universe. However, as the Universe evolved only matter remained and most of the anti-matter disappeared. An asymmetry of the order of one part in 10^{10} between matter and anti-matter at the big bang epoch is sufficient to generate the almost complete matter domination that we see in the present epoch. Three well known mechanism exist for generation of baryon asymmetry (a) Electroweak baryogenesis (b) GUT baryogenesis and (c) leptogenesis. All the above three mechanisms require Standard Model to be extended. Of the three, leptogenesis can be successfully incorporated with the most minimal extension while solving the neutrino mass problem also. More details can be found in [13].

2.1.4 Neutrino Masses

Starting from 1998, it has been increasingly established that neutrinos have masses. The corresponding mixing angles are Currently, the two mass squared differences and the three mixing angles have been well measured. Questions about whether they are Dirac or Majorana, CP-violation in the leptonic sector are still to be answered. However, the present information is sufficient to argue that discovery of neutrino masses signals existence of physics beyond Standard Model. The new physics could be in terms of new particles or new symmetries or both. More aspects of this physics are discussed in Kajitha san's lectures [14].

2.1.5 Strong CP problem

One of the questions which is not discussed in the present set of lectures is called the Strong CP problem. The strong interactions conserve CP to a high degree even though a CP violating term $\langle\theta\rangle_{QCD} G^{\mu\nu}\tilde{G}_{\mu\nu}$, where $\tilde{G}_{\mu\nu}$ is the dual field strength tensor of the Gluon Fields, is allowed by the QCD SU(3) gauge group in the lagrangian. The question is why the coefficient $\langle\theta\rangle_{QCD}$ is very small and is close to zero. One of the popular solutions to the strong CP problem require to introduce additional new particles called axions in to the Standard Model. This topic has been discussed in detail in your cosmology lectures by Moroi san [13].

In addition to the above, there are other issues like the flavour problem in the SM, which we will not elaborate here.

2.2 Nature and energy scale New Physics

After considering the various reasons for going beyond the Standard Model, let us discuss the possible nature of new physics and the energy scale of the new physics from various indications we have discussed so far. We will consider the following indications for new physics: (i) Neutrino Masses: There is a wide range of scales available from *keV* to 10^15 GeV where new physics can manifest itself to explain neutrino masses. If neutrinos are Majorana like, various kinds of seesaw mechanisms are available to explain the neutrino masses and their mixing patterns within this energy range. If they are Dirac, the right handed neutrinos are introduced at the neutrino mass scale with an extra symmetry (lepton number) protecting them. (ii) Dark Matter: Here too there is a wide range in the mass spin, and interactions of the dark matter particle available to satisfy the relic density of the universe as well as the direct and indirect experimental constraints. However, if the dark matter has weak interactions, then a particle of mass ~ 100 GeV satisfies the relic density constraint. This is the so-called WIMP miracle. (iii). Baryogensis can be explained by leptogenesis via right handed neutrinos with masses between a TeV and the GUT scale. (iv). Solutions to hierarchy problem however predict masses close to a TeV.

Let us now concentrate on solutions to the hierarchy problem as the motivation for new physics. Since the New Physics is closely related to the nature of the Higgs boson and the hierarchy problem, there can be two broad classes of solutions to be considered: (a) The Standard Model is valid only up to the scale of quantum gravity as we discussed above. However, the cut-off scale of new physics or quantum gravity, is low, *i.e*, it is no longer $\Lambda \sim M_{Pl}$ but, $\Lambda \sim (1\text{T}eV)$. This is possible in theories with extra space time dimensions where the fundamental Planck constant in α extra dimensional theory is related to the four dimensional M_{Pl} by $M_{Pl}^2 = M_\star^{2+n} R^n$. By choosing sufficiently large extra dimensions, the fundamental Planck scale can be brought close to TeV scale in a theory with n extra dimensions. If the gravity in the extra dimensions is assumed to be strongly interacting, then the radius of the extra dimensions could be much smaller.

Another possibility is that Higgs is not a elementary particle at all, but is a composite of some other tiny fundamental particles. In such a case, the Standard Model is valid only up to a scale where the compositeness of the Higgs comes in to play. A well known example of this type is the pion in ordinary QCD. The pion, a pseudo-scalar particle can be treated as an elementary particle up to energies close to a (maximum) GeV, but beyond that energy scale, the composite nature of the pion should be considered. The quarks become elementary particles from that energy scale. (b) There is a symmetry which protects the Higgs mass. Models of the type Supersymmetry, Little Higgs, Twin Higgs come in to this category. Of these, supersymmetry is interesting as a symmetry of quantum field theory. When softly broken, it preserves most of it's nice features and remains perturbative and calculable. In the following sections, we will introduce supersymmetry and construct the Minimal Supersymmetric Standard Model (MSSM). We then study the phenomenology of this model and discuss the present status of this model. A small supplement to a introduction to extra-dimensions will be provided in the appendix B.

Supersymmetries were first introduced in the context of string theories by Ramond. In quantum field theories, this symmetry is realised through fermionic generators, thus escaping the no-go theorems of Coleman and Mandula [15]. The simplest Lagrangian realising this symmetry in four dimensions was built by Wess and Zumino which contains a spin $\frac{1}{2}$ fermion and a scalar. Supersymmetry relates a particle with another particle varied by a spin 1/2. For example, spin 0 and spin 1/2 form a supersymmetric pair, similarly, spin 1/2 and spin 1 form a supersymmetric pair. Supersymmetry ensures that the within a pair, both the particles have the same mass and same couplings.

In particle physics, supersymmetry plays an important role in protecting the Higgs mass. To understand how it protects the Higgs mass, let us repeat the hierarchy problem once again. The Higgs mass enters as a bare mass parameter in the Standard Model lagrangian, eq.(A.10). Contributions from the self energy diagrams of the Higgs are quadratically divergent pushing the Higgs mass up to cut-off scale. In the absence of any new physics at the intermediate energies, the cut-off scale is typically M_{GUT} or M_{planck}. As we have seen, cancellation of these divergences with the bare mass parameter would require fine-tuning of order one part in 10^{-38} rendering the theory 'unnatural'. On the other hand, if one has additional contributions, say, for example, for the diagram with the Higgs self coupling, there is an additional contribution from a fermionic loop, with the fermion carrying the same mass as the scalar, the contribution from this additional diagram would now cancel the quadratically divergent part of the SM diagram, with the total contribution now being only logarithmically divergent. If this mechanism needs to work for all the diagrams, not just for the Higgs self-coupling and for all orders in perturbation theory, it would require a symmetry which would relate a fermion and a boson with same mass. Supersymmetry is such a symmetry.

3 Supersymmetry and Superfields

Supersymmetry is based on graded Lie algebra, which means it's generators are anti-commuting Grassman operators. For $N = 1$ supersymmetry we have

$$\{Q_\alpha, Q^\dagger_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \tag{6}$$

The supersymmetry generators change the spin of an field by 1/2 unit. For example a scalar (spin =0) is transformed to spin 1/2 field.

$$|\text{fermion} >= Q^\dagger |\text{scalar} > . \tag{7}$$

Furthermore, since

$$[Q, P^\mu] = [Q^\dagger, P^\mu] = 0 \tag{8}$$

it can be shown that, the particle and it's super-partner have the same mass. Furthermore, as long as supersymmetry is conserved, the particle and it's superpartner also share the same couplings.

We now move to study the simplest irreducible representations of $N = 1$ SUSY algebra. Two of the simplest supermultiplets will be of use are the chiral supermultiplet and the Vector supermultiplet. The supermultiplets can be expressed in terms of superfields which can be thought of as 'upgraded' versions of quantum fields. Superfields are functions (fields) written over a 'superspace' made of ordinary space (x_μ) and two fermionic 'directions' (θ,$\bar{\theta}$); they are made up of quantum fields whose spins differ by 1/2. To build interaction lagrangians one normally resorts to this formalism, originally given by Salam and Strathdee [16], as superfields simplify addition and multiplication of the representations. It should be noted however that the component fields may always be recovered from superfields by a power series expansion in grassman variable, θ.

Given that supersymmetry transforms a fermion into a boson and vice-versa, supermultiplets or superfields are multiplets which collect fermion-boson pairs which transform in to each other. We will deal with two kinds of superfields - vector superfields and chiral superfields. A chiral superfield has particle content in the off-shell formalism, contains a weyl fermion, a scalar and and an auxiliary scalar field generally denoted by F. A vector superfield contains a spin 1 boson, a spin 1/2 fermion and an auxiliary scalar field called D.

A chiral superfield has an expansion :

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta\theta F, \tag{9}$$

where ϕ is the scalar component, ψ, the two component spin 1/2 fermion and F the auxiliary field.

The second possible function of the superfields is the analytic or holomorphic function of the superfields called the superpotential, W. This would mean that W is purely a function of complex fields ($z_1 z_2 z_3$) or its conjugates ($z_1^\star z_2^\star z_3^\star$). This function essentially gives the interaction part of the lagrangian which is independent of the gauge couplings, like the Yukawa couplings. If renormalisability is demanded, the dimension of the superpotential is restricted to be less than or equal to three, $[W] \leq 3$ *i.e,* only products of three or less number of chiral superfields are allowed.

The $\theta\theta$ components of the product of three chiral superfields is given as [17]

$$\Phi_i\Phi_j\Phi_k|_{\theta\theta} = -\psi_i\psi_j\phi_k - \psi_j\psi_k\phi_i - \psi_k\psi_i\phi_j + F_i\phi_j\phi_k + F_j\phi_k\phi_i + F_k\phi_i\phi_j, \tag{10}$$

where as earlier, ψ_i represents the fermionic, ϕ_i the scalar and F_i the auxiliary component of the chiral superfield Φ_i. Similarly for the product of two superfields on has :

$$\Phi_i\Phi_j|_{\theta\theta} = -\psi_i\psi_j + F_i\phi_j + F_j\phi_i \tag{11}$$

A vector superfield in (Wess-Zumino gauge) has an expansion :

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D \tag{12}$$

Remember that in supersymmetric theories, the gauge symmetry is imposed by the transformations on matter superfields as :

$$\Phi' = e^{i\Lambda_l t_l}\Phi \tag{13}$$

where Λ_l is an arbitrary chiral superfield and t_l represent the generators of the gauge group which are l in number and the index l is summed over[1].

The gauge invariance is restored in the kinetic part by introducing a (real) vector superfield, V such that the combination

$$\Phi^\dagger e^{gV}\Phi \tag{14}$$

remains gauge invariant. For this to happen, the vector superfield V itselves transforms under the gauge symmetry as

$$\delta V = i(\Lambda - \Lambda^\dagger) \tag{15}$$

[1]To be more specific, t_l is just a number for the abelian groups. For non-abelian groups, t_l is a matrix and so is Λ_l, with $\Lambda_{ij} = t^l_{ij}\Lambda_l$ Note that V is also becomes a matrix in this case.

The supersymmetric invariant kinetic part of the lagrangian is given by:

$$\mathcal{L}_{kin} = \int d\theta^2 d\bar{\theta}^2 \Phi^\dagger e^{gV} \Phi = \Phi^\dagger e^{gV} \Phi|_{\theta\theta\bar{\theta}\bar{\theta}} \tag{16}$$

Remember that the function e^{gV} truncates at $\frac{1}{2}g^2V^2$ in the Wess-Zumino gauge. In fact, in this gauge, this function can be determined by noting:

$$\exp V_{WZ} = 1 - \theta\sigma^\mu\bar{\theta}A_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\,\theta\theta\bar{\theta}\bar{\theta}\,(D - \frac{1}{2}A^\mu A_\mu), \tag{17}$$

for an abelian Vector superfield. Here as usual λ denotes the gaugino field while A_μ represents the gauge field. D represents the auxiliary field of the Vector multiplet. The extension to the non-abelian case is straight forward.

Finally, for every vector superfield (or a set of superfields) we have an associated field strength superfield $\mathcal{W}^\alpha$, which gives the kinetic terms for the gauginos and the field strength tensors for the gauge fields. Given that it is a chiral superfield, the component expansion is given by taking the $\theta\theta$ component of 'square' of that superfield. In the Wess-Zumino gauge, $\mathcal{W}_\alpha = -\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathcal{D}_\alpha V_{WZ}$ [17] ($\mathcal{D}$ is the differential operator on superfields) and the lagrangian has the form :

$$\mathcal{L} \supset \frac{1}{4}\left(\mathcal{W}^\alpha\mathcal{W}_\alpha|_{\theta\theta} + \mathcal{W}^{\dot{\alpha}}\mathcal{W}_{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}}\right) = \frac{1}{2}D^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu\partial_\mu\bar{\lambda} \tag{18}$$

D represents the auxiliary component of the vector superfields. The extension to non-abelian vector superfields in straight forward.

3.1 How supersymmetry works

We now demonstrate with a simple example of supersymmetric QED as how in supersymmetric theories, the mass of the scalar particle is protected. The QED lagragian is $U(1)$ invariant, which is given by

$$\mathcal{L}_{\text{QED}} = \frac{-1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\left(i\partial^\mu\gamma_\mu - m_e\right)\Psi + ie\bar{\Psi}\gamma_\mu\Psi A_\mu, \tag{19}$$

where Ψ stands for the Dirac fermion field, the electron, A_μ is the photon field. We would like to construct the supersymmetric version of the lagrangian. For each 'left handed' or chiral field, we now replace it with a chiral superfield. Thus the two chiral components of the Dirac field, $\Psi = \Psi_L + \Psi_R$, where $\Psi_{L,R} = (1 \pm \gamma_5)/2\ \Psi$. In the supersymmetric version, each of the chiral components will be replaced by a corresponding chiral superfield. Thus we will have $\Psi_{L,R} \to \Phi_{L,R}$. As we have seen each chiral superfield contains, a fermion along with a spin zero partner. The $\Phi_{L,R}$ contain $\{e_{L,R}, \tilde{e}_{L,R}\}$ a left(right) electron along with it's spin zero partner, left (right) handed selectron. Note that the left/right subscripts on the scalar does not indicate the chiral structure of the scalar particles, but just to distinguish them from their chiral fermion partners - one complex scalars for each chiral fermion. The photon field is replaced by a vector superfield, V, introduced above, which contains the photon field A_μ and it's spin half fermonic partner, the photino, λ or $\tilde{A}$. We denote it by $V = \{A_\mu, \tilde{A}\}$. To construct the supersymmetric QED lagrangian, we will need to construct the superpotential , the Kahler potential and the field strength superpotential as discussed above.

The superpotential is just given by $W = m_e\Phi_L\Phi_R$. To get the lagrangian in terms of the component fields, one needs to expand the superfields in the superspace and do the two-component grassman integration, which corresponds to identifying the co-efficient of the $\theta\theta$ component. The Kähler potential is given by

$$K = \Phi_L^\dagger e^{gV}\Phi_L + \Phi_R^\dagger e^{gV}\Phi_R \tag{20}$$

The K and W functions are $U(1)$ gauge invariant. To get the lagragian in terms of the component fields, we have to integrate K over the superspace, which leaves us with the coefficient of $\theta^2\bar{\theta}^2$ term. The third function is the field strength superpotential, which leads to the kinetic terms for the photon field and photino fields. The product of two field strength superpotentials $\mathcal{W}_\alpha \mathcal{W}^\alpha$ is integrated over superspace and the coefficient of $\theta\theta$ gives the component lagrangian. Before writing down the full lagrangian, we should remember that the auxiliary fields F and D should be removed by using their (non-dynamical) equations of motion, which leads to the following definitions in terms of scalar fields:

$$F = \frac{\partial W}{\phi_i} \quad ; \quad D = g q_i \phi^\star \phi \tag{21}$$

where ϕ runs over all the scalar fields in the theory, g is the coupling constant and q_i are the charges of the field. Putting everything together we have the following lagrangian for supersymmetric QED:

$$\begin{aligned}
\mathcal{L}_{SQED} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\lambda}\bar{\sigma}^\mu \partial_\mu \lambda + (\mathcal{D}_\mu \tilde{e}_L)^\dagger (\mathcal{D}^\mu \tilde{e}_L) + (\mathcal{D}_\mu \tilde{e}_R)^\dagger (\mathcal{D}_\mu \tilde{e}_R) \\
&+ i\bar{e}_L \sigma^\mu \mathcal{D}_\mu e_L + i\bar{e}_L \bar{\sigma}^\mu \mathcal{D}_\mu e_L + \left(\sqrt{2} e e_L \lambda \tilde{e}_L^\dagger + H.c\right) + \left(\sqrt{2} e e_R \lambda \tilde{e}_R^\dagger + H.c\right) \\
&+ m_e(e_L e_R + \bar{e}_R \bar{e}_L) - m_e^2(|\tilde{e}_L|^2 - |\tilde{e}_R|^2) - \frac{e^2}{2}(\tilde{e}_L^2 - \tilde{e}_R^2)^2
\end{aligned} \tag{22}$$

The last two terms are the F^2 and D^2 terms respectively. $\mathcal{D}_\mu = \partial_\mu + ieA_\mu$, e being the electromagnetic coupling. The charges are normalised to be +1 for Φ_L and -1 for Φ_R. We can easily read of the various Feynman rules of supersymmetric QED from the above lagrangian, Eq.(22). The vertices are presented in Fig. 2. A couple of points are important to note here: (a) There s no covariant derivative for the photino. It does not interact with the gauge bosons. (b) The scalar quartic couplings are given by the gauge couplings. This is to ensure that the couplings of fermions and the scalars remain the same keeping supersymmetry intact.

The slectron, which has the same mass as the electron, does not receive large mass corrections as it is protected by supersymmetry. All the mass corrections are proportional the mass of the electron itself, protected by the so called non-renormalisable theorems of supersymmetry. The one loop corrections to the selectron mass are given by diagrams of the type shown in Fig. 3. The boson loops cancel with the fermionic loops, note that both of them have the same coupling and mass structure but with opposite sign as a consequence of supersymmetry. This cancellation holds at all orders in perturbation theory. This is how the mass of any scalar in any supersymmetry theory is predicted. We now use this theory as a stepping block to construct the full MSSM. We do so by constantly connecting with the SM lagrangian, it's particle content and gauge group, summarised in Appendix A.

4 Particle Spectrum of the MSSM

What we aim to build over the course of next few lectures is a supersymmetric version of the Standard Model, which means the lagrangian we construct should not only be gauge invariant under the Standard Model gauge group G_{SM} but also now be supersymmetric invariant. Such a model is called Minimal Supersymmetric Standard Model with the word 'Minimal' referring to minimal choice of the particle spectrum required to make it work. Furthermore, we would also like the MSSM to be renormalisable and anomaly free, just like the Standard Model is.

The minimal supersymmetric extension of the Standard Model is built by replacing every standard model matter field by a chiral superfield and every vector field by a vector superfield. Thus the existing particle spectrum of the Standard Model is doubled. The particle spectrum of the MSSM and their transformation properties under G_{SM} is given by,

$$Q_i \equiv \begin{pmatrix} u_{L_i} & \tilde{u}_{L_i} \\ d_{L_i} & \tilde{d}_{L_i} \end{pmatrix} \sim \left(3,\ 2,\ \frac{1}{6}\right) \qquad U_i^c \equiv \begin{pmatrix} u_i^c & \tilde{u}_i^c \end{pmatrix} \sim \left(\bar{3},\ 1,\ -\frac{2}{3}\right)$$

$$D_i \equiv \begin{pmatrix} d_i^c & \tilde{d}_i^c \end{pmatrix} \sim \left(\bar{3},\ 1,\ \frac{1}{3}\right)$$

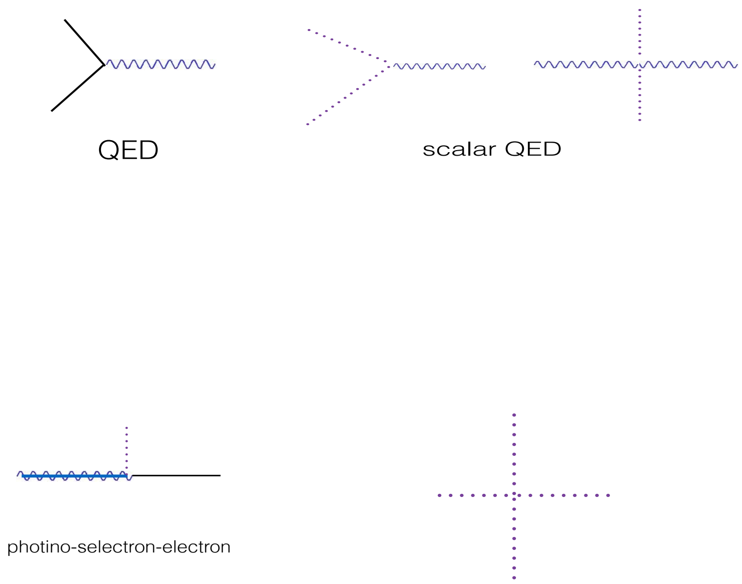

Fig. 2: Feynman Diagrams in Supersymmetric Quantum Electrodynamics contain vertices from QED, Scalar QED and further new diagrams like the last two ones.

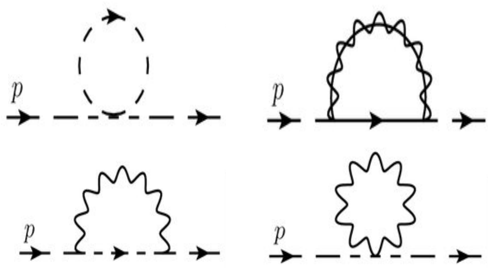

Fig. 3: Typical 1-loop corrections to the left and right selectron masses in supersymmetric QED.

$$L_i \equiv \begin{pmatrix} \nu_{L_i} & \tilde{\nu}_{L_i} \\ e_{L_i} & \tilde{e}_{L_i} \end{pmatrix} \sim \left(1,\ 2,\ -\frac{1}{2}\right) \qquad E_i \equiv \begin{pmatrix} e^c_i & \tilde{e}^c_i \end{pmatrix} \sim (1,\ 1,\ 1) \tag{23}$$

The scalar partners of the quarks and the leptons are typically named as 's'quarks and 's'leptons. Together they are called sfermions. For example, the scalar partner of the top quark is known as the 'stop'. In the above, these are represented by a 'tilde' on their SM counterparts. As in the earlier case, the index i stands for the generation index.

There are two distinct features in the spectrum of MSSM : (a) Note that we have used the conjugates of the right handed particles, instead of the right handed particles themselves. There is no additional conjugation on the superfield itselves, the c in the superscript just to remind ourselves that this chiral superfield is made up of conjugates of SM quantum fields. In eq.(8), $u^c = u_R^\dagger$ and $\tilde{u}^c = \tilde{u}_R^\star$. This way of writing down the particle spectrum is highly useful for reasons to be mentioned later in this section. Secondly (b) At least two Higgs superfields are required to complete the spectrum - one giving masses to the up-type quarks and the other giving masses to the down type quarks and charged leptons. As mentioned earlier, this is the minimal number of Higgs particles required for the model to be consistent from a quantum field theory point of view[2]. These two Higgs superfields have the following transformation properties under G_{SM}:

$$\begin{aligned} H_1 &\equiv \begin{pmatrix} H_1^0 & \tilde{H}_1^0 \\ H_1^- & \tilde{H}_1^- \end{pmatrix} \sim \left(1,\ 2,\ -\frac{1}{2}\right) \\ H_2 &\equiv \begin{pmatrix} H_2^+ & \tilde{H}_2^+ \\ H_2^0 & \tilde{H}_2^0 \end{pmatrix} \sim \left(1,\ 2,\ \frac{1}{2}\right) \end{aligned} \tag{24}$$

The Higgsinos are represented by a ˜ on them. This completes the matter spectrum of the MSSM. Then there are the gauge bosons and their super particles.

In the MSSM, corresponding to three gauge groups of the SM and for each of their corresponding gauge bosons, we need to add a vector superfield which transforms as the adjoint under the gauge group action. Each vector superfield contains the gauge boson and its corresponding super partner called gaugino. Thus in MSSM we have the following vector superfields and their corresponding transformation properties under the gauge group, completing the particle spectrum of the MSSM:

$$\begin{aligned} V_s^A &: \ \left(G^{\mu A} \quad \tilde{G}^A \right) \ \sim \ (8,1,0) \\ V_W^I &: \ \left(W^{\mu I} \quad \tilde{W}^I \right) \ \sim \ (1,3,0) \\ V_Y &: \ \left(B^\mu \quad \tilde{B} \right) \ \sim \ (1,1,0) \end{aligned} \tag{25}$$

The G's (G and $\tilde{G}$) represent the gluonic fields and their superpartners called gluinos, the index A runs from 1 to 8. The W's are the $SU(2)$ gauge bosons and their superpartners 'Winos', the index I taking values from 1 to 3 and finally Bs represents the $U(1)$ gauge boson and its superpartner 'Bino'. Together all the superpartners of the gauge bosons are called 'gauginos'. This completes the particle spectrum of the MSSM.

5 The superpotential and R-parity

The supersymmetric invariant lagrangian is constructed from functions of superfields. In general there are three functions which are: (a) The Kähler potential, K, which is a real function of the superfields (b) The superpotential W, which is a holomorphic (analytic) function of the superfields, and (c) the gauge kinetic function $f_{\alpha\beta}$ which appears in supersymmetric gauge theories. This is the coefficient of the product of field strength superfields, $\mathcal{W}_\alpha \mathcal{W}^\beta$. The field strength superfield is derived from the vector superfields contained in the model. $f_{\alpha\beta}$ determines the normalisation for the gauge kinetic terms. In MSSM, $f_{\alpha\beta} = \delta_{\alpha\beta}$. The lagrangian of the MSSM is thus given in terms of G_{SM} gauge invariant functions K, W and add the field strength superfield $\mathcal{W}$, for each of the vector superfields in the spectrum.

The gauge invariant Kähler potential has already been discussed in the eqs.(16). For the MSSM case, the Kähler potential will contain all the three vector superfields corresponding to the G_{SM} given in the eq.(25). Thus we have :

$$\mathcal{L}_{kin} = \int d\theta^2 d\bar{\theta}^2 \sum_{\text{SU(3),SU(2),U(1)}} \Phi_\beta^\dagger \, e^{gV} \, \Phi_\beta \tag{26}$$

[2]The Higgs field has a fermionic partner, higgsino which contributes to the anomalies of the SM. At least two such fields with opposite hyper-charges ($U(1)_Y$) should exist to cancel the anomalies of the Standard Model.

where the index β runs over all the matter fields $\Phi_\beta = \{Q_i, U_i^c, D_i^c, L_i, e_i^c, H_1, H_2\}$[3] in appropriate representations. Corresponding to each of the gauge groups in G_{SM}, all the matter fields which transform non-trivially under this gauge group[4] are individually taken and the grassman ($d\theta^2 d\bar{\theta}^2$) integral is evaluated with the corresponding vector superfields in the exponential

After expanding and evaluating the integral, we get the lagrangian which is supersymmetric invariant in terms of the ordinary quantum fields - the SM particles and the superparticles. This part of the lagrangian would give us the kinetic terms for the SM fermions, kinetic terms for the sfermions and their interactions with the gauge bosons and in addition also the interactions of the type: fermion-sfermion-gaugino which are structurally like the Yukawa interactions ($ff\phi$), but carry gauge couplings. Similarly, for the Higgs fields, this part of the lagrangian gives the kinetic terms for the Higgs fields and their fermionic superpartners Higgsinos and the interaction of the gauge bosons with the Higgs fields and Higgs-Higgsino-gaugino vertices.

Imposing the restriction of renormalisability the most general G_{SM} gauge invariant form of the W for the matter spectrum of MSSM (8,24) is given as

$$W = W_1 + W_2, \tag{27}$$

where

$$W_1 = h^u_{ij} Q_i U^c_j H_2 + h^d_{ij} Q_i D^c_j H_1 + h^e_{ij} L_i E^c_j H_1 + \mu H_1 H_2 \tag{28}$$
$$W_2 = \epsilon_i L_i H_2 + \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k. \tag{29}$$

Here we have arranged the entire superpotential in to two parts, W_1 and W_2 with a purpose. Though both these parts are gauge invariant, W_2 also violates the global lepton number and baryon quantum numbers. The simultaneous presence of both these set of operators can lead to rapid proton decay and thus can make the MSSM phenomenologically invalid. For these reasons, one typically imposes an additional symmetry called R-parity in MSSM which removes all the dangerous operators in W_2. We will deal with R-parity in greater detail in the next section. For the present, let us just set W_2 to be zero due to a symmetry called R-parity and just call W_1 as W. The lagrangian can be derived from the superpotential containing (mostly) gauge invariant product of the three superfields by taking the $\theta\theta$ component, which can be represented in the integral form as

$$\mathcal{L}_{yuk} = \int d\theta^2 \; W(\Phi) + \int d\bar{\theta}^2 \; \bar{W}(\bar{\Phi}) \tag{30}$$

This part gives the standard Yukawa couplings for the fermions with the Higgs, in addition also give the fermion-sfermion-higgsino couplings and scalar terms. For example, the coupling $h^u_{ij} \; Q_i u^c_j H_2$ in the superpotential has the following expansion in terms of the component fields :

$$\mathcal{L}_{yuk} \supset h^u_{ij} \; Q_i u^c_j H_2 \,|_{\theta\theta}$$
$$\supset h^u_{ij} \; (\; u_i u^c_j H^0_2 - d_i u^c_j H^+_2 \;) \,|_{\theta\theta}$$
$$\supset h^u_{ij} (\psi_{u_i} \psi_{u^c_j} \phi_{H^0_2} + \phi_{\tilde{u}_i} \psi_{u^c_j} \psi_{\tilde{H}^0_2} + \psi_{u_i} \phi_{\tilde{u}^c_j} \psi_{\tilde{H}^0_2} - \psi_{d_i} \psi_{u^c_j} \phi_{H^+_2} - \phi_{\tilde{d}_i} \psi_{u^c_j} \psi_{\tilde{H}^+_2} - \psi_{d_i} \phi_{\tilde{u}^c_j} \psi_{\tilde{H}^+_2}) \tag{31}$$
$$\equiv h^u_{ij} \; (\; u_i u^c_j H^0_2 + \; \tilde{u}_i u^c_j \tilde{H}^0_2 + u_i \tilde{u}^c_j \tilde{H}^0_2 - d_i u^c_j H^+_2 \; - \tilde{d}_i u^c_j \tilde{H}^+_2 \; - d_i \tilde{u}^c_j \tilde{H}^+_2 \;), \tag{32}$$

where in the last equation, we have used the same notation for the chiral superfield as well as for its lowest component namely the scalar component. Note that we have not written the F-terms which give rise to the scalar terms in the potential. Similarly, there is the μ term which gives 'Majorana' type mass term for the Higgsino fields.

[3]The indices i, j, k always stand for the three generations through out this notes, taking values between 1 and 3.
[4]As given in the list of representations in eqs. (8,24)

In the MSSM, we have to add the corresponding field strength $\mathcal{W}$ superfields for electroweak vector superfields, W and B as well as for the gluonic G vector superfields of eqs.(25).

So far we have kept the auxiliary fields (D and F) of various chiral and vector superfields in the component form of our lagrangian. However, given that these fields are unphysical, they have to be removed from the lagrangian to go "on-shell". To eliminate the D and F fields, we have to use the equations of motions of these fields which have simple solutions for the F and D as :

$$F_i = \frac{\partial W}{\partial \phi_i} \quad ; \quad D_A = -g_A \ \phi_i^\star \, T^A_{ij} \, \phi_j, \tag{33}$$

where ϕ_i represents all the scalar fields present in MSSM. The index A runs over all the gauge groups in the model. For example, for $U(1)_Y$, $T^A_{ij} = \ (Y^2/2)\delta_{ij}$. The F and D terms together form the scalar potential of the MSSM[5] which is given as

$$V = \sum_i |F_i|^2 \ + \ \frac{1}{2} D^A D_A \tag{34}$$

Putting together, we see that the lagrangian of the MSSM with SUSY unbroken is of the form :

$$\mathcal{L}^{(0)}_{MSSM} = \int \left(d\theta^2 \ W(\Phi) + H.c\right) + \int d\theta^2 \ d\bar{\theta}^2 \ \Phi_i^\dagger \ e^{gV} \ \Phi_i + \int \left(d\theta^2 \ \mathcal{W}^\alpha \mathcal{W}_\alpha + H.c\right). \tag{35}$$

where all the functions appearing in (35) have been discussed in eqs.(26,28) and (18).

5.1 R-parity

In the previous section, we have seen that there are terms in the superpotential, eq.(29) which are invariant under the Standard Model gauge group G_{SM} but however violate baryon (B) and individual lepton numbers ($\mathrm{L}_{e,\mu,\tau}$). At the first sight, it is bit surprising : the matter *superfields* carry the same quantum numbers under the G_{SM} just like the ordinary matter fields do in the Standard Model and B and $\mathrm{L}_{e,\mu,\tau}$ violating terms are not present in the Standard Model. The reason can be traced to the fact that in the MSSM, where matter sector is represented in terms of superfields, there is no distinction between the fermions and the bosons of the model. In the Standard Model, the Higgs field is a boson and the leptons and quarks are fermions and they are different representations of the Lorentz group. This distinction is lost in the MSSM, the Higgs superfield, H_1 and the lepton superfields L_i have the same quantum numbers under G_{SM} and given that they are both (chiral) superfields, there is no way of distinguishing them. For this reason, the second part of the superpotential W_2 makes an appearance in supersymmetric version of the Standard Model. In fact, the first three terms of eq.(29) can be achieved by replacing $H_1 \ \rightarrow \ L_i$ in the terms containing H_1 of W_1.

The first three terms of the second part of the superpotential W_2 (eq.(29)), are lepton number violating whereas the last term is baryon number violating. The simultaneous presence of both these interactions can lead to proton decay, for example, through a squark exchange. An example of such an process in given in Figure 1. Experimentally the proton is quite stable. In fact its life time is pretty large $\gtrsim \ \mathcal{O}(10^{33})$ years [18]. Thus products of these couplings (λ'' and one of (λ' , ϵ, λ) which can lead to proton decay are severely constrained to be of the order of $(O)(10^{-20})^6$. Thus to make the MSSM phenomenologically viable one should expect these couplings in W_2 to take such extremely small values.

A more natural way of dealing with such small numbers for these couplings would be to set them to be zero. This can be arrived at by imposing a discrete symmetry on the lagrangian called R-parity.

[5] Later we will see that there are also additional terms which contribute to the scalar potential which come from the supersymmetry breaking sector.

[6] The magnitude of these constraints depends also on the scale of supersymmetry breaking, which we will come to discuss only in the next section. For a list of constraints on R-violating couplings, please see G. Bhattacharyya [19].

R-parity has been originally introduced as a discrete R-symmetry [7] by Ferrar and Fayet [20] and then later realised to be of the following form by Ferrar and Weinberg [21] acting on the component fields:

$$R_p = (-1)^{3(B-L)+2s}, \tag{36}$$

where B and L represent the Baryon and Lepton number respectively and s represents the spin of the particle. Under R-parity the transformation properties of various superfields can be summarised as:

$$\begin{aligned}
\{V_s^A, V_w^I, V_y\} &\rightarrow \{V_s^A, V_w^I, V_y\} \\
\theta &\rightarrow -\theta^\star \\
\{Q_i, U_i^c, D_i^c, L_i, E_i^c\} &\rightarrow -\{Q_i, U_i^c, D_i^c, L_i, E_i^c\} \\
\{H_1, H_2\} &\rightarrow \{H_1, H_2\}
\end{aligned} \tag{37}$$

Imposing these constraints on the superfields will now set all the couplings in W_2 to zero.

Imposing R-parity has an advantage that it provides a natural candidate for dark matter. This can be seen by observing that R-parity distinguishes a particle from its superpartner. This ensures that every interaction vertex has at least two supersymmetric partners when R-parity is conserved. The lightest supersymmetric particle (LSP) cannot decay in to a pair of SM particles and remains stable. R-parity can also be thought of as a remnant symmetry theories with an additional $U(1)$ symmetry, which is natural in a large class of supersymmetric Grand Unified theories. Finally, one curious fact about R-parity : it should be noted that R-parity constraints baryon and lepton number violating couplings of dimension four or rather only at the renormalisable level. If one allows for non-renormalisable operators in the MSSM, *i.e* that is terms of dimension more than three in the superpotential, they can induce dim 6 operators which violate baryon and lepton numbers at the lagrangian level and are still allowed by R-parity. Such operators are typically suppressed by high mass scale $\sim M_{Pl}$ or M_{GUT} and thus are less dangerous. In the present set of lectures, we will always impose R-parity in the MSSM so that the proton does not decay, though there are alternatives to R-parity which can also make proton stable.

6 Supersymmetry breaking

So far, we have seen that the Supersymmetric Standard Model lagrangian can also be organised in a similar way like the Standard Model lagrangian though one uses functions of superfields now to get the lagrangian rather than the ordinary fields. In the present section we will cover the last part (term) of the total MSSM lagrangian

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{gauge/kinetic}}\left(K(\Phi, V)\right) + \mathcal{L}_{\text{yukawa}}\left(W(\Phi)\right) + \mathcal{L}_{\text{scalar}}\left(F^2, D^2\right) + \mathcal{L}_{\text{SSB}} \tag{38}$$

which we have left out so far and that concerns supersymmetry breaking (SSB). Note that the first three terms are essentially from $\mathcal{L}^{(0)}_{\text{MSSM}}$ of eq.(35). In Nature, we do not observe supersymmetry. Supersymmetry breaking has to be incorporated in the MSSM to make it realistic. In a general lagrangian, supersymmetry can be broken spontaneously if the auxiliary fields F or D appearing in the definitions of the chiral and vector superfields respectively attain a vacuum expectation value (*vev*). If the F fields get a *vev*, it is called F-breaking whereas if the D fields get a *vev*, it is called D-breaking.

Incorporation of spontaneous SUSY breaking in MSSM would mean that at least one (or more) of the F-components corresponding to one (or more) of the MSSM chiral (matter) superfields would attain a vacuum expectation value. However, this approach fails as this leads to phenomenologically unacceptable prediction that at least one of the super-partner should be lighter (in mass) than the ordinary particle. This is not valid phenomenologically as such a light super partner (of SM particle) has been ruled out experimentally. One has to think of a different approach for incorporating supersymmetry breaking in to the MSSM [23].

[7] R-symmetries are symmetries under which the θ parameter transform non-trivially.

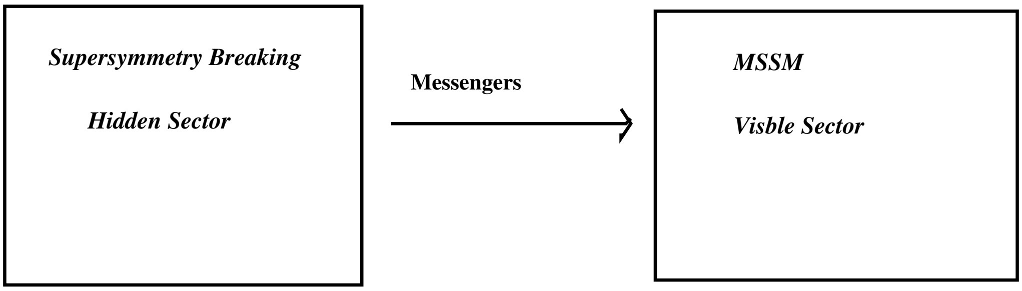

Fig. 4: A schematic diagram showing SUSY breaking using Hidden sector models

One of the most popular and successful approaches has been to assume another sector of the theory consisting of superfields which are not charged under the Standard Model gauge group. Such a sector of the theory is called 'Hidden Sector' as they cannot been "seen" like the Standard Model particles and remain hidden. Supersymmetry can be broken spontaneously in this sector. This information is communicated to the visible sector or MSSM through a messenger sector. The messenger sector can be made up of gravitational interactions or ordinary gauge interactions. The communication of supersymmetry breaking leads to supersymmetry breaking terms in MSSM. Thus, supersymmetry is not broken spontaneously within the MSSM, but *explicitly* by adding supersymmetry breaking terms in the lagrangian.

However, not all supersymmetric terms can be added. We need to add only those terms which do not re-introduce quadratic divergences back into the theory[8]. It should be noted that in most models of spontaneous supersymmetry breaking, only such terms are generated. These terms which are called "soft" supersymmetry breaking terms can be classified as follows:

- a) Mass terms for the gauginos which are a part of the various vector superfields of the MSSM.
- b) Mass terms for the scalar particles, $m^2_{\phi_{ij}}\ \phi_i^\star \phi_j$ with $\phi_{i,j}$ representing the scalar partners of chiral superfields of the MSSM.
- c) Trilinear scalar interactions, $A_{ijk}\phi_i\phi_j\phi_k$ corresponding to the cubic terms in the superpotential.
- d) Bilinear scalar interactions, $B_{ij}\phi_i\phi_j$ corresponding to the bilinear terms in the superpotential.

Note that all the above terms are dimensionful. Adding these terms would make the MSSM non-supersymmetric and thus realistic. The total MSSM lagrangian is thus equal to

$$\mathcal{L}_{total} = \mathcal{L}^{(0)}_{MSSM} + \mathcal{L}_{SSB} \tag{39}$$

with $\mathcal{L}^{(0)}_{MSSM}$ given in eq.(35). Sometimes in literature we have $\mathcal{L}_{SSB} = \mathcal{L}_{soft}$. Let us now see the complete list of all the soft SUSY breaking terms one can incorporate in the MSSM:

1. *Gaugino Mass terms*: Corresponding to the three vector superfields (for gauge groups $U(1)$, $SU(2)$ and $SU(3)$) we have $\tilde{B}, \tilde{W}$ and $\tilde{G}$) we have three gaugino mass terms which are given as $M_1\tilde{B}\tilde{B}$, $M_2\tilde{W}_I\tilde{W}_I$ and $M_3\tilde{G}_A\tilde{G}_A$, where $I(A)$ runs over all the $SU(2)(SU(3))$ group generators.
2. *Scalar Mass terms*: For every scalar in each chiral (matter) superfield , we can add a mass term of the form $m^2\ \phi_i^\star\phi_j$. Note that the generation indices i, j *need* not be the same. Thus the mass terms can violate flavour. Further, given that SUSY is broken prior to $SU(2) \times U(1)$ breaking , all these mass terms for the scalar fields should be written in terms of their 'unbroken' $SU(2) \times$

[8]Interaction terms and other couplings which do not lead to quadratically divergent (in cut-off Λ) terms in the theory once loop corrections are taken in to consideration. It essentially means we only add dimensional full couplings which are supersymmetry breaking.

$U(1)$ representations. Thus the scalar mass terms are : $m^2_{Q_{ij}} \tilde{Q}^\dagger_i \tilde{Q}_j$, $m^2_{u_{ij}} \tilde{u^c}^\star_i \tilde{u^c}_j$, $m^2_{d_{ij}} \tilde{d^c}^\star_i \tilde{d^c}_j$, $m^2_{L_{ij}} \tilde{L}^\dagger_i \tilde{L}_j$, $m^2_{e_{ij}} \tilde{e^c}^\star_i \tilde{e^c}_j$, $m^2_{H_1} H_1^\dagger H_1$ and $m^2_{H_2} H_2^\dagger H_2$.

3. *Trilinear Scalar Couplings*: As mentioned again, there are only three types of trilinear scalar couplings one can write which are G_{SM} gauge invariant. In fact, their form exactly follows from the Yukawa couplings. These are : $A^u_{ij} \tilde{Q}_i \tilde{u}^c_j H_2$, $A^d_{ij} \tilde{Q}_i \tilde{d}^c_j H_1$ and $A^e_{ij} \tilde{L}_i \tilde{e}^c_j H_1$.
4. *Bilinear Scalar Couplings*: Finally, there is only one bilinear scalar coupling (other than the mass terms) which is gauge invariant. The form of this term also follows from the superpotential. It is given as : BH_1H_2.

Adding all these terms completes the lagrangian for the MSSM. However, the particles are still not in their 'physical' basis as $SU(2) \times U(1)$ breaking is not yet incorporated. Once incorporated the physical states of the MSSM and their couplings could be derived.

7 $SU(2) \times U(1)$ breaking

As a starting point, it is important to realize that the MSSM is a two Higgs doublet model *i.e*, SM with two Higgs doublets instead of one, with a different set of couplings [24]. Just as in Standard Model, spontaneous breaking of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$ can be incorporated here too. Doing this leads to constraints relating various parameters of the model. To see this, consider the neutral Higgs part of the total scalar potential including the soft terms. It is given as

$$\begin{aligned} V_{scalar} &= (m^2_{H_1} + \mu^2)|H_1^0|^2 + (m^2_{H_2} + \mu^2)|H_2^0|^2 - (B_\mu \mu H_1^0 H_2^0 + H.c) \\ &+ \frac{1}{8}(g^2 + g'^2)(H_2^{0 2} - H_1^{0 2})^2 + \ldots, \end{aligned} \tag{40}$$

where H_1^0, H_2^0 stand for the neutral Higgs scalars and we have parameterised the bilinear soft term $B \equiv B_\mu \mu$. Firstly, we should require that the potential should be bounded from below. This gives the condition (in field configurations where the D-term goes to zero, *i.e*, the second line in eq.(40)):

$$2B_\mu < 2|\mu|^2 \ + \ m^2_{H_2} \ + m^2_{H_1} \tag{41}$$

Secondly, the existence of a minima for the above potential would require at least one of the Higgs mass squared to be negative giving the condition, (determinant of the 2×2 neutral Higgs mass squared matrix should be negative)

$$B^2_\mu > (|\mu|^2 \ + \ m^2_{H_2} \ ) \ (|\mu|^2 + m^2_{H_1}) \tag{42}$$

In addition to ensuring the existence of a minima, one would also require that the minima should be able to reproduce the standard model relations i.e, correct gauge boson masses. We insist that both the neutral Higgs attain vacuum expectation values :

$$< H_1^0 >= \frac{v_1}{\sqrt{2}} \quad ; \quad < H_2^0 >= \frac{v_2}{\sqrt{2}} \tag{43}$$

and furthermore we define

$$v_1^2 + v_2^2 \ = \ v^2 \ = \ 246^2 \text{ GeV}^2,$$

where v represents the vev of the Standard Model (SM) Higgs field. However, these *vev*s should correspond to the minima of the MSSM potential. The minima are derived by requiring $\partial V/\partial H_1^0 \ = \ 0$ and $\partial V/H_2^0 \ = \ 0$ at the minimum, where the form of V is given in eq.(40). These derivative conditions lead to relations between the various parameters of the model at the minimum of the potential. We have, using the Higgs vev (43) and the formulae for[9] $M_Z^2 = \frac{1}{4}(g^2 + g'^{\,2})v^2$, the minimisation conditions can rewritten as

$$\begin{aligned} \frac{1}{2}M_Z^2 &= \frac{m_{H_1}^2 - \tan^2\beta\, m_{H_2}^2}{\tan^2\beta - 1} - \mu^2 \\ \text{Sin}2\beta &= \frac{2B_\mu\,\mu}{m_{H_2}^2 + m_{H_1}^2 + 2\mu^2}, \end{aligned} \tag{44}$$

where we have used the definition $\tan\beta = v_2/v_1$ as the ratio of the vacuum expectation values of H_2^0 and H_1^0 respectively. Note that the parameters $m_{H_1}^2$, $m_{H_2}^2$, B_μ are all supersymmetry breaking 'soft' terms. μ is the coupling which comes in the superpotential giving the supersymmetry conserving masses to the Higgs scalars. These are related to the Standard Model parameters M_Z and a ratio of *vev*s, parameterised by an angle tanβ. Thus these conditions relate SUSY breaking soft parameters with the SUSY conserving ones and the Standard Model parameters. For any model of supersymmetry to make contact with reality, the above two conditions (44)need to be satisfied.

The above minimisation conditions are given for the 'tree level' potential only. 1-loop corrections a 'la Coleman-Weinberg can significantly modify these minima. We will discuss a part of them in later sections when we discuss the Higgs spectrum. Finally we should mention that, in a more concrete approach, one should consider the entire scalar potential including all the scalars in the theory, not just confining ourselves to the neutral Higgs scalars. For such a potential one should further demand that there are no deeper minima which are color and charge breaking (which effectively means none of the colored and charged scalar fields get vacuum expectation values). These conditions lead to additional constraints on parameters of the MSSM [25].

8 Mass spectrum

We have seen in the earlier section, supersymmetry breaking terms introduce mass-splittings between ordinary particles and their super-partners. Given that particles have zero masses in the limit of exact G_{SM}, only superpartners are given soft mass terms. After the $SU(2) \times U(1)$ breaking, ordinary particles as well as superparticles attain mass terms. For the supersymmetric partners, these mass terms are either additional contributions or mixing terms between the various super-particles. Thus, just like in the case of ordinary SM fermions, where one has to diagonalise the fermion mass matrices to write the lagrangian in the 'on-shell' format or the physical basis, a similar diagonalisation has to be done for the supersymmetric particles and their mass matrices.

8.1 The Neutralino Sector

To begin with lets start with the gauge sector. The superpartners of the neutral gauge bosons (neutral gauginos) and the fermionic partners of the neutral higgs bosons (neutral higgsinos) mix to form Neutralinos. The neutralino mass matrix in the basis

$$\mathcal{L} \supset \frac{1}{2}\,\Psi_N \mathcal{M}_N \Psi_N^T\; + H.c$$

where $\Psi_N = \{\tilde{B},\ \tilde{W}^0, \tilde{H}_1^0, \tilde{H}_2^0\}$ is given as :

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -M_Z c\beta\; s\theta_W & M_Z s\beta\; s\theta_W \\ 0 & M_2 & M_Z c\beta\; c\theta_W & M_Z s\beta\; c\theta_W \\ -M_Z c\beta\; s\theta_W & M_Z c\beta\; c\theta_W & 0 & -\mu \\ M_Z s\beta\; s\theta_W & -M_Z s\beta\; c\theta_W & -\mu & 0 \end{pmatrix}, \tag{45}$$

[9] In this lecture note, we will be using $g_2 = g = g_W$ for the SU(2) coupling, whereas $g' = g_1$ for the $U(1)_Y$ coupling and $g_s = g_3$ for the SU(3) strong coupling.

with $c\beta(s\beta)$ and $c\theta_W(s\theta_W)$ standing for $\cos\beta(\sin\beta)$ and $\cos\theta_W(\sin\theta_W)$ respectively. As mentioned earlier, M_1 and M_2 are the soft parameters, whereas μ is the superpotential parameter, thus SUSY conserving. The angle β is typically taken as a input parameter, $tan\beta = v_2/v_1$ whereas θ_W is the Weinberg angle given by the inverse tangent of the ratio of the gauge couplings as in the SM. Note that the neutralino mass matrix being a Majorana mass matrix is complex symmetric in nature. Hence it is diagonalised by a unitary matrix N,

$$N^* \cdot M_{\tilde{N}} \cdot N^\dagger = \text{Diag.}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}) \tag{46}$$

The states are rotated by $\chi_i^0 = N^\star \Psi$ to go the physical basis.

8.2 The Chargino Sector

In a similar manner to the neutralino sector, all the fermionic partners of the charged gauge bosons and of the charged Higgs bosons mix after electroweak symmetry breaking. However, they combine in a such a way that a Wino-Higgsino Weyl fermion pair forms a Dirac fermion called the chargino. This mass matrix is given as

$$\mathcal{L} \supset -\frac{1}{2} \begin{pmatrix} \tilde{W}^- & \tilde{H}_1^- \end{pmatrix} \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_2^+ \end{pmatrix}, \tag{47}$$

Given the non-symmetric (non-hermitian) matrix nature of this matrix, it is diagonalised by a bi-unitary transformation, $U^* \cdot M_C \cdot V^\dagger = \text{Diag.}(m_{\chi_1^+}, m_{\chi_2^+})$. The chargino eigenstates are typically represented by $\chi^\pm$ with mass eigenvalues $m_{\chi^\pm}$. The explicit forms for U and V can be found by the eigenvectors of $M_C M_C^\dagger$ and $M_C^\dagger M_C$ respectively [26].

8.3 The Sfermion Sector

Next let us come to the sfermion sector. Remember that we have added different scalar fields for the right and left handed fermions in the Standard Model. After electroweak symmetry breaking, the sfermions corresponding to the left fermion and the right fermion mix with each other. Furthermore while writing down the mass matrix for the sfermions, we should remember that these terms could break the flavour *i.e*, we can have mass terms which mix different generation. Thus, in general the sfermion mass matrix is a 6×6 mass matrix given as :

$$\xi^\dagger \, M_{\tilde{f}}^2 \xi \;\; ; \;\; \xi = \{\tilde{f}_{L_i}, \tilde{f}_{R_i}\}$$

From the total scalar potential, the mass matrix for these sfermions can be derived using standard definition given as

$$m_{ij}^2 = \begin{pmatrix} \frac{\partial^2 V}{\partial\phi_i \partial\phi_j^\star} & \frac{\partial^2 V}{\partial\phi_i \partial\phi_j} \\ \frac{\partial^2 V}{\partial\phi_i^\star \partial\phi_j^\star} & \frac{\partial^2 V}{\partial\phi_i^\star \partial\phi_j} \end{pmatrix} \tag{48}$$

Using this for sfermions, we have :

$$M_{\tilde{f}}^2 \; = \begin{pmatrix} m_{\tilde{f}_{\rm LL}}^2 & m_{\tilde{f}_{\rm LR}}^2 \\ m_{\tilde{f}_{\rm LR}}^{2\dagger} & m_{\tilde{f}_{\rm RR}}^2 \end{pmatrix}, \tag{49}$$

where each of the above entries represents 3×3 matrices in the generation space. More specifically, they have the form (as usual, i, j are generation indices):

$$\begin{aligned} m_{\tilde{f}_{L_i L_j}}^2 &= M_{\tilde{f}_{L_i L_j}}^2 + m_f^2 \delta_{ij} + M_Z^2 \cos 2\beta (T_3 + \sin^2\theta_W Q_{\rm em}) \delta_{ij} \\ m_{\tilde{f}_{L_i R_j}}^2 &= \left(\left(Y_f^A \cdot \tfrac{v_2}{v_1} - m_f \mu^{\tan\beta}_{\cot\beta} \right) \text{ for } f =^{e,d}_{u} \right) \delta_{ij} \end{aligned}$$

$$m^2_{\tilde{f}_{\rm RR}} \;=\; M^2_{\tilde{f}_{R_{ij}}} + \left(m_f^2 + M_Z^2 \cos 2\beta \sin^2\theta_W Q_{\rm em}\right)\delta_{ij} \tag{50}$$

In the above, $M^2_{\tilde{f}_L}$ represents the soft mass term for the corresponding fermion (L for left, R for right), T_3 is the eigenvalue of the diagonal generator of $SU(2)$, m_f is the mass of the fermion with Y and Q_{em} representing the hypercharge and electromagnetic charge (in units of the charge of the electron) respectively. The sfermion mass matrices are hermitian and are thus diagonalised by a unitary rotation, $R_{\tilde{f}} R^\dagger_{\tilde{f}} = 1$:

$$R_{\tilde{f}} \cdot M_{\tilde{f}} \cdot R^\dagger_{\tilde{f}} = \text{Diag.}(m_{\tilde{f}_1}, m_{\tilde{f}_2}, \ldots, m_{\tilde{f}_6}) \tag{51}$$

8.4 The Higgs sector

Now let us turn our attention to the Higgs fields. We will use again use the standard formula of eq.(48), to derive the Higgs mass matrices. The eight Higgs degrees of freedom form a 8×8 Higgs mass matrix which breaks down diagonally in to three 2×2 mass matrices[10].

The mass matrices are divided in to charged sector, CP odd neutral and CP even neutral. This helps us in identifying the goldstone modes and the physical spectrum in an simple manner. Before writing down the mass matrices, let us first define the following parameters :

$$m_1^2 = m_{H_1}^2 + \mu^2, \quad m_2^2 = m_{H_2}^2 + \mu^2, \quad m_3^2 = B_\mu \mu.$$

In terms of these parameters, the various mass matrices and the corresponding physical states obtained after diagonalising the mass matrices are given below:

Charged Higgs and Goldstone Modes:

$$\begin{pmatrix} H_1^+ & H_2^+ \end{pmatrix} \begin{pmatrix} m_1^2 + \frac{1}{8}(g_1^2+g_2^2)(v_1^2 - v_2^2) + \frac{1}{4} g_2^2 v_2^2 & m_3^2 + \frac{1}{4} g_2^2 v_1 v_2 \\ m_3^2 + \frac{1}{4} g_2^2 v_1 v_2 & m_2^2 - \frac{1}{8}(g_1^2+g_2^2)(v_1^2 - v_2^2) + \frac{1}{4} g_2^2 v_2^2 \end{pmatrix} \begin{pmatrix} H_1^- \\ H_2^- \end{pmatrix} \tag{52}$$

Using the minimisation conditions (44), this matrix becomes,

$$\begin{pmatrix} H_1^+ & H_2^+ \end{pmatrix} \left(\frac{m_3^2}{v_1 v_2} + \frac{1}{4} g_2^2\right) \begin{pmatrix} v_2^2 & v_1 v_2 \\ v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} H_1^- \\ H_2^- \end{pmatrix} \tag{53}$$

which has determinant zero leading to the two eigenvalues as :

$$\begin{aligned} m^2_{G^\pm} &= 0 \\ m^2_{H^\pm} &= \left(\frac{m_3^2}{v_1 v_2} + \frac{1}{4} g_2^2\right)(v_1^2 + v_2^2), && (54) \\ &= \frac{2 m_3^2}{sin 2\beta} + M_W^2 && (55) \end{aligned}$$

where $G^\pm$ represents the Goldstone mode. The physical states are obtained just by rotating the original states in terms of the $H_1,\ H_2$ fields by an mixing angle. The mixing angle in the present case (in the unitary gauge) is just tanβ:

$$\begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \begin{pmatrix} sin\beta & cos\beta \\ -cos\beta & sin\beta \end{pmatrix} \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} \tag{56}$$

CP odd Higgs and Goldstone Modes:

[10] The discussion in this section closely follows from the discussion presented in Ref. [27]

Let us now turn our attention to the CP-odd Higgs sector. The mass matrices can be written in a similar manner but this time for imaginary components of the neutral Higgs.

$$\left(\begin{array}{cc} ImH_1^0 & ImH_2^0 \end{array} \right) \left(\begin{array}{cc} m_1^2 + \frac{1}{8}(g_1^2+g_2^2)(v_1^2-v_2^2) & m_3^2 \\ m_3^2 & m_2^2 - \frac{1}{8}(g_1^2+g_2^2)(v_1^2-v_2^2) \end{array} \right) \left(\begin{array}{c} ImH_1^0 \\ ImH_2^0 \end{array} \right) \quad (57)$$

As before, again using the minimisation conditions, this matrix becomes,

$$\left(\begin{array}{cc} ImH_1^0 & ImH_2^0 \end{array} \right) m_3^2 \left(\begin{array}{cc} v_2/v_1 & 1 \\ 1 & v_1/v_2 \end{array} \right) \left(\begin{array}{c} ImH_1^0 \\ ImH_2^0 \end{array} \right) \quad (58)$$

which has determinant zero leading to the two eigenvalues as :

$$\begin{aligned} m_{G^0}^2 &= 0 \\ m_{A^0}^2 &= \left(\frac{m_3^2}{v_1 v_2} \right)(v_1^2+v_2^2) = \frac{2m_3^2}{sin2\beta} \end{aligned} \quad (59)$$

Similar to the charged sector, the mixing angle between these two states in the unitary gauge is again just tanβ.

$$\frac{1}{\sqrt{2}} \left(\begin{array}{c} A^0 \\ G^0 \end{array} \right) = \left(\begin{array}{cc} sin\beta & cos\beta \\ -cos\beta & sin\beta \end{array} \right) \left(\begin{array}{c} ImH_1^0 \\ ImH_2^0 \end{array} \right) \quad (60)$$

CP even Higgs:
Finally, let us come to the real part of the neutral Higgs sector. The mass matrix in this case is given by the following.

$$\left(\begin{array}{cc} ReH_1^0 & ReH_2^0 \end{array} \right) \frac{1}{2} \left(\begin{array}{cc} 2m_1^2 + \frac{1}{4}(g_1^2+g_2^2)(3v_1^2-v_2^2) & -2m_3^2 - \frac{1}{4}v_1v_2(g_1^2+g_2^2) \\ -2m_3^2 - \frac{1}{4}v_1v_2(g_1^2+g_2^2) & 2m_2^2 + \frac{1}{4}(g_1^2+g_2^2)(3v_2^2-v_1^2) \end{array} \right) \left(\begin{array}{c} ReH_1^0 \\ ReH_2^0 \end{array} \right) \quad (61)$$

Note that in the present case, there is no Goldstone mode. As before, we will use the minimisation conditions and further using the definition of m_A^2 from eq.(59), we have :

$$\left(\begin{array}{cc} ReH_1^0 & ReH_2^0 \end{array} \right) \left(\begin{array}{cc} m_A^2 sin^2\beta + M_z^2 cos\beta & -(m_A^2+m_Z^2) sin\beta cos\beta \\ -(m_A^2+m_Z^2) sin\beta cos\beta & m_A^2 cos^2\beta + M_z^2 sin\beta \end{array} \right) \left(\begin{array}{c} ReH_1^0 \\ ReH_2^0 \end{array} \right) \quad (62)$$

The matrix has two eigenvalues which are given by the two signs of the following equation:

$$m_{H,h}^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 \pm \{(m_A^2+m_Z^2)^2 - 4m_Z^2 m_A^2 cos^2 2\beta\}^{1/2} \right] \quad (63)$$

The heavier eigenvalue m_H^2, is obtained by taken the positive sign, whereas the lighter eigenvalue m_h^2 is obtained by taking the negative sign respectively. The mixing angle between these two states can be read out from the mass matrix of the above[11] as :

$$tan\ 2\alpha = \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}\ tan\ 2\beta \quad (64)$$

Tree Level Catastrophe:
So far we have seen that out of the eight Higgs degrees of freedom, three of them form the Goldstone modes after incorporating $SU(2) \times U(1)$ breaking and there are five *physical* Higgs bosons fields in the MSSM spectrum. These are the charged Higgs ($H^{\pm}$) a CP-odd Higgs (A) and two CP-even Higgs bosons

[11]The mixing angle for a 2×2 symmetric matrix, C_{ij} is given by

$$tan2\theta = 2C_{12}/(C_{22} - C_{11}).$$

(h, H). From the mass spectrum analysis above, we have seen that the mass eigenvalues of these Higgs bosons are related to each other. In fact, putting together all the eigenvalue equations, we summarise the relations between them as follows :

$$\begin{aligned} m_{H^\pm}^2 &= m_A^2 + m_W^2 > \max(M_W^2, m_A^2) \\ m_h^2 + m_H^2 &= m_A^2 + m_Z^2 \\ m_H &> max(m_A, m_Z) \\ m_h &< min(m_A, M_Z)|cos2\beta| < min(m_A, m_Z) \end{aligned} \tag{65}$$

Let us concentrate on the last relation of the above eq.(65). The condition on the lightest CP even Higgs mass, m_h, tell us that it should be equal to m_Z in the limit tanβ is saturated to be maximum, such that cos2$\beta \rightarrow 1$ and $m_A \rightarrow \infty$. If these limits are not saturated, it is evident that the light higgs mass is less that m_Z. This is one of main predictions of MSSM which could make it easily falsifiable from the current generation of experiments like LEP, Tevatron and the upcoming LHC. Given that present day experiments have not found a Higgs less that Z-boson mass, it is tempting to conclude that the MSSM is not realised in Nature. However caution should be exercised before taking such a route as our results are valid only at the tree level. In fact, in a series of papers in the early nineties [28], it has been shown that large one-loop corrections to the Higgs mass can easily circumvent this limit.

The light Higgs Spectrum at 1-loop

As mentioned previously, radiative corrections can significantly modify the mass relations which we have presented in the previous section. As is evident, these corrections can be very important for the light Higgs boson mass. Along with the 1-loop corrections previously, in the recent years dominant parts of two-loop corrections have also been available [29] with a more complete version recently given [30]. In the following we will present the one-loop corrections to the light Higgs mass and try to understand the implications for the condition eq.(65). Writing down the 1-loop corrections to the CP-even part of the Higgs mass matrix as :

$$M_{Re}^2 = M_{Re}^2(0) + \delta M_{Re}^2, \tag{66}$$

where $M_{Re}^2(0)$ represents the tree level mass matrix given by eq.(62) and δM_{Re}^2 represents its one-loop correction. The dominant one-loop correction comes from the top quark and stop squark loops which can be written in the following form:

$$\delta M_{Re}^2 = \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12} & \Delta_{22} \end{pmatrix}, \tag{67}$$

where

$$\begin{aligned} \Delta_{11} &= \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 sin^2\beta}\left[\frac{\mu(A_t+\mu cot\beta)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\right]^2\left(2 - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} ln\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right) \\ \Delta_{12} &= \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 sin^2\beta}\left[\frac{\mu(A_t+\mu cot\beta)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\right] ln\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + \frac{A_t}{\mu}\Delta_{11} \\ \Delta_{22} &= \frac{3G_F m_t^4}{\sqrt{2}\pi^2 sin^2\beta}\left[ln\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^2} + \frac{A_t(A_t+\mu cot\beta)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}} ln\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right] + \frac{A_t}{\mu}\Delta_{11} \end{aligned} \tag{68}$$

In the above G_F represents Fermi Decay constant, m_t, the top mass, $m_{\tilde{t}_1}^2$, $m_{\tilde{t}_1}^2$ are the eigenvalues of the stop mass matrix and A_t is the trilinear scalar coupling (corresponding to the top Yukawa coupling) in the stop mass matrix. μ and the angle β have their usual meanings. Taking in to account these corrections, the condition (65) takes the form:

$$m_h^2 \ < \ m_Z^2\cos^2 2\beta + \Delta_{11}\cos^2\beta + \Delta_{12}\sin2\beta + \Delta_{22}\sin^2\beta \tag{69}$$

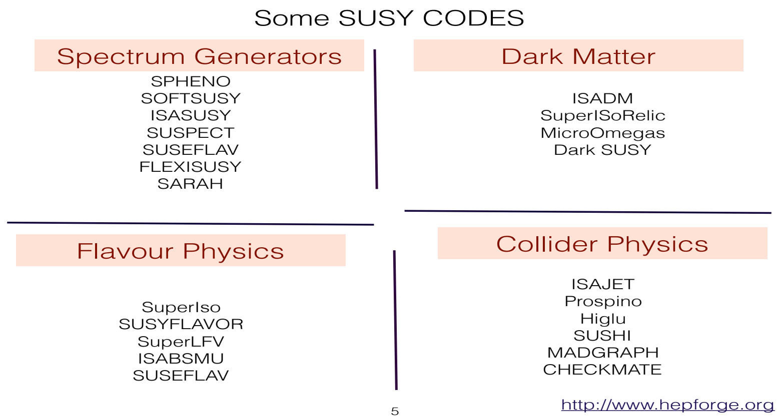

Fig. 5: Some of the computer codes relevant for studying supersymmetric phenomenology.

Given that m_t is quite large, almost twice the m_Z mass, for suitable values of the stop masses, it is clear that the tree level upper limit on the light Higgs mass is now evaded. However, a reasonable upper limit can still be got by assuming reasonable values for the stop mass. For example assuming stop masses to be around 1 TeV and maximal mixing the stop sector, one attains an upper bound on the light Higgs mass as:

$$m_h \lesssim 135 \text{ GeV}. \tag{70}$$

8.5 Feynman Rules

In this section, we have written down all the mass matrices of the superpartners, their eigenvalues and finally the eigenvectors which are required to transform the superpartners in to their physical basis. The feynman rules corresponding to the various vertices have to be written down in this basis. Thus various soft supersymmetry breaking and supersymmetry conserving parameters entering these mass matrices would now determine these couplings as well as the masses, which in turn determine the strength of various physical processes like crosssections and decay rates. A complete list of the Feynman rules in the mass basis can be found in various references like Physics Reports like Haber & Kane [26] and D Chung et. al [32] and also in textbooks like Sparticles [27] and Baer & Tata [31]. A complete set of Feynman rules is out of reach of this set of lectures. Here I will just present two examples to illustrate the points I have been making here.

Due to the mixing between the fermionic partners of the gauge bosons and the fermionic partners of the Higgs bosons, the gauge and the yukawa vertices get mixed in MSSM. We will present here the vertices of fermion-sfermion-chargino and fermion-sfermion-neutralino where this is evident.
(i) Fermion-Sfermion-Chargino :
This is the first vertex on the left of the figure. The explicit structure of this vertex is given by:

$$\tilde{C}_{iAX} = C^R_{iAX} P_R + C^L_{iAX} P_L \tag{71}$$

where $P_L(P_R)$ are the project operators[12] and C^R and C^L are given by

$$c^R_{iAX} = -g_2 (U)_{A1} R^{\nu}_{Xi} \tag{72}$$

$$C^L_{iAX} = g_2 \frac{m_{l_i}}{\sqrt{2} m_W cos\beta} (V)_{A2} R^{\nu}_{Xi} \tag{73}$$

In the above U and V are the diagonalising matrices of chargino mass matrix M_C, R^ν is the diagonalising matrix of the sneutrino mass matrix, $M^2_{\tilde{\nu}}$. And the indices A and X runs over the dimensions of the

[12] $P_L = (1-\gamma_5)/2$ and $P_R = (1+\gamma_5)/2$.

respective matrices ($A = 1, 2$ for Charginos, $X = 1, 2, 3$ for sneutrinos), whereas i as usual runs over the generations, m_{l_i} is the mass of the i th lepton and rest of the parameters carry the standard definitions.
(ii) Fermion-Sfermion-Neutralino :
In a similar manner, the fermion-sfermion-neutralino vertex is given by:

$$\tilde{D}_{iAX} = D^R_{iAX} P_R + D^L_{iAX} P_L \tag{74}$$

where D^L and D^R have the following forms:

$$D^R_{iAX} = -\frac{g_2}{\sqrt{2}} \left\{ \left[-N_{A2} - N_{A1} tan\theta_W \right] R^l_{Xi} + \frac{m_{l_i}}{m_W cos\beta} N_{A3} R^l_{X,i+3} \right\} \tag{75}$$

$$D^L_{iAX} = -\frac{g_2}{\sqrt{2}} \left\{ \frac{m_{l_i}}{m_W cos\beta} N_{A3} R^l_{Xi} + 2 N_{A1} tan\theta_W R^l_{X,i+3} \right\} \tag{76}$$

In the above N is diagonalising matrices of neutralino mass matrix M_N, R^l is the diagonalising matrix of the slepton mass matrix, $M^2_{\tilde{l}}$. And the indices A and X runs over the dimensions of the respective matrices ($A = 1, .., 4$ for neutralinos, $X = 1, .., 6$ for sleptons), whereas i as usual runs over the generations.

9 Phenomenology of Supersymmetric Models

We now have all the ingredients to discuss the phenomenology of supersymmetric theories. Before doing that, let us summarise some main features of supersymmetric theories:

- Supersymmetric theories are calculable. Compared to other extensions of SM, this is one of the most important positive characteristics of supersymmetric theories.
- The three gauge couplings unify in MSSM leading to a successful incorporation of it in a Grand Unified theory.
- As discussed before, the Higgs mass remains stable under radiative corrections.
- The MSSM with R-parity also provides a natural dark matter candidate in terms of the lightest supersymmetric particle (LSP). In most models, the LSP is the lightest neutralino providing a good WIMP candidate.

The spectrum of the supersymmetric particles discussed so far in sections 8 is defined at the tree level, except for the Higgs mass. In actual phenomenological calculations, radiative corrections to the all mass matrices and couplings are computed. The MSSM parameters are masses are typically defined at 1-loop level and the SM parameters defined at two loop level. Most of these calculations are tedious, long and are done by computer programs called spectrum generators. These programs not only compute the masses at high precision, but also compute that the electroweak symmetry breaking conditions, direct and indirect constraints on supersymmetric spectrum, supersymmetric contributions to rare processes like $b \to s\gamma$ etc are also computed. Some of these spectrum generators are SOFTSUSY, SPHENO,suspect, etc. We have constructed our own spectrum generator SUSEFLAV. While these have been the traditional spectrum generators, recently, there have been more flexible programs available which have much larger applicability like Flexisusy, SARAH etc. The reference [33] provides more detailed discussions in this regard. A small summary of available programs is available in Fig 5.

The phenomenology of supersymmetry can divided in to three small sub-areas through which we probe supersymmetry : (a) LHC searches (b) Flavour and other precision measurements like electric and magnetic dipole moments (c) Astrophysical/cosmological probes like dark matter, matter-anti-matter relic density etc. These three ‘roads’ are schematically depicted in Fig 6. We now review the present status of each of these sectors. An additional sector which has huge impact on various models of supersymmetry breaking is the Higgs sector which will be summarised in the next section. For a recent review on the present status of supersymmetric models, please see, N. Craig’s review [34].

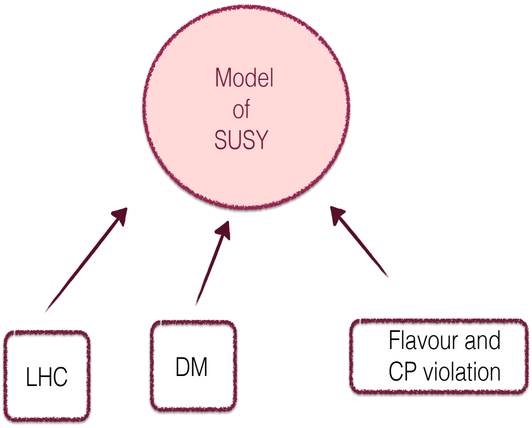

Fig. 6: A schematic representation of the three directions through which we probe supersymmetry

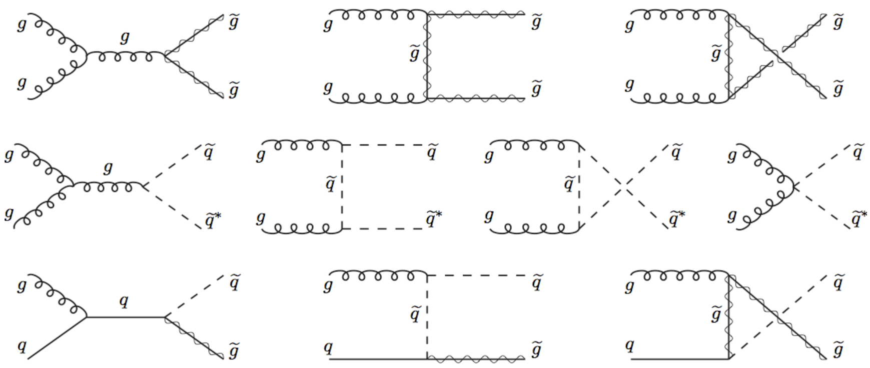

Fig. 7: A list of Feynman diagrams contributing to the production of gluinos and first two generation squarks is presented above. From Ref. [59]

9.1 LHC limits

At the LHC, the dominant processes are strong processes, which lead to the production of strongly interacting supersymmetric particles, such as gluinos and squarks. The main production channels are through qq, qg, and gg initial states as depicted in Fig 7. These production cross-sections are large about 1pb first two generations of squarks and gluinos if their masses are around a TeV. The cross-sections however fall off rapidly with increasing masses as shown in Fig. 8. As can be seen in the figure, the production cross-sections for stops are about an order of magnitude smaller for 100 GeV stops, but fall even more rapidly reaching $\sim$ 10 fb for 1 TeV stops. The backgrounds are very large, typically by several orders of magnitude as shown in Fig. 8. Inspite of these difficulties, the LHC experiments, ATLAS and CMS looking for supersymmetry have already put strong constraints on the masses of the superpartners.

As expected the strongest constraints are on the coloured supersymmetric partners such as gluinos and first two generation squarks. Gluinos are ruled out between 0.8-2.1 TeV depending on the lightest neutralino mass. Similarly the first two generation squarks are ruled out up to 1-2.0 TeV. The third generation top partners, the stops are ruled out between 200-700 GeV. The limits on weakly interacting particles such as charginos and neutralinos are steadily improving and reaching to 1 TeV in some extreme limits. It should be noted that most of these limits are within simplified models of supersymmetry and

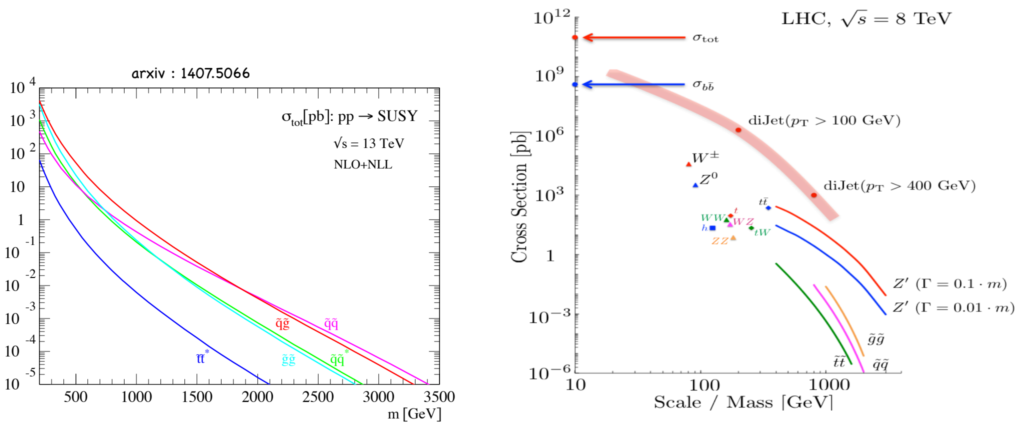

Fig. 8: A summary of production crosssection magnitudes (left) and backgrounds (right) is presented in the above figures.

therefore could lead to large variations in various other models.

A summary of these limits are presented in Fig. 9.

9.2 Flavour Constraints

Flavour physics is already covered in this school [37]. Here, I focus on the discussion relavant for MSSM. The supersymmetric soft terms introduced in the Sec. 6 contain flavour violating soft terms.

$$\begin{aligned}\mathcal{L}_{soft} &= m^2_{Q_{ii}}\tilde{Q}^\dagger_i\tilde{Q}_i + m^2_{u_{ii}}\tilde{u^c}^\star_i\tilde{u^c}_i + m^2_{d_{ii}}\tilde{d^c}^\star_i\tilde{d^c}_i + m^2_{L_{ii}}\tilde{L}^\dagger_i\tilde{L}_i + m^2_{e_{ij}}\tilde{e^c}^\star_i\tilde{e^c}_i \\ &+ \left(\Delta^{u,d}_{i\neq j}\right)_{LL}\tilde{Q}^\dagger_i\tilde{Q}_j + \left(\Delta^{u}_{i\neq j}\right)_{RR}\tilde{u^c}^\star_i\tilde{u}^c_j + \left(\Delta^{d}_{i\neq j}\right)_{RR}\tilde{d^c}^\star_i\tilde{d}^c_j \\ &+ \left(\Delta^{l}_{i\neq j}\right)_{LL}\tilde{L}^\dagger_i\tilde{L}_j + \left(\Delta^{l}_{i\neq j}\right)_{LR}\tilde{e}^\star_i\tilde{e^c}_j + \ldots\end{aligned} \tag{77}$$

As mentioned previously, the soft mass terms m^2_{ij} and the trilinear scalar couplings A_{ijk} can violate flavour. This gives us new flavour violating structures beyond the standard CKM structure of the quark sector which can also be incorporated in the MSSM. Furthermore, all these couplings can also be complex and thus could serve as new sources of CP violation in addition to the CKM phase present in the Standard Model. Given that all these terms arbitrary and could be of any magnitude close to weak scale, these terms can contribute dominantly compared to the SM amplitudes to various flavour violating processes at the weak scale, like flavour violating decays like $b \to s + \gamma$ or flavour oscillations like $K^0 \leftrightarrow \bar{K}^0$ etc and even flavour violating decays which do not have any Standard Model counterparts like $\mu \to e + \gamma$ etc. A sample Feynman diagrams are listed in Fig. 10. The CP violating phases can also contribute to electric dipole moments (EDM)s which are precisely measured at experiments.

To analyse the phenomenological impact of these processes on these terms, an useful and powerful tool is the so called Mass Insertion (MI) approximation. In this approximation, we use flavour diagonal gaugino vertices and the flavour changing is encoded in non-diagonal sfermion propagators. These propagators are then expanded assuming that the flavour changing parts are much smaller than the flavour diagonal ones. In this way we can isolate the relevant elements of the sfermion mass matrix for a given flavour changing process and it is not necessary to analyse the full 6×6 sfermion mass matrix. Using this method, the experimental limits lead to upper bounds on the parameters (or combinations of) $\delta^f_{ij} \equiv \Delta^f_{ij}/m^2_{\tilde{f}}$, known as mass insertions; where Δ^f_{ij} is the flavour-violating off-diagonal entry appearing in the $f = (u, d, l)$ sfermion mass matrices and $m^2_{\tilde{f}}$ is the average sfermion mass. In addition, the

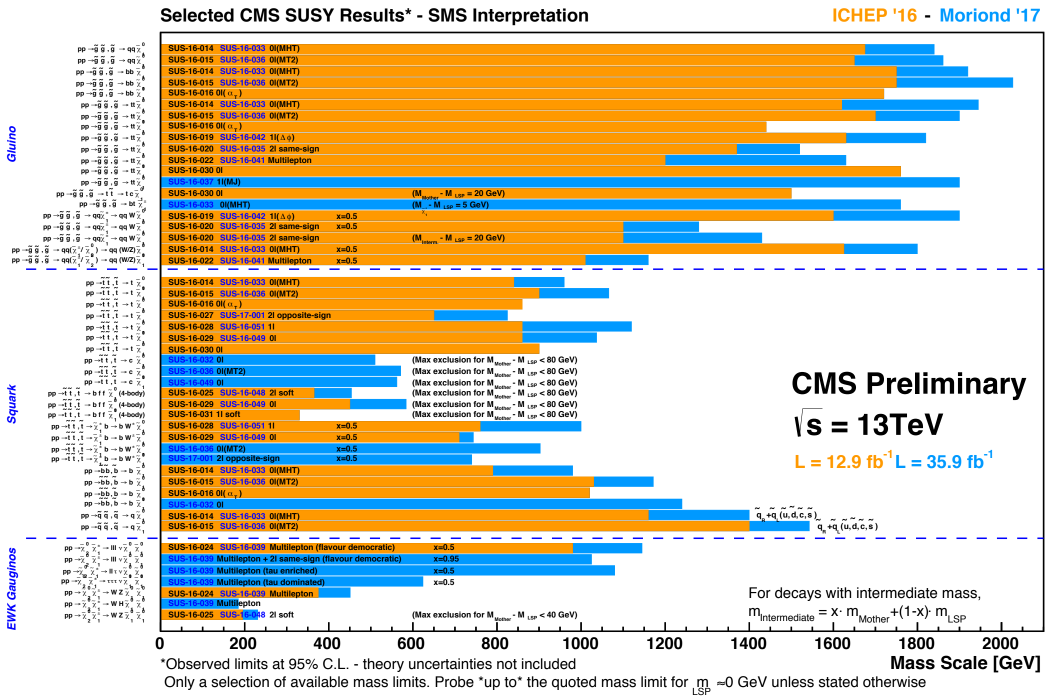

Fig. 9: A summary of current limits are presented from the CMS [35] experiment. Similar results are also presented by the ATLAS experiment. [36]

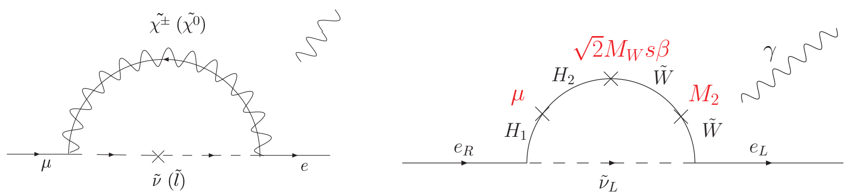

Fig. 10: Feynman Diagram contributing to the rare decay $\mu \to e + \gamma$ in mass eigenstate basis (left) and in mass insertion basis or flavour basis (right).

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mass-insertions are further sub-divided into LL/LR/RL/RR types, labeled by the chirality of the corresponding SM fermions. The latest set of results on the hadronic sector can be found in [38] and in the leptonic sector in [39]. The limits on various δ's coming from various flavour violating processes have been computed and tabulate in the literature and can be found for instance in Ref. [40, 41] (For a more recent statistical approach, see also [42]).

These limits show that the flavour violating terms should be typically at least a couple of orders of magnitude suppressed compared to the flavour conserving soft terms. The flavour problem could also be alleviated by considering decoupling soft masses or alignment mechanisms (see [40] and references there in). While this is true for the first two generations of soft terms, the recent results from B-factories have started constraining flavour violating terms involving the third generation too.

ij AB	LL	LR	RL	RR
12	1.4×10^{-2}	9.0×10^{-5}	9.0×10^{-5}	9.0×10^{-3}
13	9.0×10^{-2}	1.7×10^{-2}	1.7×10^{-2}	7.0×10^{-2}
23	1.6×10^{-1}	4.5×10^{-3}	6.0×10^{-3}	2.2×10^{-1}

Fig. 11: Bounds on $(\delta)^d_{ij})$ from Flavour data in the hadronic sector from the paper [41]. The parameter space chosen is such that the third generation squark masses are close to 500 GeV and the weakly interacting gauginos are around 200 GeV. These bounds scale inversely with the squark mass and thus can be scaled for the present limits on them.

An important point is that if we set all the flavour violating off-diagonal entries to zero through some mechanism or by choosing an appropriate supersymmetry breaking mechanism (as we will see in the next section), contribution from supersymmetric sector to flavour violation will not be completely zero. This is because CKM (Cabibbo-Kobayashi-Masakawa) matrix will induce non-trivial flavour violating interactions between the SM fermion and it's supersymmetric partner. One of the strongest constraints in this case comes from BR($b \to s + \gamma$) which has been measured very precisely by the experimental collaborations (with an error of about 5% at the one sigma level). The present numbers are as follows [43]

$$\begin{aligned} BR(b \to s + \gamma)^{exp} &= (3.43 \pm 0.21 \pm 0.07) \times 10^{-4} \\ BR(b \to s + \gamma)^{SM} &= (3.36 \pm 0.23) \times 10^{-4} \end{aligned} \tag{78}$$

Given the closeness of the Standard Model expectation to the experimental number, any new physics should either be very heavy such that it's contributions to this rare process are suppressed or should contain cancellations within its contributions such that the total SM+ New physics contribution is close to the experimental value. Both these scenarios are possible within the MSSM. If supersymmetric partners are heavy $\gtrsim$ a few TeV, then their contributions to $b \to s + \gamma$ are highly suppressed. On the other hand, it is possible that the dominant contributions from charged Higgs and the chargino diagrams cancel with each other (they come with opposite sign) for a large region of the parameter space. The general class of new physics models which do not introduce any new flavour violation other than the one originating from the CKM matrix in the Standard Model come under the umbrella of "Minimal Flavour Violation" [44].

9.3 Dark Matter

While supersymmetry offers many dark matter candidates like axino, saxion, gravitino etc, one of the most popular candidate is the lightest neutralino. For reviews, please see [45]. The neutralinos are as we have seen linear combinations of neutral gauginos and higgsinos. The composition of the lightest neutralino determines it's annihilation cross-section, which in turn determines its mass required to satisfy the relic density of the universe. The relic density has been measured very well by the satellite based experiments of the cosmic microwave background radiation (CMB), notably by WMAP and the Planck satellites. The present day relic density is given to be [46]

$$\Omega_{\text{CDM}} h^2 = 0.01199 \pm 0.0022 \tag{79}$$

Note that the lightest neutralino from the neutralino mass matrix of eq.(45) has the form

$$M_{\chi_1^0} = N_{\tilde{B}1} \tilde{B}^0 + N_{\tilde{W}1} \tilde{W}^0 + N_{\tilde{H}_u 1} \tilde{H}_u^0 + N_{\tilde{H}_d 1} \tilde{H}_d^0 \tag{80}$$

We now look at various possible compositions of the LSP to satisfy this relic density.
f (a) *Pure Bino:* If the neutralino is a pure Bino, the annihilation cross-section is given by [47]

$$\langle \sigma_\chi v \rangle = \frac{3g^4 \tan\theta_W^4 r(1+r^2)}{2\pi m_{\tilde{e}_R}^2 x(1+r)^4} \tag{81}$$

where $x = \frac{M_1}{T}$ the mass of the bino over the temperature and $r = .$ θ_W is the weak mixing angle, or the Weinberg angle. The relic density in this case is given by

$$\Omega_{\tilde{B}} h^2 = 1.3 \times 10^{-2} \left(\frac{m_{\tilde{e}_R}}{100 \text{ GeV}}\right)^2 \frac{(1+r)^4}{r^2(1+r)^2} \left(1 + 0.07 \log \frac{\sqrt{r}\ \ 100 \text{ GeV}}{m_{\tilde{e}_R}}\right) \tag{82}$$

The above relic density is typically large for reasonable range of parameters. One thus invokes typically co-annihilating partners which are very close in the mass with the bino, there by increasing the cross-section and there by bringing down the relic density to acceptable levels.
(b) *Pure Wino:* In this case the cross-section of the dark matter particle goes as g, the weak coupling and is given by

$$\langle \sigma_\chi v \rangle = \left(\frac{3g^4}{16\pi M_2^2}\right) \tag{83}$$

where M_2 stands for the Wino mass. The relic density is approximately given by

$$\Omega_{\tilde{W}} h^2 \sim 0.13 \left(\frac{M_2}{2.5 \text{ TeV}}\right)^2 \tag{84}$$

This requires heavy Neutralino of the order of 2.5 TeV.
(c) *Pure Higgsino:* In this case the cross-section of the dark matter particle is given by

$$\langle \sigma_\chi v \rangle = \frac{3g^4}{512\pi\mu^2} \left(21 + 3\tan\theta_W^2 + 11\tan\theta_W^4\right) \tag{85}$$

The relic density in this case is given by

$$\Omega_{\tilde{H}} h^2 \sim 0.10 \left(\frac{\mu}{1 \text{ TeV}}\right)^2 \tag{86}$$

A neutralino of 1 TeV would be required to satisfy the relic density. In summary a pure bino neutralino can be light but would require co-annihilating partners (or other mechanisms) to have the correct relic density, whereas both a pure Higgsino or a pure Wino would have to close to a TeV or larger. Admixtures of various components can however give the right relic density.

In addition to the relic density constraint, the WIMP dark matter is tested at the various direct detection experiments summarised in your cosmology lectures [13]. They also receive constraints from various indirect dark matter detection experiments like FERMI, AMS 02 etc. Here we present the updated constraints for various supersymmetric models from Ref. [48]. From the figures, one can see that supersymmetric neutralino dark matter is strongly constrained from the LUX results. Regions with co-annihilations are still largely allowed.

9.4 Higgs Mass Constraint

This part of the lectures might have been discussed already in the school [49]. As we have seen in the MSSM, the lightest neutral Higgs mass is a calculable quantity. It an be considered as a prediction of a supersymmetric model as it is dependent dominantly on a very few parameters such as $\tan\beta$, stop masses, and the stop mixing paramter $X_t \equiv A_t + \mu\cot\beta$. Thus the measured Higgs can be provide a strong constraint on supersymmetric models. In fact, this constraint is as strong as the constraint from

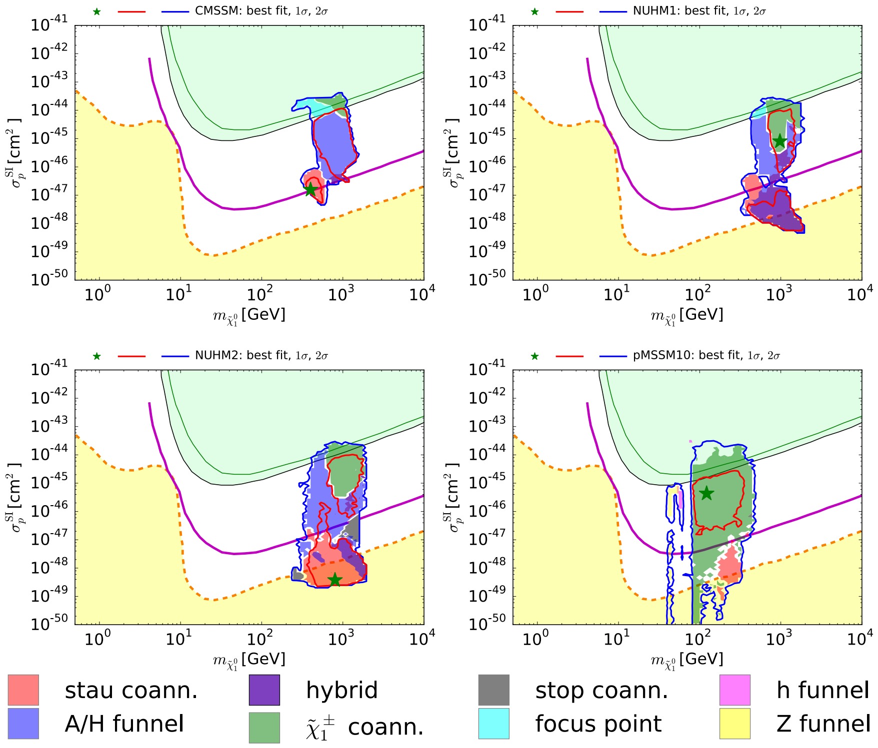

Fig. 12: Status of supersymmetric dark matter searches [48]: exclusion limits in the CMSSM (upper left), the NUHM1 (upper right), the NUHM2 (lower left) and the pMSSM10 (lower right). The red and blue solid lines are the $\Delta\chi^2 = 2.30$ and 5.99 contours, and the solid purple lines show the projected 95% exclusion sensitivity of the LUX-Zepelin (LZ) experiment. The green and black lines show the current sensitivities of the XENON100 and LUX experiments, respectively, and the dashed orange line shows the astrophysical neutrino 'floor', below which astrophysical neutrino backgrounds dominate (yellow region).

non-discovery of supersymmetry particles at the LHC, summarised in section 9.1 if not stronger. The calculation of the Higgs mass is done currently at three loop level. While the 1-loop corrections are large, two loop corrections can also be significant giving corrections to the order of 10-12 GeV in regions of parameter space. While recently full three loop calculations are being done, the efforts are towards bringing down the theoretical error in the Higgs mass computation to be around 1 GeV. For a recent review on the computation of the Higgs mass in MSSM, including various schemes, please see Patrick Draper's review [50].

To see implications of the Higgs mass measurement on supersymmetric stop parameter space, please see fig (13). We can see from the figure that for zero stop mixing $X_t \sim 0$, the stops should be above 3 TeV or heavier to give the correct Higgs mass. On the other hand, in the limit of maximal stop mixing $X_t \sim \sqrt{6} m_{susy}$, where $m_{susy} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, stop can be as light as a couple hundred GeV. This 'no-go ' theorem should be considered from the point of view of discoverability at LHC. While stops up to 1-1.5 TeV can be discovered by the time full run of LHC is completed, it is not possible produce stops of the order of 3-4 TeV at the LHC. Since typically stops are the lightest colored particles in most supersymmetric models, this has strong implications on discoverability of various supersymmetric models.

Due to the requirement that the X_t should be large to generate the right higgs mass, another important constraint comes to play. This is from the scalar potential of the MSSM. Remember that X_t is

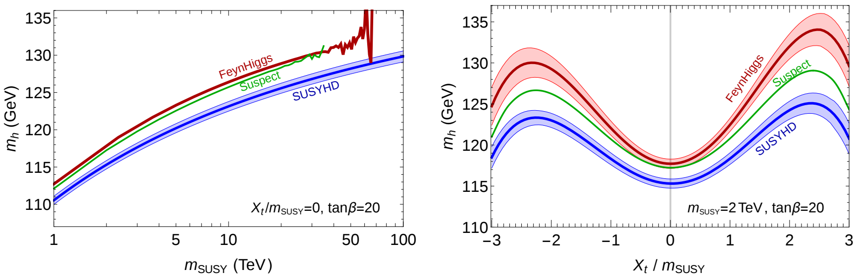

Fig. 13: A comparison [51] of the various computations of the Higgs mass in MSSM: the EFT computation (lower blue band) is compared to two existing codes; FeynHiggs and Suspect. A degenerate SUSY spectrum was used with mass m_{SUSY} in the $\overline{\mathrm{DR}}$-scheme with $\tan\beta = 20$. The plot on the left shows m_{h} vs m_{SUSY} for vanishing stop mixing. The plot on the right shows m_{h} vs X_t/m_{SUSY} for $m_{\mathrm{SUSY}} = 2$ TeV. On the left plot the instability of the non-EFT codes at large m_{SUSY} is visible.

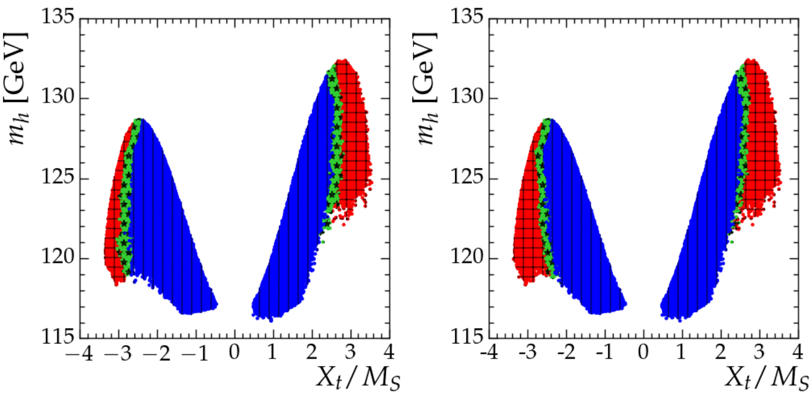

Fig. 14: Stable (blue, vertical lines), meta-stable (green, stars) and unstable (red, checkered) vacuum in the m_{h} vs X_t/M_S plane, from three-field analysis (left) and four-field analysis (right) [52].

a trilinear term of stops and the Higgs, and thus, there is a danger that the stops get a vacuum expectation value for large values of it. If stop fields get a vaccum expectation value, they break the charge and colour symmetries of the Standard Model which is unwanted and unphysical. Thus these regions of the parameter space should be avoided. In Fig. (14), we show the present constraints from charge and colour breaking minima and in the parameter space of X_t and higgs mass from the Ref. [52]. As can be seen, a significant portion of the Higgs mass region is invalid or unphysical due to constraints from charge and colour breaking minima.

10 'Standard' Models of Supersymmetry breaking

So far we have included supersymmetry breaking within the MSSM through a set of explicit supersymmetry breaking soft terms however, at a more fundamental we would like to understand the origins of these soft terms as coming from a theory where supersymmetry is spontaneously broken. In a previous section, we have mentioned that supersymmetry needs to be broken spontaneously in a hidden sector and then communicated to the visible sector through a messenger sector. In the below we will consider two main models for the messenger sector (a) the gravitational interactions and (b) the gauge interactions. But before we proceed to list problems with the general form soft supersymmetry breaking terms as discussed in the previous section. This is essential to understand what kind of constructions of supersymmetric breaking models are likely to be realised in Nature and thus are consistent with phenomenology.

The way we have parameterised supersymmetry breaking in the MSSM, using a set of gauge invariant soft terms, at the first sight, seems to be the most natural thing to do in the absence of a complete theory of supersymmetry breaking. However, this approach is itselves laden with problems as we realise once we start confronting this model with phenomenology. The two main problems can be listed as below:

(i). Large number of parameters

Compared to the SM, in MSSM, we have a set of more than 50 new particles; writing down all possible gauge invariant and supersymmetry breaking soft terms, limits the number of possible terms to about 105. All these terms are completely arbitrary, there is no theoretical input on their magnitudes, relative strengths, in short there is no theoretical guiding principle about these terms. Given that these are large in number, they can significantly effect the phenomenology. In fact, the MSSM in its softly broken form seems to have lost predictive power except to say that there are some new particles within a broad range in mass(energy) scale. The main culprit being the large dimensional parameter space $\sim$ 105 dimensional space which determines the couplings of the supersymmetric particles and their the masses. If there is a model of supersymmetry breaking which can act as a guiding principle and reduce the number of free parameters of the MSSM, it would only make MSSM more predictive.

(ii). Large Flavour and CP violations. We have seen in the previous section that generic supersymmetry breaking leads to large flavour and CP violating soft terms. The limits are very strong on these terms. In light of this stringent constraint, it is more plausible to think that the fundamental supersymmetry breaking mechanism some how suppresses these flavour violating entries. Similarly, this mechanism should also reduce the number of parameters such that the MSSM could be easily be confronted with phenomenology and make it more predictive. We will consider two such models of supersymmetry breaking below which will use two different kinds of messenger sectors.

10.1 Minimal Supergravity

In the minimal supergravity framework, gravitational interactions play the role of messenger sector. Supersymmetry is broken spontaneously in the hidden sector. This information is communicated to the MSSM sector through gravitational sector leading to the soft terms. Since gravitational interactions play an important role only at very high energies, $M_p \sim O(10^{19})$ GeV, the breaking information is passed on to the visible sector only at those scales. The strength of the soft terms is characterised roughly by, $m_{\tilde{f}}^2 \approx M_S^2/M_{planck}$, where M_S is the scale of supersymmetry breaking. These masses can be comparable to weak scale for $M_S \sim 10^{10}$ GeV. This M_S^2 can correspond to the F-term vev of the Hidden sector. The above mechanism of supersymmetry breaking is called supergravity (SUGRA) mediated supersymmetry breaking.

A particular class of supergravity mediated supersymmetry breaking models are those which go under the name of "minimal" supergravity. This model has special features that it reduces to total number of free parameters determining the entire soft spectrum to five. Furthermore, it also removes the dangerous flavour violating soft terms in the MSSM. The classic features of this model are the following boundary conditions to the soft terms at the high scale $\sim\ M_{Planck}$:

– All the gaugino mass terms are equal at the high scale.

$$M_1 = M_2 = M_3 = M_{1/2}$$

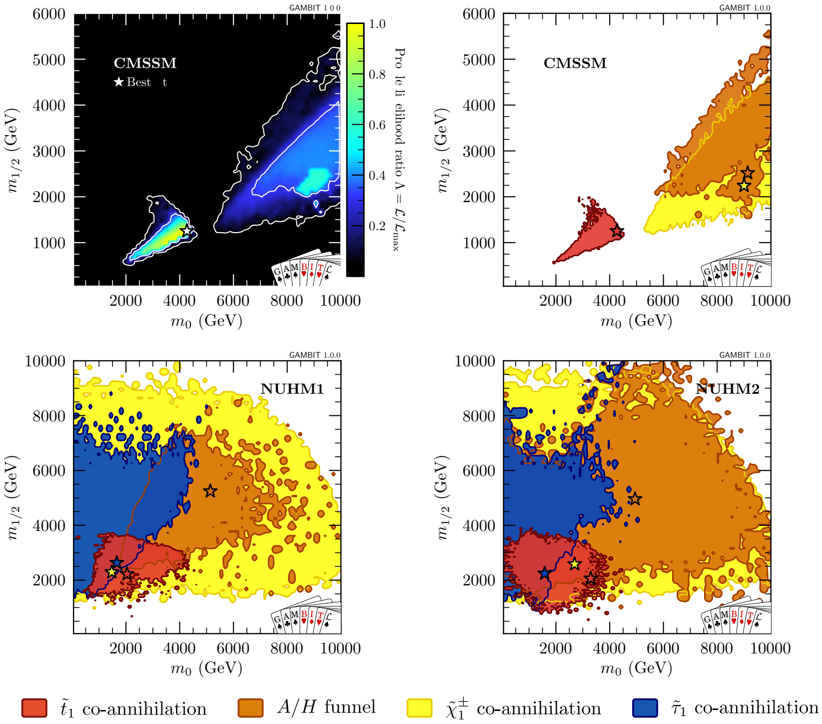

Fig. 15: Status of the CMSSM (top) and NUHM1 and NUHM2 (bottom) models according to GAMBIT [53]. Top left: the profile likelihood ratio (top left) in the $m_0 - m_{1/2}$ plane of the CMSSM. The white lines depict the 68% and 95% CL contours while the white star indicates the best fit. Top right and bottom plots: colouring of the 95% CL regions to indicate which mechanisms contribute to keeping the neutralino relic density below the observed value. Note that the colouring is not exclusive, i.e. overlapping colours indicate that multiple mechanisms may contribute in the given region.

– All the scalar mass terms at the high scale are equal.

$$m^2_{\phi_{ij}} = m_0^2 \delta_{ij}$$

– All the trilinear scalar interactions are equal at the high scale.

$$A_{ijk} = A h_{ijk}$$

– All bilinear scalar interactions are equal at the high scale.

$$B_{ij} = B$$

Using these boundary conditions, one evolves the soft terms to the weak scale using renormalisation group equations. It is possible to construct supergravity models which can give rise to such kind of strong universality in soft terms close to Planck scale. This would require the Kahler potential of the theory to be of the canonical form. As mentioned earlier, the advantage of this model is that it drastically reduces the number of parameters of the theory to about five, m_0, M (or equivalently M_2), ratio of the *vevs* of the two Higgs, tanβ, A, B. Thus, these models are also known as 'Constrained' MSSM in literature. The supersymmetric mass spectrum of these models has been extensively studied in literature. The Lightest Supersymmetric Particle (LSP) is mostly a neutralino in this case.

The present status of the CMSSM is summarised in a detailed analysis by the GAMBIT collaboration [53]. As can be seen from Fig.15, the most of the valid regions point to a very heavy supersymmetric spectrum way outside the reach of the LHC.

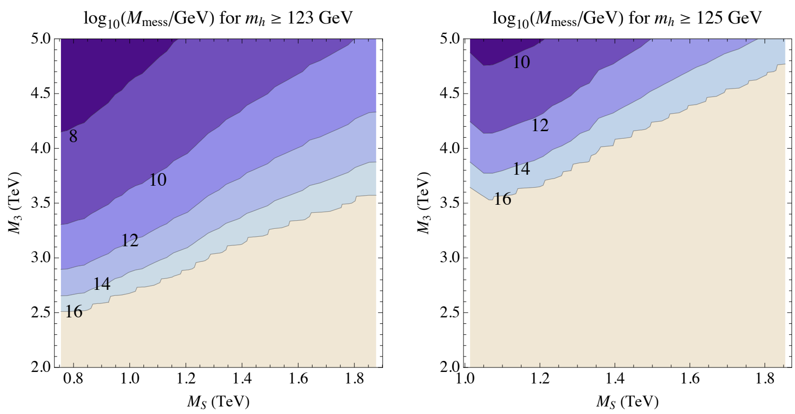

Fig. 16: Implications of the Higgs discovery on minimal GMSB models [54]: the coloured regions indicate the messenger scale required to produce a sufficiently large $|A_t|$ for $m_{\rm h} = 123$ GeV (left) and $m_{\rm h} = 125$ GeV (right) through renormalization group evolution.

10.2 Gauge Mediated Supersymmetry breaking

In a more generic case, the Kahler potential need not have the required canonical form. In particular, most low energy effective supergravities from string theories do not posses such a Kahler potential. In such a case, large FCNC's and again large number of parameters are expected from supergravity theories. An alternative mechanism has been proposed which tries to avoid these problems in a natural way. The key idea is to use gauge interactions instead of gravity to mediate the supersymmetry breaking from the hidden (also called secluded sector sometimes) to the visible MSSM sector. In this case supersymmetry breaking can be communicated at much lower energies ~ 100 TeV.

A typical model would contain a susy breaking sector called 'messenger sector' which contains a set of superfields transforming under a gauge group which 'contains' G_{SM}. Supersymmetry is broken spontaneously in this sector and this breaking information is passed on to the ordinary sector through gauge bosons and their fermionic partners in loops. The end-effect of this mechanism also is to add the soft terms in to the lagrangian. But now these soft terms are flavour diagonal as they are generated by gauge interactions. The soft terms at the messenger scale also have simple expressions in terms of the susy breaking parameters. In addition, in minimal models of gauge mediated supersymmetry breaking, only one parameter can essentially determine the entire soft spectrum.

In a similar manner as in the above, the low energy susy spectrum is determined by the RG scaling of the soft parameters. But now the high scale is around 100 TeV instead of M_{GUT} as in the previous case. The mass spectrum of these models has been studied in many papers. The lightest supersymmetric particle in this case is mostly the gravitino in contrast to the mSUGRA case.

The discovery of the Higgs boson with a mass range ~ 126 GeV has put strong constraints on the (minimal) GMSB models. In Fig. 16, we present the analysis of Ref. [54] which shows that it is not possible to generate the correct higgs mass in GMSB models unless the stop and the gluino spectrum is made very heavy, much out of the reach of LHC. Several models have been proposed since then to generate Higgs mass while keeping the stops light $\sim 1 - 2$TeV. The popular among them involve adding Yukawa interactions between the messengers and the MSSM fields in addition to the gauge interactions. A survey of these kind of models is presented in Ref. [55].

Another popular supersymmetry breaking mechanism is called Anomaly mediated supersymmetry breaking [66], which are not covered in this set of lectures.

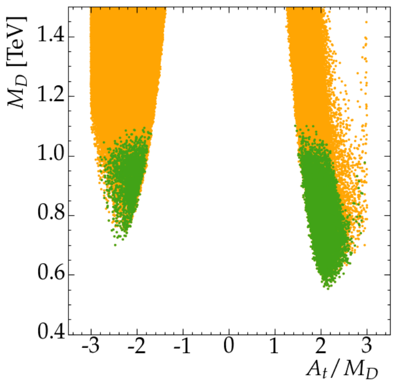

Fig. 17: Parameter space allowed for degenerate/compressed MSSM scenarios [56]. The green (orange) regions in the $A_t - M_D$ plane are consistent with the experimental value of $(g-2)_\mu$ at 2σ (3σ).

10.2.1 Radiative Electroweak symmetry breaking

In both gravity mediated as well as gauge mediated supersymmetry breaking models, we have seen that RG running effects have to included to study the soft terms at the weak scale. Typically, the soft masses which appear at those scales are positive at the high scale. But radiative corrections can significantly modify the low scale values of these parameters; in particular, making one of the Higgs mass squared to be negative at the weak scale leading to spontaneous breaking of electroweak symmetry. This mechanism is called radiative electroweak symmetry breaking.

10.3 Escaping The LHC limits

The LHC has not seen any signals of supersymmetric particles. This has put strong constraints on various supersymmetric models as we have seen. In fact, in most models, this would push all the supersymmetric particles to be very heavy $\sim$ with masses around several TeV. However, it could be that the supersymmetry particles are present within masses close to TeV, and they somehow escaped detection at the LHC. Several ideas were presented : stealth supersymmetry, compressed/degenerate supersymmetry, R-parity violation etc.

In the following we will not go in to the details of all the possible scenarios discussed above but make a few comments on the compressed/degenerate supersymmetry models. In these models, all the supersymmetric particles are almost degenerate in mass. Thus the decay chains of supersymmetric particles produced at LHC will end up leading to very soft (very low energy) final state particles that will not trigger the detectors at the LHC. Thus the only constraints would be from the Higgs mass, $b \to s + \gamma$ and other indirect constraints. In Fig.17 we show the parameter space remaining after taking in to consideration all these constraints. As you can see, the mass spectrum of MSSM can still be low to give the correct contribution to muon $g-2$. The degenerate MSSM scenarios are tested by the mono-jet, mono-photon searches at the LHC. The current limits can be found at [35].

11 Remarks

The present set of lectures are only a set of elementary introduction to the MSSM. More detailed accounts can be found in various references which we have listed at various places in the text. In preparing for these set of lectures, I have greatly benefitted from various review articles and text books. I have already listed some of them at various places in the text. Some parts of it are taken from [57, 58]. Martin's review [59] is perhaps the most comprehensive and popular references. It is also constantly updated. Another review which I strongly recommend is by Matteo Bertollini [60]. Some other excellent reviews are [61] and [62].

A concise introduction can also be found in [63]. For more formal aspects of supersymmetry including a good introduction to supergravity please have a look at [64] and [65]. Happy Susying.

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Appendices

A A lightning recap of the Standard Model

The present recap is only for completeness sake and is not considered a detailed introduction to quantum field theories and electroweak standard model can be found in lectures by Prof. Kitano [67].

The Standard Model (SM) is a spontaneously broken Yang-Mills quantum field theory describing the strong and electroweak interactions. The theoretical assumption on which the Standard Model rests on is the principle of local gauge invariance with the gauge group given by

$$G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y, \tag{A.1}$$

where the subscript c stands for color, L stands for the 'left-handed' chiral group whereas Y is the hypercharge. The particle spectrum and their transformation properties under these gauge groups are given as,

$$\begin{array}{ll} Q_i \equiv \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} \sim \left(3,\ 2,\ \frac{1}{6}\right) & U_i \equiv u_{Ri} \sim \left(\bar{3},\ 1,\ \frac{2}{3}\right) \\ & D_i \equiv d_{Ri} \sim \left(\bar{3},\ 1,\ -\frac{1}{3}\right) \\ L_i \equiv \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} \sim \left(1,\ 2,\ -\frac{1}{2}\right) & E_i \equiv e_{Ri} \sim (1,\ 1,\ -1) \end{array}$$

In the above i stands for the generation index, which runs over the there generations $i = 1, 2, 3$. Q_i represents the left handed quark doublets containing both the up and down quarks of each generation. Similarly, L_i represents left handed lepton doublet, $U_i,\ D_i,\ E_i$ represent right handed up-quark, down-quark and charged lepton singlets respectively. The numbers in the parenthesis represent the transformation properties of the particles under G_{SM} in the order given in eq.(A.1). For example, the quark doublet Q transforms a triplet (3) under $SU(3)$ of strong interactions, a doublet (2) under weak interactions gauge group and carry a hypercharge ($Y/2$) of 1/6 [13]. In addition to the fermion spectra represented above, there is also a fundamental scalar called Higgs whose transformation properties are given as

$$H \equiv \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \sim (1,\ 2,\ 1/2)\,. \tag{A.2}$$

However, the requirement of local gauge invariance will not be fulfilled unless one includes the gauge boson fields also. Including them, the total lagrangian with the above particle spectrum and gauge group can be represented as,

[13] Note that the hypercharges are fixed by the Gellman-Nishijima relation $Y/2 = Q - T_3$, where Q stands for the charge of the particle and T_3 is the eigenvalue of the third generation of the particle under $SU(2)$.

$$\mathcal{L}_{SM} = \mathcal{L}_F + \mathcal{L}_{YM} + \mathcal{L}_{yuk} + \mathcal{L}_S. \tag{A.3}$$

The fermion part $\mathcal{L}_F$ gives the kinetic terms for the fermions as well as their interactions with the gauge bosons. It is given as,

$$\mathcal{L}_F = i\bar{\Psi}\gamma^\mu \mathcal{D}_\mu \Psi, \tag{A.4}$$

where Ψ represents all the fermions in the model,

$$\Psi = (Q_i\, U_i,\ D_i,\ L_i,\ E_i) \tag{A.5}$$

where $\mathcal{D}_\mu$ represents the covariant derivative of the field given as,

$$\mathcal{D}_\mu = \partial\mu - i g_s G^A_\mu \lambda^A - i\frac{g}{2} W^I_\mu \tau^I - i g' B_\mu Y \tag{A.6}$$

Here $A = 1, .., 8$ with G^A_μ representing the $SU(3)_c$ gauge bosons, $I = 1, 2, 3$ with W^I_μ representing the $SU(2)_L$ gauge bosons. The $U(1)_Y$ gauge field is represented by B_μ. The kinetic terms for the gauge fields and their self interactions are given by,

$$\mathcal{L}_{YM} = -\frac{1}{4} G^{\mu\nu A} G^A_{\mu\nu} - \frac{1}{4} W^{\mu\nu I} W^I_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \tag{A.7}$$

with

$$\begin{aligned} G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s\, f_{ABC} G^B_\mu G^C_\nu \\ F^I_{\mu\nu} &= \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g\, f_{IJK} W^J_\mu W^K_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{A.8}$$

where $f_{ABC(IJK)}$ represent the structure constants of the $SU(3)(SU(2))$ group.

In addition to the gauge bosons, the fermions also interact with the Higgs boson, through the dimensionless Yukawa couplings given by

$$\mathcal{L}_{yuk} = h^u_{ij} \bar{Q}_i U_j \tilde{H} + h^d_{ij} \bar{Q}_i D_j H + h^e_{ij} \bar{L}_i E_j H + H.c \tag{A.9}$$

where $\tilde{H} = i\sigma^2 H^\star$. These couplings are responsible for the fermions to attain masses once the gauge symmetry is broken from $G_{SM} \rightarrow SU(3)_c \times U(1)_{em}$. This itselves is achieved by the scalar part of the lagrangian which undergoes spontaneous symmetry breakdown. The scalar part of the lagrangian is given by,

$$\mathcal{L}_S = (\mathcal{D}_\mu H)^\dagger\, \mathcal{D}_\mu H - V(H), \tag{A.10}$$

where

$$V(H) = \mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2 \tag{A.11}$$

For $\mu^2 < 0$, the Higgs field attains a vacuum expectation value (*vev*) at the minimum of the potential. The resulting goldstone bosons are 'eaten away' by the gauge bosons making them massive through the so-called Higgs mechanism. Only one degree of the Higgs field remains physical, the only scalar particle of the SM - the Higgs boson. The fermions also attain their masses through their Yukawa couplings, once the Higgs field attains a *vev*. The only exception is the neutrinos which do not attain any mass due to the absence of right handed neutrinos in the particle spectrum and thus the corresponding Yukawa couplings. Finally, the Standard Model is renormalisable and anomaly free. We would also insist that the Supersymmetric version of the Standard Model keeps these features of the Standard Model intact.

B Extra Dimensions and the hierarchy problem

Extra-dimensions with a flat geometry are simplest realizations of models with additional space dimensions. The extra-dimensions are in general compactified on an $n - torus$ thus forming a compact manifold M_n. Thus the total space-time is a $R^{(4)} \times \mathcal{M}_n$ manifold, where $R^{(4)}$ corresponds to the usual $3+1$ space-time. In earlier realizations of such theories the SM spectrum was confined on the $3+1$ manifold while only gravity was allowed to extend into the bulk. In addition to 1 massless 4D graviton, 1 massless gauge field and 1 massless scalar, we get a tower of massive gravitons called Kaluza-Klein modes.

We now consider examples where specific realizations of extra-dimensional scenarios can be useful in solving the hierarchy problem. They include (a) ADD model (b) RS model.

B.1 ADD model

The proposal to use extra-dimensional brane-world scenarios to solve the hierarchy problem was first put forward by Arkani-Hamed, Dimopoulos and Dvali [69]. The model assumes a setup with n extra spatial dimensions compactified on a $n - sphere$ with equal radius a. The metric for the $4+n$ dimensional space-time is given as

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - na^2\phi^2 \tag{B.1}$$

where $0 \leq \phi \leq 2\pi$. While the SM spectrum is assumed to be confined on the 3-brane, only gravity is allowed to propagate in all the $4+n$ dimensions. The $4+n$ dimensional gravity action is given as

$$S_{4+n} = M_\star^{2+n} \int d^{4+n} \sqrt{g^{4+n}} R^{4+n} \tag{B.2}$$

where $M_\star$ is the $4+n$ dimensional fundamental Planck scale. Integrating Eq.(B.2) over the n compact extra dimension yields

$$\begin{aligned} S_{4+n} &= M_\star^{2+n} (2\pi a)^n \int d^4x \sqrt{-g^4} R^4 \\ S_{4+n} &= M_\star^{2+n} (2\pi a)^n S_4 \end{aligned} \tag{B.3}$$

where the effective 4 dimensional Planck scale is then $M_{pl}^2 = M_\star^{2+n}(2\pi a)^n$ and S_4 effective 4D gravity action.

Putting $M_\star \sim 1$ TeV, we find the condition on the compactification radius a as

$$a \sim \times 10^{\frac{30}{n} - 17} \text{cm} \tag{B.4}$$

Putting $n = 1$, above gives a value of a which is $\sim 10^{13} cm$. This would signal deviations from Newtonian gravity at the astronomical scale and hence is ruled out. For $n \geq 2$ we get $a \leq 10^{-2}$ cm thus leading to possible modifications of Newtonian gravity at the sub-millimeter scale. Thus the ADD model links the two fundamental scales of nature by means of a large volume factor related to the size of the bulk. Hence such models are referred to as large extra-dimensional models.

B.2 Randall Sundrum model of warped extra dimension

The ADD model reduced the higher fundamental “Planck" scale to around the TeV scale from which the effective 4D scale of 10^{15} TeV resulted owing to the large volume of the extra-dimensions. This explanation of the hierarchy problem between the electroweak scale and the Planck scale in the ADD model resulted in its reintroduction between the compactification scale (10^{-2} cm) and the electroweak scale (10^{-17} cm). As a result an alternate extra-dimensional mechanism to generate the hierarchy was put forward by Randall and Sundrum [70]. The model consisted of a single extra-dimension compactified on $S1/Z_2$ orbifold. Thus the domain of the extra-dimensional coordinate is $[0, \pi R]$ where R is the compactification radius. A 3-brane[14] is introduced at each of the orbifold fixed point. The brane at $y = 0$ is referred to as the *hidden* brane while the brane at $y = \pi R$ is referred to as the *visible* brane. Introduction of a large bulk cosmological constant Λ “warps" the bulk. Brane localized sources are added to balance the effects of Λ thereby inducing a vanishing effective 4D cosmological constant. An ansatz for the line element with a warped geometry is given as

$$ds^2 = G_{MN}dx^M dx^N = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \tag{B.5}$$

where $\eta_{\mu\nu}$ is the Minkowski metric. Unlike the ADD case, the presence of the exponential factor e^{-kry} renders this metric to be non-factorizable. The metric induced at each of the orbifold fixed points are given as

$$g^{vis}_{\mu\nu} = G_{\mu\nu}(x^\mu, y = \pi R) \qquad g^{hid}_{\mu\nu} = G_{\mu\nu}(x^\mu, y = 0) \tag{B.6}$$

Thus action for the theory in the absence of any matter is as follows:

$$S = S_{Gravity} + S_{vis} + S_{hid} \tag{B.7}$$

where

$$\begin{aligned} S_{Gravity} &= \int d^4x d\phi \sqrt{-G}[2M^3 R + \Lambda] \\ S_{vis} &= \int d^4x \sqrt{-g_s}[-V_{vis}] \\ S_{hid} &= \int d^4x \sqrt{-g_p}[-V_{hid}] \end{aligned} \tag{B.8}$$

where Λ is the bulk cosmological constant. V_{vis} and V_{hid} are the brane localized potential at the corresponding branes and M is higher dimensional Planck scale. The Einstein's equations corresponding to the action in Eq.(B.8)

$$\frac{\sigma'^2}{R^2} = \frac{-\Lambda}{24M^3} \tag{B.9}$$

$$\sigma'' = \frac{V_{hid}R\delta(\phi)}{12M^3} + \frac{V_{vis}R\delta(\phi - \pi)}{12M^3} \tag{B.10}$$

Solving for σ we get

$$\sigma = R|\phi|\sqrt{\frac{-\Lambda}{24M^3}} \tag{B.11}$$

This solution is valid only for $\Lambda < 0$ implying that the space between the two $3 - branes$ is an *Anti De-Sitter* space. Differentiating σ in Eq.(B.11) twice we get

$$\sigma'' = 2R\sqrt{\frac{-\Lambda}{24M^3}}[\delta(\phi) - \delta(\phi - \pi)] \tag{B.12}$$

Thus comparing above equation with the second line in Eq.(B.10) we get $V_{hid} = -V_{vis} = 24M^3 k, \Lambda = -24M^3 k^2$ where $k = \frac{-\Lambda}{24M^3}$ being the reduced Planck scale. Thus, we see that we have two opposite tension branes. The hidden brane has positive tension and the visible brane tension is the negative of the former.

[14] 3-brane here means a 4 dimensional spacetime.

The higher dimensional fundamental Planck scale M is related to the effective 4-D Planck scale M_{Pl} by the following relation

$$M_{Pl}^2 = \frac{M^3}{k}\left[1 - e^{-2kR\pi}\right] \tag{B.13}$$

This implies a weak dependence of M_{Pl} on the compactification radius R.

The resolution to the hierarchy problem can be seen by considering the Higgs field to be localized on the visible brane. The action in this case is given as

$$S = \int d^4x\sqrt{-g_{vis}}\left[g_{vis}^{\mu\nu}(\partial_\mu H)^\dagger\partial_\nu H - m^2 H^\dagger H + \lambda(H^\dagger H)^2\right] \tag{B.14}$$

where $\sqrt{-g_{vis}} = e^{-4kR\pi}$ and $g_{vis}^{\mu\nu} = e^{2kR\pi}\eta^{\mu\nu}$. Redefining the Higgs field as $H \to e^{kR\pi}H$ the action in Eq.(B.14) reduces to

$$S = \int d^4x\left[\eta_{vis}^{\mu\nu}(\partial_\mu H)^\dagger\partial_\nu H - (e^{-kR\pi}m)^2 H^\dagger H + \lambda(H^\dagger H)^2\right] \tag{B.15}$$

We see that the effective Higgs mass is now defined as

$$m_{eff} = e^{-kR\pi}m \tag{B.16}$$

Choosing $kR \sim \mathcal{O}(10)$ electroweak scale Higgs mass can be achieved by exponential warping of scales thus solving the Hierarchy problem. The radius R in the RS setup was considered a free parameter and was appropriately adjusted to resolve the hierarchy problem. Metric fluctuations along the radial direction corresponds to the existence of a massless radion. The radius R is determined by the vev of the radion and is not included in the dynamics of the original RS setup. A proposal in this direction to was put forward by Goldberger and Wise [71]. A massive bulk scalar field with brane localized potentials is introduced. The role of the scalar field is to generate a potential for the radion. It can be shown that the radion attains a mass at the minimum of the potential thus generating an R at which the hierarchy problem is solved for reasonable choices of parameters in the radion potential. The radius of the AdS space is very small *i.e.* $R \sim \frac{1}{k}$ and hence such models are referred to as small extra-dimensional models. The observed weakness of gravity in this scenario can be very elegantly explained by the localization of the zero mode gravitons towards the UV resulting in small overlap with IR brane where the SM fields are localized.

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