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Field Theory and the Electro-Weak Standard Model

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Abstract
These lectures present an introduction to Quantum Field Theory and its particular application to the building of the Standard Model of Electro-Weak interactions.

Keywords
Quantum Field Theory, Standard Model, Electroweak interactions

"One should never underestimate the pleasure we feel from hearing something we already know"
Enrico Fermi

1 Introduction to Quantum Field Theory

Even if Quantum Mechanics was extremely successful in describing and predicting the atomic physics phenomena, it clearly appears that the inclusion of Relativity in its treatment produces serious difficulties.

We had learned that as soon as the characteristic distances of a given problem are so small as the atomic size ($1A = 10^{-8}\,cm$) or smaller, the Newton Mechanics has to be replaced by Quantum Mechanics with all its consequences. We have to remember that in these conditions we are into the validity domain of the Heisenberg Uncertain Principle, the basic brick of Quantum Theory.

$$\Delta x \Delta p \simeq \hbar \tag{1}$$

The Planck constant $\hbar$ provides the scale where quantum effects are protagonists. This principle implies that it is not possible to define a trajectory for the quantum objects because you cannot provide simultaneously the position and the velocity of the particle, necessary data for the integration of Newton equation.

The uncertain principle has still further and important consequences. In fact, if the distances implied are smaller than the Angstrom, then Δp should be larger. In other words, the velocities implied will be larger and even comparable to the light velocity. In this case it is essential to take into account the Special Relativity rules. In particular, the well known equation $E = mc^2$ has as a consequence that with sufficient energy, mass can be generated in the form of new quantum particles. The number of particles can change, is not more a constant of movement. It is necessary to build up a formalism able to treat a variable number of particles, of quantum objects. This formalism is the Quantum Field Theory (QFT). This is, at present, the best starting point to study and to describe matter and interaction at the most elementary level. It contains the possibility of creation and/or annihilation of quantum particles as for example electrons, photons and quarks.

QFT provides a set of formal strategies and mathematical tools that give rise to an image of the micro-world completely different to the classical conception of particles and fields (as the classical electromagnetic and gravitational ones).

As the paradigm of QFT is the quantum version of the electromagnetic field: Quantum Electrodynamics. Then, it is worth discussing, even briefly, the concept of a classical field that one uses for describing electricity, magnetism and the gravitational macroscopic forces.

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Let us remember the Coulomb interaction between point charges that implies that electric charges interact even if they are not in contact, situation known as an action at a distance. To avoid this situation, the concept of field was born that among its important consequences it is mandatory to mention the prediction of electromagnetic waves. Electric and magnetic fields are propagated in space and time as waves. The situation of the electromagnetic field is reproduced by the gravitational interaction with only attractive forces in this case.

We should realize that at this point we are living together with two different phenomena related to the dynamics. From one side we have the wave process that implies propagation of a perturbation without a net displacement of matter and from the other side we have the displacement of concentrated elements as the classical particles are.

From the beginning of XX century one has fundamental advances in the understanding of the intimal constitution of matter, namely, the quantum hypothesis of Planck to take care of the black body radiation and the photon concept introduced by Einstein to explain the photoelectric effect. The quantum presence is not only in the process of emission of radiation, but also in the way energy travels. On the other side, the association of a characteristic wave length, proportional to its momentum, to a particle was proposed by de Broglie. This was confirmed by the experimental detection of diffraction of electrons by crystals. These phenomena gave rise to the wave-particle duality dilemma. Today we know that this duality is not more than the result of prejudicially pretending to keep the language of classical design (appropriate for the human scale) for the description of the atomic and subatomic phenomena.

Consequently, for a coherent treatment of the observed phenomena related to a wave character connected with the probability of presence of the quantum object, that takes into account the possibility of a variable number of the quantum particles according to the energy, it was necessary to develop the quantum formalism of QFT.

Quantum Electrodynamics (QED) is the paradigm of QFT and explain with astonishing precision the interaction between electrons (the electron field) and the quantum electromagnetic field (the field of photons). The quantization of the electromagnetic field implies the presence of photons as quanta or quantum excitations of the field. On the other hand, it contains a relativistic treatment of the electron.

The process of quantizing a field contains two steps. First one produces an analysis of the classical fields in terms of normal modes, namely a Fourier analysis corresponding to infinity degrees of freedom. Then each mode (each Fourier component) is independently described as a quantum harmonic oscillator. As a result, the energy of the system is expressed as the sum of terms corresponding to the energy of each oscillator, weighted by the number of oscillators with each particular energy. This number is a quantum operator and counting the number of objects in each mode is synonymous of the quantum particles (quanta) corresponding to the field in the state characterized by each possible energy. This is clearly the interpretation of the QFT in terms of quantum particles. Notice again that these quantum particles are completely different objects to the classical particles. They only share the property of being discrete entities that can be counted.

The following step in the development of the formalism of Quantum Field Theory is to couple different fields following precise rules that are mainly based on symmetry.

2 Introduction to the Formalism of QFT

In QFT, to each quantum particle species one assigns a **field** $\chi(t,\vec{x})$ so that, this correspondence with classical mechanics is performed.

Classical Mechanics Point mass m in one dimensional space $\Rightarrow$ a generalized coordinate $q(t)$

Lagrangian: $L = L(q(t), \dot{q}(t))$

Classical Field Theory Field in three dimensions $\Rightarrow$ a generalized coordinate $\chi(t,\vec{x})$ at each space-time point

Lagrangian: $L = \int d^3x \mathscr{L}$; where the Lagrangian density is $\mathscr{L} = \mathscr{L}(\chi, \dot{\chi}, \vec{\nabla}\chi)$
Now, from the action $S = \int dt\, L \Rightarrow$ equations of motion (Euler-Lagrange):

Classical Mechanics

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

Classical Field Theory

$$\frac{\partial \mathscr{L}}{\partial \chi} - \frac{\partial}{\partial t}\frac{\partial \mathscr{L}}{\partial \dot{\chi}} + \vec{\nabla}\frac{\partial \mathscr{L}}{\partial \vec{\nabla}\chi} = 0$$

Let us first consider free fields ($\mathscr{L}_{int} = 0$). Through their behavior under Lorentz group transformations, one distinguishes i) scalar fields; ii) spinor fields; iii) vector fields; ...

i) **Scalar field**

It corresponds to spin 0. The field equation is the Klein-Gordon one and the Lagrangian density reads:

$$\mathscr{L}_0^{\phi} = \frac{1}{2}\left[\left(\frac{1}{2}\frac{\partial \phi}{\partial t}\right)^2 - \left(\vec{\nabla}\phi\right)^2\right] - \frac{1}{2} m\, \phi^2 \tag{2}$$

ii) **Spinor field**

It corresponds to spin $1/2$. The corresponding field equation is de Dirac equation and the Lagrangian density:

$$\mathscr{L}_0^{\psi} = \bar{\psi}\left(\imath \gamma^{\mu}\partial_{\mu} - m\right)\psi \tag{3}$$

iii) **Vector field**

It corresponds to spin 1. Maxwell equations are the corresponding ones. The Lagrangian density is:

$$\mathscr{L}_0^{A} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{4}$$

The next step is to take into account interactions between fields. We postpone the discussion of this point for a while.

The third step is to make the theory quantal. The process of quantizing the field theory is done, in our presentation, by "copying" Quantum Mechanics. Namely, we define the canonical momentum through the field version of

$$p(t) \equiv \frac{\partial L}{\partial \dot{q}}$$

and impose the canonical commutation relation, now with q and p read as operators, equivalent to

$$[q(t), p(t)] = \imath \hbar \tag{5}$$

that guarantees the validity of the uncertainty principle.

In this way, for

i) **Spin** 0:

$$\pi(t, \vec{x}) = \frac{\partial \mathscr{L}}{\partial \dot{\phi}(t, \vec{x})}$$

and the quantization is imposed through the equal time commutation relation

$$[\phi(t, \vec{x}), \pi(t, \vec{x})] = \imath \hbar\, \delta^{(3)}(\vec{x} - \vec{x}') \tag{6}$$

ii) **Spin** $1/2$:

$$\pi_\alpha(t,\vec{x}) = \frac{\partial \mathscr{L}}{\partial \dot{\psi}_\alpha(t,\vec{x})}$$

with the equal time anti-commutators

$$\left\{ \psi_\alpha(t,\vec{x}), \pi_\beta(t,\vec{x}) \right\} = \imath\hbar\, \delta_{\alpha\beta}\, \delta^{(3)}(\vec{x}-\vec{x}') \tag{7}$$

i) **Spin** 1:

$$\pi^\mu(t,\vec{x}) = \frac{\partial \mathscr{L}}{\partial \dot{A}_\mu(t,\vec{x})}$$

with the quantization imposed by

$$[A^\mu(t,\vec{x}), \pi^\nu(t,\vec{x})] = \imath\hbar\, g^{\mu\nu}\, \delta^{(3)}(\vec{x}-\vec{x}') \tag{8}$$

Notice that in this case there are some difficulties because gauge invariance of electromagnetism implies $A^3 = 0$ and consequently, for these components A^3 and π^3 one cannot satisfy the previous relation. This problem needs a particular treatment for the quantization of the electromagnetic field (See Bibliography).

After the quantization procedure we have ended with a series of field operators. The natural question now is where are the quantum particles?

Let us answer this question by analyzing the scalar field, whose equation of movement is

$$\left(\Box + m^2\right)\, \phi(t,\vec{x}) = 0$$

Taking profit of the fact that it is a linear equation, we write its general solution in terms of the Fourier transform

$$\phi(t,\vec{x}) \propto \int dE\, d^3p\, \delta(E^2 - \vec{p}^2 - m^2)\, \left[a(E,\vec{p})\, e^{-\imath(Et-\vec{p}.\vec{x})} + a^\dagger(E,\vec{p})\, e^{+\imath(Et-\vec{p}.\vec{x})}\right]$$

Due to the quantization condition (6), the Fourier coefficients $a(E,\vec{p})$ and $a^\dagger(E,\vec{p})$ must be operators, clearly satisfying the commutation relations

$$\left[a(E,\vec{p}), a^\dagger(E,\vec{p}')\right] = 2E\hbar\, \delta^{(3)}(\vec{p}-\vec{p}') \tag{9}$$

$$\left[a(E,\vec{p}), a(E,\vec{p}')\right] = 0 \tag{10}$$

$$\left[a^\dagger(E,\vec{p}), a^\dagger(E,\vec{p}')\right] = 0 \tag{11}$$

Consequently, the Hamiltonian of the system can be written as

$$\begin{aligned} H &= \int d^3x\, (\pi\, \dot{\phi}) \\ &\simeq \int dE\, d^3p\, \delta(E^2 - \vec{p}^2 - m^2)\, E\, a^\dagger(E,\vec{p})\, a(E,\vec{p}) a(E,\vec{p}) \end{aligned}$$

where appears the ”number operator”

$$N(p) = N(E,\vec{p}) \equiv a^\dagger(E,\vec{p})\, a(E,\vec{p}) \tag{12}$$

acting in the multiparticle-states Fock space, verifying the eigenvalue equation

$$N(E,\vec{p})\, |n(E,\vec{p})\rangle = n(E,\vec{p})\, |n(E,\vec{p})\rangle$$

that allows one to interpret $n(E,\vec{p})$ as the number of quanta with spin 0, mass m, energy between E and $E+dE$ and momentum between $\vec{p}$ and $\vec{p}+d\vec{p}$. Certainly the validity of the relationship $E = \sqrt{\vec{p}^2 + m^2}$ is implicit. The number operator has the property

$$N(p)\, a^{(\dagger)}(p)\, |n(p)\rangle = \left[n(p)\ \overset{-}{(+)}\ 1\right]\, a^{(\dagger)}(p)\, |n(p)\rangle$$

that induce the names: creation operator for $a^{(\dagger)}(p)$ and annihilation operator for $a(p)$, with the corresponding eigenvalue equations.

The previous analysis clearly shows that the quantization procedure provides the connection between quantum fields and quantum particles.

One can now consider multiparticle states. To this end one has to recall that the indistinguishability between identical quantum particles makes necessary to introduce the corresponding statistic. Namely Bose-Einstein for integer spin and Fermi-Dirac for half integer spin.

In the case of Bose-Einstein, a multi-boson state reads

$$|n_1(p_1),\cdots,n_m(p_m)\rangle \propto [a^{(\dagger)}(p_1)]^{n_1}]\cdots[a^{(\dagger)}(p_m)]^{n_m}\,|0\rangle$$

that presents a total symmetry under the interchange of any pair of particles.

For the case Fermi-Dirac one has to ensure the validity of the Pauli exclusion principle that implies using anticommutators (as we have already used in the process of quantization) instead of commutators because

$$\left\{a^{(\dagger)}(p),a^{(\dagger)}(p)\right\}=0\ \Rightarrow\ (a^{(\dagger)}(p))^2=0\ \Rightarrow\ n_i=0.1$$

and the multiparticle fermion state reads

$$|p_1,\cdots,p_m\rangle = a^{(\dagger)}(p_1)\cdots a^{(\dagger)}(p_m)|0\rangle$$

Notice that the use of anticommutators for fermions guarantees also that the energy of the system is bounded from below.

Let us now go to discuss the *parameters* and the *observables* of a quantum field theory.

In general, to specify a field theory is equivalent to give a Lagrangian. For example, for the scalar case one gives $\mathscr{L}(\phi,\partial\phi)$. Lets take the simple case with a quartic self-interaction, namely

$$\mathscr{L}=\frac{1}{2}\left[\partial_\mu\phi_B(x)\right]^2-\frac{1}{2}m_B^2\,\phi_B^2-\frac{1}{4}\,g_B\,\phi^4 \tag{13}$$

where the index B is there for "bare", the initial value before any interaction, measured by g_B has occurred. Remember that the field and the corresponding canonical momentum verify (6). If we compute now physical observables like cross-sections, or decay rates, or the physical mass, all of them result functions of the bare parameters m_B and g_B. Consequently, any perturbative calculation one can do (following the similar mechanism we use in Quantum Mechanics) will end with a series in powers of g_B. The construction of the perturbative series has a protocol based on clear rules known as Feynman diagrams. The reader can consult, for example, these specific recent references where Feynman diagrams are treated: i) K. Kumerički, "Feynman diagrams for beginners", arXiv: 1602.04182; ii) S.M. Bilenky, "Introduction to Feynman diagrams", vol.65 in International series in natural philosophy, ISBN: 978-0-08-017799-1.

Feynman diagrams are a pictorial representation of probability amplitudes at a given order of the perturbation theory. Every line and every cross in the picture has a mathematical interpretation, in a similar way as in an electrical circuit.

In any case, calculations could depend on other parameter. In fact, it is valid to perform the quantization as

$$[A^\mu(t,\vec{x}),\pi^\nu(t,\vec{x})]=\frac{\imath}{Z_\phi}\,\hbar\,g^{\mu\nu}\,\delta^{(3)}(\vec{x}-\vec{x}')$$

where Z_ϕ is an arbitrary number. Certainly, $Z_\phi=1$ corresponds to the bare situation. It is also clear that any magnitude you can compute, say the S-matrix, will start depending on Z_ϕ.

The important fact to notice is that both, m_B and g_B do not contain any physics. In fact, one can ask, for example, which is the physical mass of the quantum of the field ϕ. It is the value of p^2, the square of the momentum, where the propagator of the quantum particle has a pole. If there were no interaction ($g_B = 0$) then of course $P^2_{pole} = m^2_B$. But if interactions are present, the pole of the propagator is at a value $p^2_{pole} = m^2 \neq m^2_B$. It acquires infinite corrections!

This simple example shows the necessity of *renormalization*.

We present briefly the idea of renormalization. If m_b, g_B and $Z_\phi = 1$ are maintained fixed, the perturbative contributions, coming from the calculation of Feynman diagrams, ends with divergent integrals. Take as an example the electron selfenergy in QED. It has at the lower order a logarithmic divergence. If a cut-off λ in momentum is introduced, one gets in this example a result $I \propto e^2_B \ln \lambda$ (in this case the bare electron electric charge plays the role of g_B).

So under this cut-off intent, in general one ends with S-matrix elements $S_B = S_B(p_i, m_B, g_B, Z_\phi; \lambda)$ and for many of these contributions, the (physical) limit $\lambda \to \infty$ gives rise to the non sense result $S_B \to \infty$.

Just to try to get sensible results from the perturbation theory approach to quantum field theory, the renormalization scheme was proposed. Namely, to allow that

$$\begin{aligned} m_B &= m_B(\lambda) \\ g_B &= g_B(\lambda) \\ Z_\phi(\lambda) &\neq 1 \end{aligned}$$

and to adjust the functional dependence on λ in order to cancel the divergencies that appear when $\lambda \to \infty$. This is achieved computing some S_B and comparing the results with experimental data in order to deduce the dependence of the above functions, for example $m_B = m_B(\lambda)$. Clearly, a priori there exist a big problem, namely, in principle there is an infinite number of potentially divergent contributions and only three functions to be adjusted. But extraordinary cases exist... Some theories are certainly renormalizable and three function to adjust are enough. Those theories where the number of terms that are independently divergent (primitive divergent as they are called) is equal or less than the functions of λ to adjust.

The practice of renormalization goes through the replacement

$$\begin{aligned} \phi_B &= \sqrt{Z_\phi(\lambda)}\,\phi \\ g_B &= Z_\phi^{-2}(\lambda) Z_g(\lambda)\, g \\ m^2_B &= Z_\phi^{-1}(\lambda) Z_m(\lambda)\, m^2 \end{aligned}$$

and the adjustment of the $Z_i(\lambda)$ so to get g and m finite and independent of λ. These $Z_i(\lambda)$ are treated as power series in g. The relations before seems capricious but they are chosen in this way because the Lagrangian written in terms of the new magnitudes reads simple

$$\mathscr{L} = \frac{1}{2} Z_\phi \left[\partial_\mu \phi(x)\right]^2 - \frac{1}{2} Z_m m^2 \phi^2 - \frac{1}{4} Z_g\, g\, \phi^4 \tag{14}$$

that certainly can be treated with the Feynman diagrams technique, now in terms of 'dressed" constants.

3 Standard Model

What is what one understands for a Standard Model? It is a theoretical framework that starting from observation allows to predict and correlate new data. In general it is an excellent "approximation" at a given (energy) scale. Consequently, one is always driven to go beyond. In Physics, the Standard Model has had a nice evolution. We can first mention the proposal of Empedocles of considering the four elements: earth, water, air and fire, as the compounds of everything, linked by love and hate. The next step in the evolution of the Standard Model is Mendeleev's periodic table, the base of Chemistry. After this, Quantum Mechanics, the theory of atoms, is the following Standard Model. Today Standard Model is 3!, the Glashow-Salam-Weinberg electroweak model plus Quantum Chromodynamics that includes quantum particles and quantum fields with validity down to distances of the order of $10^{-18}\,m$.

The present Standard Model contains, in an extremely economical way, the ingredients to describe (almost) everything. It is based upon *Gauge Symmetry*. This symmetry is the engine of the present physics development allowing a unified treatment of the fundamental forces. This treatment conforms the *Gauge theories*. A Gauge theory is a synthesis of Quantum Field Theory with a particular symmetry. The idea of a gauge theory was introduced by Hermann Weyl in 1919. At that time, only the electron and the proton were known...It was an idea for future times.

4 Symmetries

Wigner, referring to Einstein's relativity, stated: "Einstein's work sets the inversion of a tendency: before it, invariance principles came from the dynamical laws. Now, is natural for us, to obtain the laws of Nature from invariance principles". This is the way our present knowledge of the fundamental interactions is obtained.

Certainly, people love symmetry and also Nature loves symmetry, or better said: our model of Nature loves symmetry. In the same way that we act in front of a framed picture that is not in the right position because we are compelled to exert a force in order to restore the broken axial symmetry, fundamental forces of Nature are present just to ensure the validity of a symmetry, the gauge symmetry. In order to arrive to this concept, let us start by recalling which are the symmetries in Physics.

One can divide the symmetries in Physics into two types: i) Geometrical symmetries, related to transformation on the space-time coordinates that have to do with observational situations and ii) Internal symmetries, acting on the dynamical variables and operators, having to do with the quality of observables. These internal symmetries could be global if they have no contact with space-time, or local when the parameters of the internal transformations are connected with space-time. These local internal symmetries are *gauge symmetries*.

The important point for the present discussion is that in order to guarantee a gauge symmetry, particularly in QFT, it is mandatory to introduce new fields into the game. These extra fields produce the interactions. This is the framework for formalizing the fundamental interactions in Nature.

In particular, it is the main ingredient in building the 3!-model, the Standard Model. We call it 3! because $3! = 3 \times 2 \times 1$, that remember us that the Standard Model is based on the requirement of the Gauge Symmetry

$$3! = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

where the symmetry $SU(3)_C$ is responsible for the strong interaction among quarks that carry color charge and $SU(2)_L \otimes U(1)_Y$ is related to weak and electromagnetic interactions that are treated in a unified way and named electroweak.

The gauge symmetry is guaranteed through the inclusion of gauge fields, mediators of the interactions. All the interaction between fermion fields (leptons or quarks) are carried by the exchange of vector-boson gauge fields (gluons, weak-bosons, photon).

The known fermions, the characters of the "drama", appear in three generations and are related among them via gauge transformations. Vertically are connected by $SU(2)_L$ and quarks, horizontally, via $SU(3)_C$. We summarize them in this table

$$\begin{bmatrix} \nu_e & | & u_1 & u_2 & u_3 \\ e & | & d_1 & d_2 & d_3 \end{bmatrix} ; \begin{bmatrix} \nu_\mu & | & c_1 & c_2 & c_3 \\ \mu & | & s_1 & s_2 & s_3 \end{bmatrix} ; \begin{bmatrix} \nu_\tau & | & t_1 & t_2 & t_3 \\ \tau & | & b_1 & b_2 & b_3 \end{bmatrix}$$

Fermion Generations or Families

We include below the mediators of the interactions that we shall relate briefly with gauge fields.

gluons	$g_1, g_2, \ldots, g_8$
photon	γ
"*weakons*"	W^+, W^-, Z

Intermediate Bosons

Notice that these are the "characters" at the level of elementarity we recognize at present and that is why they bear today the name of "elementary (?) particles (?)".

We also should add the Higgs field H, responsible, as we will see, of the masses of the massive particles.

5 Symmetries in Field Theory

Hopefully, we have convinced the audience that the natural language for Particle Physics is Quantum Field Theory (QFT). The fundamental magnitude to start with is a quantum field $\chi_\alpha(x)$ where α summarized all the internal indices needed (as the components, etc), and now symmetries need to be incorporated in the corresponding Lagrangian. For example in

$$\mathscr{L} = \mathscr{L}(\phi, \partial_\mu \phi)$$

We start considering continuous symmetries whose transformations are represented by elements of a continuous group G.Then the field χ_α will be a member of an irreducible multiplet that under a transformation a belonging to the group G transform as

$$\chi_\alpha(x) \stackrel{a}{\to} \chi'_\alpha(x) = \mathscr{R}_{\alpha\beta}(a)\,\chi_\beta(x)$$

where $\mathscr{R}_{\alpha\beta}$ is the matrix representation of G that verifies

$$\mathscr{R}_{\alpha\beta}(a')\,\mathscr{R}_{\beta\gamma}(a) = \mathscr{R}_{\alpha\gamma}(a'')$$

in order to satisfy the group product equivalence

$$\chi_\alpha(x) \stackrel{a}{\to} \chi'_\alpha(x) \stackrel{a'}{\to} \chi''_\alpha(x) \equiv \chi_\alpha(x) \stackrel{a''}{\to} \chi''_\alpha(x)$$

Going now to the Hilbert space, transformations are represented by unitary operators $U(a)$ and one has, being χ_α a member of an irreducible multiplet, that

$$U^{-1}(a)\,\chi_\alpha(x)\,U(a) = \chi'_\alpha(x) = \mathscr{R}_{\alpha\beta}(a)\,\chi_\beta(x) \tag{15}$$

and

$$U(a)\,U(a') = U(a'') \tag{16}$$

As we are treating continuous symmetries, it is possible to study their infinitesimal version and write

$$U(\delta a) = 1 + \imath\,\delta a_i\,G_i$$

with δa real. G_i are the generators of the group G, being hermitian operators. Due to the composition law (16) one has

$$[G_i, G_j] = \imath\,c_{ijk}\,G_k \tag{17}$$

namely, the Lie algebra of the group. c_{ijk} are the structure constants of the group. It is clear that the representation of the group for infinitesimal transformation can be written as

$$\mathscr{R}_{\alpha\beta}(\delta a) = \delta_{\alpha\beta} + \imath\,\delta a_i\,(g_i)_{\alpha\beta}$$

where g_i are the representations of the generators G_i and obey the Lie algebra.

By using the relation (15), one immediately gets

$$[G_i, \chi_\alpha(x)] = -(g_i)_{\alpha\beta}\,\chi_\beta(x)$$

that shows how the field transforms under the group G.

Clearly, the invariance of a field theory under the group G implies that the action is invariant. Consequently, via the Noether theorem, there are currents

$$J_i^\mu(x) = \frac{\partial \mathscr{L}}{\partial \partial_\mu \chi_\alpha(x)}\,\frac{1}{\imath}\,(g_i)_{\alpha\beta}\,\chi_\beta(x)$$

that are conserved:

$$\partial_\mu\,J_i^\mu(x) = 0$$

allowing the identification of charges

$$G_i = \int d^3x J_i^0(x)$$

that are also conserved, they commute with the Hamiltonian.

Let us now consider quantum states as $|p;\alpha\rangle$ corresponding to a one particle with $p^2 = -m^2$.

The particle states corresponding to a field $\chi_\alpha(x)$ transform as the field if and only if the vacuum is G-invariant, namily

$$U(a)\,|0\rangle = |0\rangle$$

that implies that the generators annihilate the vacuum

$$G_j|0\rangle = 0$$

In general this is not the case and for this reason one divides the realization of a given symmetry according to the behavior of the vacuum being G-invariant or not.

Wigner-Weyl realization

The vacuum is G-invariant, or in other words, it is annihilated by the generators and consequently, it has the same symmetry as the action. Then the field has vacuum expectation value equal to zero

$$\langle 0|\,\chi_\alpha(x)\,|0\rangle = 0$$

When this condition is fulfilled, there is a theorem that shows that all the states in a given multiplet have the same mass. This is the case, for example, of proton and neutron that if isospin would be a perfect symmetry, they should have the same mass.

Nambu-Goldstone realization

In this case the vacuum is not G-invariant. In other words

$$U(a)\,|0\rangle \neq |0\rangle$$

and consequently

$$\langle 0|\,\chi_\alpha(x)\,|0\rangle \neq 0$$

Consequently, the Goldstone theorem applies. It says that in a theory, for each generator G_j that does not annihilate the vacuum there is present one massless boson, called Goldstone boson. That is

$$G_j^{N.A.}\,|0\rangle = |p;j\rangle_G \;with\; m = 0$$

6 Gauge theories

From the original era of gauge invariance, started by Weyl (1919), it survived mainly as Maxwell equations' symmetry until around 1959 when Yang and Mills proposed an extension of gauge symmetry beyond electromagnetism.

The basic ideas of a theory based upon gauge invariance, a gauge theory can be mimicked by the following very simple example of a harmonic oscillator rotating in a plane with period $T = 2\pi/\omega$. Referring to standard coordinates, the motion is represented by

$$\begin{aligned} y &= A\sin(\omega t) \\ x &= B\cos(\omega t) \end{aligned}$$

that can be written together by means of a complex variable $z = x + \imath y$. The oscillator equation now reads

$$\frac{d^2z}{dt^2} + \omega^2 z = 0 \tag{18}$$

In fact, to decide measuring the instant position of the oscillator $\theta = \omega t$, from the horizontal x-axis is arbitrary. One certainly can choose an alternative orthogonal system (x', y') and measure angles from x' rotated an angle α from x. We are changing $\theta \to \theta - \alpha$ and say that θ was "regauged". Notice now that multiplying the Eq.(18) by $\exp(-\imath\alpha)$ and redefining $z' = z\exp(-\imath\alpha)$ the Eq.(18) is covariant (does not change written in terms of z'. One can conclude that the absolute value of θ is irrelevant or in more precise words, the equation is *global gauge invariant*. Global means that α is time independent.

What happens if we allow for a *local gauge transformation* $\alpha = \alpha(t)$? It is clear that $\exp(-\imath\alpha(t))$ cannot be absorbed in the redefinition of z' because $d\alpha(t)/dt \neq 0$.

One can regain the invariance, now a local one, compensating the time derivative of α by means of the replacement

$$\frac{d}{dt} \to \frac{d}{dt} - A(t)$$

with the requirement that: if

$$\theta \to \theta - \alpha(t)$$

then

$$A(t) \to A(t) - \frac{d\alpha(t)}{dt}$$

that implies that $A(t)$ works as a compensator, a *gauge field* allowing for the validity of the invariance under a local gauge transformation.

We clearly notice that when $\alpha = \alpha(t)$ one is in fact rotating the reference frame with an angular velocity $d\alpha(t)/dt$ and in a rotating frame, a non inertial frame, there are the so called "fictitious" forces (centrifuge, Coriolis). It is precisely $A(t)$ the generator of these forces. The origin of new interactions comes from the requirement of local gauge invariance.

To go ahead let us now remember for a while Maxwell electromagnetic gauge invariance and in particular its origin. The vector $\vec{A}$ is defined via its curl as $\vec{\nabla} \wedge \vec{A} = \vec{B}$, but there is no condition to be imposed to its divergence. This freedom in the election of $\vec{\nabla} \cdot \vec{A}$ is precisely the gauge symmetry of electromagnetism. Taking into account the "vectorial poem" due to Enrique Loedel (1901-1962, former professor at La Plata) that says [1]

Esto el Papa exclamó al firmar la bula

[1] This was exclaimed by the Pope/ when furious excommunicates Luther:/ The divergency of a curl is zero/ and the curl of a gradient is always zero/ The great German priest pleaded god/ and exclaimed with his usual vehemence/ the curl of a curl plus nabla two/ gives the gradient of any divergence

con que furioso excomulgó a Lutero:
La *divergencia* de un *rotor* es nula
y el *rotor* de un gradiente es siempre cero.
El gran fraile alemán invocó a dios
y exclamó con su habitual vehemencia:
El *rotor* de un *rotor* más *nabla dos*
da el *gradiente* de toda *divergencia*.

one understand that the electromagnetic equations do not change if the replacement, with Λ a scalar function

$$\begin{aligned} \vec{A} &\rightarrow \vec{A}' = \vec{A} + \vec{\nabla}\Lambda \\ \phi &\rightarrow \phi' = \phi - \frac{\partial \Lambda}{\partial t} \end{aligned}$$

is performed, because both $\vec{E}$ and $\vec{B}$ are not changed. The gauge symmetry is clear. Moreover, gauge invariance of electromagnetism guarantees the charge conservation.

The previous comments open the possibility of using the field $\vec{A}$ as compensator when one ask for local gauge invariance of a field theory. In fact, the quantum electromagnetic interaction mediated by photons, can be formalized starting from the requirement of local (abelian) gauge invariance. This framework can be generalized, as Yang and Mills proposed, to the case of a non-abelian group of symmetry.

In summary, the fundamental interactions can be described by theories with local gauge symmetry and consequently mediated by the corresponding compensator field, the corresponding gauge field.

Even if we will come back later to this important point, let us state a fundamental property of gauge fields: they are massless fields. In fact, a typical mass term $\mathscr{L}_{mass} = \frac{1}{2} m A_\mu A^\mu$ for a vector field is clearly not gauge invariant.

7 Constructing a Gauge Theory for matter fields

7.1 "Standard" way

First of all, one chooses an appropriate unitary Lie group $G\{g\}$. Then one propose an action with global invariance under G as the matter symmetry group. Having then an action that is invariant under constant phase transformation, there appears the conservation of a Noether current. After this one promotes global invariance to a local one: $g \rightarrow g(x)\, \varepsilon\, G$, i.e. phases that are space-time dependent. Now one needs to introduce gauge fields to compensate, via a gauge transformation, the changes produced by the presence of local phases. Finally, a kinetic term for gauge fields has to be included in the resulting Lagrangian.

In all this procedure, as soon as $g(x)$ are well behaved, the conserved Noether current of the global case is unchanged.

Let us consider a typical example. A complex scalar field and an one parameter group of symmetry.

$$\mathscr{L}(\phi, \partial_\mu \phi) = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

that is invariant under a global phase transformation (constant phase α)

$$\phi(x) \rightarrow \phi'(x) = e^{\imath \alpha} \phi(x)$$

corresponding to the group $U(1)$: implements rotations on a unit circle.

Now we ask for a local phase transformation $\alpha = \alpha(x)$. The motivation for this requirement was given by Yang and Mills, staten that: "The concept of field and the concept of local interactions imply a spreading of information to neighbouring points and eliminate the action at a distance. Then global phase invariance seems to contradict the generalized idea of locality".

Performing the local gauge transformation

$$\phi(x) \to \phi'(x) = e^{\imath\,\alpha(x)}\,\phi(x) \tag{19}$$

in the Lagrangian above, one sees that the mass term stays without changes when written in terms of $\phi'(x)$ while the kinetic term does not because

$$\partial^{\mu}\phi(x) \to e^{\imath\,\alpha(x)}\left[\partial_{\mu} + \imath\,\partial_{\mu}\alpha(x)\right]\,\phi(x)$$

The question is how to turn $\mathscr{L}$ local invariant. Taking profit of the freedom of the electromagnetic field A_{μ}, one replaces ∂_{μ} by the gauge covariant derivative

$$D_{\mu}\phi(x) \equiv \left(\partial_{\mu} - \imath q A_{\mu}(x)\right)\,\phi(x) \tag{20}$$

with the constraint that when $\phi(x)$ changes with the local phase $\alpha(x)$ as in (19), the gauge field A_{μ} suffer the gauge transformation

$$A_{\mu}(x) \to A'_{\mu}(x) = A_{\mu}(x) - \frac{1}{q}\,\partial_{\mu}\alpha(x) \tag{21}$$

where q clearly measures the coupling between de scalar field and the gauge field. In this way, the covariant derivative transforms exactly as the field. Consequently, the invariant theory under local phase transformations has the new Lagrangian

$$\mathscr{L} = \left[\partial_{\mu} + \imath q A_{\mu}(x)\right]\,\phi^{*}(x)\,\left[\partial_{\mu} - \imath q A^{\mu}(x)\right]\,\phi(x) - m^{2}\,\phi^{*}(x)\phi(x) \tag{22}$$

that explicitly includes interaction, exactly the so called minimal interaction. The only way to have local phase invariance is including interaction with the gauge field that plays the dual action of a compensating field to restore the symmetry and of a comparative field that distinguishes charges in the case of the original complex field. As we have said before, one has to add the kinetic term of the gauge field, namely

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{23}$$

but one cannot add a mass term for A_{μ}, because it breaks the phase symmetry.

Certainly, one can make the same treatment for fermion fields, to obtain

$$\mathscr{L}_{f} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \imath\,\bar{\psi}\gamma^{\mu}\,\left(\partial_{\mu} - \imath e A_{\mu}(x)\right)\,\psi - m\,\bar{\psi}\,\psi \tag{24}$$

and the same procedure could be performed in connection with non-abelian phase transformations. We will be back to this point below.

8 "Upsidedown" way

It is possible and "natural" to invert the process above avoiding the "ukase"[2] that promotes global to local symmetries on the matter Lagrangian. The upsidedown proposal we have presented (C.A. García Canal and F.A. Schaposnik, "Building Gauge Theories: The Natural Way", Fundamental Journal of Modern Physics, 2 (2011) 15) starts with the symmetry of the gauge field, where it is natural. We build the gauge theory of fundamental interactions starting from the interaction mediators: the gauge fields. The sources (matter) are added imposing local gauge invariance and Lorentz invariance.

[2]ukase: have the power of laws but may not alter the regulations of existing laws

Let us explain the procedure starting with electrodynamics. The Maxwell equations, without sources are contained in

$$\partial_\mu F^{\mu\nu} = 0$$

the other two equations are given by Bianchi identity. Remember that

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu$$

that under the gauge transformation (21) is invariant and of course also Maxwell equations are. The Maxwell Lagrangian (23) is gauge invariant and Lorentz invariant.

Let us include now an external (non-dynamical) source j^μ_{ext} to write

$$\partial_\mu F^{\mu\nu} = e\, j^\nu_{ext}$$

Now the natural and simplest Lorentz invariant term in the Lagrangian formalism is

$$\mathscr{L}_{int} = e A_\mu(x)\, j^\mu_{ext}(x)$$

and in order to get a gauge invariant total Lagrangian, one can require that under a $\Lambda(x)$ gauge transformation

$$j^\mu_{ext}(x) \to j^{\mu\,\Lambda}_{ext}(x) = j^\mu_{ext}(x)$$

and, forced by Maxwell

$$\partial_\mu\, j^\mu_{ext}(x) = 0$$

Notice that the Lagrangian including $j^\mu_{ext}(x)$, under a gauge transformation change at most as a total derivative.

As the structure of Maxwell equations is prescribed by Lorentz symmetry, one can anticipate the coupling of matter to the gauge field. To this end, let us consider a dynamical Dirac fermion field $\psi(x)$ at the origin of the current $j^\mu(x)$ (in principle different to $j^\mu_{ext}(x)$). Certainly the most economic is a bilinear form of fermions, that due to Lorentz requirements reads

$$j^\mu(x) = \bar{\psi}(x)\,\gamma^\mu\,\psi(x)$$

that together with the gauge invariance of $j^\mu(x)$, implies that under a gauge transformation $\Lambda(x)$

$$\psi(x) \to \psi^\Lambda(x) = e^{\imath q \Lambda(x)}\,\psi(x)$$

where q is a real number. Finally, in order to have dynamical fermions, one adds

$$\mathscr{L}_D = \imath\,\bar{\psi}(x)\,\gamma^\mu\,\partial_\mu\,\psi(x)$$

and we end with a gauge invariant total Lagrangian $\mathscr{L} = \mathscr{L}_M + \mathscr{L}_D + \mathscr{L}_{int}$, if $q \equiv e$. We can introduce the usual covariant derivative $D_\mu = \partial_\mu - e A_\mu(x)$ that allows one to write the well known minimal electromagnetic coupling

$$\mathscr{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}(x)\,\gamma^\mu\,D_\mu\,\psi(x) \tag{25}$$

Certainly, the same procedure, under the same requirements can easily be done for the non-abelian case.

Summarizing this section we remember that the usual way of building a gauge theory starts promoting a global unitary symmetry of the matter Lagrangian to a local one and this requires the inclusion of a gauge field that introduces an unavoidable, but wanted, interaction. The upsidedown way starts from the pure gauge theory that is coupled to matter by imposing Lorentz and gauge invariance in constructing the matter current.

Finally notice that the upsidedown way goes parallel to the geometrical approach that starts defining a connection in a principal fiber bundle and introduce matter fields as sections in the associated vector bundle.

9 Weak Interactions

Let us remember that the weak interaction is responsible for i) nuclear β-decay; ii) many hadron decays; iii) everything induced by neutrinos. They are characterized by an effective coupling $G_F \simeq 10^{-5}/m_p \simeq 1.4\,10^{-49}\,erg\,cm^3$ and have the property of breaking the symmetry of parity (P). They also suggested (to Pauli) the existence of neutrino.

The first phenomenological analysis due to Fermi was inspired by a comparison between the electrodynamical vertex $e \to e + \gamma$ and neutron beta decay $n \to p + e + \bar{\nu}_e$. It ends whit the Lagrangian corresponding to a point interaction

$$\mathscr{L}_F = G_F\, J^{\mu}_{np}(x) \cdot J_{\mu\, e\bar{\nu}}(x) = G_F\, \left[\bar{\psi}_p(x)\,\gamma^{\mu}\,\psi_n(x)\right]\, \left[\bar{\psi}_e(x)\,\gamma_{\mu}\,\psi_{\nu_e}(x)\right]$$

This proposal is parity conserving, but after Wu experiment to test the Lee and Yang prediction of the violation of parity in weak interactions, this Lagrangian was modified changing from a vector like interaction to a vector minus axial interaction, namely with currents of the form

$$J^{\mu}(x) = \bar{\psi}_f\, \gamma^{\mu}\, (1-\gamma^5)\, \psi_{f'}(x)$$

This $V-A$ property of the weak currents, implies that the neutrino is left handed.

It is necessary to be aware that the Fermi proposal should be considered as a phenomenological effective Lagrangian because it works well as a first order approximation and fails al higher orders and is an example of a non-renormalizable theory. It becomes harmful at the so-called limit of unitarity, around $300\,GeV$.

Consequently one tries to improve the treatment of weak interactions taking inspiration in QED. To this end one looks for a theory with a dimensionless coupling constant and a carrier of the interaction playing the role of the photon in QED.

The resulting model is the Intermediate Boson Theory that includes two charged vector bosons mediators of the weak interaction, $W^{\pm}$. This is done with the constraint that the Fermi theory is the limit of low energy ($q^2 \ll M_W^2$). Here also a problem with convergence appears because the very short range of weak interaction requires that the vector bosons must be massive (of the order of $80\,GeV$). Consequently there are contributions icluding the massive propagator of the $W^{\pm}$ that diverge for large momenta and the intermediate boson theory is again only a phenomenological model valid below its own limit of unitarity.

10 Latent Symmetry

We present now the way to the *electroweak* theory that allows one to deal simultaneously with both interactions: electromagnetic plus weak. The way to the theory is via gauge theories as the ones we have discussed previously. The problem to be solved is the necessary mass the mediators of weak interaction must have. This is in conflict with gauge symmetry that does not allow a mass term for the gauge fields. In any case, there is a way out of this conundrum, the possibility of having the symmetry in a latent version. This is no more than to have the symmetry realized à la Nambu-Goldstone. The symmetry is present in the Lagrangian but is not respected by the vacuum expectation value of the fields. This fact has non-trivial consequences in systems of infinite extension.

We start the discussion of this topic by presenting a simple mechanical example. This is the case of the bent rod.

Consider a cylindrical rod along the z-axis, charged with a force F along its axis. The system is obviously symmetric under rotations around the z-axis. The system has the symmetric solution $x = y = 0$ as soon as the force F is sufficiently small. In fact, there is a critical force F_{cr} given by $F_{cr} = \pi^2 E I/\rho^2$ where E is the modulus of elasticity, I, the moment of inertia of the rod and ρ its density. When $F > F_{cr}$, the rod is bent, given rise to an asymmetric solution from a symmetric equation of motion. Certainly one cannot predict the direction in the (x, y) plane where the rod is going to bend. However, a symmetry transformation on an asymmetric solution goes into another asymmetric solution. The symmetry is *latent*. In a language nearer to our field theory, we say that the ground state (the vacuum) is degenerate. The symmetry is said to be *spontaneously broken*, it is realized à la Nambu-Goldstone.

Another nice example is the Heisenberg model for a magnet based on the Hamiltonian with exchange interaction

$$H = K \sum_{<ij>} \vec{s}_i \cdot \vec{s}_j$$

that clearly is rotational, $O(3)$, invariant. This model presents, for a temperature below a critical one ($T < T_c$) a breaking of the $O(3)$ symmetry because elementary magnetic moments (spins if you want) tend to align. The well known ferromagnetic transition. In any case, the symmetry is maintained latent. In fact (if you can eliminate the effect of the magnetic field of Earth, or others), there is no preferred direction for the alignment. Each possible direction is connected with any other by an $O(3)$ transformation. Certainly, to this happens it is necessary to be in the thermodynamical limit, a system of infinite extension, infinite degeneracy of the angular momenta. The Goldstone mode of this spontaneously broken symmetry is the spin wave.

We go now towards the Higgs model. Let us consider first a model with:

Continuous global latent symmetry

This model is also called Goldstone model and is defined by the complex scalar field Lagrangian

$$\mathscr{L}(\phi, \partial_\mu \phi) = -\partial_\mu \phi^\dagger(x)\, \partial^\mu \phi(x) - \lambda\, [\phi^\dagger(x)\, \phi(x) - v]^2 \tag{26}$$

that is invariant anther a $U(1)$ global transformation: $\phi(x) \to \phi'(x) = e^{\imath\alpha}\, \phi(x)$ with α constant. The behavior of the interaction $V(\phi^\dagger, \phi) = \lambda\, [\phi^\dagger(x)\, \phi(x) - v]^2$ depends on the sign of v.

case i) $v < 0$

consequently, V has one minimum at $\phi = 0$ and the $U(1)$ symmetry is Wigner-Weyl realized.

case ii) $v > 0$

now V has an infinite number of minima, defined by $\phi^\dagger\, \phi = v$ or, in terms of the real ϕ_1 and the imaginary part ϕ_2 of ϕ: the circle of minima $\phi_1^2 + \phi_2^2 = v$. The picture of the interaction energy looks like a Mexican hat or a "culo de botella". Then we have a Nambu-Goldstone realization of the $U(1)$ symmetry. The symmetry is latent and there is a Goldstone boson present ($U(1)$ has one generator).

Clearly, in this case it has no sense to develop a perturbation theory around $\phi = 0$ (an unstable point). One has to develop around some point $\phi = \sqrt{v}\, e^{\imath\theta}$, with θ arbitrary due to the non-uniqueness of vacuum. One can take $\theta = 0$ and define the shift

$$\phi(x) = \sqrt{v} + \chi(x) \ \ \Rightarrow \ \ \langle 0|\chi(x)|0\rangle = 0 \tag{27}$$

It is worth writing the interaction in terms of χ

$$V = \lambda\, v\, (\chi + \chi^\dagger)^2 + 2\lambda\, \sqrt{v}\, \chi^\dagger\, \chi\, (\chi + \chi^\dagger) + \lambda\, (\chi\, \chi^\dagger)^2$$

that when expressed in terms of $\chi_1 = (\chi + \chi^\dagger)/\sqrt{2}$ and $\chi_2 = \imath\, (\chi^\dagger - \chi)/\sqrt{2}$ shows that there is a massive degree of freedom χ_1 with mass $m_1 = 4\, \lambda\, v$ and a massless one, χ_2 which is the Goldstone boson that is present because the symmetry was spontaneously broken. One can say also that this shows that the $U(1)$ symmetry is not more present in the spectrum.

We can still get more insight on the model properties by performing the change of variables: $\chi_1 = \rho \cos\theta$ and $\chi_2 = \rho \sin\theta$. It is clear that a $U(1)$ transformation implies the changes: $\rho \to \rho$ and $\theta \to \theta + \alpha$. After the displacement $\rho' = \rho - \sqrt{v}$, the Lagrangian reads

$$\mathscr{L} = \frac{1}{2}\, (\partial_\mu \rho')^2 + \frac{1}{2}\, (\rho' + \sqrt{v})^2\, (\partial_\mu \theta)^2 - v\, (\rho' + \sqrt{v})$$

and shows that ρ is a radial excitation (clearly seen on the Mexican hat) and θ corresponds to the movement around the circle without energy consumption $\hbar\omega = 0$, certainly the massless Goldstone boson excitation.

Local global latent symmetry: Higgs model

Whenever a local symmetry is Nambu-Goldstone realized, the should be Goldstone boson disappear and the gauge field acquires an effective mass. In other words, the Goldstone boson degrees of freedom are transferred to the longitudinal polarization massive vector bosons have. This is the so called Higgs mechanism.

We present the mechanism for the simple complex scalar field

$$\mathscr{L}(\phi, \partial_\mu \phi) = -\partial_\mu \phi^\dagger(x)\, \partial^\mu \phi(x) - \lambda\, [\phi^\dagger(x)\, \phi(x) - v]^2$$

that, as we saw before, when $v > 0$ the $U(1)$ symmetry is latent. Now consider that the symmetry is local. Consequently, one has to replace the derivatives by covariant ones, namely

$$\partial_\mu \to \partial_\mu - \imath g\, a_\mu(x)$$

where a_μ is the massless gauge field. It is also necessary to include the kinetic term of this field, to write

$$\mathscr{L}(\phi, \partial_\mu \phi) = -(D_\mu \phi)^\dagger D^\mu \phi - \lambda\, [\phi^\dagger \phi - v]^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

this Lagrangian includes several interactions. Among them we explicitly show

$$\begin{aligned} \mathscr{L}^{(1)} &= g\, a_\mu\, (\phi^\dagger \partial^\mu \phi - \phi\, \partial^\mu \phi^\dagger) \\ \mathscr{L}^{(2)} &= -g^2\, a_\mu\, a^\mu\, \phi^\dagger \phi \end{aligned}$$

As the symmetry is spontaneously broken, one has to shift to the field χ with zero vacuum expectation value as in Eq.(27). From the second expression above, $\mathscr{L}^{(2)}$, it results

$$\mathscr{L}_M = -g^2\, v\, a_\mu\, a^\mu = -\frac{1}{2} m_a^2\, a_\mu\, a^\mu \tag{28}$$

the gauge field a_μ acquires the mass

$$m_a = g\sqrt{\frac{v}{2}}$$

What it is important to notice is the fact that during this process of given mass to the gauge field, the symmetry was not completely lost because as we have said before, it remains latent. Moreover, it is easy to prove that during the process there is a conservation of degrees of freedom. In fact one goes from $(2+2)$ original degrees of freedom (2 of the complex scalar field and 2 from the massless gauge field) to $(3+1)$ degrees of freedom (3 of the now massive gauge field and 1 remaining of the scalar field). The should be Goldstone boson is said to be gauged away. The exchange of degrees of freedom is not a violation of the Goldstone theorem. In gauge theories the Goldstone theorem can be overcame by choosing either the unitary gauge where the initial symmetry is destroyed or a covariant gauge in which the Goldstone theorem is valid but the Goldstone bosons become unphysical particles, they uncouple from the remaining theory. There is another point to be analyzed. Namely, the fact that once the vector field acquires mass, its propagator gains a term that makes divergent the perturbative contributions, as it was discussed when the Intermediate Boson Model was presented. However, in the present case related to the Higgs mechanism there is a gauge equivalence between the original gauge invariant renormalizable Lagrangian and the Lagrangian obtained after the action of the Higgs mechanism. This last one is useful for practical purposes, while the original one, for formal (renormalization) purposes. For this reason a Yang-Mills theory with latent symmetry, with spontaneously broken symmetry, is renormalizable. It maintains sufficient Ward-like identities as to guarantee this property.

11 3!: The Standard Model

As it was mentioned before, the 3! model includes the local symmetry under the gauge group $SU_C(3) \otimes SU(2)_L \otimes U(1)_Y$. The group $SU(3)_C$ takes into account the strong interaction between quarks mediated by gluons, the corresponding gauge fields. Here we restrict ourselves to the electromagnetic and weak interactions: electroweak, included in 3! as $2! \equiv SU(2)_L \otimes U(1)_Y$.

Let us begin by quoting from one of the authors of the electroweak Standard Model, the S. Weinberg's text in the Physical Review Letters article of 1967 where the $2! = 2 \times 1$ was proposed: "Leptons interact only with photons and with the intermediate bosons which presumably mediate the weak interaction. What can be more natural than linking those spin one bosons in a multiplate of gauge fields?"

The proposal of the Salam-Weinberg-Glashow model starts by noting that it is necessary to solve several difficulties. Namely, the photon γ is massless while the intermediate boson W has to be massive. Moreover, the value of the electromagnetic coupling is very different from the weak coupling characterized by the Fermi constant G_F. In connection with this last point, it is clear that the hierarchy of couplings needs the presence of a mixing angle (sinus versus cosinus).

The first question to be answered to build up a gauge theory it is precisely to decide which gauge group to use for connecting with the weak and electromagnetic interactions. The model must include

- Charged weak current, $J_\mu^\pm$
- Electromagnetic interaction related to a group $U(1)$ that should be realized à la Wigner-Weyl to guarantee $m_\gamma = 0$
- Neutral weak current. Notice that its presence allows for a cancellation mechanism that compensate the divergent contributions coming from a massive and charged W

Consequently, the gauge group G for weak and electromagnetic interactions has to be a 4 parameter group. The proposal is

$$G \equiv SU(2)_L \otimes U(1)_Y \tag{29}$$

or

$$2! = 2 \times 1$$

where the index L stands for *left* and the index Y for *(weak)-hypercharge*.

Remember that any fermion spin 1/2 field can be decomposed into left and right components, namely

$$\psi_{L,R} = \frac{1 \mp \gamma_5}{2}\, \psi$$

The index L in $SU(2)$ is because ψ_L behaves differently of ψ_R under the transformations of this group. Then we can say that the 2!-model is a chiral theory: distinguishes between left and right.

We also remember that the charged weak current must have a $(V-A)$ structure that includes a $(1-\gamma_5)$ factor. This is why only ψ_L enters the play. In other words, Fermi phenomenological proposal plus Parity violation implies

$$J_\mu^{cc} = \bar{\psi}_i\, \gamma_\mu\, (1-\gamma_5)\, \psi_j \equiv 2\, \psi_{i,L}\, \gamma_\mu\, \psi_{j,L}$$

with $(\psi_i, \psi_j) = (\nu_e, e), (u,d), ...$

The simplest representation of $SU(2)_L$ is a doublet

$$\begin{pmatrix} \psi_i \\ \psi_j \end{pmatrix}$$

Notice that $SU(2)$ has three generators, say T_1, T_2, T_3. With T_1 and T_2 one can build $T_\pm$ related to the charged current. But T_3 cannot be identified with the electric charge Q, because the electromagnetic charge is pure vector and not $V-A$. For this reason the G group is $SU(2)_L \otimes U(1)_Y$.

Which is this $U(1)_Y$? From the currents

$$J_\mu^{e.m.} = q_i \bar{\Psi} \gamma_\mu \Psi \Rightarrow Q = \int d^3x J_0^{e.m.}$$
$$J_\mu^3 = \bar{\Psi}_L \gamma_\mu \frac{\tau_3}{2} \Psi_L \Rightarrow T_3 = \int d^3x J_0^3$$

Then, both members of any doublet as $(\nu_{e,L}\, e_L)$ have the same quantum number ($Y = Q - T_3$), the same hypercharge. $Y = Q - T_3$ commutes with the generators of $SU(2)$ and defines the group $U(1)$ of 2!.

Matter under $SU(2)_L \otimes U(1)$

In each generation of quarks and leptons, the G quantum numbers are repeated. Let us consider, as an example, the first generation: (e, ν_e, u, d). The doublets of $SU(2)_L$ are $(\nu_e, e)_L$ and $(u,d)_L$. As we already said, this election guarantees $(V-A)$ and the index L is well understood.

The good assignments of $SU(2)_L$ and $U(1)_Y$ values for the first generation is.

	$(\nu_e, e)_L$	e_R	$(u,d)_L$	u_R	d_R
$SU(2)_L$	2	1	2	1	1
$U(1)_Y$	$-1/2$	-1	$1/6$	$2/3$	$-1/3$

and as was stated above, this quantum numbers repeat for the second and the third generations. In any case, the desired electric charge of fermions verify the relation

$$Q = I_3^W + Y_W$$

where the index W was introduced just to remember that $SU(2)_L$ is in some sense the weak version of isospin while $U(1)_Y$ is seen as the weak hypercharge.

Building the Lagrangian

As it was mentioned before, the left handed components of each generation of leptons and quarks are assigned into doublets of the fundamental representation of $SU(2)_L$. In order to simplify the expressions, let us introduce the notation for the doublets

$$L_\alpha(x) = \frac{(1-\gamma_5)}{2} \begin{pmatrix} i_\alpha(x) \\ j_\alpha(x) \end{pmatrix}$$

where

$$i_\alpha(x) \equiv \nu_e(x); \nu_\mu(x); \nu_\tau(x)$$
$$\equiv u(x); c(x); t(x)$$

and

$$j_\alpha(x) \equiv e(x); \mu(x); \tau(x)$$
$$\equiv d(x); s(x); b(x)$$

and for the singlets ;

$$R_\alpha(x) = \frac{(1-\gamma_5)}{2} k_\alpha(x)$$

with $k_\alpha \equiv u(x); c(x); t(x);\ e(x); \mu(x); \tau(x);\ d(x); s(x); b(x)$

Now we go to the four gauge vector bosons needed to guarantee the local gauge invariance under $SU(2)_L \otimes U(1)_Y$. There are 3 corresponding to the weak isospin: $W_\mu^1(x)$, $W_\mu^2(x)$ and $W_\mu^3(x)$ and 1 corresponding to weak hypercharge: $B_\mu(x)$. In writing the corresponding covariant derivatives one introduces the couplings g_2 for $SU(2)_L$ and g_1 for $U(1)_Y$. The gauge invariant Lagrangian is

$$\mathscr{L}_{2!} = -\frac{1}{4} \vec{F}_{\mu\nu}(x) \vec{F}^{\mu\nu}(x) - \frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x)$$

$$
\begin{aligned}
&+\imath \sum_{l=e,\mu,\tau} \bar{L}_l(x)\,\gamma^\mu\, D_\mu L_l(x) + \imath \sum_{l=e,\mu,\tau} \bar{R}_l(x)\,\gamma^\mu\, D_\mu R_l(x) \\
&+\imath \sum_{q=(u,d),(c,s),(t,b)} \bar{L}_q(x)\,\gamma^\mu\, D_\mu L_q(x) + \imath \sum_{q^\uparrow=u,c,t} \bar{R}_{q_\uparrow}(x)\,\gamma^\mu\, D_\mu R_{q_\uparrow}(x) \\
&+\imath \sum_{q^\downarrow=d,s,b} \bar{R}_{q_\downarrow}(x)\,\gamma^\mu\, D_\mu R_{q_\downarrow}(x) \qquad (30)
\end{aligned}
$$

The corresponding Yang-Mills strength tensors are

$$
\vec{F}_{\mu\nu}(x) = \partial_\mu \vec{W}_\nu(x) - \partial_\nu \vec{W}_\mu(x) + g_2 \vec{W}_\mu(x) \wedge \vec{W}_\nu(x) \qquad (31)
$$

$$
B_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x) \qquad (32)
$$

These expressions clearly show that whenever the gauge symmetry is related to a non-abelian group, there are self interactions of gauge fields. This is an important difference with QED because the photon is the gauge field of the abelian symmetry $U(1)$.

Here we summarize the covariant derivatives. First for LEPTONS:

$$
(for\, L_l(x):)\;\; D_\mu \equiv \partial_\mu + \imath g_2 \frac{\vec{\tau}}{2}\cdot \vec{W}_\mu(x) - \imath \frac{1}{2} g_1 B_\mu(x) \qquad (33)
$$

$$
(for\, R_l(x):)\;\; D_\mu \equiv \partial_\mu - \imath g_1 B_\mu(x) \qquad (34)
$$

and then for QUARKS

$$
(for\, Lq_\uparrow(x):)\;\; D_\mu \equiv \partial_\mu + \imath g_2 \frac{\vec{\tau}}{2}\cdot \vec{W}_\mu(x) + \imath \frac{1}{6} g_1 B_\mu(x) \qquad (35)
$$

$$
(for\, R_{q_\uparrow}(x):)\;\; D_\mu \equiv \partial_\mu + \imath \frac{2}{3} g_1 B_\mu(x) \qquad (36)
$$

$$
(for\, R_{q_\downarrow}(x):)\;\; D_\mu \equiv \partial_\mu - \imath \frac{1}{3} g_1 B_\mu(x) \qquad (37)
$$

It remains the problem of assigning masses to the massive vector bosons to be identified with the mediators of weak interaction and also to give masses to the massive fermions. Remember that for the first ones, gauge symmetry does not allow for a mass term of gauge fields, and for fermions, a typical Dirac mass term

$$
\mathscr{L}_{mass} = -m\,\bar{\psi}\,\psi = -m\,(\bar{\psi}_L\,\psi_R + \bar{\psi}_R\,\psi_L)
$$

is unacceptable because ψ_L is a $SU(2)_L$ doublet while ψ_R is a singlet.

Certainly, the solution is provided by latent symmetry and the Higgs mechanism. In this way one gets massive gauge fields. But we have to remember that and the end of the play, one massless photon have to remain. In other words, the spontaneous breaking of the symmetry should go from $SU(2)_L \otimes U(1)_Y$ to a $U(1)_{e.m.}$ realized a la Wigner-Weyl, because the photon is strictly massless. The mechanism must guarantee that we go from 4 massless bosons to 3 massive and 1 massless.

The photon, the massless one, is represented by the field A^μ that results as the neutral combination of B^μ and the third component of the triplet $\vec{W}^\mu$, W_3^μ. Correspondingly, the orthogonal combination to A^μ is the weak neutral boson Z^μ. These combinations are measured by a mixing angle θ_W, called Weinberg angle. The good combinations reads

$$
A_\mu = \sin\theta_W\, W_{\mu,3} + \cos\theta_W\, B_\mu \qquad (38)
$$

$$
Z_\mu = \cos\theta_W\, W_{\mu,3} - \sin\theta_W\, B_\mu \qquad (39)
$$

On the other hand, the charged weak bosons are the combinations

$$
W_\pm^\mu = \frac{1}{\sqrt{2}}\left(W_1^\mu \mp \imath W_2^\mu\right) \qquad (40)
$$

Now we can explicitly write the weak and electromagnetic interactions contained in the Lagrangian (30)

$$
\begin{aligned}
\mathscr{L} &= \frac{g_2}{2\sqrt{2}}[J^\mu_- W_{\mu,+} + J^\mu_+ W_{\mu,-}] \\
&+ \left[(g_2 \cos\theta_W + g_1 \sin\theta_W) J^\mu_3 - g_1 \sin\theta_W J^\mu_{em}\right] Z_\mu \\
&+ \left[(g_1 \cos\theta_W J^\mu_{em} + (g_1 \cos\theta_W - g_2 \sin\theta_W) J^\mu_3\right] A_\mu
\end{aligned}
$$

where $J^\mu_\pm = 2\,(J^\mu_1 \mp \imath J^\mu_2)$ and $J^\mu_{em} = J^\mu_3 + J^\mu_Y$ while J_1 and J_2 the currents containing τ_1 and τ_2 respectively.

Now, in order to reobtain the standard electromagnetic interaction $\mathscr{L}^{em}_{int} = e\,J^\mu_{em} A_\mu$, the identification

$$e = g_1 \cos\theta_W = g_2 \sin\theta_W \tag{41}$$

is required. Consequently, the interaction Lagrangian above gets the form

$$
\begin{aligned}
\mathscr{L} &= \frac{e}{2\sqrt{2}\sin\theta_W}\left[J^\mu_- W_{\mu,+} + J^\mu_+ W_{\mu,-}\right] \\
&+ \frac{e}{2\cos\theta_W \sin\theta_W} Z^\mu J_{\mu,NC} + e A^\mu J_{\mu,em}
\end{aligned} \tag{42}
$$

where one defined the neutral current $J^\mu_{NC} = 2\,(J^\mu_3 - \sin^2\theta_W J^\mu_{em})$.

To summarize, we remark that the electromagnetic interaction is measured by e as it should be, the charged weak interactions coupling is $e/(2\sqrt{2}\sin\theta_W)$ while the neutral weak interaction one is $e/(2\cos\theta_W \sin\theta_W)$. Now the Feynman rules can easily be obtained and from them, any perturbative calculation as cross sections, decay rates, etc., follow.

It remains to discuss the problem of masses of both, the weak gauge bosons and the massive fermions. As it was stated above, the Higgs mechanis is in order. The idea is to put the 2!-symmetry in a latent version but being careful to keep a $U(1)$, corresponding to electromagnetism, realized à la Wigner-Weyl. It is necessary to introduce scalar fields being non-trivial under $SU(2)_L \otimes U(1)_Y$. The simplest election is a $SU(2)_L$ doublet

$$\Phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \tag{43}$$

with both ϕ^0 and ϕ^- complex fields. Clearly, the field Φ is a $SU(2)_L$ doublet and one chooses $Y_\Phi = -1/2$. The latent character of the symmetry is obtained by the inclusion of the interaction

$$V(\Phi^\dagger\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger\Phi - v^2\right)^2 \tag{44}$$

that implies that

$$\langle 0|\Phi|0\rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

and as we show below, this election guarantees the $U(1)_{em}$. In fact, one can perform a test of coherence of the spontaneous symmetry breaking just proposed. It is easy to compute the effect of the generators of $SU(2)_L$ and the one of $U(1)_Y$ on the vacuum expectation value of Φ, namely

$$
\begin{aligned}
G_1\,\langle 0|\Phi|0\rangle &= \frac{1}{2}\begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0 \\
G_2\,\langle 0|\Phi|0\rangle &= \frac{1}{2}\begin{pmatrix} 0 \\ \imath v \end{pmatrix} \neq 0 \\
G_3\,\langle 0|\Phi|0\rangle &= \frac{1}{2}\begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0
\end{aligned}
$$

$$G_Y \langle 0|\Phi|0\rangle \quad = \quad -\frac{1}{2}\begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

that at first sight seems to leave the complete symmetry latent. However, the electromagnetic generator of the symmetry $U(1)$, that is $Q = I_3 + Y$ certainly verifies

$$Q\langle 0|\Phi|0\rangle = 0$$

that characterizes a symmetry realized à la Wigner-Weyl. The zero mass for the photon is guaranteed.

There are three would be Goldstone bosons in (43) that decouple from the theory and provide the longitudinal polarizations that the massive W^+, W^- and Z require.

The Φ original Lagrangian reads

$$\mathscr{L} = -\frac{1}{2}(D_\mu\Phi)^\dagger D^\mu\Phi - V(\Phi^\dagger\Phi)$$

with

$$D_\mu\Phi = \left(\partial_\mu + \imath\frac{g_1}{2}Y_\mu - \imath g_2\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)$$

now one performs the standard shift $\Phi(x) = H(x) + v$ with $\langle 0|H(x)|0\rangle = 0$. The $H(x)$ is the Higgs boson field with a mass, that can be obtained from the Lagrangian after the shift, being

$$m_H^2 = 2\lambda v^2$$

The covariant derivative of the Higgs field gives rise to the gauge fields-Higgs interactions, that after the shift allows one to read the effective mass terms of $W^\pm$ and Z. In fact

$$\begin{aligned}\mathscr{L}_{GaugeMass} \quad = \quad & -\frac{1}{4}(g_2 v)^2 W_+^\mu W_{\mu,-} \\ & -\frac{1}{8}\left(\frac{g_2 v}{\cos\theta_W}\right)^2 Z^\mu Z_\mu,\end{aligned}$$

and the masses of the three massive weak gauge bosons are

$$M_{W^\pm}^2 = \frac{g_2^2 v^2}{4} \tag{45}$$

and

$$M_Z^2 = \frac{g_2^2 v^2}{4\cos^2\theta_W} = \frac{M_W^2}{\cos^2\theta_W} \tag{46}$$

Notice that the experimental determination of M_W allows to fix, once the coupling is determined, the value of $v \approx 246\, GeV$, the scale of breaking of the symmetry. This, together with the measurement of M_Z, fixes

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

that results $\sin^2\theta_W = 0.2234$ in the so called on-shell scheme.

The usual way of obtaining the value of the gauge couplings is by comparison with the low energy weak interaction phenomenology. In this regime, corresponding to $q^2 \ll M_W$, the Fermi model works satisfactorily. Moreover, under this condition, the matrix elements contributing to a weak process, say the β-decay, are identified with the Fermi prediction, as soon as

$$\frac{g_2^2}{8 M_W^2} \equiv \frac{G_F}{\sqrt{2}} \tag{47}$$

Consequently

$$v = \frac{2M_W}{g_2} = \left(\sqrt{2}G_F\right)^{-1/2} \simeq 246\,GeV \tag{48}$$

It remains to discuss the mechanism to provide mass to the leptons and quarks. One takes profit of the Higgs mechanism and starts introducing a Yukawa type of coupling with the scalar doublet. Let us begin with leptons, remembering that the Standard Model works with massless neutrinos. The Yukawa lagrangian for leptons coupled via h_l to $\Phi(x)$ reads

$$\mathscr{L}_Y(x) = -\sum_{l=(\nu_e,e),(\nu_\mu,\mu),(\nu_\tau,\tau)} h_l\,\bar{R}_l(x)\,\Phi^\dagger(x)\,L_l(x) + h.c. \tag{49}$$

Notice that this expression is correct since it implies the product of a doublet times an adjoint doublet times a singlet. Now, once the shift from $\Phi(x)$ to $H(x)$ is performed after the spontaneous breaking of the symmetry, it appears a mass like coupling

$$\mathscr{L}_Y(x) = -\sum_{l=e,\mu,\tau} h_l\,v\,\bar{\Psi}_l(x)\,\Psi_l(x) \tag{50}$$

that allows to identify the mass of each lepton

$$m_l = h_l\,v \tag{51}$$

It should be notice that the couplings h_l are not fixed by any principle in 2!. They are external parameters.

In the case of quarks, as both members of the doublets are massive, it is necessary to consider two isoscalars

$$\mathscr{L}_Y(x) = -\sum_{i,j} h_{i,j}\,\bar{L}_{q_i}(x)\,\Phi(x)\,R_{q_j}(x) - \tilde{h}_{i,j}\,\bar{L}_{q_i}(x)\,\tilde{\Phi}(x)\,R_{q_j}(x) + h.c. \tag{52}$$

where

$$\tilde{\Phi}(x) = \imath\,\tau_2\,\Phi^*(x) = \begin{pmatrix} \phi^0(x) \\ -\phi^+(x) \end{pmatrix}^*$$

Consequently, after the spontaneous breaking of the symmetry one has

$$\mathscr{L}_{masses}(x) = -\bar{q}_L^{\uparrow}(x)\,\mathscr{M}(2/3)\,q_R^{\uparrow}(x) - \bar{q}_L^{\downarrow}(x)\,\mathscr{M}(-1/3)\,q_R^{\downarrow}(x) + h.c.$$

where

$$q_{L,R}^{\uparrow}(x) \equiv \frac{1}{2}\,(1\mp\gamma_5)\begin{pmatrix} u(x) \\ c(x) \\ t(x) \end{pmatrix}$$

and

$$q_{L,R}^{\downarrow}(x) \equiv \frac{1}{2}\,(1\mp\gamma_5)\begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix}$$

Consequently, the 3×3 mass matrices are

$$\mathscr{M}(2/3) \equiv \tilde{h}_{ij}\,v \;\;;\;\; i,j = u,c,t$$

and

$$\mathscr{M}(-1/3) \equiv h_{ij}\,v \;\;;\;\; i,j = d,s,b$$

These matrices can be diagonalized via

$$U_L(Q)\,\mathscr{M}(Q)\,U_R(Q) = \hat{M}(Q)\;(diagonal)\;\;;\;\;(Q = \frac{2}{3}, -\frac{1}{3})$$

In this way, the mass Lagrangian results

$$\mathscr{L}_{masses}(x) = \bar{\psi}_L^{\uparrow}(x)\,\hat{M}(2/3)\,\psi_R^{\uparrow}(x) + \bar{\psi}_L^{\downarrow}(x)\,\hat{M}(-1/3)\,\psi_R^{\downarrow}(x)$$

where the mass eigenstates are

$$\begin{aligned}\psi_{L,R}^{\uparrow}(x) &= U_{L,R}(2/3)\,q_{L,R}^{\uparrow}(x)\\ \psi_{L,R}^{\downarrow}(x) &= U_{L,R}(-1/3)\,q_{L,R}^{\downarrow}(x)\end{aligned}$$

The process of diagonalizing the mass matrices has as a consequence that in the charged current interaction sector there is flavor mixing. In fact, the term

$$\bar{q}_L^{\uparrow}(x)\,\gamma^{\mu}\,q_L^{\downarrow}(x)\,W_{\mu}^{\dagger}(x) + h.c.$$

gives rise, after the "rotation" in the process, to

$$\bar{\psi}_L^{\uparrow}(x)\,\gamma^{\mu}\,U_L(2/3)\,U_L^{\dagger}(-1/3)\,\psi_L^{\downarrow}(x)\,W_{\mu}^{\dagger}(x) + h.c.$$

The resulting unitary matrix

$$V \equiv U_L(2/3)\,U_L^{\dagger}(-1/3) \tag{53}$$

is called Cabibbo-Kobayashi-Maskawa mixing matrix. When writing the charged weak interaction with W, that matrix acts on flavor degrees of freedom of charged $-1/3$ quarks (↓). It is a generalization of the Cabibbo matrix that was introduced when only four flavors were known and in order to keep the universality of weak interactions.

This last discussion ends the introduction of 2!, the Standard Model of electroweak interactions. It has an impressive success and a strong predictive power. We recommend the reader to consult, for example,

http://www-pdg.lbl.gov/2016/reviews/rpp2016-rev-standard-model.pdf

to have a clear idea of this success of the model and also of the constraints it implies to eventual new physics.

12 Finale

This lectures that were presented at the 2017 CERN - Latin-American School of High Energy Physics have to be considered as a mere ”opening of a door”. A door that should allows us to enter in an almost unbounded territory: the land of Contemporary Physics. Certainly, we have left aside, for time restrictions, many interesting and important topics that has to be studied and understood by the reader, consulting the vast list of references included in the Bibliography.

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