Proceedings of the 2017 CERN–Latin-American School of High-Energy Physics, San Juan del Rio, Mexico 8-21 March 2017, edited by M. Mulders and G. Zanderighi, CYR: School Proceedings, Vol. 4/2018, CERN-2018-007-SP (CERN, Geneva, 2018)

Beyond the Standard Model

M. Mondragon
Instituto de Fisica, Universidad Nacional Autonoma de Mexico UNAM, CD MX 01000, Mexico

Abstract
I present a brief outline of some of the different paths for extending the Standard Model. These include supersymmetry, Grand Unified Theories, extra dimensions, and multi-Higgs models. The aim is to give graduate students an overview of some of the theoretical motivations to go into a specific extension, and a few examples of how the experimental searches go hand in hand with these efforts.

Supersymmetry, Grand Unified Theories, Extra dimensions, Multi-Higgs models

1 Introduction

The Standard Model is an extremely successful description of the elementary particles and its interactions around the Fermi scale, but it leaves a number of unanswered questions that lead naturally to the conclusion that it is the low energy limit of a more fundamental theory. Along with these unanswered question comes a large number of free parameters whose value can only be determined experimentally. Among the unanswered questions in the Standard Model are:

- Are there more than three generations of particles?
- Why are the fundamental particles masses so different?
- How is the Higgs mass stablilized, i.e. what solves the hierarchy problem?
- Is there more than one Higgs?
- What is the nature of the neutrinos, are they Dirac or Majorana?
- What is dark matter?
- Why is there more matter than anti-matter?
- Is there more CP violation?
- Why only left-handed particles feel the electroweak interaction?
- Is it possible to unify quantum mechanics with gravity?

The collection of models and theories that make the research that extends the Standard Model to answer some of these, and many more, questions is known as physics Beyond the Standard Model (BSM). In here we will briefly explain some of the more popular efforts to go beyond the Standard Model. There are many excellent reviews devoted to each topic, in here we give only an overview of some of these extensions with an emphasis on the interplay between theory and experiment.

2 Symmetries

Modern physics is built on the observation that there are symmetries in Nature. The usual or traditional way of relating different parameters or sectors of a theory has been by adding symmetries. The Lagrangian of the theory has to respect the symmetries of the system, which means it remains invariant under symmetry transformations. This requirement means that the mathematical consistency of a proposed model can lead to new discoveries.

There are different types of symmetries: Continuous, discrete, global, local, hidden, broken, accidental... the Standard Model incorporates all of these kind of symmetries.

https://doi.org/10.23730/CYRSP-2018-004.101

Continuous symmetries are implemented by unitary transformations, since they are continuously connected to the identity. Continuous symmetries lead to conserved quantities, as stated by Noether's theorem [1], which are useful to classify or label the symmetries. These systems might have local or global conservation laws; global if the symmetry acts equally everywhere and local if the symmetry acts differently on every point.

On the other hand, discrete symmetries, as their name states, represent non-continuos transformations. In quantum mechanics they have associated a multiplicative quantum number.

The quantum field theory (QFT) that describes the SM is based on a combination of space-time symmetries and internal symmetries. Space-time symmetries act on the coordinates of space-time, they include rotations, translations, and general Lorentz and Poincaré transformations, which are global symmetries, and general coordinate transformations, which are local symmetries. Thus, the SM has all the transformations of special and general relativity.

On the other hand, internal symmetries refer to the transformations of the different fields. Internal symmetries are related to gauge invariance, which refers to the fact that different field configurations may describe the same observables. A gauge transformation on a field implies that also the kinetic term has to be transformed, so as to leave the Lagrangian invariant. This leads to the definition of the covariant derivative, where a vector or gauge potential is introduced, ensuring the invariance of the Lagrangian under gauge transformations. There are many choices of gauge potential that will describe the same physics. From the conservation of internal symmetries follows the conservation of colour and electric charge, for instance.

Often symmetries are not exact or explicit, but may be broken or hidden. A symmetry can be broken spontaneously, like in the Higgs mechanism, where the vacuum expectation value of the Higgs field is different from zero, and thus does not respect the symmetries of the original Lagrangian. Thus, a symmetry may be hidden in the sense that we may just perceive the symmetries in the vacuum state, and not the more general symmetry group of the complete Lagrangian.

A symmetry can also be broken explicitly, whereby there are terms in the Lagrangian which do not respect it, and thus are not invariant under the symmetry transformation.

The Standard Model is based on a number of symmetries: Poincaré invariance in four dimensions and gauge invariance under the group $SU(3)\times SU(2)_L\times U(1)_Y$. The electroweak part $SU(2)_L\times U(1)_Y$ is spontanously broken by the Higgs mechanism to $U(1)_{EM}$. Besides Lorentz and gauge invariance the SM exhibits some discrete symmetries, which can be exact or broken. Among these are charge conjugation (C), parity (P), and the combined effect of both CP, as well as time reversal (T), which are broken in electroweak interactions. The SM Lagrangian is invariant under the action of CPT, as required by the CPT Theorem, which states that any localized Lorentz invariant gauge theory is invariant under the combined action of C, P, and T. There are also accidental symmetries in the SM, that is, they appear without having required invariance under any particular symmetry, but as a consequence of the field content and other symmetries and properties of the Lagrangian. These include baryon and lepton number conservation, which are exact at the classical level.

3 Why go beyond the SM?

As already mentione, the SM is very successful in describing the fundamental particles and their interactions at the electroweak scale, but it leaves a number of puzzles or unanswered questions that lead to the conclusion that it is the low energy limit of a more fundamental theory. The first deviations of the SM appeared with the neutrino masses and the existence of dark matter, the latter was actually proposed even before the SM was conceived. The SM was formulated with neutrino masses equal to zero, and it might not look like a great departure from it just to add extra terms to the Lagrangian to give mass to the neutrinos, the same way as for the other matter particles. But neutrino masses are tiny, and thus the associated Yukawa coupling would be several orders of magnitude smaller than the other matter couplings, enhancing the mass hierarchy among the fundamental particles. A popular way to explain the smallness of neutrino masses is to assume the right-handed neutrinos, which are sterile, acquire a large Majorana mass. The diagonalisation of the neutrino mixing matrix which includes the Dirac mass term (of order of the electroweak scale) and the Majorana mass term leads to one very heavy and one very light eigenvalue, thus the name seesaw mechanism (see for instance [2, 3]).

Amongst the puzzles that lead to the proposal of different models BSM are some of the questions mentioned in the introduction, and more. The list is long and it grows as we go into the details. Let us look at some of the more important ones:

– The hierarchy problem refers to the fact that the we expect the fundamental scale to be the Planck scale, $M_P = 10^{19}\ GeV$, but the particle masses are "of the order" of the electroweak scale $\sim \mathcal{O}(100)\ GeV$, i.e. 17 orders of magnitude below M_P. This scale is dictated by the spontaneous electroweak symmetry breaking, where the Higgs fields acquires a vacuum expectation value (vev) different from zero, and through its self-interaction a mass, which we know now to be $\sim 125\ GeV$.

On the other hand, the Higgs field, being a scalar, has radiative corrections that grow quadratically with the energy, and not logaritmically like the other particles. The quantum corrections to the Higgs mass would drive it to the Planck scale, which is assumed to be the fundamental one

$$M_h^2 \propto M^2(\Lambda^2) + \delta m_h^2 = M^2(\Lambda^2) - Cg^2\Lambda^2 \;, \tag{1}$$

where Λ is an energy scale, g the SU(2) gauge coupling and C some constant. Thus, an incredible fine-tuning must exist between the bare mass and the radiative corrections to cancel almost completely among themselves, and render the Higgs mass $\sim 125\ GeV$. This enormous degree of fine-tuning is clearly a naturality issue. The hierarchy and naturalness (or lack thereof) problems have led to the conclusion that around $\sim \mathcal{O}(1)\ TeV$ there must appear new physics, or rather physics unknown to us.

– The cosmological constant is interpreted as the energy of the vacuum, and recent observations suggest its value is close to zero but not identically zero. In quantum field theory, assuming the fundamental theory to be at the Planck scale, the cosmological constant is predicted to be ~ 120 orders of magnitude bigger than the measured value. This is an even bigger naturaleness problem than the one of the Higgs mass.

– There are only three generations of particles, as is inferred from the Z width and from Big Bang Nucleosynthesis. The masses of the fundamental fermions are much lower than the Planck scale, as above mentioned. But there are also large differences among them, between the up and the top quarks there are five orders of magnitude in mass. The mixing in the quark sector is not very large, the CKM matrix largest elements are in the diagonal. Also in the lepton sector, if we take into account neutrino masses (which striclty speaking are zero in the SM), the discrepancy between the largest and the lightest mass is at least five orders of magnitude, if not bigger. On the other hand, the mixing angles in the PMNS matrix are fairly large. This pattern of masses and mixings is not understood, but it might suggest an underlying symmetry at work (or not).

– We have enough evidence of the existence of dark matter, and we will assume that it is a particle. This particle cannot be one of the SM ones. The neutrinos could be a fraction of the total dark matter (DM), but DM cannot be composed entirely of neutrinos, since it would be in contradiction with the large structure formation of the Universe. Thus, we need particles beyond the ones in the SM to explain dark matter (see [4, 5] and references therein). It is usually assumed that DM is only composed of one type of particle, but there is no reason that DM could be actually made of more than one type of particle.

The solution to some of these problems has been to follow the traditional path, i.e. to add symmetries, but it is also possible to add particles and/or interactions, or to add more space-time dimensions, or combinations of all of them. Of course, any attempt to extend the SM has to go hand in hand with experimental data, and the SM has to emerge as the low-energy theory of any theory beyond it. For extended recent reviews and textbooks on BSM physics from different perspectives, see for instance [6–13].

4 Grand Unified Theories – GUTs

In the context of adding more symmetries, one could think that there is a larger gauge symmetry group which contains the SM one. This is the idea behind Grand Unified Theories or GUTs, where the electroweak and strong interactions are unified in a single interaction at high energies. This larger symmetry group is realized at very high energies, which corresponds to the beginning of the Universe, and the symmetry is broken down to the SM gauge group, which is what we observe today. The unification idea is very attractive, what we perceive as three separate interactions at low energies is in reality only one, gravity is not included in this scheme. Grand unification can work because of the behaviour of the different gauge couplings as they move up in energy, which is calculated using the Renormalization Group Equations (RGEs). Whereas the inverse of the strong and weak coupling increase with energy, the inverse of the electromagnetic coupling decreases, thus they all tend to a similar value at high energies (see Fig. 4). If unified, we have two parameters, the unification scale M_{GUT} and the unification coupling α_{GUT} rather than three. But reducing one parameter is not the only achievement of a unified theory. Because now the particles are unified in a larger symetry group, they are related through the symmetry, which gives some nice surprises and predictions (see for instance [14, 15]).

The simplest, and one of the most studied, examples of a GUT is the one based in the gauge group $SU(5)$. $SU(5)$ can accomodate all the particles of the SM and can be broken exactly into its gauge group,

$$SU(5) \to SU(3) \times SU(2) \times U(1) \tag{2}$$

The left-handed quarks and leptons are accomodated in irreducible representations (irreps) of $SU(5)$ as follows $\bar{\mathbf{5}} = [\mathbf{d^c}, \mathbf{L}]$ and $\mathbf{10} = [\mathbf{Q}, \mathbf{u^c}, \mathbf{e^c}]$, and the left-handed anti-neutrinos are in a singlet irrep $\mathbf{1}$ (the original model did not include those, since there was not yet evidence of neutrino masses). In matrix notation

$$\bar{\mathbf{5}} = \begin{pmatrix} d^c \\ d^c \\ d^c \\ e \\ -\nu_e \end{pmatrix} , \qquad \mathbf{10} = \begin{pmatrix} 0 & u^c & -u^c & -u & -d \\ & 0 & u^c & -u & d \\ & & 0 & -u & -d \\ & & & 0 & -e^+ \\ & & & & \end{pmatrix} . \tag{3}$$

The breaking of the $SU(5)$ group to the SM one is achieved by the vacuum expectation value of the adjoint representation, the $\mathbf{24}$. The gauge bosons fit in the adjoint irrep $\mathbf{24}$, which also contains twelve fractionally charged bosons with both lepton and baryon number. These "exotic" bosons are called leptoquarks, and are denoted by X and Y. Since the leptons and quarks are combined in the same irreps, baryon and lepton numbers are not conserved, although the combination B-L is still conserved. Thus, unless the X and Y bosons are very heavy, the violation of B and L can lead to proton decay through exchange of leptoquarks, which is a signature of many GUTs. These decay modes obey the selection rule $\Delta(B - L) = 0$, and are mediated by effective operators of dimension 6. In Fig. 1 examples of diagrams that lead to proton decay through these dimension six operators are shown.

The Higgs boson can be accomodated in a $\mathbf{5}$ or a $\bar{\mathbf{5}}$ irrep, which will also decompose in triplets of $SU(3)$ and doublets of $SU(2)$ once the $SU(5)$ group is broken. Again, we have a coloured and an electroweak part mixed in the same irrep. These coloured Higgs triplets violate baryon and lepton number and can mediate fast proton decay via the exchange of the scalar triplet, unless they are very heavy. Thus, there must be a fine tuning to get the doublets at the electroweak scale, and at the same time leave the triplets at the GUT scale. This is referred to as the doublet-triplet splitting fine tuning problem.

The proton lifetime in these type of GUTs is

$$\tau \sim \frac{M_{GUT}^4}{\alpha_{GUT}^2 m_p^5} \sim 10^{30} - 10^{31} yrs , \tag{4}$$

which excludes them by the bound on the proton lifetime set by super-Kamiokande $\tau > 10^{34}\ yrs$, ruling out the simplest models. Moreover, letting the gauge couplings of the SM run to high energies

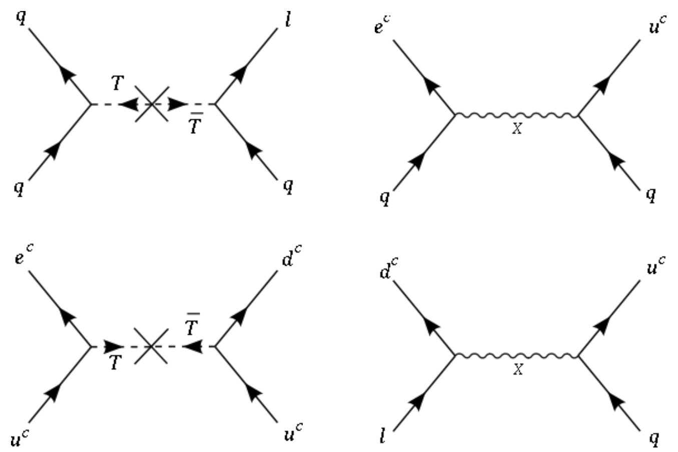

Fig. 1: Dimension six operators that mediate proton decay in $SU(5)$, the diagrams on the left are mediated by the coloured Higgs triplet and anti-triplet, the ones on the right by the X boson.

through their RGE shows that they do not quite unifiy, see upper panel of Fig. 4. If one incorporates supersymmetry in the game, then unification may be achieved, as the lower panel of Fig. 4 shows. Although in SUSY GUTs the proton decay prediction remains, the proton lifetime may be larger than in non-SUSY models, within the experimental bounds.

On the other hand, Grand Unified theories give an explanation of charge quantization. Since the charge operator is a generator of $SU(5)$, the sum of all charges in each irrep must add to zero, leading to $Q(d^c) = -1/3Q(e)$. They give an approximation to the value of the Weinberg angle, and also approximations to some quark mass ratios.

Besides the minimal $SU(5)$ sketched here, several other GUTs have been studied. For instance, another popular GUT is SO(10), which has all fermions in the same irrep $\mathbf{16}$, and a right handed neutrino may be included naturally. Other groups have different features like the Pati-Salam group $SU(4)_c \times SU(2)_L \times SU(2)$, which proposes a fourth colour charge and also predicts the existence of magnetic monopoles, or the flipped $SU(5) \times U(1)$, where the assignment of the particles to the irreps is $\bar{\mathbf{5}} = [\mathbf{u^c}, \mathbf{L}]$ and $\mathbf{10} = [\mathbf{Q}, \mathbf{d^c}, \nu^c]$, and $\mathbf{1} = \mathbf{e^c}$.

5 Supersymmetry

Particles come in two fundamental versions, bosons and fermions. Matter particles are fermions, with half-integer spin and obey Fermi-Dirac statistics. Gauge particles, which are the ones that "carry" the interactions, are vector bosons with spin one and obey Bose-Einstein statistics. The Higgs boson (or perhaps Higgs bosons) is a scalar with zero spin, and thus also obeys Bose-Einstein statistics. Supersymmetry (SUSY) relates bosons and fermions, so what in the SM are distinct types of particles, in SUSY theories are related through a symmetry transformation. There are many excellent reviews on supersymmetry, both from the formal as well as from the more phenomenological point of view (see for instance [16–18, 21]). In here we will expose the main motivation for SUSY and some of models that have been more studied.

Through SUSY it is possible to transform bosons into fermions and viceversa

$$Q|Boson>=|Fermion>; \qquad Q|Fermion>=|Boson> . \tag{5}$$

The extension of the Coleman-Mandula Theorem [19] due to Haag, Lopuszanski and Sohnius [20] tells us how to build an interacting quantum field theory with such restrictions. One has to generalize the Lie algebra to a graded Lie algebra with anti-commutators and commutators, in order to include the fermionic generators. Thus, a supersymmetric theory must have the Poincaré generators P^M for translations in space and time, and $M^{M,N}$ for the Lorentz boosts and rotations, plus the spinor generators Q^A_α ($\alpha = 1, 2$ and $A = 1, ..., \mathcal{N}$) corresponding to spins $(A,\ B) = \left(\frac{1}{2},\ 0\right)$ and $Q^{\dagger A}_{\dot\alpha}$ with $(A,\ B) = \left(0,\ \frac{1}{2}\right)$.

If O_a are operators of a Lie algebra, a graded Lie algebra staisfies the relations

$$O_a\, O_b \ -\ (-1)^{\eta_a \eta_b}\, O_b\, O_a = i C^e{}_{ab}\, O_e\ , \tag{6}$$

where the *gradings* η_a take values

$$\eta_a = \left\{ \begin{array}{ll} 0 & : O_a \text{ bosonic generator} \\ 1 & : O_a \text{ fermionic generator} \end{array} \right. . \tag{7}$$

We will only address the case of N=1 supersymmetry, i.e. only one set of Q and $Q^\dagger$. The possibility of having several copies of these generations leads to extended supersymmetries ($N = 2, N = 4$), but these theories do not have chiral fermions, which makes them unsuitable as direct extensions of the SM.

The particle states are described by irreducible representations of the SUSY algebra, called supermultiplets. The supermultiplet contains equal number of both fermionic and bosonic degrees of freedom. The particle states in each supermultiplet are called superpartners and they differ by 1/2 unit of spin. In N=1 SUSY, a chiral supermultiplet contains a Weyl fermion and two real scalars. The Weyl fermion has two degrees of freedom corresponding to each helicity state, and each real scalar has one degree of freedom, which are usually described together as one complex scalar. A gauge supermultiplet contains a massless spin-1 gauge boson and its superpartner, a gugino, which is a massless spin-1/2 Weyl fermion, both of which have two helicity states. They both transform in the adjoint representation of the gauge group which is self-conjugate, therefore the left- and right-handed components of the gauginos have the same transformation properties, thus they are Majorana in nature. The Higgs boson belongs also to a chiral supermultiplet, with a spin-1/2 superpartner, the Higgsino. But in SUSY theories one Higgs supermultiplet is not enough to guarantee an anomaly free theory. The Higgs supermultiplet has a gauge anomaly, which is cancelled if another Higgs supermultiplet with opposite hypercharge is added. The members of a supermultiplet must have the same coupling strenghts and also the same mass, thus

$$\sum m^2_{bosons} = \sum m^2_{fermions}. \tag{8}$$

In the context of the radiative corrections to the Higgs mass this means that the quadratic corrections to the Higgs mass coming from the bosons are exactly cancelled by their fermionic superpartners, order by order in perturbation theory.

One of the main motivations to study SUSY models is that they provide a solution to the hierarchy problem, in a natural way. The contributions to the radiative corrections to the Higgs mass that come from the SM particles are cancelled exactly, order by order, by their superpartners, see Fig. 2. This happens thanks to the sum rule, Eq. (8), and an extra (-1) sign that comes from the Fermi-Dirac statistics. This is exact only if SUSY is unbroken, once it is broken the squared mass difference between bosons and fermions is proportional to the susy breaking scale squared M^2_{SUSY}. This value is expected to be $\sim \mathcal{O}(1)\ TeV$, to be consistent with the scale of the Higgs mass, and it is also the value that is required to have good unification of couplings in the minimal SUSY extension of the MSSM.

5.1 The Minimal Supersymmtric Extension of the Standard Model

By far the most phenomenologically viable SUSY model studied is the Minimal Supersymmtric Standard Model (MSSM). It has N=1 SUSY, to include chiral fermions, and two Higgs supermultiplets, to be free of gauge anomalies. The gauge group is the same as the SM one, $SU(3) \times SU(2) \times U(1)$, but now the particles are arranged in supermultiplets. The matter or chiral supermultiplets contain the quarks and leptons and their respective superpartners, squarks and sleptons, which are spin-0 scalars. There are two

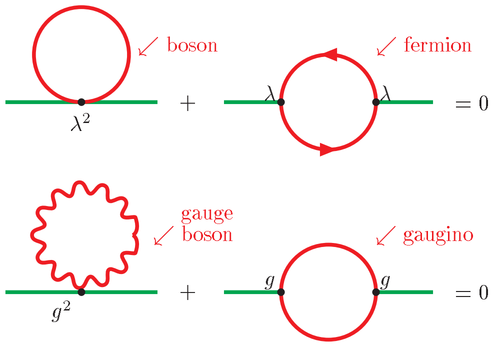

Fig. 2: Cancellation of divergencies to the Higgs boson mass from contributions from the superartners, figure from ref [21].

Particles	Bosons	Fermions	Charges under SM
quarks, squarks Q_i	$(\tilde{u}, \tilde{d})_L$	$(u, d)_L$	$(\mathbf{3}, \mathbf{2}, 1/6)$
$\bar{u}$	$\tilde{u}^*_R$	$u^\dagger_R$	$(\bar{\mathbf{3}}, \mathbf{2}, -2/3)$
$\bar{d}$	$\tilde{d}^*_R$	$d^\dagger_R$	$(\bar{\mathbf{3}}, \mathbf{2}, 1/3)$
leptons, sleptons L_i	$(\tilde{\nu}, \tilde{e})_L$	$(\nu, e)_L$	$(\mathbf{1}, \mathbf{2}, -1/2)$
$\bar{e}$	$\tilde{e}^*_R$	$e^\dagger_R$	$(\bar{\mathbf{1}}, \mathbf{1}, 1)$
Higgs bosons H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$(\bar{\mathbf{1}}, \mathbf{2}, 1/2)$
H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$(\bar{\mathbf{1}}, \mathbf{2}, -1/2)$
Bino, B	$\tilde{B}^0$	B^0	$(\mathbf{1}, \mathbf{1}, 0)$
Wino, W, Zino, Z	$(\tilde{W}^\pm, \tilde{Z})$	$(W^\pm, Z)$	$(\mathbf{1}, \mathbf{3}, 0)$
Gluino, g	$\tilde{g}$	g	$(\mathbf{8}, \mathbf{1}, 0)$

Table 1: Particle content of the MSSM

chiral supermultiplets with opposite hypercharge that contain the Higgs bosons and their corresponding superpartners, the Higgsinos. The gauge supermultiplets, which contain the spin-1 vector bosons and their superpartners the gauginos, which are spin-1/2 Majorana fermions.

In supersymmetric theories the interactions are determined by the superpotential, which is a holomorphic function of the chiral superfields. It respects SUSY and gauge invariance. The superpotential contains the Yukawa interactions and mass terms, in the MSSM it is

$$W_{MSSM} = \epsilon_{ij}(y^U_{ab}Q^j_aU^c_bH^i_2 + y^D_{ab}Q^j_aD^c_bH^i_1 + y^L_{ab}L^j_aE^c_bH^i_1 + \mu H^i_uH^j_d) \tag{9}$$

where the superfields in the potential are given in Table 5.1

In principle there could be terms in the superpotential that break baryon or lepton number, like for instance $LQ\bar{d}^c$. These terms would mediate really fast proton decay through $p \to e^+ + \pi^0$. In order to forbid these terms a new multiplicative discrete parity is introduced, R symmetry. R symmetry is defined as

$$R = (-1)^{3(B-L)+2S}\,, \tag{10}$$

thus all supersymmetry particles have $R = -1$ and the SM particles and all the Higgs bosons have $R = 1$. Besides forbidding proton decay, one immediate consequence of R parity is that the lightest supersymmetric particle (LSP) is stable. This feature may provide with a good candidate to dark matter, if the LSP is electrically neutral. Another consequence of R parity is that superparticles are created in pairs.

5.2 SUSY breaking in the MSSM

If SUSY exists in Nature it has to be a broken symmetry, since no superpartners of the SM particles have been observed. The way to break SUSY consistent with phenomenological observations, and to preserve the solution to the hierarchy problem is to add soft breaking terms to the SUSY Lagrangian. Soft breaking terms do not reintroduce quadratic divergences in the theory, they have positive mass dimension, and are super-renormalizable. The soft breaking part of the MSSM Lagrangian has a similar form to the superpotential

$$\begin{aligned}-\mathcal{L}_{Breaking} = &\sum_i m_{0i}^2 |\varphi_i|^2 + \Bigg(\frac{1}{2}\sum_\alpha M_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha + BH_1H_2 \\ &+ A_{ab}^U \tilde{Q}_a \tilde{U}_b^c H_2 + A_{ab}^D \tilde{Q}_a \tilde{D}_b^c H_1 + A_{ab}^L \tilde{L}_a \tilde{E}_b^c H_1 + h.c.\Big)\,,\end{aligned} \tag{11}$$

where φ_i are the scalar fields, $\tilde{\lambda}_\alpha$ are the gaugino fields, $\tilde{Q}, \tilde{U}, \tilde{D}$ and $\tilde{L}, \tilde{E}$ are the squark and slepton fields, respectively, and $H_{u,d}$ are the Higgs fields. In principle there can be over 100 soft breaking terms, but they are constrained by requiring the absence of Flavour Changing Neutral Currents (FCNCs) and CP violation. These constraints appear naturally if one imposes "universal" soft breaking terms, which means that all the soft scalars are proportional to the identity matrix $\mathbb{1}_{3\times3}$, the trilinear A terms are proportional to the Yukawa couplings, and there are no extra CP violating phases besides the usual CKM one

$$\tilde{m}^2_{Q,u,d,L,e} \propto \mathbb{1}_{Q,u,d,L,e}; \quad a_{u,d,e} \propto A_{u,d,e} Y_{u,d,e}\,. \tag{12}$$

Usually, universal gaugino masses at the unification scale are assumed too. These universality conditions are usually taken as boundary conditions for the RGEs at at higher scale, most commonly the GUT scale. They are assumed to originate from an underlying more fundamental theory, where SUSY is dynamically broken, allthough such theory is not known. The general assumption is that the MSSM is connected to an unknown or "hidden" sector, that communicates to the matter or "visible" sector through so-called messangers. Thus, the effective theory that is left after the hidden and visible sectors interact through the messenger has a Lagrangian with soft SUSY breaking terms. The most studied types of SUSY breaking through hidden sectors are:

- Gravity mediated, the sectors interact through gravity. The SUSY breaking scale is of the order of the gravitino mass, the superpartner of the graviton. Although it is very attractive due to the fact that gravity exists and is felt by all particles, there does not exist yet a theory of quantum gravity.
- Gauge mediated, the SUSY breaking is communicated to the observable sector through the known gauge interactions that involve the messenger particles in loop diagrams. These messanger particles are the SM particles plus particles from a unified theory. The LSP is the gravitino.
- Anomaly mediated, SUSY breaking appears at loop level through a superconformal anomaly. An interesting feature in this scenario is that the soft terms that appear are renormalization group invariant, i.e. they are valid at any renormalization scale. One drawback of the simplest anomaly mediated scenario is that the slepton is tachyonic, although there are ways out of this problem.
- Gaugino mediated, it is based on brane theory. It is assumed that the SM lives on a brane, while gravity and other field propagate in the bulk. SUSY breaking happens in a different brane and it is communicated to our brane through the gauginos. This is the least studied of the four hidden sector proposals.

All these scenarios generate soft breaking terms of the form given in Eq. (11). To each one of these scenarios correspond particular boundary conditions at the unification scale, that after evolving to the electroweak scale through the RGEs, give different predictions for the s-spectrum. Examples of such different s-spectra are shown in Fig. 3.

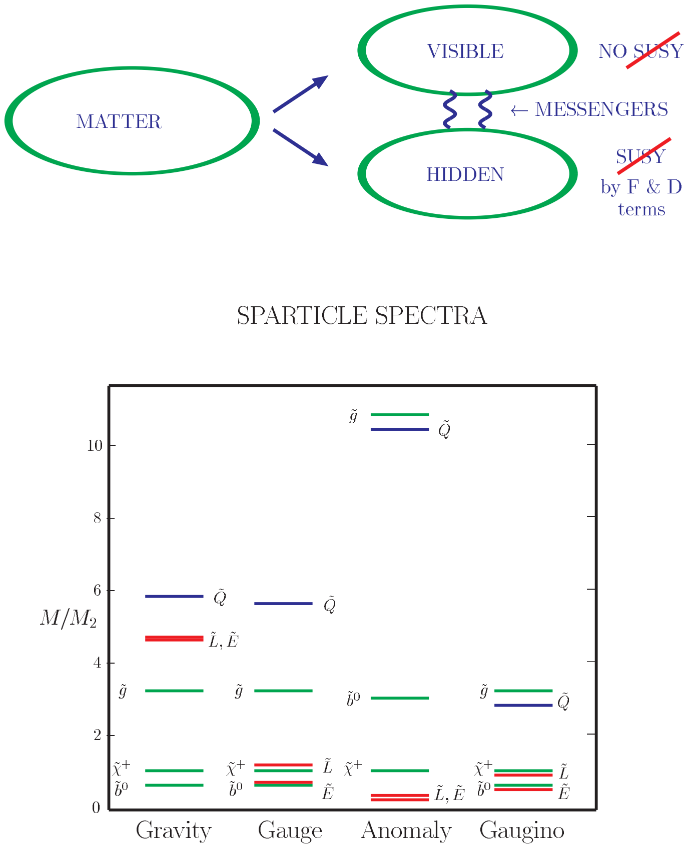

Fig. 3: The upper part shows the hidden sector scenario, whereas the lower part shows the different sparticle-spectra that come from the different scenarios, both figures from ref [21].

5.3 Higgs potential and masses in the MSSM

The tree level Higgs potential of the MSSM should be compatible also with electroweak symmetry breaking. We require that it breaks the $SU(2) \times U(1)$ group to the electromagnetic $U(1)_{EM}$. At the minimum it is always possible to rotate away the vev's of one of the Higgs fields, for instance $H_u^+ = 0$, which at the minimum $\partial V / \partial H_u^+$ implies that also $H_d^- = 0$. Thus, the potential is given by

$$V_{tree}(H_u, H_d) = (|\mu^2| + m_{H_u}^2)|H_u|^2 + (|\mu^2| + m_{H_d}^2)|H_d|^2 - b(H_u^0 H_d^0 + c.c.) + \frac{g^2 + g'^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2 \quad (13)$$

where g is the $SU(2)$ gauge coupling and g' the $U(1)$ one, $m_{H_u}^2$ and $m_{H_d}^2$ are the soft breaking mass terms for the Higgs bosons, and μ their mixing term in the superpotential. The quartic couplings are fixed in terms of the gauge couplings. The requirement that the potential is bounded from below gives positivity conditions on the potential parameters

$$m_1^2 + m_2^2 > 2|b| \ . \quad (14)$$

To guarantee electroweak symmetry breaking, a linear combination of the Higgs fields has to have a negative square mass term, which happens if

$$m_3^2 > m_1^2 m_2^2 \ . \quad (15)$$

These conditions are not satisfied at the GUT scale in general, but since they are scale dependent, i.e. "running parameters", they might be realized at at lower energies after evolving the RGEs. Thus, the symmetry breaking is driven through radiative corrections. Because of this feature the mechanism is known as radiative electroweak symmetry breaking. Furthermore, after electroweak breaking the Higgs vevs and the gauge couplings must be related to the Z boson mass through

$$m_Z^2 = (g^2 + g'^2)(v_u^2 + v_d^2)/4 \,, \tag{16}$$

where

$$< H_u^0 >= v_u/\sqrt{2} \,, \qquad < H_d^0 >= v_d/\sqrt{2}, \qquad v = v_u^2 + v_d^2 =\simeq 246.218\ GeV \,,$$

and we define $\tan\beta = v_u/v_d$. Eq. (16) imposes further conditions on the μ and b parameters

$$b \;=\; \frac{(m_{H_d}^2 - m_{H_u}^2)\tan 2\beta + M_Z^2 \sin 2\beta}{2} \,, \tag{17}$$

$$\mu^2 \;=\; \frac{m_{H_u}^2 \sin^2\beta - m_{H_d}^2 \cos^2\beta}{2} - \frac{M_Z^2}{2} \,. \tag{18}$$

To find the Higgs bosons masses, it is necessary to expand the potential around the minimum and separate it in real and imaginary parts. The two complex Higgs doublets have eight real degrees of freedom. The real parts correspond to the CP-even Higgs bosons and the imaginary parts to the CP-odd ones and to the Goldstone bosons. After electroweak symmetry breaking the three Goldstone bosons become the longitudinal modes of the $W^\pm$ and Z bosons, as in the SM. The other five degrees of freedom will give rise to four massive scalars, two of which are neutral h and H, two charged $H^\pm$, plus a massive neutral pseudoscalar A. Of these, one can be easily identified with the SM Higgs boson, namely h, which is naturally lighter than the other four.

After SUSY and electroweak symmetry breaking all the SM, Higgs bosons and the s-particles acquire their physical masses. The tree level masses of the Higgs bosons are:

$$M_Z \;=\; 2b/\sin 2\beta \tag{19}$$

$$M_{h,H} \;=\; \frac{1}{2}(M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos 2\beta}) \tag{20}$$

$$M_{H^\pm} \;=\; M_A^2 + M_W^2 \tag{21}$$

It is clear from the minimisation conditions and the expressions for the Higgs' masses that there are only two free parameters at tree level, which can be taken as M_A and $\tan\beta$. Moreover, whereas M_A, M_H, and $M_{H^\pm}$ can be very large, the lightest Higgs mass m_h is bounded from above

$$M_h \leq \min(M_A, M_Z)|\cos 2\beta| \leq M_Z \,. \tag{22}$$

Radiative corrections to M_h, particularly form the RGE running quartic coupling, which is proportional to the top quark mass, and threshold corrections from integrating out the stop quarks in the loop, are very large and change this bound to

$$M_h \lesssim 135\ GeV \,. \tag{23}$$

The actual value of M_h in any specific model will depend on the s-spectrum, particularly on the stop quark masses, and the degree of mixing among themselves. This result is encouraging for the MSSM as an extension of the SM in the sense that it has a natural decoupling limit, which reduces to the SM, but the details of the predictions depend sensibly on the details of the soft breaking terms.

Another interesting feature of the MSSM is that after electroweak symmetry breaking the neutral states can mix among themselves, the same happens to the charged states. This gives rise to four mass eigenstates known as neutralinos $\chi^0_{1,2,3,4}$ which are a mixture of the neutral gauginos ($\tilde{W}^0$, $\tilde{B}$) and neutral Higgsinos ($\tilde{H}^0_u$, $\tilde{H}^0_d$); and two charged mass eigenstates, the charginos $\chi^{\pm}_{1,2}$ which are a mixture of the charged Higgsinos ($\tilde{H}^+_u$, H^-_d) and the charged Winos ($\tilde{W}^{\pm}$).

The mass matrices of the neutralinos are

$$M^{(0)} = \begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\sin_W & M_Z\sin\beta\sin_W \\ 0 & M_2 & M_Z\cos\beta\cos_W & -M_Z\sin\beta\cos_W \\ -M_Z\cos\beta\sin_W & M_Z\cos\beta\cos_W & 0 & -\mu \\ M_Z\sin\beta\sin_W & -M_Z\sin\beta\cos_W & -\mu & 0 \end{pmatrix}, \tag{24}$$

the mass matrices of the charginos are

$$M^{(c)} = \begin{pmatrix} M_2 & \sqrt{2}M_W\sin\beta \\ \sqrt{2}M_W\cos\beta & \mu \end{pmatrix}, \tag{25}$$

and the mass matrices for the scalar quarks are of the form

$$\begin{pmatrix} \tilde{m}^2_{fL} & m_t(A_f - \mu fac\beta) \\ m_f(A_f - \mu fac\beta) & \tilde{m}^2_{fR} \end{pmatrix},$$

where f is any of the quarks and leptons, and $fac\beta = \tan\beta$ for down type quarks and leptons, and $fac\beta = \cot\beta$ for up type quarks. The mixing in the first two families is practically negligible, but can be sizeable in the third one.

As can be seen from the expressions of the mass matrices, the sparticle spectrum (s-spectrum) depends on the choice of soft breaking parameters, given in Eq. (11) and the μ Higgsino mixing term appearing in the superpotential (9). Thus, different soft breaking terms will lead to different allowed parameter space at low energies.

To make the study of the MSSM more tractable the number of free parameters is reduced in different ways. One popular way is the constrained MSSM (cMSSM), in which the universality relations Eq. (12) are taken a step further, making all the soft scalar masses at a high scale equal, $\tilde{m}_0$, all the trilinear terms equal A, and all the gaugino masses equal $m_{1/2}$. This leaves only five free parameters: $\tilde{m}_0, m_{1/2}, A, \tan\beta, \text{sign}\mu$. The cMSSM is inspired in minimal supergravity models, which have the same five parameters, but with extra relations among themselves at the unification scale. Most experimental searches are based on the cMSSM or variations of it. This model might be too constrained and even unrealistic, but opening slightly the parameter space at the unification scale can widen it at experimental scales. For instance, relaxing any of the strict universality relations changes the phenomenology at low energies.

Besides giving a solution to the hierarchy problem, there are two other widely studied features that make the MSSM and other SUSY models attractive.

- The neutralino LSP has been proposed as a good candidate for dark matter, since it is neutral and has only electroweak interactions. The neutralinos are a combination of the neutral gauginos, which means that for different values of the soft breaking terms the LSP will be different. This in turn means that the admixture of the lightest neutralino (Higgsino, Wino and Bino) will lead to different types of DM (for an extensive review on the subject see ref. [22]). The combination of experimental collider searches and direct DM detection experiments can be used to restrict the allowed parameter space of a particular model. An example of this is provided in [23], where DM is assumed to be the lightest neutralino, and regions in parameter space that agree with the dark matter relic abundance Ω and experimental constraints are studied for different SUSY models.
- The MSSM with $\mu > 0$ might provide a solution to the discrepancy between the experimental and the theoretical value of the anomalous magnetic moment of the muon $g - 2$ [24]. On the other hand, lepton flavour violating processes have been proposed also to alleviate or solve entirely this problem (see for instance [25, 26]).

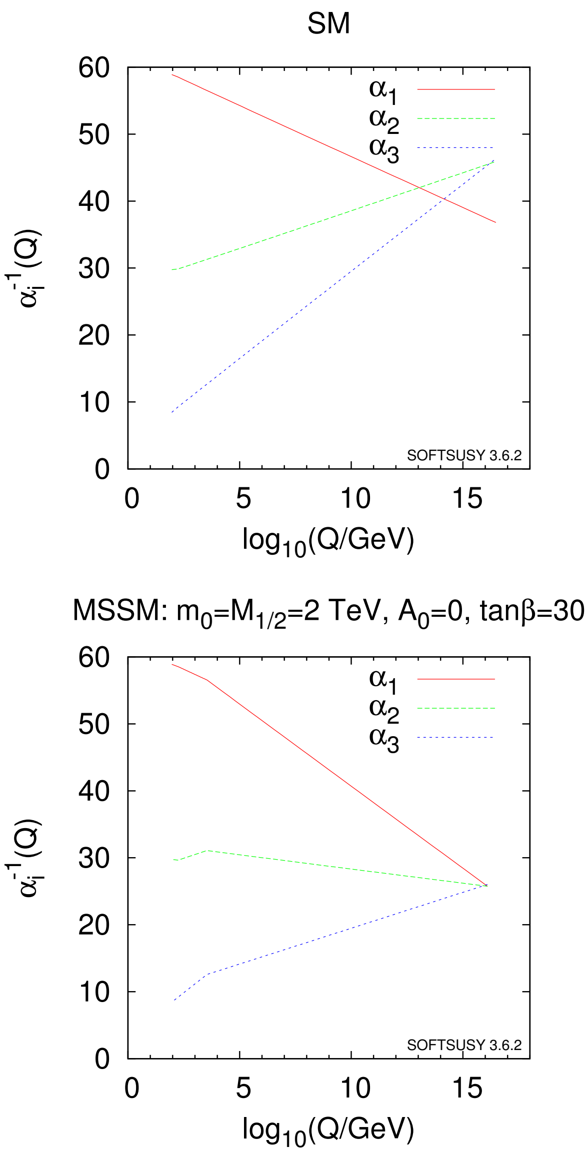

Fig. 4: Unification of couplings in the SM (up) and in the MSSM (bottom), where the SUSY breaking is assumed to be at 2 TeV. From ref. [27].

There are a number of low energy experimental constraints that reduce the parameter space for the cMSSM. The most stringent ones are the ones that come form FCNC's and flavour physics. Among the most used constraints are:

- The branching ratio $b \to s\gamma$.
- The branching ratio $B \to \mu^+\mu^-$.
- Constraints on direct searches on SUSY particles.
- Dark matter constraints, if the dark matter is composed 100% of the LSP, which is usually the neutralino.

5.4 SUSY GUTs

The unification of couplings, which was not good in the SM, turns out to work rather well in SUSY GUT models, see Fig. 4, assuming the SUSY breaking happens around a few TeV (for recent reviews see [28] and references therein). The idea is basically the same as in GUTs, but with supersymmetry added. The matter and gauge contents are assigned to the same irreps as in non-SUSY GUTs, but now they refer to the superfields, where the SM particles appear in the supermultiplets together with their superpartners.

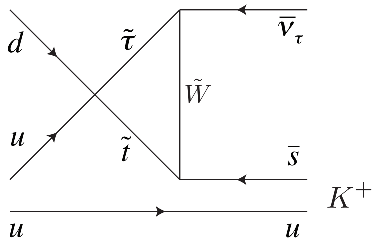

Fig. 5: Dimension five operators that mediate proton decay in SUSY GUTs, from [29].

In SUSY GUTs there are more ways for the proton to decay through dimension five operators, see Fig. 5. These decay modes are the dominant ones, with $\tau_p \sim 10^{34}\ yrs$. The coloured Higgsino triplets in SUSY GUTs can give rise to dimension five operators also, thus they have to be heavier than the GUT scale to suppress proton decay.

Dimension six operators exists, like in ordinary GUTs, but since the unification scale is large $M_{GUT} \sim 10^{16}$ the proton lifetime coming from these processes is $\tau_p \sim 10^{35}\ yrs$. Dimension four operators do not appear in the models considered here, since R parity is conserved.

Many of the constraints on the parameter space in the MSSM come from the assumption that there is an underlying unified theory behind it. That is the origin of the hidden sector SUSY breaking assumption and of the universality conditions that were discussed in the previous subsection.

An intriguing feature of SUSY GUTs is that they can be made finite to all-loops in perturbation theory, which can lead to good predictions for the third generation of quark masses, the Higgs mass, and a prediction for a relatively heavy s-spectrum (see for instance [30] and references therein).

5.5 Experimental searches

No direct or indirect evidence for supersymmetry has been found so far. Indirect evidence could come from contributions at loop level to some rare processes in the SM, like $b \to s\gamma$ or electron dipole moments, for instance. Direct production could happen through quark-antiquark annihilation, gluino fusion, gluino-quark interaction, and quark-quark scattering, which would lead to pairs of sparticles. The sparticles then would decay into SM particles and neutralinos, the latter escape detection carrying with them some missing energy. The LHC experiments, CMS and ATLAS, have put bounds on the SUSY particles, and excluded some regions of the parameter space of the cMSSM. In general, these analysis are done in simplified models that exhibit generic features, since it is difficult to test simultaneously all of the free parameters of the MSSM. We present two examples of how such searches are presented, for a more detailed description the reader should go to the experiments public pages [] and papers therein.

The results for the searches for gluinos and first and second generation squarks from proton proton collisions at 8 TeV are presented in [31]. Limits in simplified models with gluinos and squarks of the first and second generations are derived for direct and one- or two-step decays of squarks and gluinos, and gluino decays via third-generation squarks are derived. They considered simplified models with R-parity conservation, and in all of them the limit on the gluino mass exceeds 1150 GeV at 95% CL, for an LSP mass smaller than 100 GeV. They also derive exclusion limits for a number of simplified

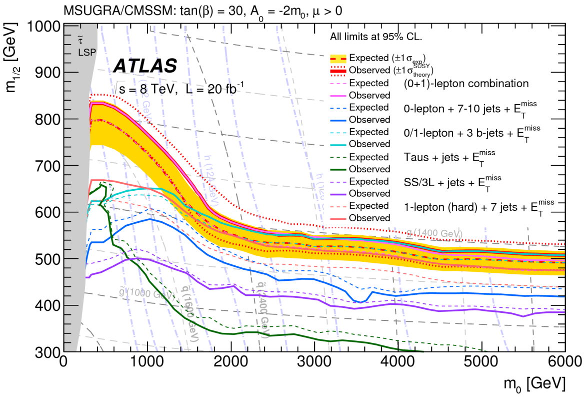

Fig. 6: Exclusion limits for MSUGRA /cMSSM models in the $(m_0, m_{1/2})$ plane, from [31, 32].

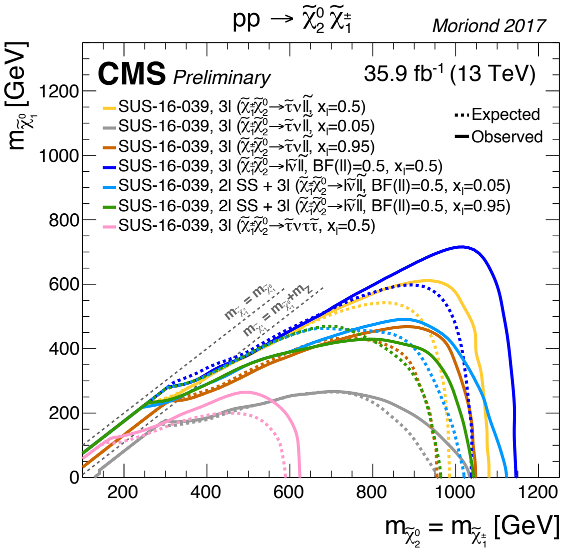

Fig. 7: CMS exclusion limits in the chargino-lightest neutralino plane are show, from [33, 34]

models, like the cMSSM and a model with non-universal Higgs mass model with gaugino mediation, among others. Fig. 6 the exclusion limits at 95% CL for 8 TeV analyses are shown in the $(m_0, m_{1/2})$ plane for the MSUGRA/cMSSM model. The other three parameters have been set to $\tan(\beta) = 30, A_0 = -2m_0, \mu > 0$ [31]. The Higgs mass is $\sim 125\ GeV$ in a large part of the parameter space. The search excludes gluino masses $< 1280\ GeV$.

The results of the searches for chargino-neutralino pair production in proton-proton collisions with sleptons as final decay modes were performed in [33]. Chargino-neutralino pair production is expected to have the largest cross section from all the electroweak processes. Higgsino pair production in a gauge mediated SUSY breaking scenario, was also studied. Different simplified models lead to different final states, where limits on the charginos and neutralino masses can be set, these range from 180 GeV to 1150 GeV at 95% C.L. For instance, models with light left-handed leptons lead to the stringest bounds, with the mass limit for charginos and neutralinos up to 1150 GeV at 95% C.L. In 7 exclusion limits for this study in the chargino/lightest neutralino plane are shown.

5.6 The Next to Minimal Supersymmetric Standard Model

Another widely studied extension of the SM is the Next to Minimal Standard Model (NMSSM). The model has the same matter content than the MSSM plus a chiral singlet superfield $\hat{S}$, i.e. an extra Higgs singlet. It provides for a solution of the so-called "μ problem" in the MSSM, which refers to the fact that the the μ term in the Higgs potential is of the order of the SUSY breaking scale, while the fundamental scale is high (see for instance [35, 36] and references therein).

The NMSSM solves the μ problem by generating this term dynamically through the new chiral singlet. The superpotential adds two terms to the MSSM usual one

$$W_{NMSSM} = W_{MSSM} + \lambda \hat{S} H^u H^d + \frac{\kappa}{3} \hat{S}^3 \tag{26}$$

and the Higgs potential is given by

$$V_{NMSSM} = m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2 + m^2_S|S|^2 + (\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa \hat{S}^3 + h.c.)\ . \tag{27}$$

When the new singlet field acquires a vev $< S >= v_S$, it generates an effective μ term $\mu_{eff} = \lambda v_S$, which appears then naturally at the electroweak scale. The NMSSM shares also the nice features of the MSSM in terms of candidates to dark matter and unification of couplings, besides solving the hierarchy problem.

The NMSSM has two more degrees of freedom than the MSSM, and after electroweak symmetry breaking there are seven physical massive scalars. If there is no extra CP breaking these states are three neutral CP even scalars $H_{1,2,3}$, two charged ones $H^\pm$, and two neutral pseudoscalars $A_[1, 2]$. The SM-like Higgs boson can be either the lightest scalar, or the second lightest scalar. In the latter it is possible to have a very light neutral Higgs that has escaped detection so far, although this situation is very constrained by LCH searches of scalares decaying into τ pairs, as well as by flavour observables. The heavier one is very similar to the heavy neutral Higgs of the MSSM.

In the NMSSM there is no upper bound for the tree-level mass, as in the MSSM, where large stop masses or large mass splittings are necessary to lift the Higgs mass through radiative corrections. Thus, the NMSSM can achieve more naturally a Higgs mass of 125 GeV, and still retain the good features of the MSSM: solution to the hierarchy problem, good unification of couplings, and good dark matter candidates. Also, in the NMSSM, the extended Higgs sector allows for Higgs to Higgs decays that are not present in the MSSM. The results for LHC searches for light Higgses, i.e. with mass below 125 GeV in the diphoton channel are presented in refs. [37, 38], and a review on the LHC NMSSM Higgs boson searches with emphasis on the mono-Higgs signature in [39].

6 Extra Dimensions

Another way to go beyond the SM is to add extra space-time dimensions, namely space ones. The idea of having extra space dimensions predates the SM by around half a century. It was proposed first by Nï£¡rdstrom in 1914 to unifiy electromagnetism and scalar gravity. Independently T. Kaluza proposed to extend general relativiy in five dimensions (1921), and then O. Klein proposed to compactify the extra dimension in 1926. Thus it is possible to describe four-dimensional gravity and electromagnetism. Although this original proposal does not work, it is the basis for modern superstring theories and the brane world scenario. For reviews on extra dimensions see for instance [13, 40].

Consider first that the fifth dimension is compactified in a circle of radius R, and the other dimensions are extended, see Fig. fig:KK-circle

The action of a massless field in five dimensions is

$$S_5 = \int d^5x \frac{1}{2} \partial_M \phi(x^\mu, y) \partial^M \phi(x^\mu, y)\ , \tag{28}$$

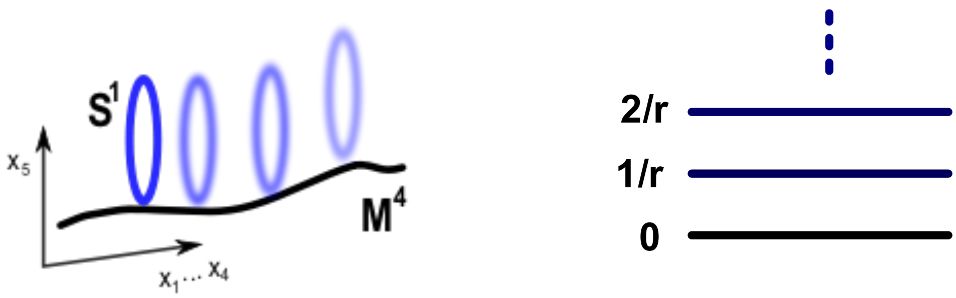

Fig. 8: A string compactification of one dimension in a circle S^1 times four dimensional Minkowski space M^4 is shown to the left, to the right is shown the associated tower of massive Kaluza-Klein modes $m_n = |n|/r$, both figures from ref. [11].

where $x^M = (x^\mu, y),\ M = 1, ...5, \mu = 0, ...4$ and y is the coordinate in the fifth dimension, which is compact. This means that it has periodic boundary conditions of the form

$$\phi(x^\mu, y) = \phi(x^\mu, y + 2\pi R) \ .$$

Since it is periodic in y it is possible to make a Fourier expansion

$$\phi(x^\mu, y) = \phi_n(x^\mu) \exp(iny/R) \ , \tag{29}$$

upon substituting the Fourier expansion in the 5-D Eq. (28) action we obtain

$$S_4 = \int d^4x \left(\partial_\mu \phi^0 \partial^\mu \phi^0 + \sum_n^\infty (\partial_\mu \phi^{n\dagger} \partial^\mu \phi^n - \frac{n^2}{R^2} \phi^{n\dagger} \phi^n) \right) \ . \tag{30}$$

The last term in the action represents an infinite tower of massive particles $m_n = n/R$. These are the so-called Kaluza-Klein modes, and a signature of theories with compactified extra dimensions.

The 5-dim action is given by

$$S_{5D} = \int d^5x \, \frac{1}{g_{5D}^2} \, F_{MN} \, F^{MN} \ , \tag{31}$$

where

$$F_{MN} \ = \ d_M A_N \ - \ d_N A_M \ . \tag{32}$$

Then, decomposing again in four dimensional fields we have the usual vector field A_μ, and a scalar $\rho = A_4$. In a similar way as the scalar one, it is possible to expand in a Fourier series along the compact dimension the gauge fields and find an expression for the 4-dim action,

$$A_\mu = \sum_{n=-\infty}^{\infty} A_\mu^n \exp\left(\frac{iny}{R}\right) \ ; \qquad \rho = \sum_{n=-\infty}^{\infty} \rho_n \exp\left(\frac{iny}{R}\right) \ . \tag{33}$$

Choosing a traverse gauge to remove the mixed terms

$$\partial^M A_M = 0, \quad A_0 = 0 \quad \rightarrow \partial^M \partial_M A_N = 0$$

leads to

$$S_4^{gauge} = \int d^4x \left(\frac{2\pi R^2}{g_{5D}} F_0^{\mu\nu} F_{0\ \mu\nu} + \frac{2\pi R}{g_{5D}^2} \partial_\mu \rho_0 \partial^\mu \rho_0 + ... \right) \tag{34}$$

This action corresponds to a massless gauge particle, a massles scalar, plus infinite towers of massive scalar and vector fields. The gauge couplings of the 5-dim and the 4-dim theory are related through

$$\frac{1}{g_4^2} = \frac{2\pi R}{g_5^2} \; .$$

In the gravity sector, we denote the graviton as G_{MN}, where $G_{\mu\nu}$ is the graviton, $G_{\mu n}$ gravivectors, and G_{mn} graviscalars. After a similar treatment to the gauge and vector fields, the 4-dim Einstein-Hilbert action reads

$$S_{4D}^{EH} = \int d^4x \sqrt{|g|} \left(M_1^{2\;(4)}R - \frac{1}{4}\phi^{(0)}\, F_{\mu\nu}^{(0)}\, F^{(0)\mu\nu} + \frac{1}{6}\,\frac{\partial^\mu \phi^{(0)}\, \partial_\mu \phi^{(0)}}{(\phi^{(0)})^2} + ... \right) \tag{35}$$

This way gravity, scalar fields, and electromagnetism get unified, but the Planck mass is not fundamental, it is a derived quantity

$$M_{Pl}^2 \sim M_D^{D-2} V_{D-4} \sim M_D^{D-2} R^{D-4} \; . \tag{36}$$

In order to include non-Abelian gauge fields in this kind of formalism one needs to go to higher dimensions, it is not possible to include strong and weak interactions only in five dimensions. Kaluza-Klein theories are the inspiration for more modern theories like superstrings and brane worlds, as well as universal extra dimensions models.

6.1 Brane Worlds

String theory was formulated originally as a theory of the strong interacion, but it turned out to provide a description of quantum gravity, with the realisation that it contains a massless spin 2 state which can be identified with the graviton. There is actually and infinte number of excitations of the strings, and the massless ones are interpreted as non-Abelian gauge bosons and matter fermions. At large distances these excitations appear like point-like objects, whose properties, like mass and charge depend on the vibrational modes of the string.

In superstring theory, instead of point particles the fundamental objects are one-dimensional: either open or closed strings. The mathematical consistency requires that the theory is supersymmetric and formulated in six extra space dimensions. Usually, these extra dimensions are assumed to be compactified with extremely small compactification radii, of the order of the Planck scale, and that is why they would be unobservable.

String theory is not formulated as a quantum field theory, it is a geometrical theoy where particles and interactionas, upon compactification of the extra dimensions it may lead to a quantum field theory, many different vacua appear as possibilities, the number can be as large as 10^{500}. This makes it very difficult to find the vacuum where the SM could come from, since the fundamental scale of string theory is the Planck scale, and we cannot make experiments that reach that scale in order to probe the different possibilities.

Another possibility, inspired in string theory and extra dimensions, was the realization that different fields might live in different dimensions. Then, the SM fields would be described by open strings whose ends are attached to a brane, which is a 3-dimensional hypersurface, whereas the gravitons are described by closed strings that can travel through all D dimensions. For reviews on string theory, branes, and extra dimensions see for instance [41, 42].

The following scenarios have been proposed as possible solutions to the hierarchy problem, where the hierarchy problem has been now turned into the problem of finding the number and size of the extra dimensions.

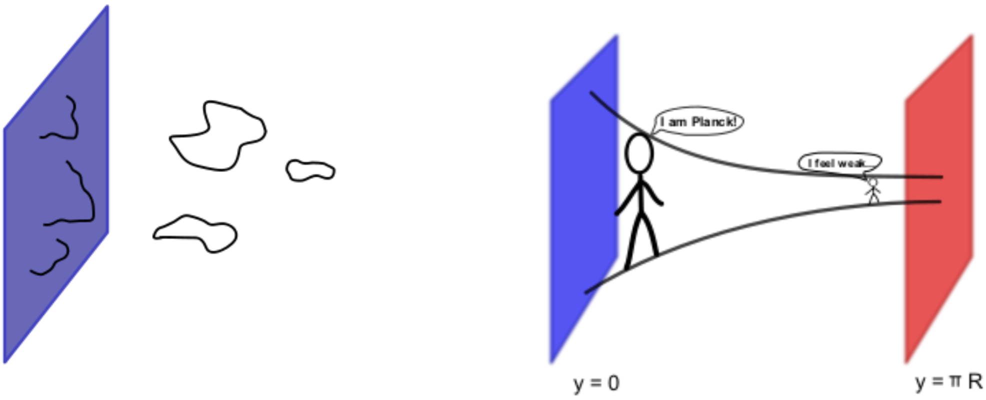

Fig. 9: The left figure depicts a brane, where the SM fields are open strings that start and end on the brane, whereas gravitons are closed strings moving in the bulk. The right figure deptics the Randall-SUndrum setup, with the Planck brane on the left, warped down to the weak brane in the right, both figures from [11].

6.2 Large extra dimensions

Large extra dimensions have been proposed as a solution, or a way to go around, the hierarchy problem. Rather than asking why the electroweak scale is so much smaller than the Planck scale, the question is why is gravity so much weaker than the other interactions. The idea is that the Planck scale is not the fundamental one, but it appears so because of the existence of extra dimensions, it is a derived quantity. The actual fundamental scale is around $\mathcal{O}(1)\ TeV$. From Eq. (36) we can figure out the number and size of the extra dimensions needed. From experimental tests the radius has to be smaller than $R \geq 1\ TeV$, since no K-K tower or its effects have been found. For d extra dimensions,

$$d = 1 \quad R \sim 10^9\ km \tag{37}$$
$$d = 2 \quad R \sim 0.5mm \tag{38}$$
$$d = 3 \quad R \sim 1^{-6}cm \tag{39}$$

the first possibility, $d = 1$ is clearly ruled out, $d = 2$ is currently being tested, and larger values of d are still allowed.

6.3 Warped extra dimensions

The warped extra dimension or Randall-Sundrum (RS) simplest scenario consists also of a five dimensional theory, which is an interval bounded by two three dimensional branes. This interval has a warped geometry, that is the metric is exponentially warped along the y direction

$$ds^2 = \exp{(\kappa y)}\eta_{\mu\nu}dx^\mu d^\nu + dy^2\ . \tag{40}$$

In this scenario the Planck scale is the fundamental one, and sits in one of the branes at $y = 0$, the other is the SM brane at $y = \pi R$. So, from Eq. (40), we can see that the metric changes exponentially as $\eta_{\mu\nu} \to \exp -\kappa\pi R\eta_{\mu\nu}$ from the SM to the Planck brane. The change in the metric implies a change in energy and length scales, for the electroweak scale this implies

$$\Lambda_{Ew} \sim M_{Pl} \exp{(\pi\kappa R)} \sim 1\ TeV\ , \tag{41}$$

with a small dimension $R \gtrsim\ 50 l_{Pl}$, just slightly bigger than the Planck length. In this scenario $d = 1$, i.e. only one extra dimension, is still allowed.

A very attractive feature of the Randall-Sundrum scenario is that it can be tested experimentally. The interaction Lagrangian is

$$\mathcal{L}_I = -G^{\mu\nu}T_{\mu\nu}/\Lambda_{Ew} \ , \tag{42}$$

where $T_{\mu\nu}$ is the energy-stress tensor, which involves the SM fields. These interactions are at the order of the electroweak scale, and in principle can be produced at the LHC. If only gravity travels through the warped extra dimension the first resonance will be the RS graviton. In more modern variants of the RS scenario, it is necessary to allow more particles to travel in the bulk, but leaving the Higgs boson attached at the SM brane, in order to avoid FCNCs.

7 Multi-Higgs Models

It is possible to extend the SM by just adding more particles, but preserving the gauge group of the SM. This is the case of some multi-Higgs models (see for instance [43, 44] and references therein). In general, adding N Higgs doublets to a Lagrangian leads to the Higgs potential of an N-Higgs doublet model (NHDM),

$$V(\phi) = Y_{ij}\phi_i^\dagger\phi_j + Z_{ijkl}(\phi_i^\dagger\phi_j)(\phi_k^\dagger\phi_l) \ , \tag{43}$$

which is Hermitian,thus $Y_{ij} = Y*_{ji}, Z_{ijkl} = Z_{jilk}*, Z_{ijkl} = Z_{kilj}$. For N Higgs complex doublets, each with four degrees of freedom, the potential Eq. (43) has $N^2 + N^2(N^2+1)/2$ real parameters. So, for a general two Higgs doublet model there are 14 real parameters, as compared to the Higgs potential of the MSSM, which has only four. For three Higgs doublet models there are in principle 54 real parameters. Because of the increase in the number of parameters, phenomenological studies of NHDM have mainly focused on two and three Higgs doublet models.

By far the most widely studied NHDM are two Higgs doublet models (2HDM) [43]. The Higgs potential for a general 2HDM is

$$\begin{aligned} V = {} & m_{11}\phi_1^\dagger\phi_1 + m_{22}\phi_2^\dagger\phi_2 - (m_{12}\phi_1^\dagger\phi_2 + h.c.) + \\ & \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ & + \left[\frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + h.c.\right] \ , \end{aligned} \tag{44}$$

where $m_{11}, m_{22}, \lambda_{1234}$ are real, and the rest $m_{12}, \lambda_{5,6,7}$ are complex. These are 14 parameters, but only eleven of which are physical. as can be seen by a change of basis. This general potential allows for charge breaking minima, which is usually avoided, as well as CP conserving and CP violating minima. The possibility of CP violating minima is quite interesting, since it allows for new sources of CP violation to address baryogenesis, and many models with this feature have been studied.

One of the main challenges of any NHDM is to avoid FCNCs. In the case of 2HDM there are two models that naturally avoid this problem. Type I 2HDM couples only one of the Higgs fields to the quarks, by convention it is taken to be ϕ_2. This can be achieved by requiring a Z_2 symmetry that acts like $\phi_1 \to -\phi_1$. The type II 2HDM is similar to the MSSM in that the right-handed up-type quarks couple to one of the Higgs fields, and the right-handed bottom-type quark to the other. In this case, besides the $\phi_1 \to -\phi_1$ discrete symmetry, it is required that $d_R^i \to -d_R^i$. In both models it is assumed the right-handed leptons couple to the Higgs field in the same way as the right-handed down quarks. From these two models, type II has been more studied, due to its similarities with the MSSM. There are variations of these two models, like the Lepton-specific or the flipped one, which will not be discussed here. In the Higgs sector, the Z_2 symmetry means there is no CP violation. Thus, in some models a term like $m_{12}^2(\phi_1^\dagger\phi_2)$ is added to allow for CP violating terms. In type III models both Higgs fields are allowed to couple to the matter sector, and in this case care has to be taken to avoid FCNCs.

In general, phenomenological studies of 2HDMs usually make a number of simplyfing assumptions, to get rid of some of the parameters. The Higgs sector is assumed to be CP conserving, no explicit and no spontaneous breaking is allowed, quartic terms odd in any of the doublets are zero (can be achieved with a discrete symmetry). To exemplify this, lets look at the following potential, which is gauge invariant, CP conserving, and that includes a term that breaks softly the symmetries, to allow for more diverse possibilities

$$\begin{aligned} V = m_{11}\phi_1^\dagger\phi_1 + m_{22}\phi_2^\dagger\phi_2 - m_{12}(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1) + \\ \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ + \frac{\lambda_5}{2}\left[(\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_1)^2\right] . \end{aligned} \tag{45}$$

The procedure to minimise the potential follows the one of the MSSM, where the fields have to be expanded in terms of their neutral and charged parts, and requiring that the minimisation gives the correct electroweak breaking. In a similar way, one has to check that the potential minimum is really bounded from below, and that perturbative unitarity is maintained.

The Higgs fields are denoted by

$$< \phi_i >= \begin{pmatrix} \varphi_i^+ \\ \frac{v_i+\rho_i+\eta_i}{\sqrt{2}} \end{pmatrix} \tag{46}$$

with $j = 1, 2$.

At the minimum the Higgs fields are

$$< \phi_1 >_0= \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix} \qquad < \phi_2 >_0= \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix} \tag{47}$$

In the CP conserving model, CP-odd and CP-even states do not mix, and the term which will become the pseudoscalar decouples from the mass matrix. The CP neutral states mix to give two neutral states, the mixing angle is denoted by α

$$h = \sin\alpha\rho_1 + \cos\alpha\rho_2 , \tag{48}$$

$$H = -\cos\alpha\rho_1 + \sin\alpha\rho_2 . \tag{49}$$

As in the MSSM the ratio of the vevs $\tan\beta = v_2/v_1$ is an important parameter, since it is the mixing angle for the charged scalars, which are are given by

$$H^\pm = -\sin\beta\varphi_1^\pm + \cos\beta\varphi_2^\pm , \tag{50}$$

where $v_1 = v\cos\beta$ and $v_2 = v\sin\beta$. After electroweak symmetry breaking there are 5 massive scalar bosons, $h, H, H^\pm$, and A. From the two neutral ones, h is assumed to be the lightest. In a general 2HDM, the five masses, together with the two angles α and β are free parameters.

In general, it is hard to minimise potential Eq. (45), so often the strategy is to fix the vevs, and then fix the mass parameters m_{ij} through the tadpole equations. In this kind of models, one has to be aware that there might be more than one neutral minima, and it is difficult to find the deeper one. It is possible to find whether the studied minimum is the deeper one by calculating a discriminant. This quantity is built from a combination of the potential parameters and vevs, a positive sign means that the potential is stable at tree-level, and a negative sign that there is a deeper minimum.

Because of the increased number of free parameters it is common to add more symmetries to multi-Higgs models in general, and to the 2HDMs in particular, to be able to do phenomenological studies.

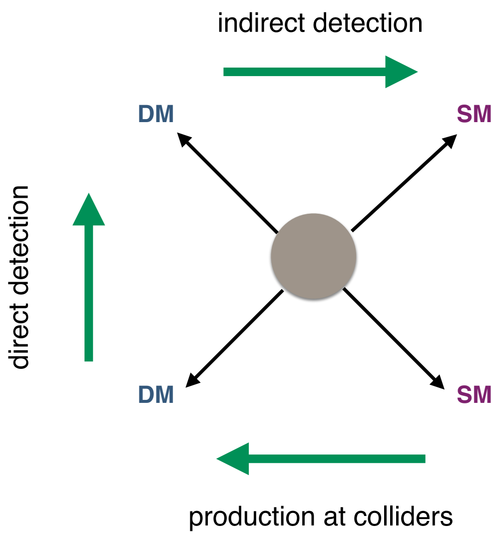

Fig. 10: The figure shows possible ways of detecting dark matter.

7.1 Searches for exotic Higgs scalars

To test multi-Higgs models one can look directly to the production of the extra scalars, decays that are not possible in the SM or deviations from processes of the SM.

Again, searches have been focused mainly on two Higgs doublet models. The 2HDM has a natural decoupling limit, where the massive exotic Higgs states are much heavier than the SM one. This makes the type I 2HDM very difficult to distinguish phenomenologically from the SM, unless we go to high energies.

The main way to distinguish different 2HDMs from the SM and among themselves, is through the branching ratios of their Higgs decays (see for instance [43, 44] and references therein). These studies are usually done assuming the decoupling limit, where the extra Higgs bosons are very heavy and thus decoupled from the SM, although there are also searches for low mass Higgs scalars and pseudoscalars [45–47]. What can also be measured, besides the branching ratios, are corrections to the different SM couplings due to the heavy states. Higgs production has also been extensively studied in 2HDMs.

There are experimental bounds which are generic to 2HDM, for instance the couplings to gauge bosons like ZHA or $\gamma H^+ H_-$, which appear already at tree level and would have been already produced at the LHC, imply that the charged scalares are heavy. The same as with SUSY models, flavour observables can place stringent bounds on 2HDMs. The decay $b \to s\gamma$ puts a bound on the charged Higgs mass $m_{H\pm\pm} > 480\ GeV$ at 95% C.L. [48], and in type II models flavour physics puts a bount of $m_{H\pm\pm} \gtrsim 600\ GeV$ [49].

7.2 Scalar Dark Matter

Extra Higgs scalars can provide also with good candidates to dark matter. They appear in multi-Higgs models with global symmetries, under which the SM particles do not transform. Of particular interest are the inert models, in which at least one extra Higgs doublet is added, with no couplings to the matter fields and with zero vacuum expectation value. This is achieved by adding an extra discrete symmetry, usually $\mathbb{Z}_2$. The combination of the symmetry and the zero vev guarantee the stability of the dark matter scalar. The new heavier scalars decay into the lightest one, which is the DM candidate, which cannot decay further. This type of models are referred to as inert doublet models [50, 51].

The inert 2HDM has been extensively studied, since it is very predictive and can be tested in colliders, as well as in direct and indirect DM searches. The extra scalars can still have pair interactions among themselves, and also pair interactions with the gauge and Higgs bosons of the SM are possible. It also has cosmological consequences, in particular a sequence of strong first order phase transitions in the early Universe, which restricts considerably the allowed parameter space (see for instance [52] and references therein).

The searches for dark matter, be it dark scalars, neutralinos or other candidates, will go on via direct production from DM particles by collision of SM ones in colliders, indirect detection which can be DM annihilation or decays, or direct detection through collision with a target nucleus, as depticted in Fig. 10 (see for instance [4, 5] and references therein).

A review on searches for new particles excluding supersymmetry, for instance leptoquarks, higgses, heavy leptons, can be found in ref [53].

8 Conclusions

There is experimental and observational evidence that there is physics beyond the Standard Model in the neutrino masses and the existence of dark matter. The need to go beyond the SM is backed also by sound theoretical motivations. We have not found yet any other evidence of new, or rather unknown to us, physics beyond the Standard Model, and it is not for want of models or experimental searches. Since physics BSM impacts particle physics and cosmology, as well as some astrophysical processes, the search has to continue in all three fronts, both from the theoretical as from the experimental point of view. The continuous feedback between these three areas will guide our future searches.

Acknowledgements

I thank the organizers of CLASHEP 2017 for the warm hospitality and the students for their attention. This work was partly supported by DGAPA-UNAM project PAPIIT IN111518.

References

[1] E. Noether, Gott. Nachr. **1918**, 235 (1918), arXiv:physics/0503066, [Transp. Theory Statist. Phys.1,186(1971)].

[2] R. N. Mohapatra, Seesaw mechanism and its implications, in *Seesaw mechanism. Proceedings, International Conference, SEESAW25, Paris, France, June 10-11, 2004*, pp. 29–44, 2004, arXiv:hep-ph/0412379.

[3] P. Hernandez, Neutrino Physics, in *Proceedings, 8th CERN-Latin-American School of High-Energy Physics (CLASHEP2015): Ibarra, Ecuador, March 05-17, 2015*, pp. 85–142, 2016, arXiv:1708.01046.

[4] K. Garrett and G. Duda, Adv. Astron. **2011**, 968283 (2011), arXiv:1006.2483.

[5] M. Lisanti, Lectures on Dark Matter Physics, in *Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: New Frontiers in Fields and Strings (TASI 2015): Boulder, CO, USA, June 1-26, 2015*, pp. 399–446, 2017, arXiv:1603.03797.

[6] C. Csï£¡ki and P. Tanedo, Beyond the Standard Model, in *Proceedings, 2013 European School of High-Energy Physics (ESHEP 2013): Paradfurdo, Hungary, June 5-18, 2013*, pp. 169–268, 2015, arXiv:1602.04228.

[7] F. Ellinghaus, K. Hoepfner, and T. Ohl, Searches for Physics Beyond the Standard Model, in *The Large Hadron Collider: Harvest of Run 1*, edited by T. Schï£¡rner-Sadenius, pp. 463–513, 2015.

[8] M. Dine, *Supersymmetry and String Theory* (Cambridge University Press, 2016).

[9] R. Rosenfeld, Physics Beyond the Standard Model, in *Proceedings, 8th CERN-Latin-American School of High-Energy Physics (CLASHEP2015): Ibarra, Ecuador, March 05-17, 2015*, pp. 159–164, 2016, arXiv:1708.00800.

[10] S. Willenbrock and C. Zhang, Ann. Rev. Nucl. Part. Sci. **64**, 83 (2014), arXiv:1401.0470.

[11] B. C. Allanach, *Beyond the Standard Model Lectures for the 2016 European School of High-Energy Physics*, PhD thesis, Cambridge U., DAMTP, 2016, arXiv:1609.02015.

[12] H. E. Logan, (2014), arXiv:1406.1786.

[13] F. Quevedo, S. Krippendorf, and O. Schlotterer, (2010), arXiv:1011.1491.

[14] G. G. Ross, *GRAND UNIFIED THEORIES* (, 1985).

[15] R. N. Mohapatra, *UNIFICATION AND SUPERSYMMETRY. THE FRONTIERS OF QUARK - LEPTON PHYSICS* (Springer, Berlin, 1986).

[16] M. F. Sohnius, Phys. Rept. **128**, 39 (1985).

[17] S. P. Martin, (1997), arXiv:hep-ph/9709356, [Adv. Ser. Direct. High Energy Phys.18,1(1998)].

[18] H. E. Haber and L. Stephenson Haskins, Supersymmetric Theory and Models, in *Theoretical Advanced Study Institute in Elementary Particle Physics: Anticipating the Next Discoveries in Particle Physics (TASI 2016) Boulder, CO, USA, June 6-July 1, 2016*, 2017, arXiv:1712.05926.

[19] S. R. Coleman and J. Mandula, Phys. Rev. **159**, 1251 (1967).

[20] R. Haag, J. T. Lopuszanski, and M. Sohnius, Nucl. Phys. **B88**, 257 (1975), [,257(1974)].

[21] D. I. Kazakov, Beyond the standard model: In search of supersymmetry, in *2000 European School of high-energy physics, Caramulo, Portugal, 20 Aug-2 Sep 2000: Proceedings*, pp. 125–199, 2000, arXiv:hep-ph/0012288.

[22] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rept. **267**, 195 (1996), arXiv:hep-ph/9506380.

[23] E. A. Bagnaschi *et al.*, Eur. Phys. J. **C75**, 500 (2015), arXiv:1508.01173.

[24] F. Jegerlehner, Springer Tracts Mod. Phys. **274**, pp.1 (2017).

[25] F. V. Flores-Baez, M. Gï£¡mez Bock, and M. Mondragï£¡n, Eur. Phys. J. **C76**, 561 (2016), arXiv:1512.00902.

[26] M. Lindner, M. Platscher, and F. S. Queiroz, Phys. Rept. **731**, 1 (2018), arXiv:1610.06587.

[27] Particle Data Group, C. Patrignani *et al.*, Chin. Phys. **C40**, 100001 (2016).

[28] S. Raby, Lect. Notes Phys. **939**, 1 (2017).

[29] J. L. Hewett *et al.*, Planning the Future of U.S. Particle Physics (Snowmass 2013): Chapter 2: Intensity Frontier, in *Proceedings, 2013 Community Summer Study on the Future of U.S. Particle Physics: Snowmass on the Mississippi (CSS2013): Minneapolis, MN, USA, July 29-August 6, 2013*, 2014, arXiv:1401.6077.

[30] S. Heinemeyer, M. Mondragï£¡n, G. Patellis, N. Tracas, and G. Zoupanos, Symmetry **10**, 62 (2018), arXiv:1802.04666.

[31] ATLAS, G. Aad *et al.*, JHEP **10**, 054 (2015), arXiv:1507.05525.

[32] ATLAS, https://twiki.cern.ch/twiki/bin/view/AtlasPublic/SupersymmetryPublicResults.

[33] CMS, A. M. Sirunyan *et al.*, JHEP **03**, 166 (2018), arXiv:1709.05406.

[34] CMS, https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSUS.

[35] U. Ellwanger, C. Hugonie, and A. M. Teixeira, Phys. Rept. **496**, 1 (2010), arXiv:0910.1785.

[36] M. Maniatis, Int. J. Mod. Phys. **A25**, 3505 (2010), arXiv:0906.0777.

[37] CMS, C. Collaboration, (2015).

[38] ATLAS, G. Aad *et al.*, Phys. Rev. Lett. **113**, 171801 (2014), arXiv:1407.6583.

[39] S. Baum, K. Freese, N. R. Shah, and B. Shakya, Phys. Rev. **D95**, 115036 (2017), arXiv:1703.07800.

[40] H.-C. Cheng, Introduction to Extra Dimensions, in *Physics of the large and the small, TASI 09, proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, USA, 1-26 June 2009*, pp. 125–162, 2011, arXiv:1003.1162.

[41] M. J. Duff, Supermembranes, in *26th British Universities Summer School in Theoretical Elementary Particle Physics (BUSSTEPP 1996) Swansea, Wales, September 3-18, 1996*, 1996, arXiv:hep-th/9611203.

[42] S. Forste, Fortsch. Phys. **50**, 221 (2002), arXiv:hep-th/0110055.

[43] G. C. Branco *et al.*, Phys. Rept. **516**, 1 (2012), arXiv:1106.0034.

[44] I. P. Ivanov, Prog. Part. Nucl. Phys. **95**, 160 (2017), arXiv:1702.03776.

[45] R. Aggleton, D. Barducci, N.-E. Bomark, S. Moretti, and C. Shepherd-Themistocleous, JHEP **02**, 035 (2017), arXiv:1609.06089.

[46] CMS, V. Khachatryan *et al.*, JHEP **10**, 076 (2017), arXiv:1701.02032.

[47] ATLAS, M. Aaboud *et al.*, Submitted to: Phys. Lett. (2018), arXiv:1803.11145.

[48] M. Misiak *et al.*, Phys. Rev. Lett. **114**, 221801 (2015), arXiv:1503.01789.

[49] A. Arbey, F. Mahmoudi, O. Stal, and T. Stefaniak, Eur. Phys. J. **C78**, 182 (2018), arXiv:1706.07414.

[50] L. Lopez Honorez, E. Nezri, J. F. Oliver, and M. H. G. Tytgat, JCAP **0702**, 028 (2007), arXiv:hep-ph/0612275.

[51] L. Lopez Honorez and C. E. Yaguna, JCAP **1101**, 002 (2011), arXiv:1011.1411.

[52] A. Belyaev, G. Cacciapaglia, I. P. Ivanov, F. Rojas-Abatte, and M. Thomas, Phys. Rev. **D97**, 035011 (2018), arXiv:1612.00511.

[53] T. Golling, Prog. Part. Nucl. Phys. **90**, 156 (2016).