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A Sketchy Introduction to Cosmology

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1 Introduction

Why should a particle physicist learn cosmology? I can think of several reasons:
• the main evidence from new physics beyond the Standard Model comes from cosmology: eg dark matter, dark energy and inflation;
• particle physics affect cosmology: eg origin of matter- anti-matter asymmetry (needs baryon number violation, CP violation, non-equlibrium processes), the Higgs field can have a role in inflation, neutrinos have a role in the formation of structures (such as galaxies and galaxy clusters) in the Universe, cosmological phase transitions due to particle physics (electroweak breaking, QCD confinement, etc) can occur in the early Universe with observable consequences (such as the generation of gravitational waves, topological defects, etc) ;
• cosmology affects particle physics: eg evolution of the Universe may be responsible for electroweak symmetry breaking (eg relaxion idea);
• early Universe is a testbed for SM and BSM: eg stability or metastability of SM vacuum;
• gravity (geometry) may play an important role in particle physics: eg models with warped extra dimensions
• new particles from geometry: KK excitations, radion, etc;
• models with extra dimensions can change the evolution of the Universe (and hence be tested).

Now that I hope to have convinced you of the importance to study cosmology with all these very interesting topics I'll unavoidably disappoint you since I will not be able to cover in much detail any of them - but my intention is to give a short start so that you can continue on your own.

I must say right away that in my opinion a full-fledged write-up of these lectures, which still need much improvement, is not essential at this point. In fact there are several excellent books and lectures on Cosmology among which I list:

– My favourite cosmology book, even if outdated, still is "The Early Universe" by Kolb and Turner [1]. I guess this is for sentimental reasons, since I learned the subject from it. Dodelson's "Modern Cosmology" is more recent and highly recommended [2];
– Lectures by Daniel Baumann at Cambridge [3] - I borrowed extensively from these lectures;
– Lectures by Pierre Binétruy at the 2012 European CERN School [4];
– Lectures by Toni Riotto at the 2010 European CERN School [5];
– Lectures on Dark Matter by Graciela Gelmini [6] and Mariangela Lisanti [7].

Hence these notes are intended as just a brief guide to what was discussed in the lectures. In the following I will present a simple sketch of my lectures pointing to some references where more details can be found.The slides of my lectures, as well as for the other lectures of the School can be found in [8].

Cosmology has become a precision, data-driven science in the last 20 years or so. I remember that when I started to read about it (admittedly a long time ago) the age of the universe was written as $t_0 = 10^{9\pm1}$ years. The uncertainty was in the exponent! It is fantastic how measurements coming from different observables now determine $t_0 = (13.799 \pm 0.021) \times 10^9$ years.

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There are several observational probes that are used to find out what is the best cosmological model that describes our universe. These include:
• the cosmic microwave background (CMB);
• the abundance of light elements, as described by Big Bang Nucleosynthesis (BBN);
• the use of supernovae of type Ia as standard candles to measure distances in the universe;
• the study of the Large Scale Structure (LSS) of the Unverse, particularly in the use of a feature in the distribution of galaxies called Baryon Acoustic Oscillation (BAO) as a standard ruler;
• the use of weak gravitational lensing, small distortions in the shape of galaxies, for the determination of the distribution of matter in the universe;
• the use of counts of galaxy clusters as a measure of the growth of perturbations in the universe.

The picture that has emerged from all these observations is consistent and somewhat disturbing: we know that we don't know what 95% of the universe is made of. Of the cosmic energy-density budget, roughly only 5% is matter that we know and love - atoms. The rest we believe is dark matter (roughly 25%) and dark energy (roughly 70%). The best model that describes our universe is a model where dark matter is made of nonrelativistic particles (called cold dark matter) and dark energy is described by a simple cosmological constant (denoted by Λ). The so-called ΛCDM model became the Standard Cosmological Model.

As you all know, particle physics also has a Standard Model (SM) - the $SU(3)_C \times SU(2)_L \times U(1)_Y$ Glashow-Weinberg-Salam model, where the $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken to electromagnetism $U(1)_{EM}$ through the Higgs mechanism. The last missing piece of the model, the Higgs boson was finally detected at the LHC in 2012. The SM of particle physics has been tremendously successful - it can actually explain all the data measured at accelerators given some input parameters. However, it is not satisfactory since it does not describe neutrino masses, it has no candidate for cold dark matter, it suffers from the so-called hierarchy problem (simply stated, why is the Higgs mass much smaller than a high energy scale where new physics reside when quantum corrections are taken into account), among other issues. This has led to the development of new physics models, extensions of the SM called Beyond SM (BSM), where these problems can be addressed.

The Standard Cosmological Model is also unsatisfactory. We do not know what dark matter and dark energy are. There are several alternatives to the cold dark matter paradigm: warm dark matter, fuzzy dark matter, self-interacting dark matter, modified newtonian dynamics, TeVeS, etc. The cosmological constant suffers from the same hierarchy problem as the Higgs mass, only worse since the sensitivity to a new physics scale is quartic instead of quadratic. For dark energy there are even more exotic alternatives that go by names such as quintessence, Horndesky, massive gravity, clustered dark energy, interacting dark energy, etc.

One important development that happened after the School was the observation by LIGO and LISA on August 17, 2017 of the gravitational waves produced by the fusion of a pair of neutron stars, with an electromagnetic counterpart identified by several observatories in different wavelengths. The time difference between the gravitational and electromagnetic waves from the event was less than 1.7 seconds. Given the distance of 130 million of light-years this measurement puts a very stringent bound on the difference of the velocities of the two types of waves of one part in 10^{15} - we can say that gravitational waves propagate with the speed of light. This simple new observational fact has eliminated several models of dark energy (see, eg [9]). I also should mention that the 2017 Nobel prize was awarded to Weiss, Barish and Thorne “for decisive contributions to the LIGO detector and the observation of gravitational waves".

For these lectures I mostly concentrate on the Standard Comological Model.

2 First lecture: The averaged Universe

2.1 The basics

The Universe is ruled primordially by gravitation. Gravity is described by Einsten's General Relativity, which relates a geometry determined by a metric to the content of the Universe as described by an energy-momentum tensor. In 1917 Einstein applied his new theory to describe the whole Universe. It's complexity can be domesticated under the assumption that the Universe is on average homogeneous and isotropic. This assumption, sometimes called the "cosmological principle", leads to a considerable simplification since the metric in this case is the so-called Friedmann-Lemaitre-Robertson-Walker (FLWR) metric:

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2. \tag{1}$$

The FLRW metric is determined by the scale factor $a(t)$ which controls the evolution of the averaged Universe. By definition the scale factor is set to one today ($a(t_0) = 1$). In fact the FLRW allows for the introduction of a spatial curvature that also impacts the evolution of the Universe. It is remarkable that observations in the last 20 years or so have measured that the curvature is consistent with zero with small errors: our Universe is on average spatially flat to a very good degree and hence I'm not considering curvature in these lectures.

The expansion rate of the Universe is defined as the Hubble factor:

$$H(t) = \frac{\dot{a}}{a}, \tag{2}$$

where the dot denotes time derivatives. The Hubble constant is the Hubble factor today ($H_0 = H(t_0)$). The acceleration of the Universe is set by $\ddot{a}$. The redshift z, given by the relative change in the position of spectral lines due to the expansion of the Universe, is related to the scale factor by:

$$a = \frac{1}{1+z}. \tag{3}$$

One describes the matter-energy content of the average Universe by a perfect homogeneous fluid energy-momentum tensor which in a rest-frame is characterized only by its energy density (ρ) and the pressure (p) can be written as

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & \text{-p} & 0 & 0 \\ 0 & 0 & \text{-p} & 0 \\ 0 & 0 & 0 & \text{-p} \end{pmatrix} \tag{4}$$

then Einstein's equation reduces to the two well-known Friedmann's equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \tag{5}$$

and

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}\left(\rho + 3p\right) \tag{6}$$

One can obtain a continuity equation by taking the time derivative of the first equation and substituting in the second one:

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{7}$$

In order to study the evolution of a fluid in the Universe we must assume an *equation of state*: a relation between its pressure and energy density. The simplest form is

$$p = \omega\rho, \tag{8}$$

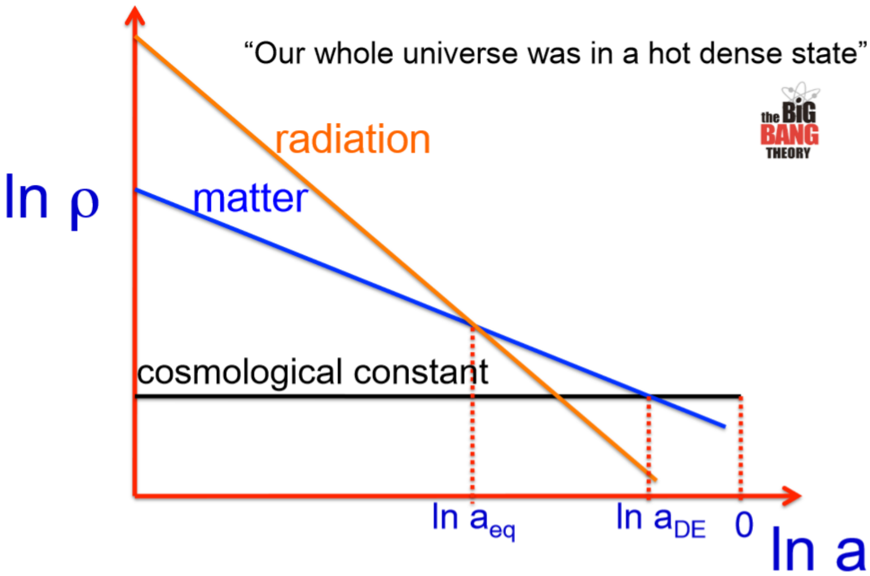

Fig. 1: Evolution of densities in the Universe as a function of the scale factor for different components.

where ω goes by the name of equation of state parameter. For a constant ω it is easy to show that the energy density changes as

$$\rho(t) = \rho(t_i) \left(\frac{a(t)}{a(t_i)} \right)^{-3(1+\omega)} \tag{9}$$

and therefore for nonrelativistic fluid ($\omega = 0$), relativistic fluid ($\omega = 1/3$) and for a cosmological constant ($\omega = -1$) one has $\rho \propto a^{-3}$, $\rho \propto a^{-4}$ and $\rho \propto a^0$ respectively.

These simple considerations lead to the conclusion that the early universe is very dense and hot (dominated by radiation) and the late universe is dominated by the cosmological constant. A sketch of this behaviour is shown in Figure 1.

It's also easy to show that since $\dot{a}/a \propto \sqrt{\rho}$ the scale factor grows with time as:

$$a(t) \propto \begin{cases} t^{2/3} \text{ for matter} \\ t^{1/2} \text{ for radiaton} \\ e^{Ht} \text{ for a cosmological constant} \end{cases} \tag{10}$$

The Universe is now dominated by a cosmological constant and hence it is expanding exponentially. We don't feel it for the same reason that we don't feel our money growing exponentially in a savings account.

The average Universe being spatially flat is a consequence of its density being equal to the so-called critical density:

$$\rho_c = \frac{3H_0^2}{8\pi G}. \tag{11}$$

The Hubble constant has been measured with a few percent accuracy recently and there is a 3σ tension between the value measured in the local Universe and the value extracted from CMB measurements [10]:

$$\begin{aligned} H_0 &= 73.24 \pm 1.74 \ \text{ km/s/Mpc} \ \text{ (local)} \\ H_0 &= 69.3 \pm 0.7 \ \text{ km/s/Mpc} \ \text{ (Planck).} \end{aligned} \tag{12}$$

The jury is still out whether this tension merits some explanation from new physics. In any case, the critical density amounts to something equivalent to only five hydrogen atoms per cubic meter. The average Universe is a pretty empty place.

One quantifies the composition of the Universe by the ratios Ω_i between the energy density in a given component i to the total critical density: $\Omega_i = \rho_i/\rho_c$. For a spatially flat Universe:

$$\sum_i \Omega_i = 1 \tag{13}$$

These quantities are time-dependent and one can re-write the first Friedmann equation as:

$$\left(\frac{H}{H_0}\right)^2 = \sum_i \Omega_i^{(0)} a^{-3(1+w_i)}. \tag{14}$$

It's worth remarking again that the equations derived so far are valid for constant ω's - it is not difficult to generalize them for time-varying equation of state. This is the case for the so-called models of Dynamical Dark Energy (DDE), where for instance Dark Energy can be modelled by a dynamical scalar field. The energy density and pressure for a spatially homogeneous, time-dependent, canonically normalized scalar field ϕ with a potential $V(\phi)$ is easily computed from its energy-momentum tensor and the equation of state is given by:

$$\omega = \frac{p}{\rho} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \tag{15}$$

and hence $-1 \leq \omega \leq 1$. Given initial conditions and a potential one can devise DDE models that can easily mimick the background expansion.

2.2 Distances in the Universe

In order to get observational information about the evolution of the Universe it is of fundamental importance to be able to measure distances. There are basically two ways to measure distances in the Universe: using standard candles or using standard rulers. Standard candles are objects of known intrisic brightness, or luminosity. Cepheid stars and supernovas of type Ia are objects that can be calibrated into standard candles. Standard rulers are features in the Universe that have a well-defined physical length, such as the scale of the so-called Baryon Acoustic Oscillation (BAO), which I will discuss later on.

Let's introduce some different distances that will be useful in the following.

(a) Comoving distance between us and an object at redshift z ($\chi(z)$):
This is defined as a distance $\chi(z)$ that a light ray would travel

$$ds^2 = 0 \Longrightarrow dt^2 = a(t)^2 d\chi^2 \tag{16}$$

and with some change of variables it is easy to show that (we are setting $c = 1$)

$$\chi(z) = \int_0^z \frac{dz}{H(z)} \tag{17}$$

(b) Comoving particle horizon ($\chi(z)_{hor}$):
Largest region in causal contact since the Big Bang - it's given by

$$\chi(z)_{hor} = \int_0^t \frac{dt'}{a(t')} \tag{18}$$

(c) Luminosity distance (d_L):

This is the distance measured using standard candles and is given in terms of the flux of photons received in a detector and the intrinsic luminosity of the source

$$F = \frac{L}{4\pi d_L^2} \tag{19}$$

and it can be shown that it is related to the comoving distance by

$$d_L = (1+z)\chi(z) \tag{20}$$

The luminosity distance is larger for a given redshift for larger values of the dark energy density. This can be traced back to the accelerated expansion of the Universe due to dark energy. Therefore SNIa's look dimmer in a Universe with dark energy. This is how the accelerated expansion of the Universe was discovered in 1998, which was a big surprise (actually a shock) for the community. The 2011 Nobel prize in Physis was awarded for "the discovery of the accelerated expansion of the Universe through observations of distant supernovae".

(d) Angular diameter distance (d_A):

This distance is related to the angle $\delta\theta$ subentended by a physical length l ($d_A = l/\delta\theta$) and it can be shown that

$$d_A = \frac{1}{1+z}\chi(z). \tag{21}$$

(e) Comoving Hubble radius (r_H):

Comoving distance that particles can travel in a Hubble time, sometimes known as the Hubble comoving horizon

$$r_H = \frac{1}{aH} \tag{22}$$

and therefore

$$r_H(t) \propto \begin{cases} a^{1/2} \text{ for matter} \\ a \text{ for radiaton} \\ 1/a \text{ for a cosmological constant} \end{cases} \tag{23}$$

It's interesting to study how a comoving scale compares with the comoving Hubble radius - we sketch in Figure 2 how a given scale can enter and exit the Hubble horizon during the evolution of the Universe.

2.3 The thermal history of the Universe

I showed above that for radiation the energy density goes as $\rho_{rad} \propto a^{-4}$. However, from the Stefan-Boltzmann law $\rho_{rad} \propto T^4$, where T is the temperature of the radiation. Therefore one finds a relation between the scale factor and the temperature of the radiation: $a \propto T^{-1}$ - the Universe cools down as it expands. Since we also know that $a \propto t^{1/2}$ in a radiation dominated Universe, one finds that

$$T(\mathrm{M}eV) \approx 1.5 g_*^{-1/4} t(s)^{-1/2} \tag{24}$$

where g_* measures the number of relativistic degrees of freedom in thermal equilibrium, the temperature is given in MeV and the time is given in seconds.

For a gas of nonrelativistic matter with mass m and temperature T ($T \ll m$) the energy density is exponentially suppressed (we are using unity for the Boltzmann constant):

$$\rho \propto m(mT)^{3/2} e^{-m/T}. \tag{25}$$

When the Universe cools down and particles become nonrelativistic the abundance of these particles is exponentially suppressed and they drop out of thermal equilibrium. The reason is the rate of interactions (Γ) which is given by

$$\Gamma(T) = n(T)\langle\sigma v\rangle, \tag{26}$$

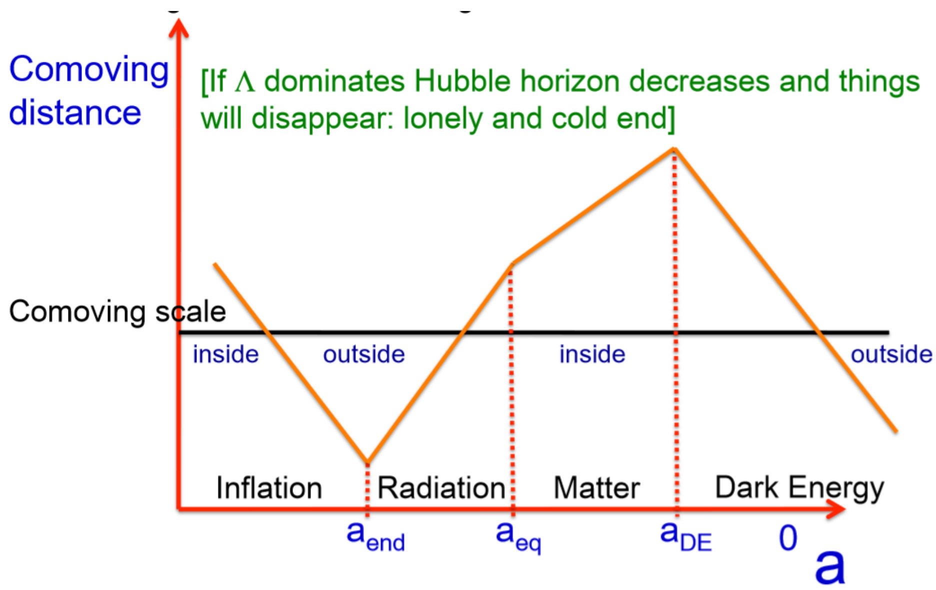

Fig. 2: Comoving Hubble radius during the evolution of the Universe compared to a given comoving scale.

where $n(T) = \rho(t)/m$ is the number density of the particles at temperature T and $\langle \sigma v \rangle$ is the thermally averaged cross section times velocity. This interaction rate should be compared with the expansion rate of the Universe given by the Hubble parameter $H(T)$. When the interaction rate is much larger than the expansion rate the interactions have time to bring the particles to thermal equilibrium. But as the Universe cools down, the interaction rate drops and eventually the particles will get out of thermal equilibrium and their number will then freeze since the interactions are no longer efficient. This happens at the so-called freeze-out or decoupling temperature. A rough estimate of the freeze-out temperature $T_{f.o.}$ is obtained from equating $\Gamma(T_{f.o.}) = H(T_{f.o.})$. A more precise estimate follows from an explicit solution of the Boltzmann equation.

A simple example is the decoupling of neutrinos in the early Universe. Neutrinos are kept in equilibrium by the weak interactions, eg $\nu_e \bar{\nu}_e \leftrightarrow e^+ e^-$ and the cross section is roughly given by $\sigma \approx G_F T^2$ from dimensional analysis ($G_F \approx 10^{-5}$ GeV^{-2} is the Fermi constant). Their number density decreases as $n_\nu \propto a^{-3} \propto T^3$ and therefore $\Gamma \approx G_F T^5$. On the other hand, the Hubble parameter is given by

$$H(T) \approx T^2/M_{Pl}, \tag{27}$$

where we used Friedmann's equation with $\rho \propto T^4$ and that $G = 1/M_{Pl}^2$. Therefore the freeze-out temperature for neutrinos can be estimated as

$$T_{f.o.} \approx \left(\frac{1}{G_F^2 M_{Pl}} \right)^{1/3} \approx 1 \text{ MeV} \tag{28}$$

After decoupling neutrinos cool down as $T \propto 1/a$. They would have the same temperature as photons except for the fact that photons get heated up by the annihilation of e^+e^- at around $T \approx 0.5$ MeV. Hence neutrinos are a bit cooler than photons today ($T_\nu = 1.95$ K).

This relic low energy neutrino background has not been directly detected yet. There is a planned experiment called PTOLOMY designed to detect it [11].

In the SM there are only left-handed neutrinos and they are massless. However, we now know that neutrinos oscillate among different flavors and must be massive. Therefore the SM should be augmented, most plausibly with the inclusion of a new degree of freedom, the right-handed neutrino, which

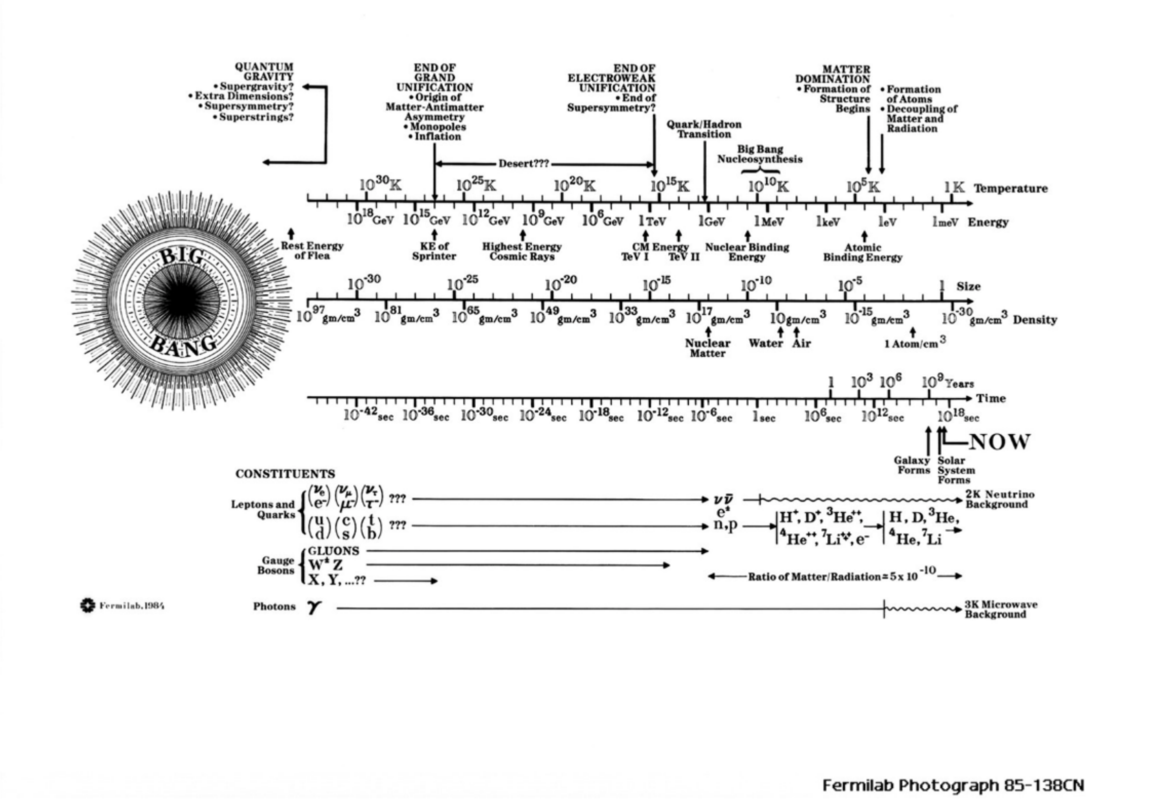

Fig. 3: Thermal history of the Universe. Figure from Kolb & Turner "The Early Universe".

is a gauge singlet. Right-handed neutrinos may have cosmological consequences, acting as warm dark matter. Neutrinos could be dark matter but because they are light they would be relativistic at the time of decoupling - this is the definition of hot dark matter. Hot dark matter is already ruled out by cosmology, more specifically due to the fact that it would erase small scale structure in the Universe because of its free streaming.

We now understand that neutrinos contribute today a very small amount to the energy density budget of the Universe, just like photons, but bounds on their masses can still be derived from cosmological observations related to the formation of structures in the Universe and from the CMB (for a recent review, see [12]). Using a recent combination of CMB and LSS observables results in the 95% upper bound $\sum_i m_{\nu_i} \leq 0.16$ eV [13].

The thermal history of the Universe is a very rich and dramatic one, with several events taking place during its evolution, such as inflation, phase transitions (electroweak symmetry breaking, QCD confinement, etc), the changes in the dominant component of the Universe, decoupling and annihilation of whole species and finally the take over of Dark Energy. There are many illustrative figures of this history but my favourite one is of course from the book by Kolb and Turner, which I reproduce in Figure 3.

2.4 The cosmological constant: the elephant in the room

The cosmological constant, first introduced by Einstein in 1917 in order to allow for a static Universe that was then consistent with observations, has been rejected by him as his biggest blunder after the discovery of the expansion of the Universe in 1929. However, as George Gamow puts it in his 1970 autobiography [14], “But this “blunder", rejected by Einstein, is still sometimes used by cosmologists even today and the cosmological constant rears its ugly head again, again and again."

The cosmological constant was always something that we, theoretical physicists, wanted to put aside. But the discovery of the accelerated expansion of the Universe blew the cosmological constant into our faces. It is too much of an embarrassment. The main reason for this embarrassment is the fact that its value is uncontrollable: it suffers from quadratic divergences when quantum corrections are taken into account, much worse than the quadratic divergences that afflicts the mass of the Higgs boson in the SM. The easiest way to see the origin of this divergence is to consider the contribution of the zero-point (vacuum) energy to a scalar quantum field of mass m (the $\hbar\omega/2$ factor, where $\omega^2 = k^2 + m^2$), which can be thought of as an infinite number of harmonic oscillators:

$$\rho_{vac} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2}\sqrt{k^2 + m^2}. \tag{29}$$

This integral diverges as an energy cut-off scale E^4_{cut}. If one uses the Planck scale as the cut-off the well known discrepancy of 10^{120} with respect to observations is obtained. This may very well be largest discrepancy in the history of Physics and it is referred to as the cosmological constant problem [15]. However, it only reflects the fact that we do not understand what is going on. To this date there is not a good solution to this problem. Therefore, most of us chose to continue ignoring it.

3 Second lecture: Origins

In this second lecture I briefly discuss how some things came to existence in the Universe:
- Origin of light elements;
- Origin of baryons;
- Origin of dark matter;
- Origin of inhomogeneities.

3.1 Origin of light elements: BBN

Big Bang Nucleosynthesis (BBN) is one of the pillars of the Standard Cosmological Model (for a recent review see [16]) [1]. It is the earliest cosmological probe that we have so far (couple of minutes after the Bang). The idea goes back to George Gamow and his students in the 1940’s.

The details involve a complicated set of nuclear reactions which can be studied with sophisticated codes but here I will present a very simplified picture of BBN developed in five easy steps:
(1) When $T \gg 1$ MeV ($t \ll 1$ s), the Universe is made out of neutrons, protons, electrons and photons. Neutrons and protons are in thermal equilibrium due to the weak force. When $T = \mathcal{O}(1$ MeV$)$, protons and neutrons are non-relativistic with the ratio of their number density (denoted by n_n/n_p) given by

$$\frac{n_n}{n_p} = e^{-Q/T}, \tag{30}$$

where $Q = (m_n - m_p)$ is the mass difference between them. Notice that the neutron-proton mass difference ($Q = 1.3$ MeV) is very small compared to their masses ($m_n \approx m_p \approx 1000$ MeV).
(2) Neutrons and protons freeze out at $T = 0.8$ MeV. Their number remains constant afterwards (number densities are $\propto a^{-3}$), except for neutron decay which we will take into account in step (3). The neutron fraction (X_n) then becomes

$$X_n = \frac{n_n}{n_n + n_p} = \left.\frac{e^{-Q/T}}{1 + e^{-Q/T}}\right|_{T=0.8\ MeV} \approx \frac{1}{6}. \tag{31}$$

[1] I learned BBN many years ago from an excellent review by Gary Steigman, a pioneer in the development of the connection between cosmology and particle physics and an expert on BBN [17]. Gary used to come to Brazil frequently and we became good friends. I was very saddened to learn about his untimely passing last year a bit after the School took place.

(3) The neutron fraction is almost frozen except for the fact that free neutrons decay with a lifetime $\tau_n \approx 900$s. Hence, after freeze-out

$$X_n(t) = e^{-t/\tau_n} X_n. \tag{32}$$

(4) Helium can only be formed by nuclear reactions when deuterium is present. Hence the temperature of the Universe has to be smaller than the deuterium binding energy - $E_D \approx 0.06$ MeV ($t_D \approx 330$s). At this time

$$X_n(330\ s) \approx 1/8. \tag{33}$$

(5) At this point we can approximate that all neutrons present are used to form helium-4 ($n_{He} = 2n_n$). Then we can compute the fraction in mass of the Universe in ${}^4He$ as:

$$Y_{He} = \frac{4n_{He}}{n_p} \approx 2X_n \approx 1/4. \tag{34}$$

Therefore after the first few minutes of the Universe approximately 25% of the mass of atoms are in the form of ^{4}He. There are also small quantities of D, ^{3}He, and ^{6}Li that can be computed by dedicated computer codes that take into account the dynamics of the many nuclear reactions involved in the background of an expanding Universe. The results from these computations depend crucially on the amount of photons in the Universe or, more precisely, on the ratio between the number of protons (or baryons, more generally) and photons. This ratio, denoted by η, determined from BBN is in good agreement with an independent measurement from the CMB:

$$\eta = \frac{n_b}{n_\gamma} = (6.10 \pm 0.04) \times 10^{-10}. \tag{35}$$

The determination of η coupled to the measurement of CMB provides an estimate of the baryonic content of the Universe, Ω_b.

The agreement between the measurements of the abundances of the light elements (with some extrapolation to their primordial abundance) and the predictions from BBN is one of the great successes of the Standard Cosmological Model.

3.2 Origin of baryons: baryogenesis

One can try to estimate what would be the relic amount of baryons using arguments similar to the ones discussed above: baryons can be kept in thermal equilibrium by the strong interactions until their density drop sufficiently when they become nonrelativistic so that they leave thermal equilibrium and the corresponding baryon number gets frozen. A simple calculation that I sketched in the lectures lead to the disturbing result of $\eta \approx 10^{-19}$. It is not a typo: there is a disagreement of 9 orders of magnitude with the observed value!

What is the catch? We started with the same number of baryons and anti-baryons, a very reasonable assumption given that they are in thermal equilibrium. But that can't work. It turns out that we need to generate a tiny asymmetry between baryons and anti-baryons. In fact, a difference of one extra baryon in 1 billion baryons would be sufficient.

How can such an asymmetry be generated in particle physics? Andrei Sakharov laid the conditions for this to happen in 1967 [18]. These are:
- presence of baryon number violating processes;
- presence of C and CP violation in these processes;
- these processes to be out-of-equilibrium.

It is fair to say that there is no standard model of baryogenesis. All these conditions are met in the SM (even baryon number violation which occurs non-perturbatively) but the amount of CP violation turns out to be too small. This is one of the motivations for searching new sources of CP violation (eg, neutrino sector). Models with Grand Unified Theories, baryogenesis through leptogenesis and other BSM possibilities are also being currently considered. This is certainly one of the hottest topics of research today and the jury is still out.

3.3 Origin of dark matter

3.3.1 Evidences

Evidence for dark matter (DM) arises from different observations at different scales. So far they are unfortunately all based on observations in the heavens and not in laboratories. Among them I can list:
- Dynamics of clusters of galaxies
- Rotational curves of galaxies
- Gravitational lensing
- Cosmic microwave background
- Big bang nucleosynthesis
- Structure formation in the universe
- Baryon acoustic oscillations
- Bullet cluster

I'll not have time to describe them in any detail here. For a review see the lectures [6, 7].

DM is most possible a neutral, long-lived particle. I don't know any viable alternative. In the SM the neutrino could have the role of DM but they are ruled out since their contribution to the energy budget of the Universe is very small. Hence, DM implies *new physics beyond the SM*.[2] Structure formation tells us that dark matter must be cold (ie, nonrelativistic at decoupling) or a most warm (there are bounds from the so-called Lyman-α absorption lines from distant quasars setting $m \geq 5.3$ keV at 2σ for the mass of the warm dark matter particle [21]).

There are several candidates for dark matter: weakly interacting massive particles (WIMPs), new scalars (phion, inert Higgs models), ν_R, axions, primordial black holes, lightest KK particle, self-interacting dark matter, etc. Usually one needs a symmetry (most times discrete, such as Z_2) to ensure the stability of the lightest particle that is odd under it. WIMPs in particular are well-motivated candidates since they are predicted in SUSY extensions of the SM (with R parity conservation): the lightests supersymetric particle (LSP), usually a neutralino (a given combination of gauginos and higgsinos). Candidates must pass several observational constraints. In [22] a ten-point test is proposed and 16 candidates are scrutinized.

3.3.2 Thermal production of dark matter: the "miracle"

Dark matter can have other interactions in addition to the gravitational one. Hence it could have been in thermal equilibrium in the early Universe - in this case it is called thermal DM.[3] In the lectures I presented a very simple way to estimate the relic abundance of thermal DM particles. For accurate estimates one must solve the appropriate Boltzmann equation with the correct thermally averaged cross section. There are specialized and sophisticated codes such as `MicroOMEGAs`[4] and others that compute DM relic densities.

We already saw in the first lecture that particles get out of thermal equilibrium and their abundance freeze-out roughly when the interaction rate becomes of the order of the expansion rate. Using Eqs. (26)

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[2] The lack of signals for new physics and the detection of gravitational waves have led people to think harder in DM within the SM, either in the form of primordial black holes or new quark states, see eg [19, 20].

[3] There is also the possibility that DM is non-thermally produced, eg from the non-equilibrium decay of other particles or by coherent field oscillations (as in the case of axions).

[4] https://lapth.cnrs.fr/micromegas/

and (27) one obtains for its number density at freeze-out:

$$n^{\chi}_{f.o.} \approx \frac{T_{f.o.}}{\langle \sigma v \rangle M_{Pl}}. \tag{36}$$

On the other hand, if the DM is cold it is nonrelativistic at freeze-out and its number density is given by a Boltzmann distribution:

$$n^{\chi}_{f.o.} = (m_\chi T_{f.o.})^{3/2} e^{-m_\chi/T_{f.o.}}, \tag{37}$$

where I'm using m_χ for the DM mass. Introducing $x = m_\chi/T$ it can be shown that equating Eqs (36) and (37) results in $x_{f.o.} \approx 30$ for typical values of $m_\chi = 100$ GeV, $v = 0.3$ and $\sigma = G_F^2 m_\chi^2$. One can now compute the relic DM abundance

$$\Omega_\chi = \frac{m_\chi n^\chi(T=0)}{\rho_c^{(0)}} = \mathcal{O}(1) \tag{38}$$

This is the so-called WIMP miracle: a thermal relic with weak cross section results in a relic abundance with order of magnitude of the observed one. This mechanism of thermal relic production leads to the *survival of the weakest*: the weaker the cross section the earlier the particle freezes out and consequently the larger its abundance.

There are several experiments looking for DM. There are basically three types of searches: direct production at the LHC (with signatures such as monojets), direct detection of DM particles that surround us in our galaxy in underground laboratories (one can estimate that of the order of a billion particles of DM passes through a typical person per second) and indirect detection through the annihilation of DM into SM particles occurring in dense DM rich regions of the Universe (center of our galaxy, satellite dwarf galaxies, etc). It is beyond the scope of these lectures to discuss these searches but many details can be found in the suggested reading. I just can't resist to mention the recent lower bound of 70 GeV on the mass of thermal DM coming from indirect searches from annihilation (assumed to be exclusively into $\bar{b}b$) in the Milky Way halo [23].

3.4 Origin of inhomogeneities

This is another sketchy subsection. Again I refer to the excellent lectures mentioned in the introduction.

3.4.1 The causality problem

CMB is originated at the "last scattering surface" when atoms are formed and the Universe becomes transparent to radiation. That's when radiation decouples from matter. This happened at $z \approx 1100$ ($t \approx 380,000$ years after the bang).

There are regions in the last scattering surface that were never in causal contact - and hence no reason to have the same temperature. Nevertheless the CMB is very uniform over the whole sky, with small variations of 1 part in 10^5. This problem is sketched in Figure 4.

3.4.2 Inflation

Inflation is a period of very fast (exponential) expansion of the Universe. It is similar to the period of dark energy domination we are in now. A single small patch can fill the whole horizon at decoupling. Hence inflation also predicts that the Universe is spatially flat - as observed. Inflation also provides quantum fluctuations that are the seeds for inhomogeneities in the Universe.

The basic idea is simple: the very early Universe is dominated by the energy density of a (surprise!) scalar field - called inflaton - that is slowly rolling down in a potential. There are a plethora of models used to implement inflation and most of them are listed in the "Encyclopedia Inflationaris" [24].

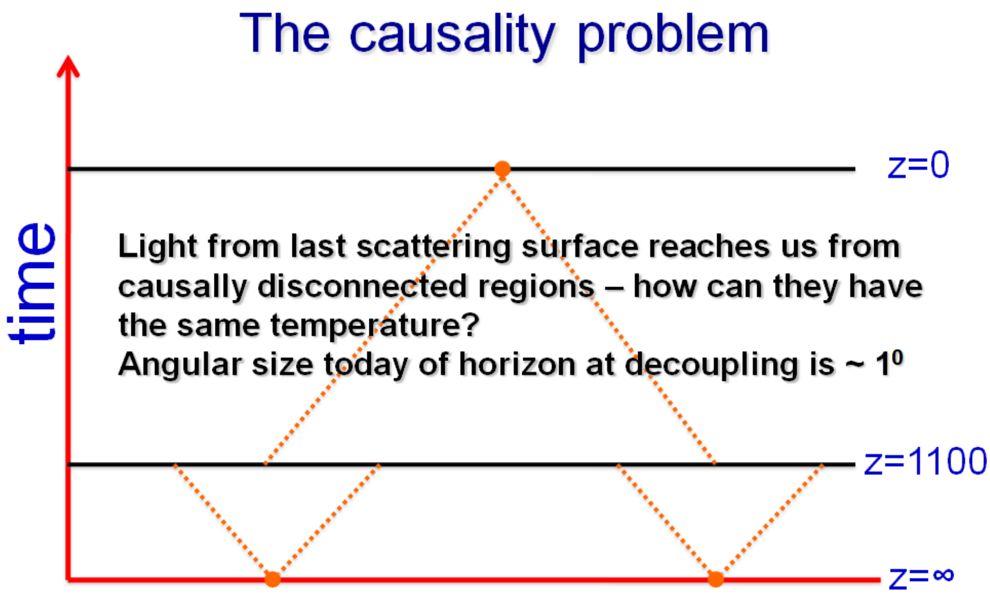

Fig. 4: The causality problem.

Inflation must end otherwise we wouldn't be here. And at the end of the inflationary period the Universe is empty and cold. Something has to jump-start the Universe again to the usual hot and dense phase that we know it should have existed in the past. Usually the end of inflation is identified with the end of the so-called slow-roll period when the scalar field is rolling down in a part of the potential that is relatively flat. After the slow-roll phase, the potential becomes steeper and usually has a minimum. The field then starts to oscillate around the minimum of the potential. It is not difficult to show that an oscillating field is equivalent to a nonrelativistic gas of inflaton particles (the mass of the inflaton is set at $m \approx 10^{12}$ GeV from the amplitude of perturbations in the CMB). The inflaton then decays into radiation (this is model dependent) and results in the so-called reheating of the Universe. A sketch of the potential is shown in Figure 5.

The actual reheating process is more complicated and can be simulated numerically. There is a possibility of preheating, when instabilities in the scalar field perturbations can occur before the coherent oscillation period. The only bound on the reheating temperature T_R is that it must be larger than temperatures required by BBN (1 GeV).

In order to solve the causality problem, one should require that the observable Universe today was to fit in the comoving Hubble radius at the beginning of inflation. A simple calculation that I did in the lectures shows that the amount of inflation needed, characterized by the number of e-foldings denoted by N is:

$$N = \ln\left(\frac{a_e}{a_i}\right) = 26 - 64, \tag{39}$$

where a_i and a_e are the scale factor and the beginning and end of the inflation and the range in N arises from the uncertainty in the reheating temperature (from 1 to 10^{15} GeV). One usually assumes $N \approx 50$.

3.4.3 Perturbations generated during inflation

The origin of the inhomogeneities we observe today are quantum fluctuations in the inflaton field $\delta\phi$ generated during inflation. The size of quantum fluctuations of the inflaton field during inflation is set by

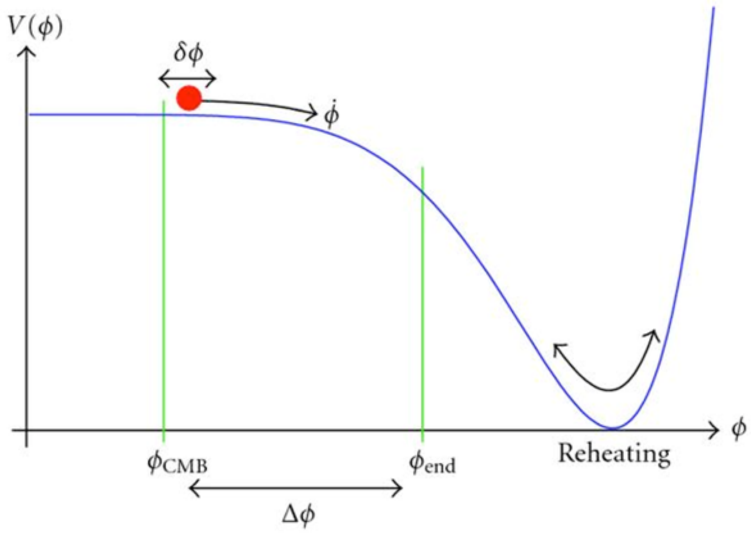

Fig. 5: A cartoon of the inflaton potential.

H:

$$\langle(\delta\phi)^2\rangle = \frac{H^2}{2\pi} \tag{40}$$

In general, perturbations $\delta(\vec{x}, t)$ can be decomposed in Fourier modes:

$$\delta(\vec{x}, t) = \int d^3k\, \delta_k(t)\, e^{i\vec{k}\cdot\vec{x}} \tag{41}$$

and the *power spectrum* $P(k)$ is defined by

$$\langle\delta_{k'}\delta_k\rangle = (2\pi)^3\delta^3(\vec{k'} - \vec{k})P(k). \tag{42}$$

Inflation *predicts* a power-law primordial power spectrum for scalar and tensor (gravitational waves) perturbations

$$\begin{aligned} P_s(k) &= A_s k^{n_s - 1} \\ P_t(k) &= A_t k^{n_t} \end{aligned} \tag{43}$$

where $A_{s,t}$ and $n_{s,t}$ are respectively the amplitudes and spectral indices of the perturbations. They can be computed in a given model of inflation.

In particular, CMB bounds on the ratio of the tensor-to-scalar amplitudes, denoted by r, have been used to eliminate several models of inflation. There were some claims that a non-zero value of r had been detected, which would imply in the detection (albeit indirect) of primordial gravitational waves generated during inflation. However, the claim turned out to be just background and the latest value is $r < 0.12$ at 95% confidence level [25]. But this upper bound is enough to rule out well-know models for the inflaton field, such as models with a ϕ^4 potential.

Even if there is no fully satisfactory model, inflation is great because it

– explain why the Universe is spatially flat;
– solve the causality problem;
– generate almost gaussian, almost scale invariant fluctuations;

- generate both scalar and tensor fluctuations;
- given a inflation potential one can predict the spectrum of scalar and tensor perturbations;
- the scalar (density) perturbations gives rise to the large scale structure of the Universe.

4 Third lecture: The perturbed Universe

In this lecture I briefly describe how the tiny perturbations generated during inflation grow to give rise to the structures in the Universe that we measure today with large surveys of galaxies such as the Dark Energy Survey.

4.1 Growth of perturbations

Tiny fluctuations of the order $\delta \approx 10^{-5}$ were detected in the CMB around 1991. These fluctuations grew due to gravity in the evolution of the Universe. Their growth is determined by solving the perturbed Einstein's equations. While this can be done analytically in a linearized way, ie keeping only linear terms in the perturbations, when the perturbations become large enough ($\delta \approx 1$) this approximation ceases to be valid. In this case one has to resort to full numerical simulations, such as N-body simulations. These simulations are becoming more and more realistic, with the inclusion of baryons.[5]

In principle one has to use full GR to study the evolution of perturbations. However, at scales smaller than the Hubble radius and for non-relativistic matter one can simplify the problem and use Newtonian physics. Hence one has to study fluid dynamics in an expanding Universe. In this case we have to consider three coupled equations: the continuity equation, the Euler equation and the Poisson equation. As I described in the lecture (you can see the slides), in the linearized approximation we can derive a single equation for the time evolution of the matter density perturbation δ_m:

$$\ddot{\delta}_m + 2H(t)\dot{\delta}_m - \frac{3}{3}H(t)^2\Omega_m(t)\delta_m = 0 \tag{44}$$

where

$$\delta_m = \frac{\rho_m - \bar{\rho}_m}{\bar{\rho}_m} \tag{45}$$

Using this simple equation it is easy to show that in a matter-dominated Universe ($\Omega_m \approx 1$, $a \propto t^{2/3}$, $H \propto 2/(3t)$) the matter density perturbation grow as the scale factor $\delta \propto a$. On the other hand, in a dark energy dominated Universe ($\Omega_m \approx 0$, $a \propto e^{Ht}$, $H =$const.) a solution to the equation is $\delta =$const., ie dark energy prevents matter perturbations from growing. This led to the famous anthropic argument for an upper limit in the cosmological constant put forward by Weinberg: the energy density of the cosmological constant can't be too large otherwise galaxies would not form and we would not exist.

4.1.1 The matter power spectrum in recent times

We already introduced the power spectrum in Eq.(42). Here we discuss some observables related to it after the evolution of perturbations. It can be easily shown that the power spectrum is the Fourier transform of the 2-point correlation function of the density perturbations:

$$P(k) = \int d^3k\xi(r)e^{i\vec{k}\cdot\vec{r}} \tag{46}$$

where the two-point correlation function

$$\xi(r) = \langle\delta(\vec{x}_1)\delta(\vec{x}_2)\rangle \tag{47}$$

[5] See eg the Eagle Simulation, http://icc.dur.ac.uk/Eagle/

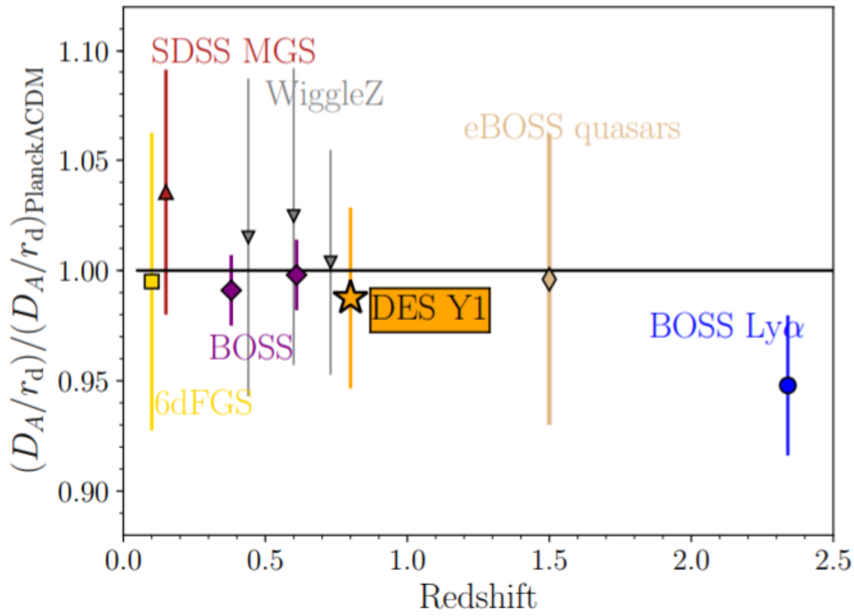

Fig. 6: Ratio of angular distance measurements in BAO to values from Planck fiducial cosmology [27].

depends only on $r = |\vec{x}_1 - \vec{x}_2|$ due to homogeneity and isotropy. Therefore features such as peaks in the 2-point correlation function are transformed into oscillatory features in the power spectrum (think that the Fourier transform of a Dirac δ-function $\delta^3(r - r_*)$ results in $P(k) = e^{ikr_*}$).

Are there peaks in the correlation function of galaxies? Is there a preferred scale in the sky? The answer is affirmative and comes from physics related to the decoupling of matter and radiation at recombination ($z_{rec} \approx 1100$). It is the sound horizon at decoupling.

Before recombination, baryons and photons were strongly coupled, forming a single fluid with pressure and speed. Dark matter, neutrinos and other forms were decoupled. After decoupling, baryons are left behind at a characteristic distance given by the sound horizon at decoupling, which is called the baryon acoustic oscillation (BAO) scale r_{BAO}:

$$r_{BAO} = \int_{z_{rec}}^{\infty} dz \frac{c_s(z)}{H(z)} \approx 150\,\text{Mpc}, \tag{48}$$

where $c_s^2 \approx 1/3$ is the speed of propagation of the photon-baryon fluid. This characteristic scale in the distribution of baryons in real space gives rise to oscillations in the matter power spectrum.

The BAO feature in the matter power spectrum and in the real space 2-point correlation function was first detected in 2005. The position of the BAO peak provides a standard ruler in the sky that can be used to determine cosmological parameters. The ratio of an angular distance measure in BAO data to its theoretical value in a ΛCDM cosmology with Planck cosmology (ie, cosmological parameters as derived by the Planck collaboration) is shown in Figure 6 from [27], where one can see the good agreement.

4.2 If the Universe is the answer what is the question?

The question we want to answer is: given our Universe what is the best model that describes it? And at this moment the flat ΛCDM Standard Cosmological model describes all the observations so far. This model is characterized by six parameters: the Hubble constant (H_0), the baryon abundance (Ω_b), the abundance of cold dark matter (Ω_{CDM}), the amplitude of the initial scalar perturbations (A_s), the spectral index of the scalar perturbations(n_s) and the so-called optical depth (τ, related to the ionization history of the Universe). In addition to these baseline parameters, one could add neutrino masses, nonzero spatial curvature and a constant equation of state $\omega \neq -1$.

Cosmology is sensitive to the sum of the neutrino masses ($\Sigma = \sum_i m_i$), as we already mentioned in Section 2.3 - it is interesting to notice that the value of Σ depends on the hierarchy of neutrino masses. Using data from neutrino oscillations:

$$\Sigma \geq \begin{cases} (58.5 \pm 0.48) \text{ meV for normal hierarchy} \\ (98.6 \pm 0.85) \text{ meV for inverted hierarchy} \end{cases} \tag{49}$$

The equality is attained when the lightest mass is zero. Therefore if from cosmology one finds $\Sigma < 0.098$ eV then one can say that the inverted hierarchy is excluded. There are claims in the literature of strong evidence (in the bayesian sense) for normal hierarchy [26].

Large scale surveys are important for the determination of cosmological parameters. I'm a member of the Dark Energy Survey (DES), which uses photometric techniques to estimate the redshift of galaxies. DES uses a 570 Mpixel digital camera installed in the Blanco 4-meter telescope at the Cerro Tololo International Observatory in Chile. It will finish its 5 years observational period in 2018. The data for the first year has already been mostly analysed and we are now in the process of analysing the data of the three first years. Around 300 million galaxies have already been detected and catalogued. The DES BAO result for the first year of data is shown in Fig. 6.

The distribution of galaxies in the universe provide information about growth of perturbations (and hence is sensitive to Dark Energy or Modified Gravity), information about dark matter (eg hot DM is already ruled out) and a standard ruler (BAO scale). In addition, DES can measure weak gravitational lensing through the small distortions in the shapes of galaxies, can measure the distribution of galaxy clusters and can also measure SNIa. All these probes will be combined to derive the best constraints on cosmological parameters.

I would like to finish this subsection by drawing the following analogy between high energy accelerators and large scale galaxy survey:

- Energy $\leftrightarrow$ Redshift
- Luminosity $\leftrightarrow$ Area & observation time
- Energy resolution $\leftrightarrow$ Redshift errors
- Energy calibration $\leftrightarrow$ Redshift calibration using objects with known redshifts
- p_T cuts, etc $\leftrightarrow$ Magnitude cuts, mask, etc
- Final data set $\leftrightarrow$ Value added catalogs
- Higgs bump hunting $\leftrightarrow$ BAO bump hunting
- Perturbation theory in QCD is ok at high energies $\leftrightarrow$ Perturbation theory in GR is ok at high z

4.3 Finding out the best model from data

The way we estimate the best model from data is through the likelihood function, which can generically be written as

$$\mathcal{L}(\vec{p}) \propto e^{\left[(\vec{x}^{obs}-\vec{x}^{th}(\vec{p}))^t \,\text{Cov}\,(\vec{x}^{obs}-\vec{x}^{th}(\vec{p}))\right]} \tag{50}$$

where $\vec{p}$ is a set of parameters (it turns out that several so-called nuisance parameters must be introduced to model the theoretical prediction), $\vec{x}^{obs}$ is the data vector, $\vec{x}^{th}$ is the theoretical modelling of the observable and Cov denotes de covariance matrix. The covariance matrix can be either theoretically modelled, measure from subsets of data (jackknife, bootstrap or subsampling methods) or measured from a set of

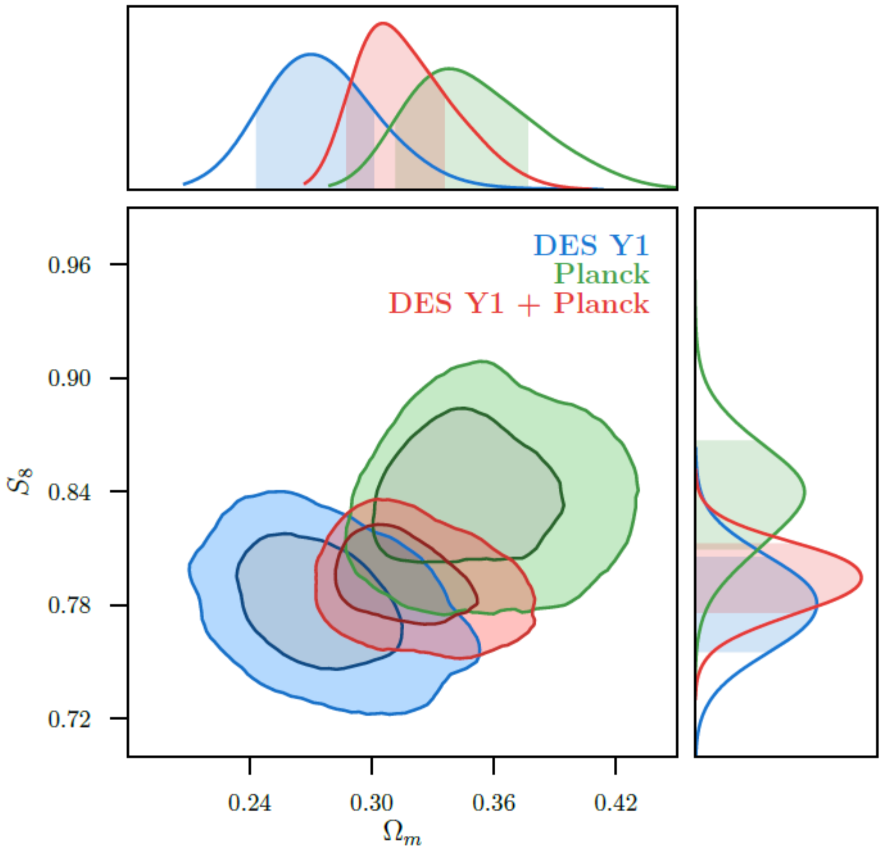

Fig. 7: ΛCDM constraints from the three combined probes in DES Y1 (blue), Planck with no lensing (green), and their combination (red) [28].

mock data. One can then use Bayes theorem to turn the likelihood function into a probability distribution for the parameters given the data using a prior probability for the parameters. The parameters can then be estimated using the Markov Chain Monte Carlo method to sample the space of parameters with the known probability distribution.

In Figure 7 I show the money plot for the DES analyses of the first year of data using a combination of galaxy distribution and weak lensing data [28]. It shows constraints on only 2 parameters, S_8 (related to the amplitude of the scalar power spectrum A_s) and Ω_m. All other parameters (including 20 nuisance parameters related to redshift uncertainties, galaxy bias, etc) have been marginalized over. In the same plot it is shown the constraints from CMB obtained by the Planck collaboration. It is amazing that for the first time the two constraints are comparable (usually CMB results are much more constraining that LSS ones) and compatible, given that they come from signals produced with billions of years of difference. We have also analysed a model with constant equation of state denoted by ωCDM. Combining DES data with several external datasets resulted in $\omega = -1.00^{+0.04}_{-0.05}$. Therefore this analysis confirms that dark energy is compatible with the cosmological constant.

5 Coda

I have given a brief tour on some selected topics in cosmology. The current situation in cosmology is somewhat akin to the one in particle physics. As in particle physics, we have a Standard Cosmological Model, ΛCDM, that explains all the cosmological observations so far. It has only 6 parameters in its minimal version (much less than the 20 or so parameters in the SM). However, in contrast to the SM where all the building blocks have been found and studied, in cosmology we don't know much about the 95% of the Universe that is comprised of dark energy and dark matter. Surveys like DES and the future Large Synoptic Survey Telescope (LSST), of which I'm also a member, will hopefully shed some light on the dark side of the Universe.

6 Acknowledgements

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